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Convective Instability in a Hele Shaw Cell with the Effect of Through Flow and Magnetic Field

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Abstract:
In this paper, an analytical investigation of the combined effect of through flow and magnetic field on the convective instability in an electrically conducting fluid layer, bounded in a Hele-Shaw cell is presented within the context of linear stability theory. The Galarkin method is utilized to solve the eigenvalue problem. The outcome of the important parameters on the stability of the system is examined analytically as well as graphically. It is observed that the through flow and magnetic field have both stabilizing effects, while the Hele-Shaw number has destabilizing effect on the stability of system. It is also found that the oscillatory mode of convection possible only when the magnetic Prandtl number takes the values less than unity.
Keywords: Linear stability theory; Magneto convection; Through flow; Hele-Shaw cell; Galarkin method.

1. Introduction

This paper concerns the joint effect of magnetic field and through flow on the convective instability of an electrically conducting fluid layer circumscribed vertically by two thermally insulated planes and horizontally by two perfect heat conducting planes. This configuration is called the Hele-Shaw cell. A range of applications of fluid mechanics goes to the Hele-Shaw flows. Nowadays, this flow is applied in numerous fields of physics and engineering, in particular, material processing and crystal growth owing to manufacturing procedures [1-3]. Also, during the last years, convection in a porous medium has gained significant interest because of its important in geothermal, petroleum production, oil reservoir, building of thermal insulations and nuclear reactor [4]. The problem in a porous medium can be easily solved by taking an appropriate permeability of the Hele-Shaw cell. Hele–Shaw [5] was the first who gave the similarity between the two-dimensional flow in a porous medium and Hele-Shaw cell by taking an equivalent permeability $b^2/12$ for the Hele-Shaw cell where b is the width of the cell. He showed that the Hele-Shaw cell can be a controlling device for quantitative study of two-dimensional flow in porous media by suitably recognizing the Hele-Shaw permeability. The similarity among flow in a porous medium and flow in a Hele-Shaw cell has commonly been applied to study convection in porous medium [6-8].

Convective instability of electrically conducting fluid in the existence of a magnetic field has drawn great attention as a consequence of its many real-world applications for instance in electrical machineries, chemical apparatus, plasmas, MHD accelerators and power generation systems. The study of magnetic field on the onset of convection yields a range of activities when the ratio of the magnetic to thermal diffusivity is small; the governing system then allows both stationary and oscillatory mode of convections. The strength of magneto convection is indicated by Chandrasekhar number which is the relation between the Lorentz force and the viscosity. If the Lorentz force was smaller than viscous force then the convective motions twist and stretch the magnetic field. If the Lorentz force was larger than the viscous force, then the magnetic field sets the plasma flows along the field direction and constrains the convection. The large numbers of investigations related to magneto convection are recognized by Chandrasekhar [9] and Nield and Bejan [4]. Thompson [10] and Chandrasekhar [9] were the first to examine the effect of magnetic field on the convective instability. Rudraiah and Shivakumara [11] studied convection with an imposed magnetic field. They observed that the magnetic field, under some situations creates the system unstable. The interplay between magnetic fields and convection by considering the effect of a solid rotor on an otherwise uniform magnetic field was investigated by Weiss [12]. Abd-el-Malek and Helal [13] studied the problem of an unsteady convective laminar flow under the effect of a magnetic field. They found that the velocity boundary-layer thickness becomes smaller for the increase in the magnetic influence number. Very recently, the effect of magnetic field on the convective instability in nanofluids was considered by Yadav et al. [14-18], Chand and Rana [19, 20], Sheikholeslami et al. [21-23], Al-Zamily [24], Gupta et al. [25] and Hamada et al. [26].

Through flow effect on the convective instability in an electrically conducting fluid layer with magnetic field is important concept because of its applications in engineering, geophysics and magnetohydrodynamics. In situ processing of electronic components, chemical equipment, cooling of nuclear reactors, energy assets such as coal, geothermal energy, oil shale and many real-world problems frequently occupies the through flow in a Hele-Shaw cell. The significance of buoyancy driven convective instability in such circumstances may become important when specific processing is needed. Besides, the through flow effect in such situations may be of attention due to the opportunity of controlling the convective instability by regulating the through flow in accumulation to the gravity. Through flow changes the basic profile for temperature from linear to nonlinear with layer height, which marks the stability of the system considerably. The effect of through flow on the onset of convection was studied by Jones and Persichetti [27]. Addition to porous medium was made by Wooding [28], Sutton [29] and Nield [30]. They observed that, the effect of through flow is not always stabilizing and depends on the character of the boundaries. Khalili and Shivakumara [31] examined the effect of through flow on the onset of convection in a porous medium with internal heat generation. They observed that through flow destabilizes the system in the present of internal heat source, even if the boundaries are of the same type. Later on many investigators studied the effect of through flow on convective instability for different types of fluids [32-36].

