

Principles of Instability and Stability in Digital PID Control Strategies and Analysis for a Continuous Alcoholic Fermentation Tank Process Start-up

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Abstract

Some professionals in the industrial community are not as familiar with digital PID controllers as the analog one. So the art work of this present paper is to show properly conditions and aspects to reach stability in the process control, as also to avoid instability, through the use of digital PID controllers, with the objective of help the industrial community. The discussed control strategy used to reach stability is to manipulate variables that are directly proportional to their controlled variables. To validate and to put the exposed principles into practice, it was done two simulations of a continuous fermentation tank process start-up with the yeast strain *S. Cerevisiae* NRRL-Y-132, in which the substrate feed stream contains different types of sugar derived from cane bagasse hydrolysate. These tests were carried out through the resolution of conservation laws, more specifically mass and energy balances, in which the fluid temperature and level inside the tank were the controlled variables. The results showed that if the exposed proceedings are followed, the system stabilize without difficulties.

Keywords: Digital PID; PID controller; Instability; Stability; Alcoholic fermentation; Process start-up.

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1. Introduction

Analogic PID controllers are used in the industry because of its facility in about tuning the parameters (proportional, integral and derivative) and other advantages, but two of the negative points of these controllers are the manipulated variable's kick and oscillation, hindering the practical plant control, increasing maintenance costs and promoting undesired oscillations.

Another weak point of using analogic controllers is that they are not able to make plants start-up without difficulties, since they depend of fixed setpoint values for the manipulated variables. One example is the start-up of an empty tank, in which if the outlet flow rate have a setpoint value equal to the inlet flow rate, the system will not accumulate fluid, so it will remain empty.

Digital PID controllers may lead to instability in plenty situations if they are bad tuned, but one thing that also harms the process control is a wrong chosen strategy, which can lead to instability even if the process design itself naturally tends to be stable. Despite of these situations, these controllers lead to a cheap, simple and efficient process control if well implemented.

So the main objectives of the present paper are to help the community by showing a properly configuration for a digital PID control strategy to execute a stable process control and also to explain a principle of instability. These objectives were validated with two simulations, in which it is controlled, the temperature and the fluid level inside an alcoholic fermentation tank that is being fed with yeast strain *Saccharomyces Cerevisiae* NRRL-Y-132 and a substrate stream of sugar coming from cane bagasse hydrolysate. Both simulations relate to the process start-up, where the first simulation lead to instability and the second to stability.

The first tank in a traditional continuous alcoholic fermentation (Figure 1) process must have more attention than the others due to the higher liberated heat, being a reasonable justification to be the object on scope.

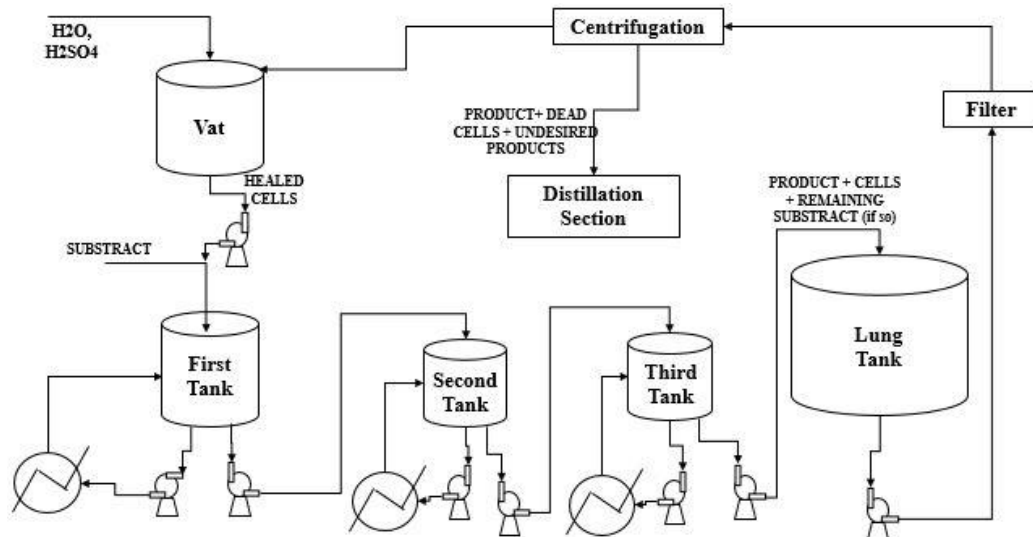


Figure 1: Traditional continuous alcoholic fermentation process.

The growth medium in most likely *S. Cerevisiae* strains, must have the temperature between 301 and 307 K, so they can reproduce without stress and denaturation. The heat liberated during alcoholic fermentation process was first studied by Berthelot, and then by Rechenberg and others authors, but Bouffard made directly experiments with dextrose and studied it more precisely, finding a more real liberated heat value during alcoholic fermentation, and so then used here in the simulations [1]. So a bad control strategy or a bad tuned PID controller can overheat the system, killing yeast cells and thus destroying the process. The first fermentation tank design with its streams is shown in Figure 2:

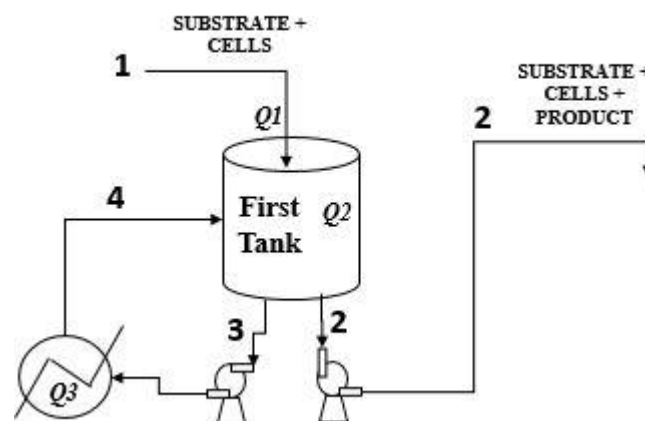


Figure 2: First tank's design.

