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Field Equations in C^4 space-time

D.Mastoridis – 1

Amaliados 17, Athens, Greece, P.O.Box 11523, d.mastoridis@prv.ypeka.gr

K.Kalogirou – 2

Amaliados 17, Athens, Greece, P.O.Box 11523, k.kalogirou@prv.ypeka.gr

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Abstract

We explore the field equations in a 4-d complex space-time, in the same way, that general relativity does for our usual 4-d real space-time, forming this way, a new "general relativity" in C^4 space-time, free of sources. Afterwards, by embedding our usual 4-d real space-time in C^4 space-time, we describe geometrically the energy-momentum tensor $T_{\mu\nu}$ as the lost geometric information of this embedding. We further give possible explanation of dark field and dark energy.

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1 Introduction

This is the second paper of a series of papers [?] [?] [?] [?] [?], concerning a physical theory in an extended C^4 spacetime. It is natural after we have derived the geodesic equations, firstly in the symplectic R^8 space and afterwards in the embedded R^4 in R^8, to proceed to the field equations in the same spirit as general relativity works. So, we will proceed with an action similar to

$$S_\pi = \int R\sqrt{g}d\Omega \tag{1}$$

but in C^4 where will give us equations of fields not only for gravity as Eq. (1) but for the unified field G_{ij} which represents much more than gravity as we have seen in the first paper [?]. In order to proceed, we must give some important definitions concerning the tensor calculus in C^4.

2 Tensor calculus in C^4

If G_{ij} the Hermitian metric tensor and v^i a contra-variant complex vector, the covariant complex vector is defined as $v_j = G_{ij}v^i$ and the following relations holds

$$v^j = G^{ij}v_i = G^{ij}G_{pi}v^p = G_{pi}G^{ij}v^p = \delta^j_p v^p = v^j \tag{2}$$

$$v_j = G_{ij}v^i = G_{ij}G^{pi}v_p = G^{pi}G_{ij}v_p = \delta^p_j v_p = v_j \tag{3}$$

and the measure is

$$|| \ v \ ||^2 = G_{ij}v^i\overline{v}^j = v_j\overline{v}^j \tag{4}$$

but

$$v^j = G^{pj}v_p \longrightarrow \overline{v}^j = \overline{G^{pj}}\overline{v}_p = G^{jp}\overline{v}_p \tag{5}$$

so Eq. (4) can be written also as

$$|| \ v \ ||^2 = v_j\overline{v}^j = v_j G^{jp}\overline{v}_j = G^{pj}v_j\overline{v}_p \tag{6}$$

Now, let us consider in general a complex tensor A^{ij}. If we wish to lower the indice j we must use the relation $A^i_j = G_{pj}A^{ip}$ and in the same spirit $A^{ij} = G^{pj}A^i_p$ and as a consequence the following relation holds

$$A^{ij} = G^{pj}(G_{kp}A^{ik}) = G_{kp}G^{pj}A^{ik} = \delta^j_k A^{ik} = A^{ij} \tag{7}$$

We can further proceed with a complex tensor with three indices B_{ijk} where the mixed tensor can be written as $B^i_{jk} = G^{pi}B_{pjk}$ and $B_{ijk} = G_{pi}B^p_{jk}$ and moreover

$$B_{ijk} = G_{pi}B^p_{jk} = G_{pi}(G^{lp}B_{ljk}) = G^{lp}G_{pi}B_{ljk} = \delta^l_i B_{ljk} = B_{ijk} \tag{8}$$

and as a conclusion the above relations holds. We can also define the pseudo-product between the contra- variant complex vectors as

$$< u \mid v >= G_{ij}u^i\overline{v}^j \tag{9}$$

If we consider $u = \lambda_1 u_1 + \lambda_1 u_2$, then we have

$$< \lambda_1 u_1 + \lambda_1 u_2 \mid v >= G_{ij}(\lambda_1 u_1^i + \lambda_1 u_2^2)\overline{v}^j = \lambda_1 < u_1 \mid v > +\lambda_2 < u_2 \mid v > \tag{10}$$

