Relative contribution of PIN-containing secretory vesicles and plasma membrane PINs to the directed auxin transport: theoretical estimation

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Abstract: Intercellular transport of auxin is driven by PIN-formed (PIN) proteins. PINs are localized at the plasma membrane (PM) and on constitutively recycling endomembrane vesicles. Therefore, PINs can mediate auxin transport either by direct translocation across the PM or by pumping it into secretory vesicles (SVs), leading to its secretory release upon fusion with the PM. Which of these two mechanisms dominates is a matter of debate. Here we addressed the issue with a mathematical modeling approach. We demonstrate that the efficiency of secretory transport depends on SV size, half-life of PINs on the PM, pH, exocytosis frequency and PIN density. 3D-SIM microscopy was used to determine PIN density on the PM. Combing this data with published values of the other parameters, we show that the transport activity of PINs in SVs would have to be at least 1000x greater than on the PM in order to produce a comparable macroscopic auxin transport. If both transport mechanisms operated simultaneously and PINs were equally active on SVs and PM, the contribution of secretion to the total auxin flux would be negligible. In conclusion, while secretory vesicle-mediated transport of auxin is intriguing and theoretically possible model, it unlikely to be a major mechanism of auxin transport in planta.

Keywords: Auxin; Polar Auxin Transport; PIN transporters; Secretion; 3D-SIM microscopy; Mathematical modeling

1. Introduction

The plant hormone auxin (IAA) is subject to intercellular Polar Auxin Transport (PAT) mediated by diffusion and the action of efflux and influx carriers. PAT generates local auxin maxima that are crucial for a plethora of developmental processes [1]. Therefore, studies into the mechanism of auxin transport and its regulation have had a prominent place in the auxin field. Auxin maxima-driven developmental events depend on the activity of the PIN-FORMED (PIN) IAA efflux carriers, which provide directionality to intercellular IAA transport through their asymmetric subcellular localization [1,2]. The PIN proteins constitutively cycle between the plasma membrane (PM) and endosomal compartments [3–5], and the developmental importance of this energetically demanding phenomenon has not been unequivocally explained. Three hypotheses have been suggested to explain the requirement of PIN recycling: 1) Recycling enables rapid relocation of PINs and thereby the rapid redirection of auxin transport; 2) PINs serve as IAA transceptors (transporters and receptors at the same time) and their recycling is important for the process of signal transduction; and 3) the PIN-containing secretory vesicles (SVs) are filled with auxin, which is released into the apoplast upon fusion of the SV with the PM. This process is analogous to neurotransmitter release in animals, and is important for the transport of auxin as suggested by previous reports [6,7] (Figure 1).
Several studies claimed to have proven the vesicular transport of IAA [8–10]. However, the validity of the evidence presented in most of these studies has been questioned by many researchers in the field, and thus the hypothesis of vesicular transport of auxin remains controversial [11,12]. A major reason this controversy exists is the inability to uncouple vesicular trafficking from PIN occurrence at the PM using existing cell biology methods. For example, if one could genetically or pharmacologically completely and specifically block the movement of PIN-containing secretory vesicles, it would be impossible to conclude whether the resulting reduction in auxin transport was caused by the lack of IAA secreting vesicles or by the lack of PINs at the PM.

Here, we have constructed a simple mathematical model to estimate the parameters under which vesicular transport could explain the measured net flux values of PAT. We show that vesicular transport model would require at least 1000 times greater PIN activity than the conventional PM transport model to generate the same net flux values.

2. Results

2.1 Model assumptions: PINs can be active on the endomembrane vesicles exclusively or in addition to their activity on the PM

PAT is characterized by the sustained velocity of auxin over long distances (several millimeters, which is much greater than a typical cell length). The magnitude of this velocity is in the range of $1 \div 10 \frac{\text{m}}{\text{s}}$ for different species and different types of plant tissue [13]. Arabidopsis root epidermal cells serve as a useful example because many of the physiological parameters which influence the PAT, and thus used in our calculations, have been experimentally measured in this cell type. Furthermore, vesicular auxin transport has been proposed to play a role in the transition zone of the root [8]. Thus, we chose to apply our model to this tissue. In the roots auxin is transported towards the root tip inside the central cells (stele), whereas in the outer cell layer (lateral root cap cells and epidermal cells) it is transported in the reverse direction: from the root tip towards the shoot [14]. The velocity of this directed transport in Arabidopsis roots was found to be $v_{\text{PAT}} = 2 \div 3 \frac{\text{m}}{\text{s}}$ [15]. We therefore assume that $v_{\text{PAT}} \geq 1 \frac{\text{m}}{\text{s}}$ in the epidermal cells in the root transition zone and estimated the permeability of PINs needed to yield this value.

In our model we consider only secretory/recycling vesicles (SVs) that fuse with the PM, and consider these secretory vesicles as the method of PIN delivery from the endosome to the PM. For such auxin-transporting vesicles we considered two hypothetical scenarios: 1) PINs transport auxin exclusively in these endomembrane vesicles; and 2) PINs transport auxin both in the vesicles and on the PM. The auxin permeability of a PIN-containing membrane ($P_{\text{PIN}}$) remains undetermined. However, $P_{\text{PIN}}$ has been calculated previously for scenario when PINs are active solely on PM ($P_{\text{PIN}}^M$) [16,17]. Therefore, the permeability of PINs in vesicles $P_{\text{PIN}}^V$ is a readout rather than input to our
calculations. We calculated 1) the relative values of permeability $P_{PIN}^V$ and the individual activity of PINs proteins needed for the vesicular transport model to generate the same net flux as the PM model and 2) the relative contribution of the vesicular transport to the net flux, assuming that PINs activity is the same in the vesicles and in the PM.

2.2 Short summary of model results

1) For the case when only vesicular PINs are actively transporting auxin, we derived equations for auxin accumulation inside the vesicles. The auxin accumulation ratio depends on the permeability of the vesicle membrane to auxin due to PINs ($P_{PIN}^V$). At the same time, it governs the amount of auxin released by this mechanism. Therefore, we relate macroscopic auxin transport velocity ($v_{PAT}$) to $P_{PIN}^V$ and calculated the coefficient of proportionality between these variables for the physiological values of the other parameters: $P_{PIN}^{\text{v-only}} \geq 40 \cdot v_{PAT}$.

However, if PINs are only active on the PM, then PINs permeability $P_{PIN}^{\text{PM-only}} \approx v_{PAT}$ [16]. Thus, for the minimal $v_{PAT} = \frac{1 \mu m}{s}$, as observed in epidermal root cells [13], the lower bound of permeability is $P_{PIN}^{\text{v-only}} \approx 40 \mu m/s$ for vesicular transport and $P_{PIN}^{\text{PM-only}} \approx \frac{1 \mu m}{s}$ for PM transport.