2. Notable Principles in Digital PID Controllers

It is stated a **principle of instability** in digital PID controllers here, which means that when a variable is controlled (C.V.) by a manipulated variable (M.V.) in such a manner that if M.V. grows C.V. decreases, or when even the counter is true, it implies on saying that the C.V. will diverge in direction of plus infinite or minus infinite. The only way to avoid instability and to reach stability in the setpoint value, is to choose a M.V., in which when it grows, the C.V. also grows (directly proportional), or when the counter is also true.

When PID parameters are positive and C.V. is inversely proportional to M.V., as says the mentioned principle of instability, the exact moment that the C.V. goes through instability (will never stabilize in setpoint value) is when the error (setpoint value subtracted from dynamic value) start to have a negative value. This happen because M.V. decreases, while it should increase to realize the control. In the case that PID parameters are negative, the instability point will happen when the error start to have a positive value, due to the fact that M.V. increases while it should decrease to realize the control. In the present paper this is proved through the simulation, and the instability is clearly noted in the graphs of temperature and fluid level.

So it is not secure to implement a project with a digital PID control strategy that depends of only positives values for the error (if PID parameters are positive) or neither of only negative values for the error (if PID parameters are negative). If the PID parameters are badly tuned, the error is more likely to get both negative and positive values during the process execution, implying in the need of a good control strategy, which is, M.V. must be directly proportional to C.V..

3. Modeling the System

Since stream 2 of the Figure 2 is an outlet flow rate, it does not exchange heat with the content inside the tank, because they are not mixing. The variation of temperature occur due to heat exchange and is expressed by the energy balance written in the form of heat balance for a perfect mixture tank and is represented by Equation (1):

$$\frac{dT_{T-1}}{dt} = \frac{Q_1 + Q_2 + Q_3}{\rho_{T-1} \cdot cp_{T-1} \cdot h_{T-1} \cdot A_{T-1}} \quad (1)$$

In which:

$$Q_1 = v_1 \cdot \rho_{T-1} \cdot cp_{T-1} \cdot (T_1 - T_{T-1}) \quad (2)$$

$$Q_2 = \alpha \cdot R_{S_{T-1}} \cdot h_{T-1} \cdot A_{T-1} \quad (3)$$

$$Q_3 = v_3 \cdot \rho_{T-1} \cdot cp_{T-1} \cdot (T_4 - T_{T-1}) \quad (4)$$

Overall mass balance:

$$\frac{dh_{T-1}}{dt} = \frac{v_1 - v_2}{A_{T-1}} \quad (5)$$

Mass balance for cells:

$$\frac{dC_{C_{T-1}}}{dt} = R_{C_{T-1}} + \frac{v_1 \cdot C_{C1} - v_2 \cdot C_{C_{T-1}}}{h_{T-1} \cdot A_{T-1}} \quad (6)$$

Mass balance for substrate:

$$\frac{dC_{S_{T-1}}}{dt} = -R_{S_{T-1}} + \frac{v_1 \cdot C_{S1} - v_2 \cdot C_{S_{T-1}}}{h_{T-1} \cdot A_{T-1}} \quad (7)$$

Mass balance for product:

$$\frac{dC_{P_{T-1}}}{dt} = R_{P_{T-1}} - \frac{v_2 \cdot C_{P_{T-1}}}{h_{T-1} \cdot A_{T-1}} \quad (8)$$

Xylose, cellobiose and others types of sugars are present in the medium, but they act like inhibitors and do not take part in the fermentation. See [2] for more details about the kinetic model of yeast strain *S. Cerevisiae* NRRL-Y-132 fermenting cane bagasse hydrolysate to produce ethanol, which is used in this paper simulation. The kinetic rates are:

$$R_{C_{T-1}} = (\mu - K_d) \cdot C_{C_{T-1}} \quad (9)$$

$$R_{S_{T-1}} = \mu \cdot C_{C_{T-1}} \cdot \left(\frac{1}{Y_{CS}} \right) \quad (10)$$

$$R_{PT-1} = \mu \cdot C_{CT-1} \cdot \left(\frac{Y_{PS}}{Y_{CS}}\right) \quad (11)$$

In which:

$$\mu = \left(\frac{\mu_{max} \cdot C_{ST-1}}{\left(K_S + C_{ST-1} + \frac{(C_{ST-1})^2}{K_I} \right)} \right) \cdot \left(1 - \frac{C_{PT-1}}{C_{PI}} \right) \quad (12)$$

Temperature error:

$$eT_k = T_{setpoint} - T_{k,T-1} \quad (13)$$

Fluid level error:

$$eh_k = h_{setpoint} - h_{k,T-1} \quad (14)$$

Average heat capacity:

$$cp_{T-1} = \frac{cp_{CO2} + cp_{H2O} + cp_{yeast} + cp_{ethanol} + cp_{sugar}}{5} \quad (15)$$

The control strategy used in the first start-up simulation was to manipulate variables that are inversely proportional to their controlling variables:

- 1- Manipulate v_2 to control the fluid level;
- 2- Manipulate v_3 to control the temperature of the fluid inside the tank.