which tells us that it is linear with respect to the first variable. If we consider now as $v = \mu_1 u_1 + \mu_1 u_2$

$$< u \mid \mu_1 u_1 + \mu_1 u_2 >= \overline{\mu_1} < u \mid v_1 > +\overline{\mu_2} < u \mid v_2 > \tag{11}$$

which tells us that it is not linear with respect to the second variable.
Moreover, the determinant of a complex matrix exists and it is a real number.
Our next step is to define the "Christoffel symbols" in this complex geometry. We have already define in the first paper

$$\widehat{\Gamma}_{k,ij} = \Lambda_{k,ij} = \left(\Gamma^{(x)}_{k,ij} + \Delta^{(x)}_{k,ij}, -\Gamma^{(y)}_{k,ij} + \Delta^{(y)}_{k,ij}\right) \tag{12}$$

with respect to the Cauchy derivative. But we can also define the $\Lambda_{\overline{k},\overline{i},\overline{j}}$ symbols as

$$\Lambda_{\overline{k},\overline{i},\overline{j}} = \frac{1}{2}\left(\frac{\partial G_{jk}}{\partial \overline{z}^i} + \frac{\partial G_{jk}}{\partial \overline{z}^j} - \frac{\partial G_{jk}}{\partial \overline{z}^k}\right) \tag{13}$$

but we can also prove that

$$\overline{\Lambda_{\bar{k},\bar{i},\bar{j}}} = \Lambda_{k,ji} \tag{14}$$

The "Christoffel symbols" of the second kind will be respectfully

$$\Lambda^k_{ij} = G^{lk}\Lambda_{l,ij} \tag{15}$$

Now, we can define the associate "Riemmann- Christoffel" curvature tensor in the complex geometry as

$$Z_{ijkl} = \begin{vmatrix} \dfrac{\partial}{\partial z^k} & \dfrac{\partial}{\partial z^l} \\ \Lambda_{i,jk} & \Lambda_{i,jl} \end{vmatrix} + \begin{vmatrix} \Lambda^b_{jk} & \Lambda^b_{jl} \\ \Lambda_{i,jk} & \Lambda_{i,jl} \end{vmatrix}$$

After some calculation Z_{ijkl} can be written also as

$$Z_{ijkl} = \big(\overset{\text{xx}}{R}_{ijkl} - \overset{\text{yy}}{R}_{ijkl} + \overset{\text{xy}}{M}_{ijkl} + \overset{\text{yx}}{M}_{ijkl}\big) + i\big(\overset{\text{xx}}{M}_{ijkl} - \overset{\text{yy}}{M}_{ijkl} - \overset{\text{xy}}{R}_{ijkl} - \overset{\text{yx}}{R}_{ijkl}\big) \tag{16}$$

where with $R_{....}$ we symbolise the "Riemmann-Christoffel" tensor with respect to the symmetric tensor g_{ij} and with $M_{....}$ the "Riemmann-Christoffel" tensor with respect to the anti-symmetric tensor I_{ij} and the symbols over shows us the kind of the partials i.e xx stands for $\dfrac{\partial}{\partial x^i}\dfrac{\partial}{\partial x^i}$. In the same spirit the mixed one is

$$Z^i_{jkl} = G^{pi}Z_{pjkl} = \big(\overset{\text{xx}}{R^i_{jkl}} - \overset{\text{yy}}{R^i_{jkl}} + \overset{\text{xy}}{M^i_{jkl}} + \overset{\text{xy}}{M^i_{jkl}}\big) + i\big(\overset{\text{xx}}{M^i_{jkl}} - \overset{\text{yy}}{M^i_{jkl}} + \overset{\text{xy}}{R^i_{jkl}} + \overset{\text{yx}}{R^i_{jkl}}\big) \tag{17}$$