Furthermore, by reducing permeability values by the density of PINs we show that the activity of individual PINs on the vesicle membrane has to be 3 orders of magnitude higher than in the scenario where they are active only on the PM, in order to produce the same PAT velocity: $\frac{p_{v-only}}{p_{PM-only}} > 4200$.

2) In the case that PINs are active both in vesicles and on the PM, the auxin flux ratio through these two mechanisms reads:

$$\frac{\Phi_{PM}}{\Phi_v} = 4200 \cdot \frac{p_{PM}}{p_v},$$

where $p_{PM}$ is the ratio of individual activities of PIN molecules in these domains. If activities are equal, $p_{PM} = p_v$, auxin flux through SVs contributes no more than 0.02% of the total flux.

Conclusion: The transport of auxin by vesicles is ~1000 times less efficient than through PINs active on the PM. Permeability values of PINs ($P_{PIN}^V$ and $P_{PIN}^{\text{PM}}$) still await direct measurements, but values as high as $P_{PIN}^V \approx 40 \mu m/s$, as estimated in our study, are unlikely. This therefore argues against vesicular transport of auxin in SVs as a major mechanism of directional auxin transport.

2.3 Detailed model description.

2.3.1. PINs active only in vesicles.

1. How much auxin should a vesicle with active PIN contain to produce the PAT?

1.1 Size of a vesicle.

The size of exocytotic vesicles that are considered to perform auxin transport are well known, where the dimeter of these vesicles are reported to be $d = 0.06 \div 0.08 \mu m$ [18], and are spherical in shape (see Table 2 for corresponding surface area and volume).

<table>
<thead>
<tr>
<th>Table 1. Values of parameters used in this study.</th>
</tr>
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<tbody>
<tr>
<td>Parameter</td>
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1.2 Biochemical constituents and transport processes. Equation for auxin concentration inside the vesicle.

The total concentration of auxin (IAA) is the mixture of anions IAA⁻ and protonated IAAH, and the ratio between which depends on pH of the solution:

- \( pH_c = 7 \) in cytoplasm, \( \sim 99\% \) of IAA⁻, \( 1\% \) of IAAH
- \( pH_v = 5.5 \div 6.5 \) in vesicle, \( \sim 83\%-98\% \) of IAA⁻, \( 17\%-2\% \) of IAAH
- \( pK_a = 4.8 \) for IAA.

The fractions of IAA in anion form in cytosol and vesicle are denoted by \( f_{ac} \) and \( f_{av} \) respectively. The values were computed using

\[
\begin{align*}
  f_{av} &= \frac{1}{1 + 10^{pK_a - pH}} \\
  f_{ac} &= \frac{1}{1 + 10^{pK_a - pH}} .
\end{align*}
\]

Thus, \( (1 - f_{av}) \) and \( (1 - f_{ac}) \) are fractions of IAAH in vesicle and cytosol respectively (see Figure 2 and Table 2 for IAA⁻ fractions depending on pH).

We denote
- \( A_c \) - cytoplasmic concentration of auxin [mol/l],
- \( A_v \) - concentration of auxin inside the vesicle [mol/l].
**Table 2.** Dependence of the accumulation ratio $R$ (eq.8) and maximum number of molecules in the vesicle $N_v^{\text{max}}$ (eq.7) on pH in the vesicle. The last column provides lower bounds for $\frac{p_{\text{PIN}}^{\text{v-only}}}{P_{\text{diff}}} $ required to yield directional transport of auxin $v_{\text{PAT}}$ depending on $pH_v$. $f_{av/c}$ - fraction of IAA in the vesicle/cytoplasm, $(1 - f_{av/c})$ -fraction of IAAH in the vesicle/cytoplasm.

<table>
<thead>
<tr>
<th>$pH_v$</th>
<th>$f_{av}$</th>
<th>$1 - f_{av}$</th>
<th>$R^*$</th>
<th>$N_v^{\text{max}}$</th>
<th>$\frac{p_{\text{PIN}}^{\text{v-only}}}{P_{\text{diff}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>0.833</td>
<td>0.167</td>
<td>6.0</td>
<td>0.096</td>
<td>$\geq 2330 \frac{S}{\mu m} \cdot v_{\text{PAT}}$</td>
</tr>
<tr>
<td>6.2</td>
<td>0.962</td>
<td>0.038</td>
<td>26.2</td>
<td>0.42</td>
<td>$\geq 535 \frac{S}{\mu m} \cdot v_{\text{PAT}}$</td>
</tr>
<tr>
<td>6.5</td>
<td>0.980</td>
<td>0.020</td>
<td>49.7</td>
<td>0.80</td>
<td>$\geq 280 \frac{S}{\mu m} \cdot v_{\text{PAT}}$</td>
</tr>
<tr>
<td>7.0</td>
<td>0.994</td>
<td>0.006</td>
<td>166</td>
<td>2.7</td>
<td>$\geq 80 \frac{S}{\mu m} \cdot v_{\text{PAT}}$</td>
</tr>
</tbody>
</table>

* For lower $pH_v$ accumulation ratios $R$ have lower coefficient in front of $\frac{p_{\text{PIN}}^{\text{v-only}}}{P_{\text{diff}}}$ because diffusion from the vesicle, that balances PIN-mediated influx, is higher for lower $pH_v$.

** Terms in gray are always much smaller than left terms and can be neglected.

Fast diffusion in the cytoplasm ($D_c = 600 \text{ } \mu m^2/s$) ensures fast mixing of IAA inside the small vesicle, that takes $\tau_{\text{diff}} \leq 10^{-6} \text{ s}$ in vesicles of diameter $d \leq 100 \text{ nm }$. Thus, IAA concentration inside the vesicle can be considered homogenous and increases proportionally to the flux through the membrane:

$$ \frac{dA_v}{dt} = \frac{1}{V_v} j_{\text{net}}^{\text{in}} \quad \text{(Eq.1)} $$

where $j_{\text{net}}^{\text{in}}$ is the net influx of IAA into vesicle (units [mol/s]), which has two components: transport of anions $\text{IAA}^-$ by PINs and transport of protonated neutral form $\text{IAAH}$ by diffusion through membrane:

$$ j_{\text{net}}^{\text{in}} = j_{\text{PIN}}^{\text{IAA}^-} + j_{\text{diff}}^{\text{IAAH}} \quad \text{(Eq.2)} $$

$$ j_{\text{diff}}^{\text{IAAH}} = \frac{P_{\text{diff}} S_v}{(1 - f_{ac}) A_c - (1 - f_{av}) A_v} \quad \text{(Eq.3)} $$
Diffusional permeability is known: $P_{\text{diff}}(\text{IAAH}) = 0.5 \mu\text{m/s}$ \cite{17,20}; $P_{\text{diff}}(\text{IAA}^-) \approx \frac{P_{\text{diff}}(\text{IAAH})}{100} = 0.005 \mu\text{m/s}$ and is considered negligible. IAAH has the same diffusional permeability $P_{\text{diff}}$ in both directions, as seen in Eq.3.