The digital PID temperature (Equation 16) and level (Equation 17) controllers of the first simulation uses the velocity form equation of expanded PID controller [3]:

$$v_{3k} = v_{3k-1} + K_{P1}(eT_k - eT_{k-1}) + K_{I1}\Delta t \cdot eT_k + \frac{K_{D1}}{\Delta t}(eT_k - 2eT_{k-1} + eT_{k-2}) \quad (16)$$

$$v_{2k} = v_{2k-1} + K_{P2}(eh_k - eh_{k-1}) + K_{I2}\Delta t \cdot eh_k + \frac{K_{D2}}{\Delta t}(eh_k - 2eh_{k-1} + eh_{k-2}) \quad (17)$$

The control strategy used in the second start-up simulation was to manipulate variables that are directly proportional to their controlling variables:

- 1- Manipulate v_1 to control the fluid level;
- 2- Manipulate Q_3 to control the temperature of the fluid inside the tank.

The plant commonly does not have a final control element that manipulate heat (Q_3), and since T_4 is specified because in the practice this temperature will be controlled by the heat exchanger showed in Figures 1 and 2, v_3 will be manipulated to control the tank temperature. Since Q_3 depends of v_3 , as showed in Equation (4), the controller in the plant must have an equation to calculate the value of v_3 that will be manipulated, in which the equation is derived from Equation (4):

$$v_3 = \left| \frac{Q_3}{\rho_{T-1} \cdot cp_{T-1} \cdot (T_4 - T_{T-1})} \right| \quad (18)$$

In the second simulation it has also a condition, which is to equalize the value of v_2 into a project value when the fluid level first reach the setpoint value. In terms of process control, the system interprets this as a step gain in v_2 . The digital PID temperature (Equation 19) and level (Equation 20) controllers of the second simulation also uses the velocity form equation of expanded PID controller:

$$Q_{3k} = Q_{3k-1} + K_{P3}(eT_k - eT_{k-1}) + K_{I3}\Delta t \cdot eT_k + \frac{K_{D3}}{\Delta t}(eT_k - 2eT_{k-1} + eT_{k-2}) \quad (19)$$

$$v_{1k} = v_{1k-1} + K_{P4}(eh_k - eh_{k-1}) + K_{I4}\Delta t \cdot eh_k + \frac{K_{D4}}{\Delta t}(eh_k - 2eh_{k-1} + eh_{k-2}) \quad (20)$$

Table 1 below shows the parameters values used in the simulations:

Table 1: Parameters values used in the simulations.

Parameter	Value	Unit	Reference
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ρ_{T-1}	1082.97 (Brix=20)	Kg.m^{-3}	[4]
C_{PI}	90	Kg.m^{-3}	[2]
K_I	203.5	Kg.m^{-3}	[2]
K_S	0.48	Kg.m^{-3}	[2]
K_d	2.035×10^{-6}	s^{-1}	[2]
μ_{max}	10^{-4}	s^{-1}	[2]
Y_{CS}	0.035	$\text{Kg}_{\text{cell}} \cdot \text{Kg}_{\text{Sugar}}^{-1}$	[2]
Y_{PS}	0.48	$\text{Kg}_{\text{product}} \cdot \text{Kg}_{\text{Sugar}}^{-1}$	[2]
cp_{CO2}	0.86×10^3	$\text{J.Kg}^{-1} \cdot \text{K}^{-1}$	[5]
cp_{H2O}	4.1828×10^3	$\text{J.Kg}^{-1} \cdot \text{K}^{-1}$	[5]
cp_{yeast}	1.32×10^3	$\text{J.Kg}^{-1} \cdot \text{K}^{-1}$	[6]
$cp_{ethanol}$	2.84×10^3	$\text{J.Kg}^{-1} \cdot \text{K}^{-1}$	[7]
cp_{sugar}	3.7×10^3	$\text{J.Kg}^{-1} \cdot \text{K}^{-1}$	[8]
α	0.98324×10^5	$\text{J.Kg}_{\text{sugar}}^{-1}$	[1]
A_{T-1}	$180/7 = 25.7143$	m^2	-
$T_{setpoint}$	302	K	-
$h_{setpoint}$	5	M	-
T_1	301	K	-
T_4	301.5	K	-
C_{S1}	85	Kg.m^{-3}	-
C_{C1}	5	Kg.m^{-3}	-

Table 2 shows the parameters values of the digital PID controllers used in the simulations:

Table 2: Parameters values of the digital PID controllers used in the simulations.

Parameter	Value
K_{P1}	10^{-5}
K_{I1}	5×10^{-6}
K_{D1}	0
K_{P2}	10^{-5}
K_{I2}	0.7×10^{-6}
K_{D2}	0
K_{P3}	5×10^{-2}
K_{I3}	20
K_{D3}	0
K_{P4}	5×10^{-3}
K_{I4}	2×10^{-6}
K_{D4}	0

It was tested values other than zero for all the derivative terms constant and the principle of instability remains with the same philosophy, theoretical basis and validity.

4. Simulating the System

The system was simulated two times in MATLAB® (R2015a, Mathworks, Natick, MA, USA): The first was the one that caused instability, and the second caused stability. In both simulations, the fermentation has a lag phase of 10800 seconds (3 hours), which is commonly the real lag phase time in alcoholic fermentation tanks with *S. Cerevisiae*. In this phase it was specified that both kinetic rate of product formation and cell growth had 5% of usual value, while substrate kinetic rate had 10%. So after lag phase, all kinetic rates had 100% of their usual value, represented by Equations (9), (10) and (11).

For simulation purpose, it was chosen a temperature setpoint equal to 302 K as an optimum temperature of cell growth, like it happens i.e. for the yeast strain *S. Cerevisiae* CICC 1302 or 1308. Table 3 below shows the initial conditions values for the execution of the first simulation:

Table 3: Initial conditions values for the execution of the first simulation.