We must keep in mind that all the tensors of type R and M in the covariant form are all real, while in the mixed form they are all complex due to the fact that we raise by the complex-Hermitian metric tensor G^{pj}. We can continue with the "Ricci tensor" $Z_{jk} = Z^l_{jkl} = G^{pl}Z_{pjkl}$

$$Z_{jk} = \big(\overset{\text{xx}}{R}_{jk} - \overset{\text{yy}}{R}_{jk} + \overset{\text{xy}}{M}_{jk} + \overset{\text{yx}}{M}_{jk}\big) + i\big(\overset{\text{xx}}{M}_{jk} - \overset{\text{yy}}{M}_{jk} + \overset{\text{xY}}{R}_{jk} + \overset{\text{Yx}}{R}_{jk}\big) \tag{18}$$

and finally the "Ricci scalar quantity" $Z = G^{jk} = Z_{jk}$

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$$Z = \big(\overset{xx}{R} - \overset{yy}{R} + \overset{xy}{M} + \overset{yx}{M}\big) + i\big(\overset{xx}{M} - \overset{yy}{M} + \overset{xy}{R} + \overset{yx}{R}\big) \tag{19}$$

all the scalar quantities R and M are complex. Furthermore, in the usual geometry of R^4 there exists the relation

$$\frac{\partial ln\sqrt{|g\,|}}{\partial x^l} = \Gamma^i_{il} \tag{20}$$

which takes the form for the Hermitian metric tensor G_{ij}

$$\frac{\partial ln\sqrt{|G\,|}}{\partial x^l} = G^{ji}\Lambda_{i,jl} + \Lambda^i_{lj} \tag{21}$$

3 Field equations in C^4

We can now proceed with the field equations in the same spirit of general relativity by an action of the form

$$S_\pi = \int Z\sqrt{G}d\Omega \tag{22}$$

where $d\Omega$ volume of C^4. The variation of the action will lead us to the equations

$$Z_{\mu\nu} - \frac{1}{2}ZG_{\mu\nu} = 0 \tag{23}$$

$$\nabla_l G^{\mu\nu} = 0 \tag{24}$$

Eq. (24) expresses the metric compatibility with respect to the Hermitian metric tensor. Moreover, Eq. (23) can be written as a pair with respect to R^8 as

$$\begin{aligned} Re\big(Z_{\mu\nu} - \frac{1}{2}ZG_{\mu\nu}\big) &= 0 \\ Im\big(Z_{\mu\nu} - \frac{1}{2}ZG_{\mu\nu}\big) &= 0 \end{aligned} \tag{25}$$

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These equations must represent the unified geometric theory for all fields if our consideration is valid. It necessary to say that we will not need to add an energy-momentum tensor $T_{\mu\nu}$! We will create the energy-momentum tensor $T_{\mu\nu}$ by embedding R^4 in R^8 or C^4. This way we will find how the energy-momentum tensor $T_{\mu\nu}$ is formed, plus from what quantities it consists of. In order to understand, we must go back to the embedded metric tensor N_{ij} for the simplest case $y^{\alpha'} = \lambda\delta_{\varrho}^{\alpha'} x^{\varrho}$ for $\alpha' = 1, 2, 3$ and $y^0 = y^0(x^0)$ as it was investigated in our first paper

$$N_{ij} = (1-\lambda^2)g_{ij} + \lambda D_{ij}\frac{\partial y^0}{\partial x^0} - 2E_{ij}\Big(\frac{\partial y^0}{\partial x^0}\Big)^2 - M_{ij}\frac{\partial y^0}{\partial x^0} \tag{26}$$

This tells us that the pair of Eq. (25) will become only one equation where all quantities will be refereed to N_{ij} instead G_{ij}. Thus, the "Ricci tensor" with respect to N_{ij} will break to pieces as

$$\overset{\mathrm{N}}{R}_{\mu\nu} = (1-\lambda^2)^2\overset{\mathrm{g}}{R}_{\mu\nu} + \lambda^2\kappa^2\overset{\mathrm{D}}{R}_{\mu\nu} - 2\kappa^4\overset{\mathrm{E}}{R}_{\mu\nu} - \kappa^2\overset{\mathrm{M}}{R}_{\mu\nu} \tag{27}$$

where we have set $\kappa = \dfrac{\partial y^0}{\partial x^0}$. It is important to observe and note the factor k^4 in front of the term $\overset{\mathrm{E}}{R}_{\mu\nu}$ which is actually a scalar quantity (only g_{44} exists) and connected to dark energy term. The field equations will be then

$$\overset{\mathrm{N}}{R}_{\mu\nu} - \frac{1}{2}\overset{\mathrm{N}}{R}N_{\mu\nu} = 0 \tag{28}$$