PINs transport anions IAA- and are presumed to be permeable only in one direction (into vesicle) resulting in the unknown permeability $P_{\text{PIN}}^v$ of the vesicular membrane. We neglect dependence of PIN-mediated influx on intravesicular auxin concentration and assume that reverse permeability is zero. By doing so, we make PINs more efficient in our considerations than they can possibly be in reality. This approach is acceptable because our aim is to find the lower bound of the permeability of PINs which is able to produce the physiologically observed PAT transport.

Thus, PIN-mediated transport is simplified to:

$$J_{\text{PIN}}^{\text{IAA}^-} = P_{\text{PIN}}^v S_v f_{\text{ac}} A_c $$

(Eq.4)

Net total flux has thus the following form:

$$J_{\text{in}}^{\text{net}} = P_{\text{PIN}}^v S_v f_{\text{ac}} A_c + P_{\text{diff}} S_v \left((1 - f_{\text{ac}})A_c - (1 - f_{\text{av}})A_v\right).$$

(Eq.5)

Eq.1 can be rewritten as:

$$\frac{dA_v}{dt} = A_c \cdot \alpha - \lambda \cdot A_v,$$

(Eq.6)

where

$$\alpha = \frac{P_{\text{PIN}}^v S_v}{V_v} f_{\text{ac}} + \frac{P_{\text{diff}} S_v}{V_v} \left(1 - f_{\text{ac}}\right),$$

$$\lambda = \frac{P_{\text{diff}} S_v}{V_v} \left(1 - f_{\text{av}}\right).$$

1.3 Maximum loading of vesicles with auxin is proportional to the permeability of PINs.

We assume that vesicles exist in the cytoplasm long enough that the vesicular internal auxin concentration reaches its maximum: steady state concentration $A_v$. It is reached at time $t \gg \tau_{\text{load}} = \frac{1}{\lambda} = 4.6s$ for $pH_v = 7$ (and faster for $pH_v < 7$). We consider fully filled vesicles because our aim is to find the minimal requirements for the vesicular transport, and partly filled vesicles would require higher permeability values to produce the PAT.

At steady state, when $\frac{dA_v}{dt} = 0$, Eq.6 simplifies to:

$$A_c \cdot \alpha = \lambda \cdot A_v,$$

(Eq.6')

Thus, number of IAA molecules (in moles) in one vesicle:

$$N_v^{\text{max}} = A_v^{\text{max}} \cdot V_v = A_c \cdot V_v \frac{\alpha}{\lambda},$$

(Eq.7)

Substituting $\alpha$ and $\lambda$ by their expressions gives:

$$N_v^{\text{max}} = A_c \cdot V_v \left[\frac{P_{\text{PIN}}^v S_v}{P_{\text{diff}} S_v} \cdot \frac{f_{\text{ac}}}{1 - f_{\text{av}}} + \frac{1 - f_{\text{ac}}}{1 - f_{\text{av}}} \right],$$

(Eq.7')

that shows that number of auxin molecules loaded inside the vesicle is proportional to $P_{\text{PIN}}^v$.

We can also rewrite this equation in the form:

$$N_v^{\text{max}} = A_c \cdot V_v \cdot R,$$

(Eq.7'')

where

$$R = \frac{A_v^{\text{max}}}{A_c} = \frac{P_{\text{PIN}}^v}{P_{\text{diff}}} \cdot \frac{f_{\text{ac}}}{1 - f_{\text{av}}} + \frac{1 - f_{\text{ac}}}{1 - f_{\text{av}}}$$

(Eq.8)

is the accumulation ratio – the ratio of intravesicular IAA concentration to that in the cytoplasm surrounding the vesicle. Expressions for accumulation ratio and maximum loading of vesicles for different $pH_v$ are presented in Table 2.
1.4. Lower bound for accumulation ratio $R$ necessary to produce the PAT.

In this section we estimated the auxin accumulation ratio $R$ in vesicles, which is necessary to produce $v_{PAT}$ if auxin in transported via vesicles only. Auxin flux density that corresponds to $v_{PAT}$ velocity is given by

$$
\Phi_{\text{cell--apoplast}} = v_{PAT} \cdot A_c \left[ \text{mol} \cdot \mu\text{m}^{-2} \cdot \text{s} \right].
$$

(Eq.9)

Maximum flux of auxin, that vesicles can carry through the PM is

$$
\Phi_v = N_v^\text{max} \cdot F_v^\text{max} \left[ \text{mol} \cdot \mu\text{m}^{-2} \cdot \text{s} \right],
$$

(Eq.10)

where $F_v^\text{max}$ is maximum exocytosis frequency (vesicles per second fusing with the unit area of cell face). This flux should be not lower than the yielded flux:

$$
\Phi_v \geq \Phi_{\text{cell--apoplast}}.
$$

(Eq.11)

Thus,

$$
N_v^\text{max} \cdot F_v^\text{max} \geq v_{PAT} \cdot A_c
$$

(Eq.12)

which gives, using Eq. 7'':

$$
R \cdot A_c \cdot v_v \cdot F_v^\text{max} \geq v_{PAT} \cdot A_c
$$

(Eq.12')

Consequently,

$$
R \geq \frac{v_{PAT}}{v_v F_v^\text{max}}.
$$

(Eq.13)

Substituting $F_v^\text{max}$ with its expression (see Eq.1.17 in Box 1) $F_v^\text{max} = \frac{\ln 2}{S_v \cdot \tau_{1/2} \left( \frac{\rho_{PM}}{\rho_v} \right)_{\text{max}}}$, we can find the lower bound of the accumulation ratio, that is required for $v_{PAT}$:

$$
R \geq \frac{v_{PAT} S_v \cdot \tau_{1/2} \left( \frac{\rho_v}{\rho_{PM}} \right)_{\text{min}}}{\ln 2}.
$$

(Eq.14)

Note, that intracellular auxin concentration $A_c$ cancels (Eq.12'), that means that the equations are valid for any $A_c$. Nevertheless, we checked that vesicles will contain at least one molecule of auxin for the physiological value of $A_c$ – a condition required for auxin transport to be theoretically possible (see Box 3). We also calculated minimum accumulation ratio required for $v_{PAT} = 1 \mu\text{m}/\text{s}$ (Box 3).