Variable	Initial value
v_1	Fixed value of $0.0278 \text{ m}^3 \cdot \text{s}^{-1}$ (supposed production volumetric flow rate)
v_2	$0 \text{ m}^3 \cdot \text{s}^{-1}$
v_3	$0 \text{ m}^3 \cdot \text{s}^{-1}$
h	0 m

Table 4 below shows the initial conditions values for the execution of the second simulation:

Table 4: Initial conditions values for the execution of the second simulation.

Variable	Initial value
v_1	$0 \text{ m}^3 \cdot \text{s}^{-1}$
v_2	$0 \text{ m}^3 \cdot \text{s}^{-1}$
v_3	$0 \text{ m}^3 \cdot \text{s}^{-1}$
h	0 m
Q_3	$0 \text{ J} \cdot \text{s}^{-1}$
Project value for v_2 when the fluid level first reach the setpoint value	$0.0278 \text{ m}^3 \cdot \text{s}^{-1}$

In both simulations were put disturbances in C_{S1} value, which happens constantly, i.e. each second, and is represented by Equation (21), in which $RAND$ is a random number between 0 and 1:

$$C_{S1} = 0.1 \cdot RAND + 85 \quad (21)$$

And for each ten seconds, it were put disturbances in C_{C1} and T_1 values (Equations 22 and 23):

$$C_{C1} = 0.3 \cdot RAND + 5 \quad (22)$$

$$T_1 = 0.45 \cdot RAND + 301 \quad (23)$$

4.1. Instability in the First Simulation

As it is mentioned in the principle of instability, the exact moment that the temperature (Figure 3) starts to go through instability and diverges to the direction of minus infinite is when the error starts to get negative values. This happen because in the heat balance equation, the term $(T_4 - T_{T-1})$ and the term $(T_1 - T_{T-1})$ can get negative values if $T_{T-1} > T_4, T_1$, and when the error also starts to get negative values, v_3 value starts to decrease (calculated by Equation 16), while it should increase to control the temperature inside the tank. This happen with same analogy for the fluid level (Figure 6), and v_2 (Figure 4) starts to decrease when the error starts to get negative values, while it should increase to control the level. That is, the fluid level is naturally inversely proportional to v_2 , and the fluid temperature inside the tank (T_{T-1}) is inversely proportional to v_3 (Figure 5), composing the reasons to reach instability and divergence as says the principle exposed in this paper.

Choosing this strategy for the digital PID controllers, the C.V. will never reach stability in the setpoint value, because the chosen manipulated variables are not directly proportional to their controlled variables (fluid level and temperature).

Since the system is instable, the fluid temperature and level, the cell (Figure 7), substrate (Figure 8) and product (Figure 9) concentrations will diverge and the results are not physically real. v_3 and v_2 also diverge and they get negative values also, which is physically impossible, and the temperature graph violates the second law of thermodynamics, that is, most of these instability problems would not occur in the practice, but the system is certain to be instable and to cause several critical errors i.e. tank inundation, pump cavitation, null production etc.

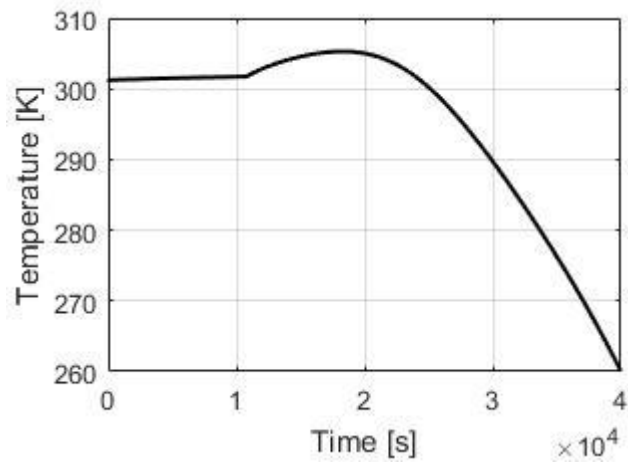


Figure 3: Fluid temperature (T_{T-1}) inside the tank in the first simulation (instable).

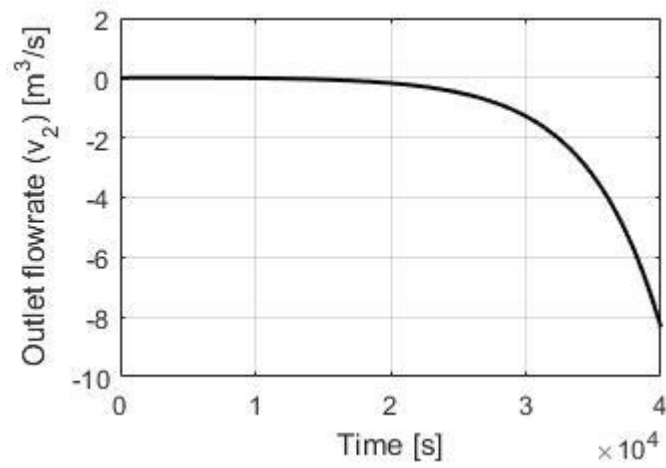


Figure 4: Outlet flow rate (v_2) in the first simulation (instable).