From the literature no study has been found which considers the effect of through flow on magneto convection confined within a Hele-Shaw cell. Therefore, it would be of importance here to examine the combined effect of through flow and magnetic field on the convective instability in an electrically conducting fluid layer, bounded within a Hele-Shaw cell. By linear stability theory, the critical conditions for stationary and oscillatory convections are derived analytically and discussed graphically.

2. Mathematical Model

In this work, an infinitely extended horizontal incompressible electrically conducting fluid layer of height d is taken. The fluid layer confines between two parallel boundaries at $z^*=0$ and $z^*=d$, which are preserved at uniform but different temperature T_l^* and T_u^* ($T_l^* > T_u^*$), respectively. The fluid shall be infinitely extended in the x- direction, but restricted in the y- direction by sidewalls at $y^*=0$ and $y^*=b$. For a suitably small thickness, $b << d$, the flow can be estimated as a 2-dimensional Stokes flow in the X–Z-plane, usually stated to as Hele-Shaw flow. A constant magnetic field $\mathbf{H}^* = (0, 0, H_0^*)$ applies on the system. The physical configuration of the system is as shown in Fig. 1. Asterisks are used to differentiate the dimensional variables from the non-dimensional variables (without asterisks). On considering the Hele-Shaw approximation and using the Boussinesq approximation, the governing equations under this model are:

$$\nabla^* \cdot \vec{\mathbf{v}}^* = 0, \tag{1}$$

$$\frac{\mu}{K}\vec{\mathbf{v}}^* = -\nabla^* p^* + \rho_0\left[1-\beta\left(T^* - T_u^*\right)\right]\vec{\mathbf{g}} + \mu_e(\vec{\mathbf{H}}^* \cdot \nabla^*)\vec{\mathbf{H}}^*, \tag{2}$$

$$\left[\frac{\partial}{\partial t^*} + \left(\vec{\mathbf{v}}^* \cdot \nabla^*\right)\right]T^* = \alpha \nabla^{*2} T^*, \tag{3}$$

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$$\left[\frac{\partial}{\partial t^*}+\left(\vec{\mathbf{v}}^*\cdot\nabla^*\right)\right]\vec{\mathbf{H}}^*=\nu_m\nabla^{*2}\vec{\mathbf{H}}^*+\left(\vec{\mathbf{H}}^*\cdot\nabla^*\right)\vec{\mathbf{v}}^*, \tag{4}$$

$$\nabla^*\cdot\vec{\mathbf{H}}^*=0\ . \tag{5}$$

Here, $\vec{\mathbf{v}}^*$ is the velocity of the fluid, t^* is the time, ρ_0 is the fluid density at reference temperature T_u^*, p^* is the pressure, $\vec{\mathbf{H}}^*$ is the magnetic field, β is the thermal expansion coefficient, α is the thermal diffusivity, $K=b^2/12$ is the permeability of the fluid flow in Hele Shaw cell, μ, ν_m, μ_e and k are the viscosity, magnetic viscosity, magnetic permeability and thermal conductivity of the fluid, respectively.

We suppose that there is upward through flow with constant mean value w_u^*. Thus the boundary situation are:

$$w^*=w_c^*,\ T^*=T_l^*,\quad \text{at}\ z^*=0, \tag{6a}$$

$$w^*=w_c^*,\ T^*=T_u^*,\quad \text{at}\ z^*=d\ . \tag{6b}$$

We present the following non dimensional variables by defining:

$$\mathbf{x}=\frac{\mathbf{x}^*}{d},\ t=\frac{t^*}{d^2}\alpha,\ p=\frac{p^*d^2}{\mu\alpha},\ T=\frac{T^*-T_u^*}{T_l^*-T_u^*},\ \vec{\mathbf{v}}=\frac{d}{\alpha}\vec{\mathbf{v}}^*,\ \vec{\mathbf{H}}=\frac{\vec{\mathbf{H}}^*}{H_0^*}. \tag{7}$$