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**Box.1. Expression for $F^+$, frequency of secretory vesicles fusing with PM, and estimation of its maximum value $F_v^\text{max}$.**

In the following text we will consider only the polar domain of PM, which is the front membrane in the direction of PAT. It contains more PINs than the neighboring sides of the PM, and is visible by fluorescence microscopy [19]. The density of PINs depends on the rate of their delivery to PM by vesicles and on the rate of their removal. We assume that within the polar domain PINs are homogeneously distributed. Mass conservation for number of PINs on a polar domain of PM membrane of area $S_{PM}$ then reads:

$$
\frac{dn}{dt} = n_v F^+ S_{PM} - \beta \cdot n,
$$

(Eq.1.1)

where $n$ - number of PINs on PM, $n_v$ - number of PINs in one vesicle, $F^+$ is frequency of exocytosis. PINs come to PM via $F^+ S_{PM}$ vesicles per second. Removal of PINs is proportional to $n$ with the decay coefficient $\beta$, which describes any possible mechanism (through endocytosis, diffusion to other PM domains, degradation etc.).

If exocytosis is stopped, $F^+ = 0$, PINs are only removed:
\[ \frac{dn}{dt} = -\beta \cdot n. \]  
(Eq.1.2)

The solution of this equation reads:

\[ n(t) = n(0)e^{-\beta t} \]  
(Eq.1.3)

Introducing half-life \( \tau_{1/2} \), time when 50% of PINs has been removed \( (n(\tau_{1/2}) = n(0)/2) \), gives \( \beta \):

\[ \beta = \frac{\ln 2}{\tau_{1/2}}. \]  
(Eq.1.4)

At steady state, when removal and arrival of PINs are balanced, \( \frac{dn}{dt} = 0 \),

\[ n \cdot F^+ S_{PM} = \beta \cdot n. \]  
(Eq.1.5)

Eq.1.4 and Eq.1.5 give us expression for \( F^+ \), the frequency of vesicle fusion with the PM:

\[ F^+ = \frac{\ln 2}{S_{PM} \cdot \tau_{1/2} \cdot \rho_v} \]  
(Eq.1.6)

We rewrite it using \( \rho_{PM} = \frac{n}{S_{PM}} \cdot \rho_v = \frac{n_v}{S_v} \) - densities of PINs on the PM and vesicle membrane respectively:

\[ F^+ = \frac{\ln 2}{S_{PM} \cdot \tau_{1/2} \cdot \rho_v} = \frac{\ln 2}{S_{PM} \cdot \tau_{1/2} \cdot \rho_{PM}} \cdot \frac{S_{PM}}{S_v} \cdot \frac{\rho_{PM}}{\rho_v} = \frac{\ln 2}{S_v \cdot \tau_{1/2} \cdot \rho_v} \]  
(Eq.1.7)

\( \tau_{1/2} = 1.3 \cdot 10^4 \text{s} \), which has been measured by Jásik et al. ([19]) for PIN2 in epidermal cells of Arabidopsis root. Finally, introducing values for \( S_v \) and \( \tau_{1/2} \) gives:

\[ F^+ = \frac{0.69}{1.1 \cdot 10^{-2} \mu m^2 \cdot 1.3 \cdot 10^4 \text{s}} \cdot \frac{\rho_{PM}}{\rho_v} \cdot 0.005 \cdot \frac{1}{\mu m^2 \text{s}}. \]  
(Eq.1.8)

Introducing estimation of minimum value for densities ratio \( \left( \frac{\rho_v}{\rho_{PM}} \right)_{\text{min}} = 0.01 \) (see Box.2) into Eq.1.8 gives upper bound for \( F^+ \):

\[ F^+_{\text{max}} = 0.5 \frac{\text{vesicles}}{\mu m^2 \text{s}}. \]  
(Eq.1.9)

**Experimentally measured endocytosis rate gives alternative estimate of exocytosis rate.**

In accordance with the above result, the measured rate of endocytosis is \( \sim 0.5 \frac{\text{vesicles}}{\mu m^2 \text{s}} \) (Table 3). The total area of vesicles fusing with the PM in one cell has to be balanced by the area of endocytosed vesicles. Given that sizes of exocytotic and endocytotic vesicles are the same, maximal rate of endocytosis gives a rough estimate of maximal possible rate of exocytosis, which is in the same order of magnitude as our theoretical estimate.

It is worth to note that the half-life of PINs \( \tau_{1/2} \) was measured for “old” PINs and doesn’t account for possibility of “old” PINs being removed and brought back to the membrane via vesicles. This process would effectively reduce \( \tau_{1/2} \) and allow for higher \( F^+_{\text{max}} \). However, based on our comparison with experimental values of endocytosis, we argue that \( F^+_{\text{max}} \) can not be much higher than \( 0.5 \frac{\text{vesicles}}{\mu m^2 \text{s}} \). This restriction comes from physiological \textit{in vivo} rate of endocytic vesicle formation (taking \( \sim 18-22 \text{ seconds per vesicle, see Table 3 and [21]} \) and limited area of the cell membrane.

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**Box.2. Estimation of minimal PINs density ratio on vesicles and PM:** \( \left( \frac{\rho_v}{\rho_{PM}} \right)_{\text{min}} \).

The lowest possible density of PINs on the vesicular membrane is one PIN molecule per vesicle: \( \left( \rho_v \right)_{\text{min}} = \frac{1}{S_v} = 50 \frac{1}{\mu m^2} \).
The maximum PIN density can be calculated as follows: diameter of the globular 60kDa protein is ~ 8nm. If PINs on the PM are densely packed, their density is at most \( \rho_{PM}^{\max} \approx 2 \times 10^4 \ \frac{1}{\mu m^2} \), which is the maximum value of PIN density on PM.

Thus,

\[
\frac{\rho_{PM}}{\rho_{PM}^\text{min}} = \frac{\rho_{PM}^{\text{min}}}{\rho_{PM}^{\text{max}}} = 0.002.
\]  

(Eq.2.1)

However, PINs most probably cannot reach this maximum density in vivo, for the reason that numerous other proteins occupy space in the membrane. Also quantification of PIN2-GFP in epidermal cells of Arabidopsis root cells, observed using 3D structure illumination microscopy (SIM) with a resolution of 110 nm (Figure 3a), showed that separated source-spots of GFP signal are always resolved with maximum density of \( 5.6 \ \frac{\text{spots}}{\mu m^2} \) (Figure 3b). Each of the spot contains at least one PIN-GFP protein. However, most spots are large: mean area of the spots is \( 0.034 \pm 0.002 \ \mu m^2 \), max area \( 0.098 \ \mu m^2 \), min area \( 0.01 \ \mu m^2 \) (corresponds to resolution limit). Thus, most spots are likely to contain many PINs, which all together fill at most 1/5 part of the membrane area (as \( 5.6 \ \frac{1}{\mu m^2} \times 0.034 \ \mu m^2 = 0.19 \)). This result argues that upper bound for the PIN density on PM is less than close-packing: \( \rho_{PM}^{\max} \leq 4 \times 10^3 \ \frac{1}{\mu m^2} \). Correspondingly,

\[
\frac{\rho_{PM}}{\rho_{PM}^\text{min}} = 0.01
\]  

(Eq.2.2)

is a more realistic value of minimal PINs density ratio for epidermal root cells.