Finally, the desired form of our usual general relativity will be

$$\overset{\mathrm{g}}{R}_{\mu\nu} - \frac{1}{2}\overset{\mathrm{g}}{R}g_{\mu\nu} = T_{\mu\nu} \tag{29}$$

where $T_{\mu\nu}$ consists of all the other parts that are left from Eq. (28) except the terms of the first part $\overset{\mathrm{g}}{R}_{\mu\nu} - \frac{1}{2}\overset{\mathrm{g}}{R}g_{\mu\nu}$ of Eq. (29). As a result we suggest that all the terms consist of the $T_{\mu\nu}$ have the form

$$2 \text{ indices "curvature tensor"} - \text{scalar tensor} \times \text{"metric tensor"}$$

This will be more simple if we remember what happens in the usual context of electromagnetism. Particularly, the $T_{\mu\nu}$ of electromagnetism is

$$T_{ik} = \frac{1}{4\pi}\big(- F_{il}F^l_k + \frac{1}{4}F_{lm}F^{lm}g_{ik}\big) \tag{30}$$

which follows the above mentioned scheme and must be compared to the term formulated by M. The same behaviour happens with the case of perfect fluid. The big difference is that in our consideration the variation is always with respect to a metric tensor, in contrast with the usual variation of electromagnetism which is with respect to the "field" A_μ. But the context of this different pictures must be finally equivalent, as we have seen in the first paper [?]. If we write the "Ricci tensor"with respect to I_{ij}, we have

$$\overset{\mathrm{I}}{R}_{ij} = \begin{vmatrix} \frac{\partial}{\partial z^l} & \frac{\partial}{\partial z^j} \\ \Delta^m_{ij} & \Delta^l_{lm} \end{vmatrix} + \begin{vmatrix} \Delta^m_{ij} & \Delta^m_{il} \\ \Delta^l_{jm} & \Delta^l_{lm} \end{vmatrix}$$

the second determinant contains terms of the form $\Delta^m_{ij}\Delta^l_{lm} - \Delta^m_{il}\Delta^l_{lm}$. But, we have to remember that Δ symbols where connected with the K_{ij} which is finally in the sub case (and after embedded) our usual F_{ij}. This way, the first determinant represents (after embedded) our usual currents! If we repeat this step for the g_{ij} part, we can form a 2 tensor similar to K_{ij}, using now the Γ symbols (let us symbolise it B_{ij}), breaking this way $\overset{\mathrm{g}}{R}_{ij}$ into products of B_{ij} as we do for electromagnetism. Specifically, after we embedded, we can have a gravitation field tensor Gr_{ij}, a dark field tensor Dm_{ij} and a dark energy field tensor (actually a scalar) DE_{ij}. This way we can form Lagrangians containing terms of the form

$$L = \int \Omega_{ij}\Omega^{ij} \tag{31}$$

where Ω_{ij} is the unified field tensor with respect to the Hermitian metric tensor G_{ij} which will break after embedded to terms as

$$Gr_{ij}Gr^{ij}, Dm_{ij}Dm^{ij}, DE_{ij}DE^{ij}, F_{ij}F^{ij}, W_{ij}W^{ij}, G_{ij}G^{ij} \tag{32}$$

where the variation must be with respect to the broken "fields" which follows the relation

$$\Omega_i = G_{ij}V^j \tag{33}$$

plus the terms associated with currents and are produced by the first determinant.

