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MATHEMATICAL MODELLING OF RLC CIRCUIT THROUGH MIXED QUADRATURE RULE

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Abstract

In this study, a mixed rule of degree of precision nine has been developed and implemented in the field of electrical sciences to obtain the instantaneous current in the RLC- circuit for particular value $(R=1, L=1, C=1)$.The linearity has been performed with the Volterra's integral equation of second kind with particular kernel $(1+(t-x))$. Then the definite integral has been evaluated through the mixed quadrature to obtain the numerical result which is very effective. A polynomial has been used to evaluate Volterra's integral equation in the place of unknown functions. The accuracy of the proposed method has been tested taking different electromotive force in the circuit and absolute error has been estimated.

Keywords

Kirchhoff's voltage law, Volterra integral equation, Mixed quadrature, Initial value problem, Error bound, Maclaurin's theorem.

M SC 2010: 65D30, 65D32, 45D05, 31C15 .

1. Introduction

Various problems of Mathematical Science are often resolved within reformulation of different mathematical problems like integral equations. So the study of integral equations and the ways of finding methods for it are quite helpful.

Now–a-days, Volterra's equations have been implemented in the fields of Mathematics and Technology. Some valid methods, for solving Volterra's equations have been developed by many researchers. In particular, Huang[2] has solved the unknown function by using the Taylor's expansion. The chebyshev polynomials is one of the method to find the solution of integral equation by Rahman[1]. Piessen[3] developed a method for computing integral transforms by Chebyshev polynomial approximations.

Mixed quadrature has faster convergence due to higher degree of precision for evaluation of real definite integrals. The mixed quadrature of higher precision has been established with linear convex combination of Gaussian and Newtonian type rule of lower precision for single variables. Recently mixed quadrature have been successfully used for the numerical solution of integral equations as well as finite

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element methods. A step has been carried out by Jena and Nayak [11] for numerical solution of non linear Fredholm equation of second kind. Dash and Das [10] proposed identification of some Clenshaw- Curtis rule with Fejer rules. Motivated by the excellent performance of these methods, we have applied the resolvent kernel to find the solution of Volterra's integral equation for numerical integration of real definite integrals for single variables which has been applied in the field of RLC circuit (Wazwaz[4]).

The proposed work to find instantaneous current in RLC circuit with the help of Volterra's equations with mixed rule for the integral equation of type

$$u(p)=f(p)+\lambda\int_0^p k(p-t)u(t)dt, \qquad (1)$$

$$p\in[0,t]$$

where $f(p)$ is the source function and k is the kernel which are given and $u(p)$ is the unknown function and λ is a non zero real or complex parameter (Jerri[5]). Applying Kirchhoff's Voltage law to the differential equation of RLC circuit has been transformed to Volterra's integral equation of second kind then it has been reduced by mixed quadrature rule of Clenshaw Cutis seven point rule and Lobatto five point rule since each of are higher precision rule for higher rate of convergence towards the exact value of the integral.

This proposed work is organized as follows. Section-1 meant for the introduction. In section-2, Kirchhoff's Voltage law and an approximated solution for the Volterra integral equation of second kind has been established. In section-3, a mixed quadrature rule has been constructed which is approximately convergence to the exact solution and error bound has been estimated for the suggested method. In section-4 approximate solution for various problems has been done and section-5 has found some conclusions .

2. Basic properties of Kirchhoff's Voltage law and Volterra integral equation

In this section, some basic properties have been used for definitions of Kirchhoff's Voltage law and Volterra integral equation of second kind of (1).

According to KVL, the Integro-differential equation for RLC circuit (Fig.-1) is

$$L\,I'+R\,I+\frac{1}{C}\int I\,dt=E(t) \qquad (2)$$

where $E(t)$ is the electromotive force

E_0 = constant

L = Inductance

R = Resistance

C = Capacitance and $I(t)$ = The instantaneous current

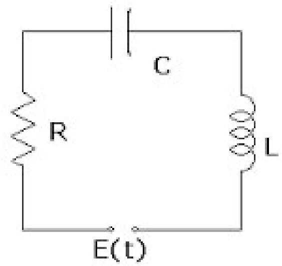

Fig.1 (RLC circuit)

The integral equation(1) after differentiation becomes

$$I'' + \frac{R}{L} I' + \frac{1}{CL} I = \frac{E'(t)}{L} \tag{3}$$

The equation (3) is a non homogeneous second order differential equation (Kreyszig [6]).