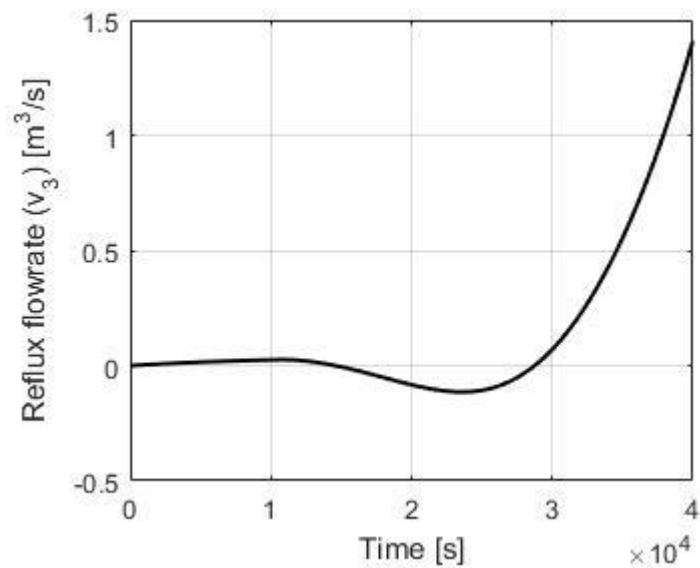


Figure 5: Reflux flow rate (v_3) in the first simulation (instable).

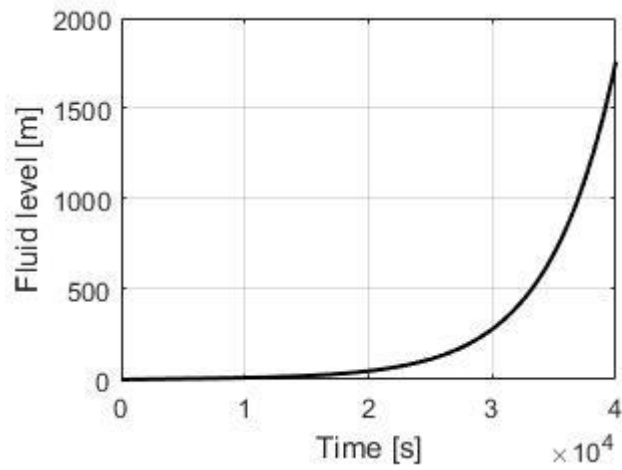


Figure 6: Fluid level in the first simulation (instable).

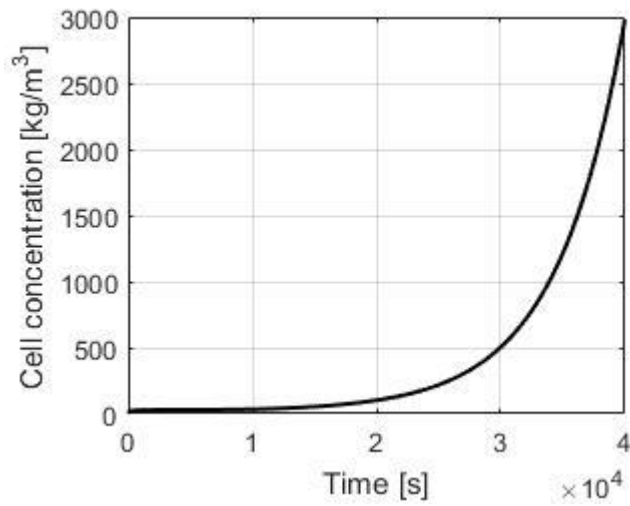


Figure 7: Cell concentration in the first simulation (instable).

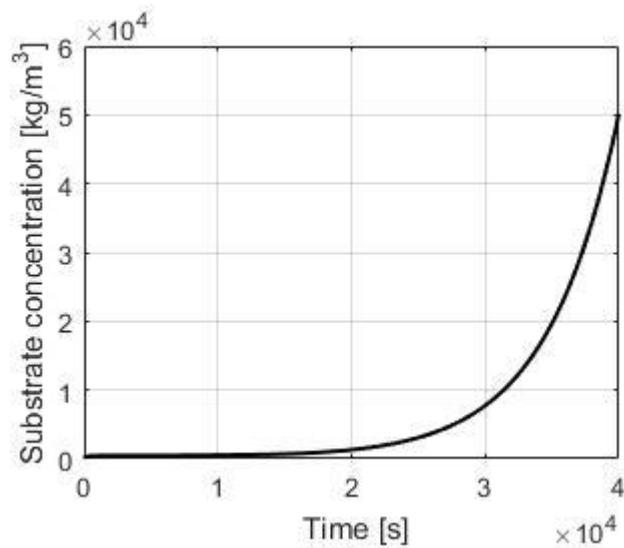


Figure 8: Substrate concentration in the first simulation (instable).

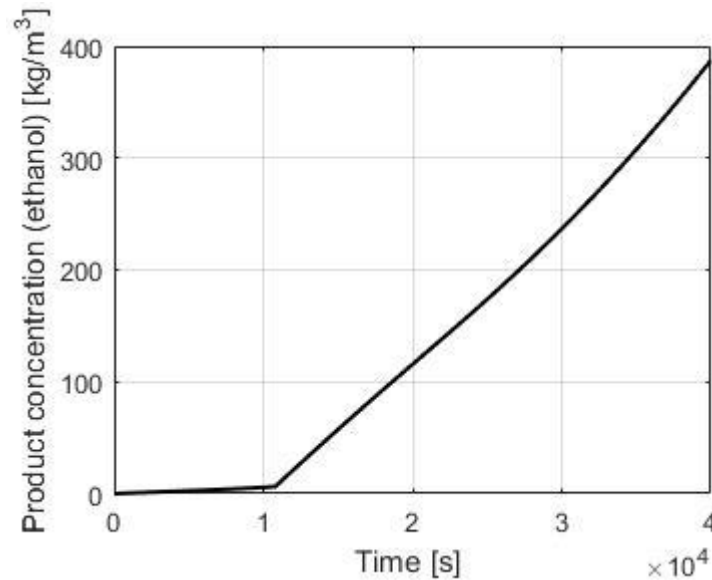


Figure 9: Product concentration in the first simulation (instable).