The governing equations then become:

$$\nabla\cdot\vec{\mathbf{v}}=0, \tag{8}$$

$$\frac{\vec{\mathbf{v}}}{H_S}=-\nabla p+R_aT\hat{\mathbf{e}}_Z+QP_m(\vec{\mathbf{H}}\cdot\nabla)\vec{\mathbf{H}}, \tag{9}$$

$$\left[\frac{\partial}{\partial t}+\left(\vec{\mathbf{v}}\cdot\nabla\right)\right]T=\nabla^2T, \tag{10}$$

$$\left[\frac{\partial}{\partial t}+\left(\vec{\mathbf{v}}\cdot\nabla\right)\right]\vec{\mathbf{H}}=\left(\vec{\mathbf{H}}\cdot\nabla\right)\vec{\mathbf{v}}+P_m\nabla^2\vec{\mathbf{H}}, \tag{11}$$

$$\nabla\cdot\vec{\mathbf{H}}=0. \tag{12}$$

In the non-dimensional form, the boundary conditions become:

$$\begin{aligned} w=\lambda,\ T=1\ \ \text{at}\ z=0,\\ w=\lambda,\ T=0\ \ \text{at}\ z=0, \end{aligned} \tag{13a,b}$$

The non-dimensional parameters that come in Eqs. (8)-(13) are defined as:

$R_a=\dfrac{gd^3\beta\Delta T^*}{\alpha\nu}$ is the Rayleigh number, $H_S=\dfrac{K}{d^2}$ is the Hele-Shaw number, $P_m=\dfrac{\nu_m}{\alpha}$ is the magnetic Prandtl number, $Q=\dfrac{\mu_eH_0^{*2}d^2}{\rho_0\nu\nu_m}$ is the Chandrasekhar number, $\nu=\dfrac{\mu}{\rho_0}$ is the kinematic viscosity, $\nu_m=\dfrac{\mu_e}{\rho_0}$ is the magnetic viscosity and $\lambda=\dfrac{dw_c^*}{\alpha}$ is the Péclet number.

2.1 Basic State

The basic state of the fluid is considered to be time independent and is given by

$$\vec{\mathbf{v}}_b=\lambda\hat{\mathbf{e}}_z,\ T=T_b,\ p=p_b,\ \vec{\mathbf{H}}_b=\hat{\mathbf{e}}_z. \tag{14}$$

Then Eq. (10) gives:

$$\frac{d^2T_b}{dz^2}-\lambda\frac{dT_b}{dz}=0 \tag{15}$$

The boundary conditions for $T_b(z)$ are:

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$$T_b = 1 \text{ at } z = 0,\ T_b = 0 \text{ at } z = 1. \tag{16}$$

With the application of the boundary conditions (16), the solution of Eq. (15) is

$$T_b(z) = \frac{e^{\lambda} - e^{\lambda z}}{e^{\lambda} - 1}. \tag{17}$$

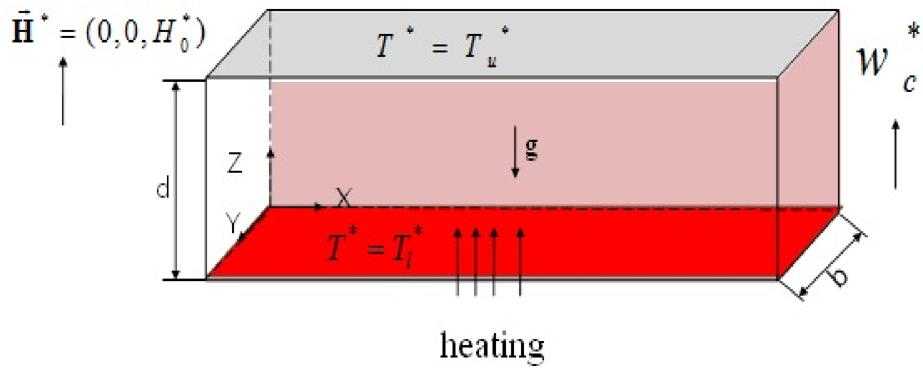

Fig. 1. The physical configuration of the system.

2.2 Perturbation Equation

We now apply to small perturbations on this basic state as:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_b + \vec{\mathbf{v}}',\ T = T_b + T',\ p = p_b + p',\ \vec{\mathbf{H}} = \hat{\mathbf{e}}_{\mathbf{z}} + \vec{\mathbf{H}}', \tag{18}$$

where the primed quantities are functions of $\mathbf{x}$ and t.

On substituting the Eq. (18) into Eqs. (8)–(13), and linearizing the equations, we have:

$$\nabla \cdot \vec{\mathbf{v}}' = 0, \tag{19}$$

$$\frac{\vec{\mathbf{v}}'}{H_S} = -\nabla p' + R_a T' \hat{\mathbf{e}}_Z + Q P_m \frac{\partial \vec{\mathbf{H}}'}{\partial z}, \tag{20}$$

$$\frac{\partial T'}{\partial t} + \vec{\mathbf{v}}' \cdot \nabla T_b + \vec{\mathbf{v}}_b \cdot \nabla T' = \nabla^2 T', \tag{21}$$