2.1 Volterra's integral equation

Here some basic theorems are given below to solve the Volterra's integral equation of second kind which is of the form (1).

Theorem-2.1

The initial value problem which is of the $\frac{d^2I}{dt^2} + C(t)\frac{dI}{dt} + D(t)I = f(t)$ with the initial condition $I(c) = I_0$, $I'(c) = I'_0$ is transformed to Volterra's integral equation of second kind (Pundir and Pundir[7]) is of the form

$$I(t) = \left[\begin{array}{l} I_0 + [I'_0 + I_0 C(c)](t-c) + \int_a^t (t-p) f(p) dp - \\ \int_a^t [C(p) + (t-p)\{D(p) - C'(p)\}] I(p) dp \end{array} \right]$$

Proof:

Here we have given the differential equation $\frac{d^2I}{dt^2}+C(t)\frac{dI}{dt}+D(t)I=f(t)$ with the initial condition $I(c)=I_0,\ \ I'(c)=I'_0$.Then integrating twice with respect to t from $c\ to\ t$ we get

$$I(t)=\begin{bmatrix} I_0+[I'_0+I_0C(c)](t-c)+\int_a^t(t-p)f(p)dp- \\ \int_a^t[C(p)+(t-p)\{D(p)-C'(p)\}]I(p)dp \end{bmatrix}$$

which is the required Volterra's Integral equation of second kind.

Theorem-2.2

The resolvent kernel of the kernel $k(p,t)$is a polynomial of degree $(n-1)$ in t which is express in the form

$k(p,t)=D_0(t)+D_1(t)(t-p)+D_2(t)(t-p)^2+....\frac{D_{n-1}(t)}{(n-1)!}(t-p)^{n-1}+....$ where the coefficient

$D_n(t)$ are continuous in the interval $[0,c]$ is $R(p,t;\lambda)=-\frac{1}{\lambda}\frac{d^n}{dp^n}\phi(t,p;\lambda)$.

Proof:

Consider $R(p,t;\lambda)=-\frac{1}{\lambda}\frac{d^n}{dp^n}\phi(t,p;\lambda)$ (4)

The auxiliary function ϕ satisfies the following conditions $\phi=\frac{d\phi}{dp}=....=\frac{d^{n-2}\phi}{dp^{n-2}}=0$ at

$t=p\quad \frac{d^{n-1}\phi}{dp^{n-1}}=1$ at $t=p$ (5)

Therefore, the functional relation reduces to

$$\frac{d^n\phi}{dp^n}=-\lambda\, k(p,t)+\lambda\int_s^t k(z,p)\frac{d^n}{ds^n}\phi(t,z;\lambda)ds \tag{6}$$

Integrating by parts in equation (4) and (5)

$$\phi=\frac{d^n\phi}{dp^n}+\lambda\left[D_0(t)\frac{d^{n-1}\phi}{dp^{n-1}}+D_1(t)\frac{d^{n-2}\phi}{dp^{n-2}}+....D^{n-1}(t)\phi\right]=0$$

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Hence the resolvent kernel is of the form

$$R(p,t;\lambda) = -\frac{1}{\lambda}\frac{d^n}{dt^n}\phi(t,p;\lambda)$$

In this case study, we have used a polynomial in the place of the unknown function $I(p)$ for the transcendental function for making

$$I(p) = \begin{cases} \dfrac{f(t)}{2^i}, & for \quad i=1,2 \quad (Trigonometric \quad function) \\ \begin{cases} \dfrac{f(t)}{2}, & if \quad t=1 \\ f(t), & if \quad t=2 \\ \dfrac{f(t)}{4}, & if \quad t=3 \end{cases} & (Exponential \quad function) \end{cases} \tag{7}$$

exponential function (oscillatory function) to convert smooth function.

3. Construction of mixed quadrature rule

In this section, a mixed quadrature rule has been constructed by the convex combination of Clenshaw-Curtis seven point rule $R_{CC7}(f)$ and Gauss Lobatto-five point rule $R_{L5}(f)$ and also the error bound estimated which has been given by the following theorems.