But, it is well known that in order to define the equation of general relativity we need the equation of states also. The equation of states must be derived from the volumes of R^8 or C^4 as they transformed to volumes of the embedded R^4. A theory of "primitive" thermodynamics must be formulated in R^8 or C^4, in order to fully understand the definitions of entropy and pressure as we meet them in our usual context of thermodynamics. But, we can proceed with some speculations, even if we do not accomplish such a task at this point. We know that ordinary matter varies as $\frac{1}{R^3}$ (in our consideration connected to g_{ij}) and radiation as $\frac{1}{R^4}$(in our consideration connected to M_{ij}). The energy density of ordinary matter is

$$\varepsilon_{matter} \simeq N\frac{mc^2}{R^3} \sim R^{-3}$$

on the other hand, wavelength behaves as $\lambda \sim R$ and photons as $\frac{hc}{\lambda} \sim \frac{1}{R}$ and as a result

$$\varepsilon_{photons} \sim n_{ph} \times \frac{hc}{\lambda} \sim \frac{1}{R^4}$$

Moreover, dark energy varies as $\frac{1}{R^0}$ (in our consideration connected to E_{ij} which is eventually a scalar quantity). The only choice left, is a quantity that varies as $\frac{1}{R}$ and must be connected (through our consideration) to dark matter! If the usual equation of states $\rho = wpc^2$ behaves as

$$\rho \sim R^{-3(1+w)}$$

we can have the following scheme

- radiation (R^{-4}) :$w = \frac{1}{3}$
- matter-dust(R^{-3}) :$w = 0$
- dark-matter (R^{-1}) :$w = -\frac{2}{3}$
- dark-energy (R^{0}) :$w = -1$

This way the energy density of dark matter will behave as

$$\varepsilon_{darkmatter} \sim \frac{l}{mR} \sim \frac{1}{R} \tag{34}$$

where l is a constant in order to recover the energy units. There is a peculiar linearity because the power of R is one. Can this linearity remind us something? The most peculiar image that we have about Galaxies is that they are like pancakes, they look from a far point of view as two dimensional objects or that something stretches them in two than three axis. Another picture is that galaxy looks like a set of hair or fibres. We believe that the linear behaviour of dark matter is responsible for these pictures. Of course, these thoughts are in a preliminary basis, but we could write a relation of the form

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$$H^2 = H_0^2(\Omega_{\Lambda 0} + \Omega_{d0}(1+z) + \Omega_{k0}(1+z)^2 + \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4) \tag{35}$$

where the terms of the right side of the equation are the current values for dark energy, "dark matter", curvature, ordinary matter and radiation. Generally, in a series the power terms behave analogous to the powers, meaning for instance that the second power term contributes much more than the third one. This way, we could presume

$$\text{dark energy} > \text{dark matter} > \text{curvature} > \text{ordinary matter} > \text{radiation}$$

As a final comment, we think that all the above mentioned considerations should be investigated as an expanded manifold of C^4 with respect to the dynamic parameter $\mathcal{T} = T + it$ in the literature of Poincare's conjecture by implying the Ricci flow with respect to the Hermitian tensor G_{ij} and keep up with the proof of Poincare's conjecture in Hermitian manifolds. In our next paper we will continue with the material of the first paper. Specifically, we will consider actions with free end point, in order to define the associated Hamilton-Jacobi equation with respect to the unified field Ω_μ as

$$\Omega_\mu = G_{\mu\nu}V^\nu = g_{\mu\nu}V^\nu + iI_{\mu\nu}V^\nu = \mathcal{B}_\mu + i\mathcal{K}_\mu \tag{36}$$

where $\mathcal{B}_\mu$ is the unified field with respect to the symmetric tensor $g_{\mu\nu}$ and $\mathcal{K}_\mu$ the unified field with respect to the antisymmetric tensor $I_{\mu\nu}$. Eventually, we will show that our usual quantum theories are nothing else than ordinary (classic) theory, after embed our usual space-time directly in C^4. All axioms and demands of quantum theories will be just properties of this procedure!

4 Conclusion

We have shown, that the embedding of our usual 4-d real space-time in C^4, give us the ordinary equation of general relativity (where it describes gravity), plus new information. The energy-momentum tensor $T_{\mu\nu}$ was described geometrically. This extended "general relativity" includes not only gravity, but electromagnetism, as well. The extra terms that are included in these equations, where interpreted as dark matter and dark energy fields. In the third paper [?] of this series, we will continue with the introduction of a quantum theory, as a simple classic mechanics theory in C^4.

5 References

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