$$\frac{\partial \vec{\mathbf{H}}'}{\partial t} + \vec{\mathbf{v}}' \cdot \nabla \vec{\mathbf{H}}_b + \vec{\mathbf{v}}_b \cdot \nabla \vec{\mathbf{H}}' = \vec{\mathbf{H}}' \cdot \nabla \vec{\mathbf{v}}_b + \vec{\mathbf{H}}_b \cdot \nabla \vec{\mathbf{v}}' + P_m \nabla^2 \vec{\mathbf{H}}', \tag{22}$$

$$\nabla \cdot \vec{\mathbf{H}}' = 0, \tag{23}$$

Operating on Eq. (21) with $\hat{\mathbf{e}}_{\mathbf{z}} \cdot \nabla \times \nabla \times$ and using the Eqs. (19) and (23), we obtain z-component of the momentum equation as:

$$\frac{\nabla^2 w'}{H_S} - R_a \nabla_p{}^2 T' - Q P_m \nabla^2 \left(\frac{\partial H_z'}{\partial z} \right) = 0. \tag{24}$$

The z- component of the Eq. (22) is

$$\frac{\partial H_z'}{\partial t} + \lambda \frac{\partial H_z'}{\partial z} = \frac{\partial w'}{\partial z} + P_m \nabla^2 H_z', \tag{25}$$

Eliminating H_z' from Eqs. (24) and (25), we get:

$$\left(P_m \nabla^2 - \frac{\partial}{\partial t} - \lambda \frac{\partial}{\partial z} \right) \left[\frac{\nabla^2 w'}{H_S} - R_a \nabla_p{}^2 T' \right] + Q P_m \nabla^2 \left(\frac{\partial^2 w'}{\partial z^2} \right) = 0. \tag{26}$$

Taking the perturbation quantities of the form:

$$\left(w',\ T'\right)=\left[W\left(z\right),\ \Theta\left(z\right)\right]\exp\left[ik_x x+ik_y y+st\right], \tag{27}$$

where k_x and k_y are the wave numbers in the x and y directions, respectively and s is the growth rate of volatility. The growth rate s is in commonly a complex number such that $s=s_r+is_i$. The classification with $s_r<0$ is for all time stable, while is unstable when $s_r>0$. For neutral stability, the real part of s is zero. Hence, we consider $s=is_i$, where s_i is real and is a dimensionless frequency. On replacing Eq. (27) into Eqs. (26) and (21), we have:

$$\left[P_m\left(D^2-a^2\right)-is_i-\lambda D\right]\left[\left(D^2-a^2\right)\frac{W}{H_S}+a^2R_a\Theta\right]+QP_m\left(D^2-a^2\right)D^2W=0, \tag{28}$$

$$\left(D^2-a^2-\lambda D-is_i\right)\Theta-fW=0, \tag{29}$$

where $\frac{d}{dz}\equiv D$, $f\left(z\right)=\frac{\lambda e^{\lambda z}}{1-e^{\lambda}}$ and $a=\sqrt{k_x^2+k_y^2}$ is the resultant dimensionless wave number.

In the perturbation dimensionless form, the boundary conditions become:

$$W=0,\ \Theta=0\ \text{ at } z=0,1. \tag{30}$$

In order to solve the system of equations (28)-(30), the Galerkin weighted residuals method is applied. Accordingly, the support functions for W and Θ are assumed as:

$$W=\sum_{P=1}^{N}A_pW_p,\quad \Theta=\sum_{P=1}^{N}B_p\Theta_p, \tag{31}$$

where, $W_p=\Theta_p=\sin p\pi z$ (fulfilling the boundary conditions), A_p and B_p are unknown coefficients, and $p=1,2,3,...,N$. On putting the above expression for W and Θ into Eqs. (28)-(29), we get a system of $2N$ linear algebraic equations in the $2N$ unknowns A_p and B_p, $p=1,2,3,...,N$. For the occurrence of non-trivial solution, the determinant of coefficients matrix must be zero, which provides the characteristic equation for the system with Rayleigh number R_a as the eigenvalue.

3. Results and Discussion

For analytical result, we choose $N=1$, and then the Darcy-Rayleigh number R_a is given as

$$R_a=\Delta_1+is_i\Delta_2, \tag{32}$$

where,

$$\Delta_1=\frac{J\left(\lambda^2+4\pi^2\right)\left\{J\left(s_i^2+J^2P_m^2\right)+H_S\pi^2P_m\left(s_i^2+J^2P_m\right)Q\right\}}{4a^2H_S\pi^2\left(s_i^2+J^2P_m^2\right)}, \tag{33}$$

$$\Delta_2=\frac{J\left(\lambda^2+4\pi^2\right)\left[s_i^2+JP_m\left\{JP_m+H_S\pi^2\left(P_m-1\right)Q\right\}\right]}{4a^2H_S\pi^2\left(s_i^2+J^2P_m^2\right)}. \tag{34}$$

Here, $J=\left(a^2+\pi^2\right)$.