Figure 3. PIN2-GFP density measured by 3D SIM. (a) Example max projection of a 3D SIM image of the lateral membrane of root epidermal cells. (b) Left panel; magnified of the yellow rectangle in (a). Middle panel; the image is made binary and subjected to watershed segmentation. Right panel; pink circles denote detected PIN2-GFP spots. Scale bars; a, 5 \( \mu m \), b, 2 \( \mu m \).
Table 3. Calculation of rate of endocytosis (based on values from [22,23])

<table>
<thead>
<tr>
<th>Endocytic marker</th>
<th>DRP1C-GFP</th>
<th>CLC-GFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>foci per µm²</td>
<td>3.54</td>
<td>3.48</td>
</tr>
<tr>
<td>SD</td>
<td>0.62</td>
<td>0.55</td>
</tr>
<tr>
<td>average lifetime (s)</td>
<td>17.7</td>
<td>19.7</td>
</tr>
<tr>
<td>SD</td>
<td>8.8</td>
<td>6.8</td>
</tr>
<tr>
<td>foci per model cell (15x15 micron)</td>
<td>796.5</td>
<td>783</td>
</tr>
<tr>
<td>SD</td>
<td>139.5</td>
<td>123.75</td>
</tr>
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</table>

Endocytosis events per cell per second

<table>
<thead>
<tr>
<th></th>
<th>DRP1C-GFP</th>
<th>CLC-GFP</th>
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</tr>
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<td>70.3</td>
</tr>
<tr>
<td>min</td>
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<td>24.9</td>
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Maximum rate of endocytosis per second per µm²

<table>
<thead>
<tr>
<th></th>
<th>DRP1C-GFP</th>
<th>CLC-GFP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.467</td>
<td>0.312</td>
</tr>
</tbody>
</table>

Box 3. How many IAA molecules should be inside of one vesicle?

To find the accumulation ratio $R$, which is necessary for minimal PAT: $v_{PAT} = 1 \mu m/s$.

Rewriting Eq.14 as inequality gives:

$$R \geq \frac{v_{min} S_v \cdot \tau_1}{V_c \cdot \ln 2 \cdot \frac{\rho_v}{\rho_{PM}}} = \frac{1 \mu m/s \cdot 2 \cdot 1 \cdot 10^{-2} \mu m^2 \cdot 1.3 \cdot 10^4 s}{2.7 \cdot 10^{-4} \mu m^3 \cdot 0.69} = 0.002 > 2800 \quad \text{(Eq.3.1)}$$

This minimal required accumulation ratio is valid for any $A_c$. Nevertheless, we have to check, that vesicles will contain at least one molecule. In the volume of a vesicle (maximum $2.7 \times 10^5$ nm$^3$) there will be initially $N_v^{z=0} = A_c \cdot V_c = 0.005$ molecules if cytoplasmic auxin concentration $A_c = 30 nM$, which is a plausible estimate for the root epidermis [24]. For the lowest possible accumulation ratio $R = 2800$ number of molecules in one vesicle will be $N_v = A_c \cdot V_c \cdot R \approx 14$, which is a minimal requirement for vesicular transport.

2. Minimal permeability of PINs in vesicles necessary for vesicular auxin transport.

By combining Eq.8 and Eq. 14, we can find the relation between vesicular PINs permeability $P_{PIN}^{v-only}$ and the transport velocity $v_{PAT}$ in the case that PINs are only active in vesicles and thus all PAT is produced by vesicles:

$$\frac{P_{PIN}^{v-only}}{P_{diff}} \cdot \frac{f_{ac}}{1 - f_{av}} + \frac{1 - f_{ac}}{1 - f_{av}} \geq v_{PAT} \cdot S_v \cdot \frac{\tau_{1/2}}{V_c \cdot \ln 2 \cdot \frac{\rho_v}{\rho_{PM}}}$$

Consequently,

$$P_{PIN}^{v-only} \geq P_{diff} \left( \frac{v_{PAT} \cdot S_v \cdot \tau_{1/2}}{V_c \cdot \ln 2 \cdot \frac{\rho_v}{\rho_{PM}}} \right)_{min} \frac{1 - f_{ac}}{1 - f_{av}} \left( \frac{f_{ac}}{1 - f_{av}} \right)$$
Thus, because \( \frac{1-f_{ac}}{1-f_{av}} \) can be neglected being much smaller than the first term,

\[
P_{\text{PIN}}^{v-only} \geq P_{\text{diff}} \cdot \frac{v_{\text{PAT}} \cdot S_v \cdot \tau_{1/2} (1 - f_{av})}{V_v \cdot \ln2} \left( \frac{\rho_v}{\rho_{\text{PM}}} \right)_{\text{min}} \quad (\text{Eq.15})
\]

From the Table 2 we find \( \min \left( \frac{1-f_{ac}}{1-f_{av}} \right) = 0.006 \) for \( pH_v = 7.0 \), and using \( \frac{\sigma_v}{\rho_{\text{PM}}} = 0.01 \) from Box.2, and parameter values from Table 1 we reduce inequality to

\[
P_{\text{PIN}}^{v-only} \geq 80 \cdot \frac{\sigma}{\mu\text{m}} \cdot v_{\text{PAT}} \cdot P_{\text{diff}} \quad (\text{Eq.16})
\]

Diffusional permeability has been measured: \( P_{\text{diff}} = 0.5 \mu\text{m/s} \). This gives an estimate of the lower bound of PIN permeability in vesicles:

\[
P_{\text{PIN}}^{v-only} \geq 40 \cdot v_{\text{PAT}} \quad (\text{Eq.17})
\]

For \( pH_v < 7.0 \) bounds for \( P_{\text{PIN}}^{v-only} \) are provided in Table 2. For pH value that we consider realistic, \( pH_v = 6.2 \) permeability has to be higher: \( P_{\text{PIN}}^{v-only} \geq 260 \cdot v_{\text{PAT}} \).

3. Membrane permeability due to PINs has to be much greater on the vesicles than on PM to produce the same auxin transport velocity.

It would be informative to compare our estimate of the permeability value (Eq.17) \( P_{\text{PIN}}^{v-only} \geq 40 \cdot v_{\text{PAT}} \) with experimental measurements. Unfortunately, the permeability of PINs has never been measured directly in the intact tissues. Also measurements of the influx carrier permeability are not possible in explants like protoplasts, as PINs do not stay on the PM, but are instead internalized [25], resulting in no contribution to the efflux of auxin as in vivo systems.