Theorem-3.1

The quadrature rule and error for the smooth function $f(p)$ which is defined on $0 \le p \le 1$ is obtained by convex combination of Clenshaw-Curtis seven point rule $R_{CC7}(f)$ and the Gauss Lobatto-five point rule $R_{L5}(f)$ rules formed mixed quadrature rule of degree of precision nine $R_{CC7L5}(f)$, then

$$I = \int_{-1}^{1} f\,dx \cong \frac{1}{135}\left[128R_{CC7}(f) + 7R_{L5}(f)\right] + \frac{1}{15}\left[128E_{CC7}(f) + 7E_{L5}(f)\right]$$

where $R_{CC7L5}(f) = \frac{1}{135}\left[128R_{CC7}(f) + 7R_{L5}(f)\right]$ and $E_{CC7L5}(f) = \frac{1}{15}\left[E_{CC7}(f) + 7E_{L5}(f)\right]$

Proof:

$$R_{CC7}(f) \cong \int_{-1}^{1} f\,dx = \frac{1}{315}\left[\begin{array}{l} 9\{f(-1) + f(1)\} + 80\left\{f\left(-\frac{\sqrt{3}}{2}\right) + f\left(\frac{\sqrt{3}}{2}\right)\right\} \\ +144\left\{f\left(-\frac{1}{2}\right) + f\left(\frac{1}{2}\right)\right\} + 164f(0) \end{array}\right]$$

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$$R_{L5}(f) \cong \int_{-1}^{1} f\,dx = \frac{1}{90}\left[9\{f(-1)+f(1)\}+49\left\{f\left(-\sqrt{\frac{3}{7}}\right)+f\left(\sqrt{\frac{3}{7}}\right)\right\}+64f(0)\right]$$

Each of the rule $R_{CC7}(f)$ and $R_{L5}(f)$ is of degree of precision seven where $E_{CC7}(f)$ is the error due to $R_{CC7}(f)$ and $E_{L5}(f)$ is the error due to $R_{L5}(f)$

$$I(f) = R_{CC7}(f) + E_{CC7}(f) \tag{8}$$

$$I(f) = R_{L5}(f) + E_{L5}(f) \tag{9}$$

Expanding equation (8) and (9) with the help of Taylor's series, we get

$$E_{CC7}(f) \cong \frac{1}{1260\times 8!} f^{viii}(0) + \frac{1}{308\times 10!} f^{x}(0) + \frac{177}{29120\times 12!} f^{xii}(0)........$$

$$E_{L5}(f) \cong -\frac{32}{2205\times 8!} f^{viii}(0) - \frac{128}{3773\times 10!} f^{x}(0) - \frac{8256}{156065\times 12!} f^{xii}(0).......$$

Multiplying equation $\left(\frac{32}{7}\right)$ with equation (8) and $\left(\frac{1}{4}\right)$ with equation (9) and adding them respectively.

We have

$$I = \int_{-1}^{1} f\,dx \cong \frac{1}{135}\left[128R_{CC7}(f) + 7R_{L5}(f)\right] +$$
$$\frac{1}{135}\left[128E_{CC7}(f) + 7E_{L5}(f)\right]$$

Then $\quad I(f) = R_{CC7L5}(f) + E_{CC7L5}(f)$

where $\quad R_{CC7L5}(f) = \frac{1}{135}\left[128R_{CC7}(f) + 7R_{L5}(f)\right] \qquad (10)$

and $E_{CC7L5}(f) = \frac{1}{135}\left[128E_{CC7}(f) + 7E_{L5}(f)\right] \qquad (11)$

Equation (10) is our proposed mixed rule

Theorem-3.2

Let the smooth function $f(p)$ is defined on $-1 \le p \le 1$, then the error $E_{CC7L5}(f)$ is associated with the mixed quadrature rule $R_{CC7L5}(f)$ is given by

$$\left|E_{CC7L5}(f)\right| \cong \frac{463}{15092\times 10!}\left|f^{x}(0)\right|$$

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Proof:

The proof follows from equation (11).