Since R_a is a physical quantity, it needs to be compulsorily real. Thus, it follow from Eq. (32) that either $s_i=0$ (stationary convection) or $\Delta_2=0$ ($s_i\neq 0$ non-oscillatory convection).

3.1. Stationary Mode of Convection

Stationary convection occurs when $s_i=0$. In this case, from Eq. (32), the stationary Rayleigh number R_a^S can be obtained as

$$R_a^S=\frac{\left(a^2+\pi^2\right)\left(\lambda^2+4\pi^2\right)\left\{\left(a^2+\pi^2\right)+H_S\pi^2Q\right\}}{4a^2H_S\pi^2}, \tag{35}$$

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From the Eq. (35), it is clear that the critical Rayleigh number increases with an increase in Q and λ, while decreases with H_S. Thus, the magnetic field and the through flow have stabilizing effect, while the Hele Shaw number has destabilizing effect on the stability of the system.
The critical wave number a_c can be obtained as

$$a_c = \pi\left(1+H_s Q\right)^{1/4}. \tag{36}$$

For the case of porous medium $\left(H_S = 1\right)$, Eqs. (35) and (36) become:

$$R_a^S = \frac{\left(a^2+\pi^2\right)\left(\lambda^2+4\pi^2\right)\left\{\left(a^2+\pi^2\right)+\pi^2 Q\right\}}{4a^2\pi^2}, \tag{37}$$

$$a_c = \pi\left(1+Q\right)^{1/4}. \tag{38}$$

In the nonexistence of magnetic field $\left(Q=0\right)$, Eqs. (37) and (38) become:

$$R_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2}\left[1+\frac{\lambda^2}{4\pi^2}\right], \tag{39}$$

$$a_c = \pi. \tag{40}$$

This result is identical with the result of Nield and Kuznetsov [37].
From Eqs. (39) and (40), when through flow is equal to one, i.e. $\lambda = 1$, the value of critical Rayleigh number is 40.4784. Recently, Barletta et al. (38) obtained a more exact value by using a different mythology and it is equal to 40.8751. Hence the approximation formula used in this paper gives a value accurate to 1%. This shows that the approximation formula used in this paper is satisfactory for the case when through flow is equal to one.
In the absence of through flow, i.e. $\lambda = 0$, Eq. (37) gives

$$R_a^S = \frac{\left(a^2+\pi^2\right)\left\{\left(a^2+\pi^2\right)+\pi^2 Q\right\}}{a^2}. \tag{41}$$

Eq. (41) coincides with that of Kiran et al. [39].

3.2. Oscillatory Mode of Convection

For oscillatory convection $\Delta_2 = 0$ and $s_i \neq 0$. Using these in Eq. (32), the expressions for oscillatory Rayleigh number R_a^{Osc} and the frequency of oscillations s_i can be written as:

$$R_a^{Osc} = \frac{J\left(\lambda^2+4\pi^2\right)\left[J\left(s_i^{\,2}+J^2P_m^{\,2}\right)+H_S\pi^2P_m\left(s_i^{\,2}+J^2P_m\right)Q\right]}{4a^2H_S\pi^2\left(s_i^{\,2}+J^2P_m^{\,2}\right)}, \tag{42}$$

$$s_i^{\,2} = -JP_m\left[JP_m + H_S\pi^2\left(P_m-1\right)Q\right]. \tag{43}$$

The above Eq. (43) shows that, the necessary condition for the occurrence of oscillatory mode of convection is:

$$\frac{JP_m}{H_S\pi^2\left(1-P_m\right)} < Q. \tag{44}$$

In order to build Q positive, the magnetic Prandtl number P_m must acquire the values less than unity. From Eq. (43), it is also found that the oscillatory mode of convection is not likely in the absence of magnetic field.

The graphical investigations of the stability of the system in $\left(R_a,\, a\right)$ plane are made in Figs 2-5 for various parameter values. The values of the parameters used in the figures are taken as determined by various researchers in their studies [4,9]. The linear stability theory gives the condition of stability in terms of the critical Rayleigh number, below which the system is stable and unstable above.

Fig.2 represents the effect of the Hele-Shaw number H_S on the stability of the system. From this figure, it is observed that the critical Rayleigh number increases with a decrease in the Hele-Shaw number H_S. Hence, the Hele-Shaw number H_S has a destabilizing effect on the system.