4.2. Stability in the Second Simulation

When Q_3 (Figure 11) and v_1 (Figure 13) are manipulated to control respectively the fluid temperature (Figure 10) and level (Figure 15), the system easily reach stability. Even if the controller is bad tuned and PID parameters are positive, v_2 (Figure 14) and v_3 (Figure 12) will not have negative values, different case compared to the first simulation control strategy.

The second simulation will never lead to negative values for v_1 and v_3 , because v_1 and Q_3 is directly proportional to their response variables (h and T_{T-1}), leading to stability and convergence as already justified. The simulation has converged to the setpoint values, both in temperature and fluid level, eliminating then the offset value commonly generated in some cases, especially when analog PID control strategies are used.

The temperature has an absolute value of overshoot equal to 1.2 K, reaching a tolerable temperature of 303.02 K and becoming stable right after. As early mentioned, and as part of the control strategy, v_2 obtain a step gain that would cause instability in the system if the control strategy was badly chosen and if the digital PID controller was badly tuned, which did not happen.

As the system is stabilized in temperature and fluid level setpoint values, the substrate (Figure 16), cell (Figure 17) and product (Figure 18) concentrations reach the steady-state with values equal to 28.668, 7.052 and 27.055 Kg.m⁻³, respectively, which are the same value of when calculated through the given cell and ethanol yield factor.

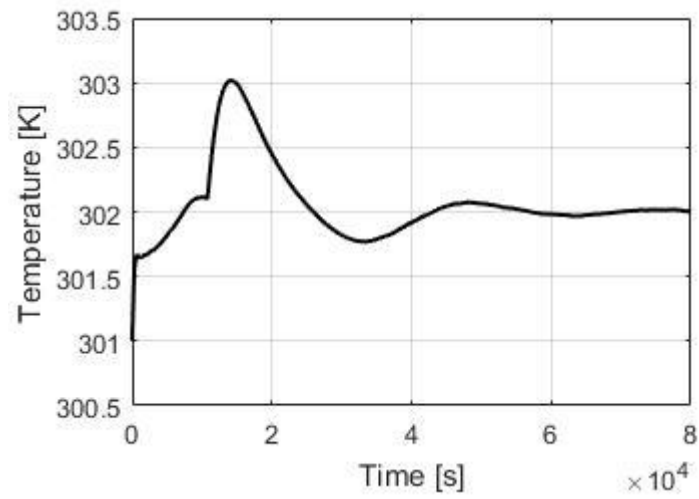


Figure 10: Fluid temperature inside the tank in the second simulation (stable).

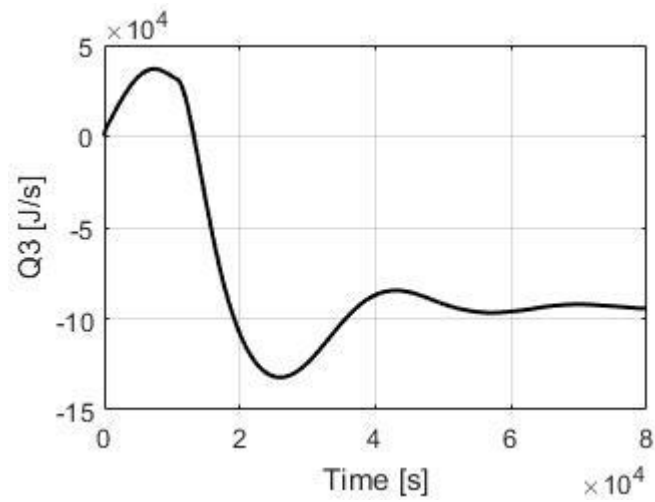


Figure 11: Heat Q_3 in the second simulation (stable).

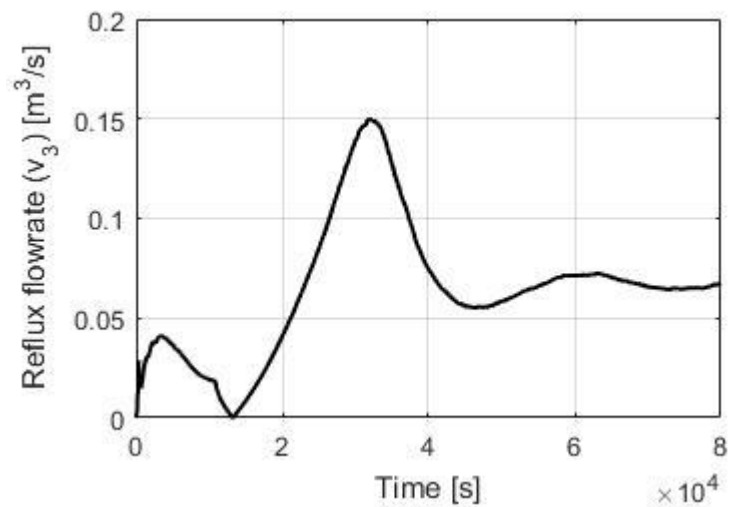


Figure 12: Reflux flow rate (v_3) in the second simulation (stable).