Fortunately, analogous estimates for PIN permeability have been done for the case when PINs are active only on PM (see Box.4, [17]), which have been shown that:

\[
P_{\text{PIN}}^{PM-only} \approx v_{\text{PAT}} \quad (\text{Eq.18})
\]

This relation was proved in theoretical studies using simple mathematical models and confirmed by computational models of multicellular tissues (see Box.4). It is determined under the assumption that PINs are active only on the PMs and facilitate auxin transport without any action from vesicles. Consequently, in epidermal cells of Arabidopsis root \( P_{\text{PIN}}^{PM} \approx 1 \mu\text{m/s} \).

We conclude from Eq.18 and Eq.19 that if all other parameters are held in physiological range, to yield the same PAT velocity as PM PINs, vesicular PINs permeability has to be much higher:

\[
P_{\text{PIN}}^{v-only} \geq 40. \quad \text{However, } P_{\text{PIN}}^{v-only} \text{ and } P_{\text{PIN}}^{PM-only} \text{ also depend on density of PINs on the membranes.}
\]

To clarify this issue, in the next section we calculated the ratio of PINs activity, which is characteristic of individual transporters and which does not depend on the densities of PINs.

---

**Box.4.** The permeability of PINs on the plasma membrane \( P_{\text{PIN}}^{PM} \) equals the directional auxin transport velocity \( v_{\text{PAT}} \).

\( P_{\text{PIN}}^{PM} \) can be derived from the following considerations, analogous to Eq.9-11. Auxin flux density through plasma membrane has the following expression because PINs are transporting anions IAA· and depend only on its intracellular concentration \( f_{ac} \cdot A_c \) (=Eq.21):

\[
\Phi_{\text{PM}} = P_{\text{PIN}}^{PM} \cdot f_{ac} \cdot A_c \quad (\text{Eq.4.1})
\]

It should be equal to PAT flux (Eq.9): \( \Phi_{\text{PM}} = \Phi_{\text{cell–apoplast}} \). Thus,
\[ P_{PIN} \cdot f_{ac} \cdot A_c = v_{PAT} \cdot A_c \]  
(Eq. 4.2)

Consequently,

\[ P_{PIN}^M \equiv v_{PAT} \]  
(Eq. 4.3)

Thus, for \( v_{PAT} = 1 \ \mu m/s \), \( P_{PIN}^M = 1 \ \mu m/s \).

In fact, [16] and [26] have shown that for a file of cells transport speed is comparable to the efflux permeability, and this conclusion was confirmed by computer simulations of multilayered tissues [16],[17],[26]. In these classical works, it has been proven that “advection” of auxin can be just a result of combined polar membrane transport and cytoplasmic diffusion. Thus, macroscopic advection velocity is limited either by polar membrane transport or by the rate of auxin’s transfer along the cell length; whichever value is lower. One can calculate that diffusion along the longest cell length (~100 \( \mu m \)) is faster than the measured velocity, proving that cytoplasmic transport does not limit the “macroscopic advection velocity \( v_{PAT} \)” [13]. In this case, polar membrane transport governs macroscopic velocity [16],[26].

### Derivation of \( P_{PIN}^M \) that would yield \( v_{PAT} = 1 \ \mu m/s \) from theory developed by Mitchison [16].

Equation (2) from reads [16]:

\[
\frac{1}{v_{PAT}} = \frac{1}{p} + \left( 1 + \frac{2q}{p} \right) \frac{L}{2D}
\]

where \( v_{PAT} \) – macroscopic auxin velocity, \( p \) – polar efflux permeability, \( q \) – nonpolar permeability (by diffusion), \( L \)- cell length, \( D \)- diffusion coefficient of auxin inside the cell. Because

\[
\frac{L}{2D} \leq \frac{1 \cdot 10^{-4} m}{2 \cdot 6.7 \cdot 10^{-1} m^2/s} = 7.46 \cdot 10^{-4} s/m,
\]

\( v_{PAT} = 1 \cdot 10^{-6} m/s \) and \( q \geq 0 \),

\[
\Rightarrow p = \frac{1 + 2q L}{v_{PAT}} \geq 1.1 \cdot 10^{-6} m/s .
\]

This equation also shows that the lower bound of efflux permeability \( p \equiv v_{PAT} \), because \( \frac{L}{2D} \ll 1 \). Note, that condition \( \frac{1}{v_{PAT}} - \frac{L}{2D} \geq 0 \) must hold, so that value of permeability is positive, which is true for \( v_{PAT} \leq \frac{2D}{L} = 13.4 \cdot 10^{-6} m \). Estimates of permeability values provided by [16] are in the same order of magnitude.

In principle, auxin-containing vesicles can also contribute to directional transport within the cell. However, as noted above, diffusion is already sufficient to transport auxin inside the cell, so such additional “acceleration” is not relevant for the macroscopic transport rate.

4. Individual activity of PINs has to be much higher on the vesicles than on the PM to produce the same auxin transport velocity.

To compare the efficiency of transport it is necessary to normalize permeabilities \( P_{PIN}^v-only \) and \( P_{PM}^v-only \) to the corresponding density of PINs. The ratio of normalized permeabilities equals the ratio of individual activity of PIN transporters situated in vesicles \( (p_{v-only}) \) and on the PM \( (P_{PM-only}) \) (see Box.5 for explanation):

\[
P_{v-only}^{P_{PIN}} = \rho_{v} \cdot \frac{P_{PIN}^{P_{PM-only}} \cdot v_{PAT} \cdot S_{v} \cdot \tau_{1/2} (1 - f_{av})}{V_{o} \cdot \ln 2 \cdot f_{ac} \cdot \rho_{PM}}.
\]

Combining this equation (=Eq. 5.1), and Eq. 15(equality form)

\[
P_{v-only}^{P_{PIN}} = P_{diff} \cdot \frac{v_{PAT} \cdot S_{v} \cdot \tau_{1/2} (1 - f_{av}) \rho_{v}}{V_{o} \cdot \ln 2 \cdot f_{ac} \cdot \rho_{PM}}.
\]
gives:
\[
\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} = \frac{v_{PAT} \cdot S_v \cdot \tau_{1/2}}{V_v \cdot \ln 2} \cdot \frac{P_{diff}}{P_{PM\text{-only}}} \cdot \frac{P_{PIN}}{P_{PM\text{-only}}} \cdot \frac{1 - f_{av}}{f_{ac}}.
\]

Note, that density ratio \(\frac{P_v}{P_{PM}}\) cancels in this equation:
\[
\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} = \frac{v_{PAT} \cdot S_v \cdot \tau_{1/2}}{V_v \cdot \ln 2} \cdot \frac{P_{diff}}{P_{PM\text{-only}}} \cdot \frac{1 - f_{av}}{f_{ac}}
\]

By calculating the minimum of the right-hand side we can find the lower bound for PINs activity ratio.
\[
\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} \geq \left( \frac{v_{PAT} \cdot S_v \cdot \tau_{1/2}}{V_v \cdot \ln 2} \cdot \frac{P_{diff}}{P_{PM\text{-only}}} \cdot \frac{1 - f_{av}}{f_{ac}} \right)_{\text{min}} \quad (\text{Eq.19})
\]

Using the fact that \(v_{PAT} \approx P_{PIN}^{PM\text{-only}}\) (see Box.4) we find\(^1\):
\[
\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} > 4200.
\] (Eq.20)

For \(pH_v = 6.2\), (see Table 2), the ratio is even higher: \(\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} > 26600\).