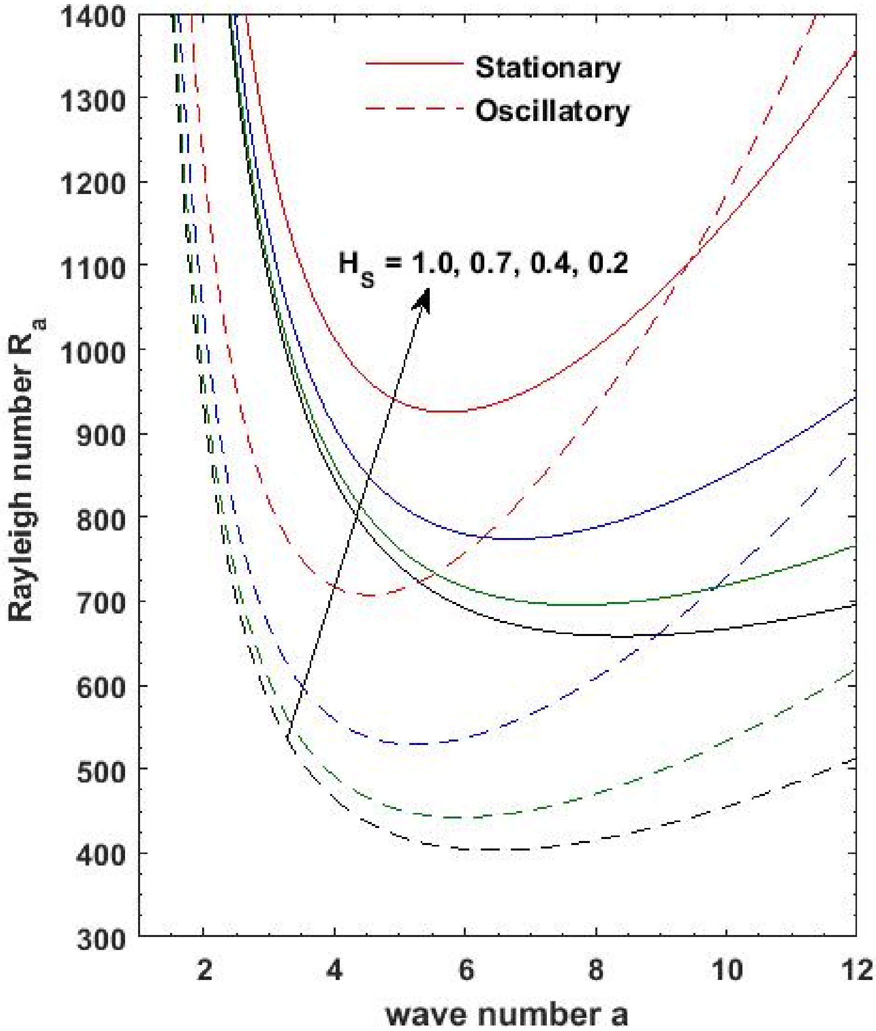

Fig. 2. The effect of the Hele-Shaw number H_S on the stationary and oscillatory convection curves at $\lambda = 0.5$, $Q = 50$ and $P_m = 0.5$.

The effect of the magnetic field parameter Q on the onset of stationary and oscillatory convection curves are displayed in Fig. 3. This figure shows that a decrease in the value of Q decreases the critical stationary and oscillatory Rayleigh number. Hence the magnetic field parameter Q delays the onset of convection. This is because the increase of Q increase to the Lorentz force and the Lorentz force gives more resistance to transport phenomenon. Hence, magnetic field has stabilizing effect on the system.