Theorem-3.3

The truncated error bound for $E_{CC7L5}(f)=I(f)-R_{CC7L5}(f)$ is evaluated by $\left|E_{CC7L5}(f)\right|\le\frac{64M}{42525\times 8!}$ where $M=\max\limits_{-1\le x\le 1}\left|f^{ix}(p)\right|$.

Proof:

$$E_{CC7}(f)=\frac{1}{1260\times 8!}f^{viii}(\eta_1),\qquad \eta_1\in[-1,1]$$

$$E_{L5}(f)=-\left(\frac{32}{2205\times 8!}\right)f^{viii}(\eta_2),\qquad \eta_2\in[-1,1]$$

$$E_{CC7L5}(f)=\frac{32}{42525\times 8!}\left[f^{viii}(\eta_2)-f^{viii}(\eta_1)\right]$$

where $K=\max\limits_{-1\le x\le 1}\left|f^{viii}(x)\right|$ and $S=\min\limits_{-1\le x\le 1}\left|f^{viii}(x)\right|$. Hence there exists points d and c in the interval $[-1,1]$ such that $S=f^{viii}(c)$. Thus by Conte & Boor [9]

$$\left|E_{CC7L5}(f)\right|\cong\frac{32}{42525\times 8!}\left[f^{viii}(d)-f^{viii}(c)\right]$$

$$=\frac{32}{42525\times 8!}\int_{-1}^{1}f^{ix}(p)\,dp$$

$$=\frac{32}{42525\times 8!}(d-c)f^{ix}(\gamma)\quad for\ \ some\ \gamma\in[-1,1]$$

where $|d-c|\le 2$

$$\left|E_{CC7L5}(f)\right|\le\frac{64}{42525\times 8!}f^{ix}(\gamma)$$

Hence $\left|E_{CC7L5}(f)\right|\le\frac{64M}{42525\times 8!}$ where $M=\max\limits_{-1\le x\le 1}\left|f^{ix}(p)\right|$.

4. Numerical verifications

In this section, a Table-1 and Fig-2 for the numerical approximation of RLC-circuit for the mixed quadrature rule $R_{CC7L5}(f)$ with the initial conditions $I(0)=0,\ I'(0)=1$ has been made.

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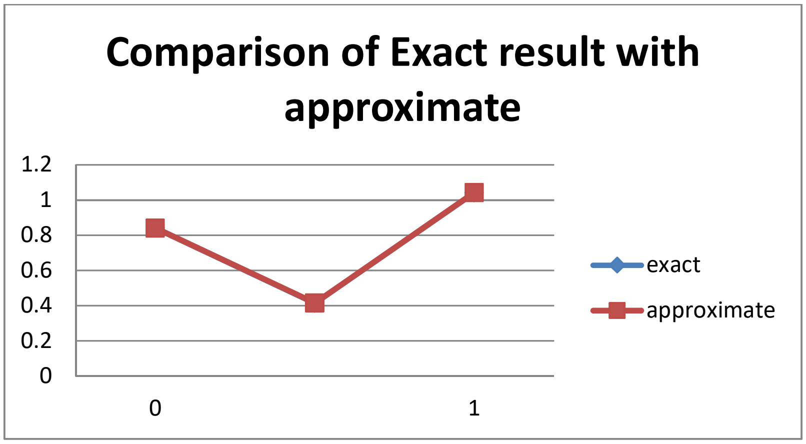
Table-1 (Comparison of Exact result with approximate result)

$f(t)$	I_{EXACT}	I_{CC7L5}	$\lvert Error \rvert$
$\sin t$	0.841470984807897 amp	0.841492252269964 amp	0.000021267462067
$\cos t$	0.414109347591131 amp	0.415287660821751 amp	0.001178313230620
e^t	1.041865355098910 amp	1.041834401921017 amp	0.000030953177893



5. Conclusion

The RLC circuit problem has been resolved through integral equation of second kind with suitable kernel which is Volterra in nature. The approximate solution can be achieved by taking a few calculations which is much better. The results obtained by the presented method to introduce KVL for solving the said integral equation reveals that our work is very effective and convenient. The efficiency of the method has verified numerically by taking some test problems in Table-1 and also observed graphically in Fig.-2. The extensive work for finding instantaneous current for any value of R, L and C and for any function $f(t)$ can be carried out .

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