**Conclusion I:** For vesicle transport to be able to produce all of the observed IAA flux, the activity of PINs on the vesicle membrane has to be at least three orders of magnitude greater than that estimated for the case when PINs are only active on the PM. Our calculations show that transporting auxin directly through the PM is 1000 times more effective than by means of SVs. This, in our opinion, is an argument against SVs-mediated transport of auxin as a major mechanism of directional auxin transport.

**Box. 5. Individual activity (specific permeability) of PIN transporters.**

Membrane permeability due to PINs is proportional to PINs density and can be expressed as a product of 1) density of transporters on the membrane \(\rho_{PIN}[\text{mol/µm}^2]\) and 2) individual activity of one transporter protein \(p_{PIN}[\text{µm/s/µmol/µm}^2]\), which depends on affinity to auxin, electrical potential across the membrane*, phosphorylation status and any other parameters.

\[
P_{PIN} = \rho_{PIN} \cdot p_{PIN} \quad (\text{Eq.5.1})
\]

* Dependence on membrane potential has been derived in [26]: \(p_{PIN}(\Delta V) \sim \frac{\Delta V \cdot F / R T}{\exp(F / R T) - 1}\), where \(F\) is the Faraday constant, \(R\) is the gas constant, \(T\) is the temperature. For the plasma membrane \(\Delta V \approx 160 \pm 100\text{mV}\). For exocytotic vesicles it is unknown.

2.3.2. PINs active both in vesicles and on PM

Next, we find the ratio of auxin fluxes through both mechanisms in the case that PINs are active on vesicles and the PM. Auxin efflux caused by (active) PINs on the PM:

\[
\Phi_{PM} = \frac{P_{PM}}{P_{PIN}} \cdot f_{ac} \cdot L_C. \quad (\text{Eq.21})
\]

Auxin efflux caused by arriving PIN delivery vesicles (using Eq.7'):

\[
\left( \frac{v_{PAT} \cdot S_v \cdot \tau_{1/2}}{V_v \cdot \ln 2} \cdot \frac{P_{diff}}{P_{PM\text{-only}}} \cdot \frac{1 - f_{av}}{f_{ac}} \right)_{\text{min}} = \frac{10^3 \cdot 6 \cdot 10^8 \cdot 6 \cdot 10^{-3}}{\frac{r}{0.89} 2} \approx 4200.
\]
\[ \Phi_v = N_v \cdot F^+ \cong \frac{P_{PIN}^v f_{ac}}{P_{diff}} \cdot \frac{f_{av}}{1 - f_{av}} \cdot V_o \cdot F^+ \cdot A_c. \]  
(Eq.22)

Dividing Eq.21 by Eq. 22 gives flux ratio:

\[ \frac{\Phi_{PM}}{\Phi_v} = \frac{P_{PM}}{P_{PIN}} \cdot \frac{f_{ac} A_c}{f_{av} V_o F^+} = \frac{P_{PM}}{P_{PIN}} \cdot \frac{P_{diff} (1 - f_{av})}{V_o} \cdot \frac{1}{P_{PIN}}. \]  
(Eq.23)

Substituting \( F^+ \) by its expression from Eq.1.7 gives:

\[ \frac{\Phi_{PM}}{\Phi_v} = \frac{P_{PM}}{P_{PIN}} \cdot \frac{P_{diff} (1 - f_{av})}{V_o} \cdot \frac{P_{PM}}{V_{PM}} \cdot \frac{P_{PIN}}{P_{PIN}} \cdot \frac{1}{P_{PIN}}. \]  
(Eq.23')

Using Eq.5.1 we reduce it further:

\[ \frac{\Phi_{PM}}{\Phi_v} = \frac{P_{diff} (1 - f_{av})}{V_o} \cdot \frac{P_{PM}}{V_{PM}} \cdot \frac{P_{PIN}}{P_{PIN}} \cdot \frac{1}{P_{PIN}}. \]  
(Eq.24)

Introducing parameter values for pH=7 and \( d = 0.08 \) \( \mu \)m gives a lower bound on the ratio:

\[ \frac{\Phi_{PM}}{\Phi_v} \geq 4200 \cdot \frac{P_{PM}}{P_{PIN}}. \]  
(Eq.25)

where \( \frac{P_{PM}}{P_{PIN}} \) is the ratio of individual PINs activity (see Box.5) on the PM and vesicles if PINs are active on both membranes. Note, that the densities of PINs cancel, and flux distributions between the two mechanisms depends only on the activity of PINs.

If the individual activity of PINs on the vesicle and on membrane are equal (\( p_v = P_{PM} \)), then

\[ \frac{\Phi_{PM}}{\Phi_v} \geq 4200, \]  
(Eq.26)

which means that the flux through the vesicle mechanism does not contribute more than 0.02% to the total flux.

For \( pH_v = 6.2 \) the lower bound of the ratio is higher: \( \frac{\Phi_{PM}}{\Phi_v} \geq 26600 \), which implies that the contribution of the vesicular mechanism is less than 0.004%, which is negligibly small compared to PM flux.

The flux ratio \( \frac{\Phi_{PM}}{\Phi_v} \) would be higher (favoring PM flux over SV-mediated flux) if:

- PINs half-life on PM is higher
- pH in vesicle is lower
- vesicles do not stay in the cytoplasm for enough time before fusing with the PM to be fully filled with IAA
- size of vesicles is smaller

The opposite changes of parameter values would increase SV-mediated contribution to the auxin flux (see Eq.24).

**Conclusion II:** The activity of PIN transporters in the vesicle has to be at least 1000 times greater than on the PM to make a substantial contribution to the total directional auxin transport. When realistic physiological parameter values were used in the model, a factor greater than \( 10^4 \) was determined.

3. Discussion

The hypothesis of vesicular secretion of auxin was postulated more than 15 years ago [6,7]. While several studies have claimed to provide experimental evidence to support this concept since [8–10], their conclusions are currently greatly debated in the field [11,12]. Hence the question whether vesicular secretion significantly contributes to intercellular auxin transport remains unresolved. Here, we took a modeling approach to estimate whether such a mode of auxin transport is even theoretically possible.