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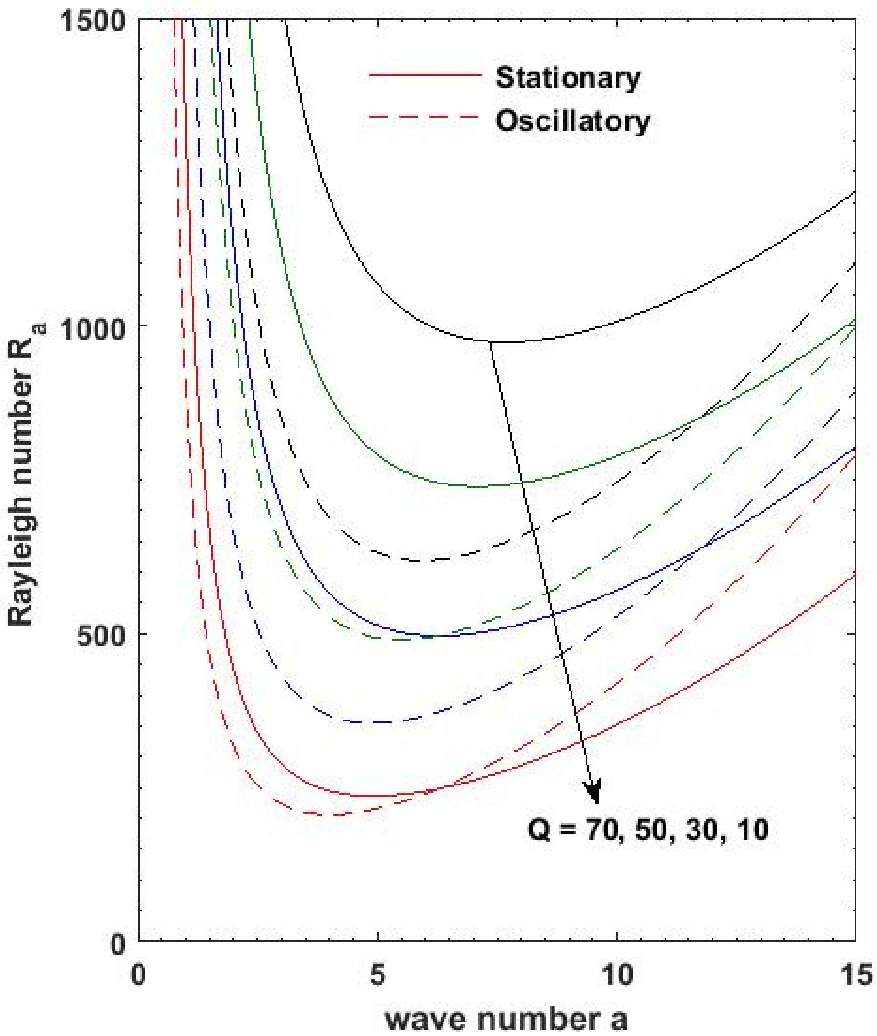

Fig. 3. The effect of the magnetic field Q on the stationary and oscillatory convection curves at $\lambda=0.5$, $H_S = 0.5$ and $P_m = 0.5$.

The effect of through flow parameter λ on the stationary and oscillatory mode of convections is shown in Fig.4. The minima on each plot give the critical Rayleigh number for the exchange of stabilities. This critical Rayleigh number decreases with decreasing value of the through flow λ and hence their effect is to delay the onset of convection.

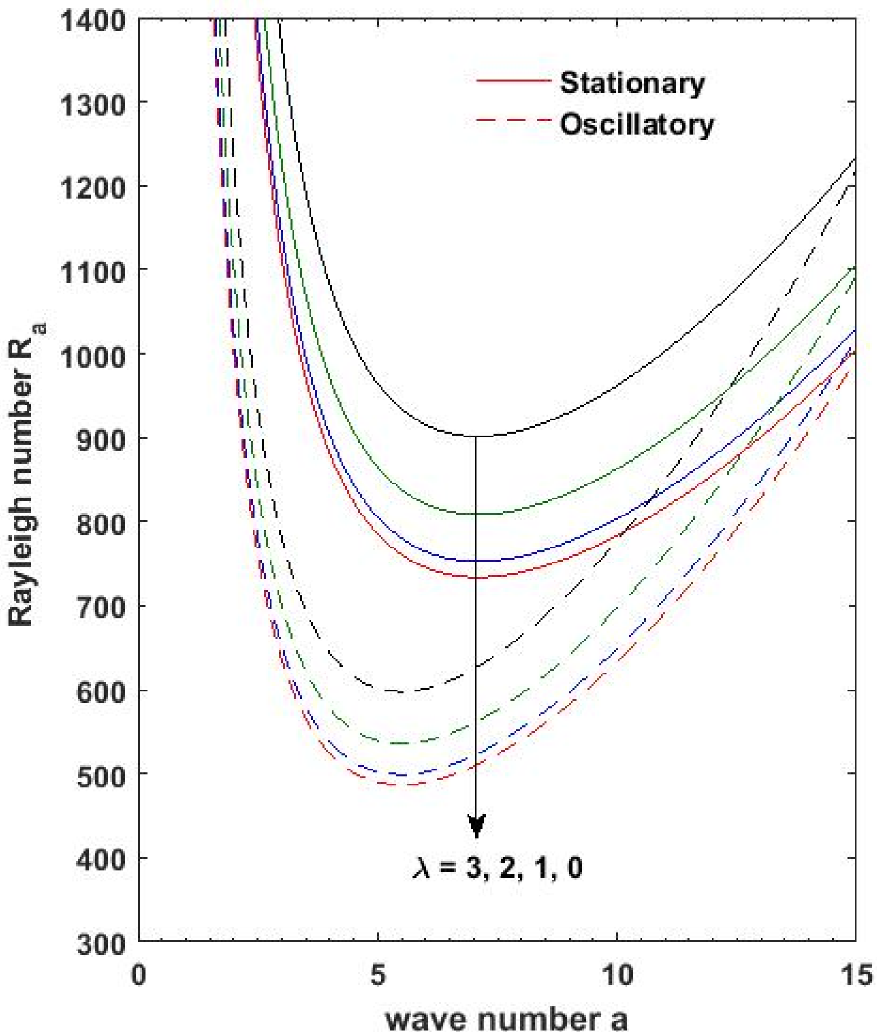

Fig. 4. The effect of the through flow λ on the stationary and oscillatory convection curves at Q=50, $H_S = 0.5$ and P_m =0.5.