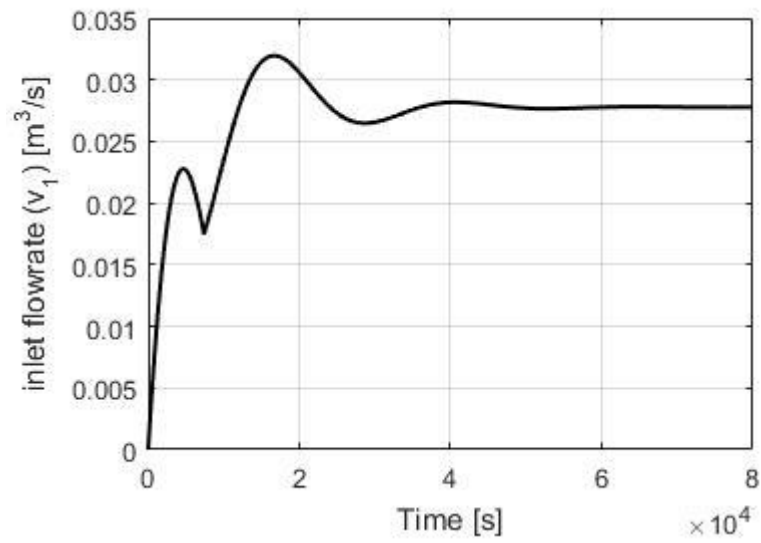


Figure 13: Inlet flow rate (v_1) in the second simulation (stable).

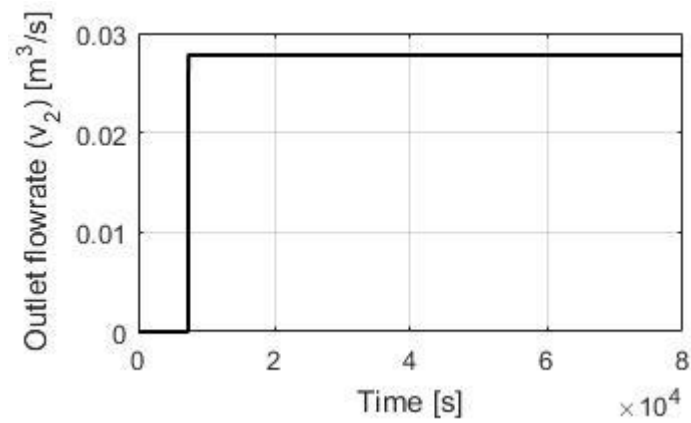


Figure 14: Outlet flow rate (v_2) in the second simulation (stable).

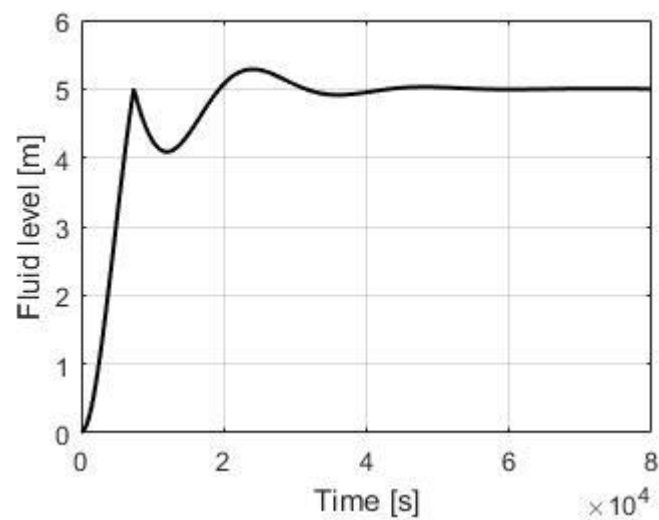


Figure 15: Fluid level (h) in the second simulation (stable).

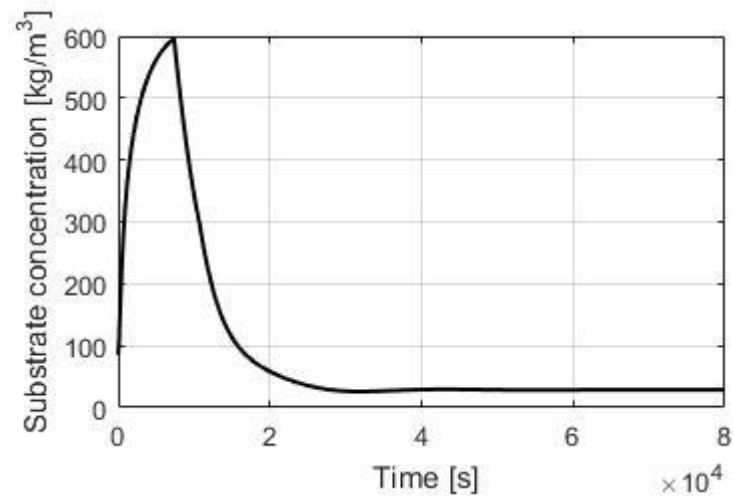


Figure 16: Substrate concentration (C_{ST-1}) in the second simulation (stable).

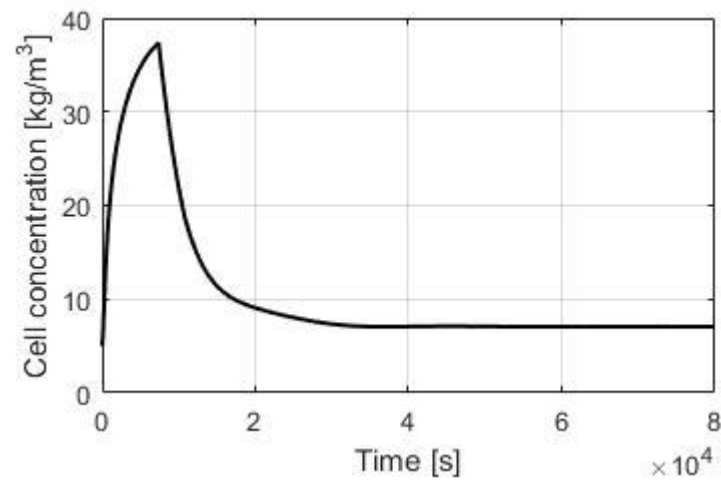


Figure 17: Cell concentration (C_{CT-1}) in the second simulation (stable).