First, we compared individual activities of PINs active exclusively on vesicles or on the PM, that are necessary to yield the same PAT velocity (Eq.20):
\[
\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} \geq P_{diff} \left( \frac{6 \cdot \tau_{1/2}}{d \cdot n^2} \right) \left( 1 - f_{av} \right) f_{ac} \min .
\]  
(Eq. 27)

This ratio is valid for any PAT velocity and depends on the following measurable parameters: diameter of vesicles \(d\), half-life time of PINs on the PM \(\tau_{1/2}\), pH-dependent fraction of IAAH in a vesicle \(1 - f_{av}\), fraction of IAA in cytoplasm. Experimental parameter values (Table 1) gave us \(\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} > 4200\), meaning that individual activity of PINs on SVs \(p_{PIN}\) needs to be at least three orders of magnitude greater than the activity of PINs on the PM \(p_{PM}\) in order to produce the same auxin transport.

We also provide an estimate for the permeability of PINs in the vesicles required to yield \(v_{PAT} = 1 \text{ m} \text{ s}^{-1}\) in epidermal root cells: \(P_{v\text{PIN}}^v \geq 40 \text{ m} \text{ s}^{-1}\), which is not the measure of individual transporters, but characteristic of the unit area of membrane (explained in Box 5). This value is much higher than any measured permeabilities to date [17,20]. The measurements of permeability due to PINs either on the PM, in vesicles or in both domains will allow one to draw precise conclusions from our model.

To extend these findings further to a more general case, we derived an expression that relates auxin flux driven by PINs on the polar domain of PM and auxin flux driven by PINs at the vesicles, if they are both active (Eq. 24):

\[
\frac{\Phi_{PM}}{\Phi_v} = \frac{P_{diff} (1 - f_{av})}{d} \frac{6 \cdot \tau_{1/2}}{n^2} \frac{P_{PM}}{P_{v}}
\]  
(Eq. 28)

It shows that relative auxin fluxes through both mechanisms depend on: diameter of vesicles \(d\), half-life time of PINs on the PM \(\tau_{1/2}\), pH-dependent fraction of IAAH in a vesicle \(1 - f_{av}\) and individual activities of PIN proteins \(p_{PM}\) and \(p_{v}\).

The main parameter, namely PIN activity, has not been experimentally determined on the PM or in vesicles, which makes it impossible to find actual ratio of fluxes in vivo. Nevertheless, by estimating the extreme values of the rest of parameters from available experimental data (Table 1), we were able to show that \(\frac{\Phi_{PM}}{\Phi_v} > 4200 \cdot \frac{P_{PM}}{P_{v}}\). This expression suggests that if PINs were equally active both at the PM and on the SVs, the contribution of secreted IAA to the overall net IAA flux would be less than 0.02% at pH=7 inside vesicles of maximal size, which is the maximal possible contribution in that case.

Moreover, instead of the half-life time of PINs on the PM \(\tau_{1/2}\) and vesicle diameter, other measurable parameters can be put in this equation (using Eq. 1.7): exocytosis frequency \(F^+\), densities of PINs \(\rho_{v,PM}\) and vesicular volume \(v_v\). Then Eq. 28 takes the following form:

\[
\frac{\rho_{PM}}{\rho_v} = \frac{P_{diff} (1 - f_{av})}{V_v} \frac{6 \cdot \tau_{1/2}}{P_{PM} P_{v}} \frac{\rho_{v,PM}}{P_{PM} P_{v}}
\]  
(Eq. 29)

We provide measurement of density of PINs \(\rho_{PM} \leq 4 \cdot 10^3 \frac{1}{\mu \text{ m}^2}\), which gives a plausible estimate of \(\frac{\rho_{PM}}{\rho_{v,PM}} = 0.01\) (Box 2). We also used published experimental data to estimate \(F_{max}^+ = 0.5 \frac{\text{vesicles}}{\mu \text{ m}^2 \text{ s}}\) (end of Box 1). These parameter values give the lower bound for flux ratios: \(\frac{\rho_{PM}}{\rho_v} > 4200 \cdot \frac{P_{PM}}{P_{v}}\) which coincide with the result in the previous paragraph. Thus, by acquiring the same minimal flux ratio using two different sets of parameters we confirm that our conclusions are plausible.

The fact that the parameter values can depend on cell type, PIN type or biochemical status of the cell, it is important to consider if in different tissue types vesicular transport can become dominant. Our equations can be used to resolve this question by calculating the auxin flux ratio for any other tissue type once experimental values are available; especially if the activity of individual PINs on the PM and on vesicles can be measured in the future. If it is found that PINs transporters are active exclusively in vesicles and are inactive on the PM \(p_{PM} = 0\) Eq. 28&29 would become meaningless. This however remains a technically demanding value to determine experimentally. If it was found that PINs are active only on the vesicles, the testable relation between the individual activity of PINs and the measurable parameters is (from Eq. 15):

\[
\frac{P_{v\text{-only}}}{P_{PM\text{-only}}} = \frac{P_{PIN}}{P_{PM}} = P_{diff} \frac{v_{PAT} \cdot 6 \cdot \tau_{1/2} \cdot (1 - f_{av})}{d \cdot n^2} \frac{f_{ac}}{f_{ac}}
\]  
(Eq. 30)
Notably, other parameters in Eq.28&29 cannot have zero values, thus, their change by orders of magnitude is needed to make vesicular auxin transport a dominant mechanism. Methods for separate perturbation of those parameters are needed to distinguish between vesicular-mediated and PM-mediated auxin fluxes.

In conclusion, we have created a simple mathematical model to calculate the efficiency of PIN-mediated vesicular secretion of auxin compared to transport across the PM. Even under the most “vesicular transport-favoring” values of the parameters the vesicular transport of auxin were still determined to be several orders of magnitude less efficient compared to the membrane transport. Our calculations showed that PINs on the PM can produce auxin transport having much less individual activity than required for PINs in the secretory vesicles (SVs). Therefore, we consider it unlikely that PINs are active only in the secretory vesicles; and in case they are active on both PM and SVs, vesicular transport would play a negligible role in PAT.

4. Materials and Methods

4.1. Measurement of PIN density using SIM

Samples were prepared as previously described by Johnson & Vert [21], where the imaging media was supplemented with 30% opti-prep and the coverslips were fixed on to the microscope slide. Cells in the elongation zone were imaged using an OMX v4 3D SIM. A z-stack was created to image the lateral membrane of the PIN2 polar domain and a maximum projection of this was used for analysis. Images were made binary and subjected to watershed segmentation using Fiji [27]. PIN2 spots were then detected using TrackMate [28]. Density of PIN2, mean, maximum and minimum area of the PIN2 spots was calculated from on 12343 spots in 9 different cells from 3 independent roots.

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Abbreviations

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<tr>
<th>Acronym</th>
<th>Definition</th>
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<td>PM</td>
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<tr>
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</tr>
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<td>IAA</td>
<td>3-indole acetic acid = auxin</td>
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References