To measure the effect of magnetic Prandtl number P_m on the stability of the system, the deviation of Rayleigh number for stationary and oscillatory mode of convection is plotted in Fig. 5 as a function of wave number a for different values of the magnetic Prandtl number P_m. From this figure it is observed that the magnetic Prandtl number P_m has no effect on the stationary convection, while for oscillatory convection it has a stabilizing effect on the stability of the system.

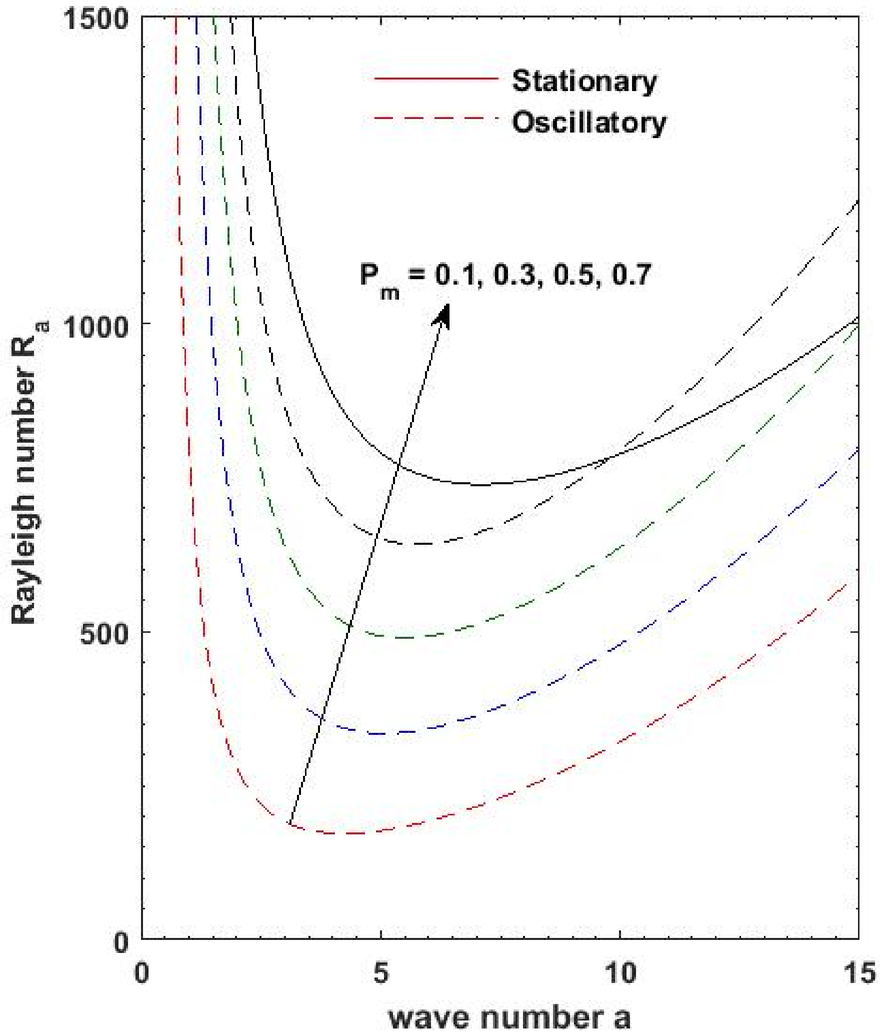

Fig. 5. The effect of the magnetic Prandtl number P_m on the stationary and oscillatory convection curves at Q=50, $H_S = 0.5$ and λ=0.5.

4. Conclusions:

In this paper, the combined effect of through flow and magnetic field on the instability in a fluid occupying within a Hele-Shaw cell heated from below was investigated using the linear stability theory. The behaviour of the magnetic field, the through flow and the Hele-Shaw number on the onset of convection was analysed analytically and discussed graphically. The results show that the increase in magnetic field and strength of throw flow tends to stabilize the system. Increase in the Hele-Shaw number was found to have a destabilizing effect on the system. It was also observed that the oscillatory mode of convection possible only when the magnetic Prandtl number acquires the values less than unity.

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