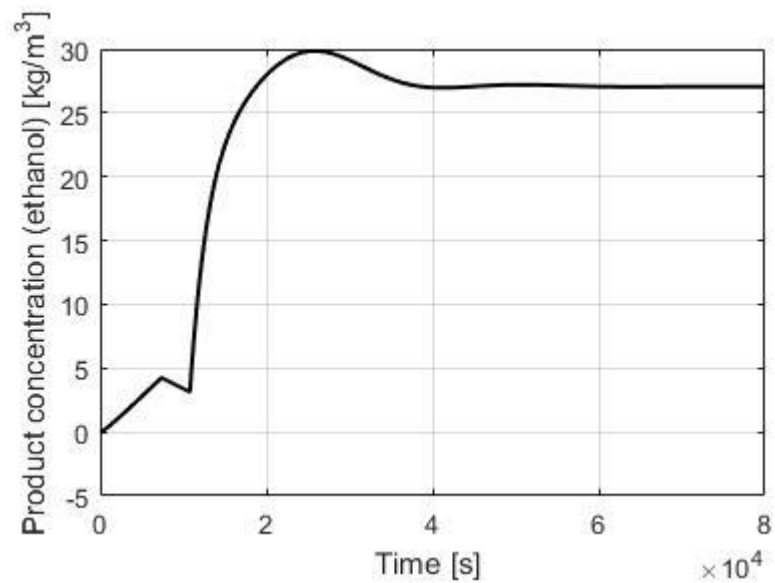


Figure 18: Product concentration (C_{PT-1}) in the second simulation (stable).

5. Conclusions

It was discussed digital PID control strategies to state principles of instability and stability that lead the process to divergence or convergence in the setpoint value of a controlled variable. The instability and stability promoted by these principles were studied in two simulations of a continuous alcoholic fermentation tank process start-up being fed with yeast strain *S. Cerevisiae* NRRL-Y-132 and a stream of sugar derived from cane bagasse hydrolysate. It is clearly visible from the results of both process start-up simulations that manipulating variables that are directly proportional to their controlled variables, makes the system get stable in the setpoint values. Since how to reach stability using digital PID controllers is now well explained and justified, these strategies can be used in all industrial branches, with the objective of control the process by reaching the desired specifications and dodging analog controller's weak point (avoiding the use of them).

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Nomenclature (in order of appearance)

M.V.	Manipulated variable
C.V.	Controlled variable
T_{T-1}	Fluid temperature inside the tank
Q_1	Heat exchanged between Stream 1 and the fluid inside the tank
Q_2	Heat liberated from substrate consumption by the yeasts
Q_3	Heat exchanged between Stream 4 and the fluid inside the tank
ρ_{T-1}	Fluid density inside the tank

cp_{T-1}	Heat capacity of the fluid inside the tank
h_{T-1}	Fluid level
A_{T-1}	Circular area of the cylindrical tank's base
v_1	Volumetric flow rate of stream 1
T_1	Temperature of stream 1
α	Heat liberated per kilogram of processed sugar
$R_{S_{T-1}}$	Substrate consumption rate
v_3	Volumetric flow rate of stream 3
T_4	Temperature of stream 4
v_2	Volumetric flow rate of stream 2
$C_{C_{T-1}}$	Cell concentration inside the tank
$R_{C_{T-1}}$	Cell formation rate
C_{C1}	Cell concentration in Stream 1
$C_{S_{T-1}}$	Substrate concentration inside the tank
C_{S1}	Substrate concentration in Stream 1
$C_{P_{T-1}}$	Product concentration inside the tank
$R_{P_{T-1}}$	Product formation rate
μ	Specific growth rate
K_d	Cell death rate
Y_{CS}	Cell yield factor
Y_{PS}	Ethanol yield factor
μ_{max}	Maximum specific growth rate
K_S	Half-velocity constant
K_I	Constant of substrate inhibition
C_{PI}	Constant of product inhibition
eT_k	Temperature error in iteration "k"
$T_{setpoint}$	Temperature setpoint
$T_{k,T-1}$	Fluid temperature inside the tank in iteration "k"
eh_k	Fluid level error in iteration "k"

$h_{setpoint}$	Fluid level setpoint
$h_{k,T-1}$	Fluid level in iteration “k”
cp_{CO2}	Heat capacity of carbon dioxide
cp_{H2O}	Heat capacity of water
cp_{yeast}	Heat capacity of yeast <i>S. Cerevisiae</i>
$cp_{ethanol}$	Heat capacity of ethanol
cp_{sugar}	Heat capacity of sugar
v_{3k}	Volumetric flow rate of stream 3 in iteration “k”
v_{3k-1}	Volumetric flow rate of stream 3 in iteration “k-1”
eT_{k-1}	Temperature error in iteration “k-1”
eT_{k-2}	Temperature error in iteration “k-2”
Δt	Measuring signal time interval
eh_{k-1}	Fluid level error in iteration “k-1”
eh_{k-2}	Fluid level error in iteration “k-2”
Q_{3k}	Heat exchanged between Stream 4 and the fluid inside the tank in iteration “k”
Q_{3k-1}	Heat exchanged between Stream 4 and the fluid inside the tank in iteration “k-1”
v_{1k}	Volumetric flow rate of stream 1 in iteration “k”
v_{1k-1}	Volumetric flow rate of stream 1 in iteration “k-1”

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