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Soliton solutions,Kink and antikink of the Gerdjikov-Ivanov equation

Alphonse Houwe [a,b] , MIBAILE Justin [c] , DIKWA Jerome [d] , GAMBO Betchewe [b], , Serge Y. Doka [d] , KOFANE Timoleon Crepin [e]

[a] *Department of Marine engineering, Limbe Nautical Arts and Fisheries Institute, P.O. Box. 485 Limbe, Cameroon*

[b] *Department of Physics, Faculty of Science, the University of Maroua, P.O Box 814, Cameroon*

[c] *Higher Teachers' Training College of Maroua, the University of Maroua, P.O Box. 55, Cameroon*

[d] *Department of Physics, Faculty of Science, the University of Ngaoundere, P.O Box 454, Cameroon*

[e] *Department of Physics, Faculty of Science, the University of Yaounde I, P.O Box 812, Cameroon*

Abstract

This paper secures the analytical soliton solutions of the perturbed NLSE gives to (1). The existence criteria have been demonstrated and the wave speed of the soliton befall out. To achieve all the results obtained we used the mathematical technique. In view of the results obtained, some new additional one have been reported in left-handed metamaterials.

Key words: Gerdjikov Ivanov equation;Integrability; Chirped soliton.

Email addresses: `ahouw220@yahoo.fr` (Alphonse Houwe), `mibailethejust7@yahoo.fr` (MIBAILE Justin), `jeromedikwa@gmail.com` (DIKWA Jerome), `gambobetch@yahoo.fr` (GAMBO Betchewe), `numami@gmail.com` (Serge Y. Doka), `tckofane@yahoo.com` (KOFANE Timoleon Crepin).

1 Introduction

There are various nonlinear equations that govern propagation of the wave [1,2]. For this purpose, the nonlinear partial differential equation (PDEs) are widely used to investigate exact solutions and soliton solutions in various scientific and engineering fields, such as metamaterials with negative index,fluid mechanics,plasma physics, optical fibers, biology, solid state physics, etc.[3–10]. Recently,to secure optical solitons, the Gerdjikov Ivanov model have been studied by Anjan Biwas et al.[1,12], H. Yilmaz[13] and various effective approaches have been used to construct the exact traveling-wave solution of this equation. In this direction, Zhou et al. [4] studied solitons in MMs with parabolic law nonlinearity. Xu et al. [5] reported Raman solitons in MMs having polynomial law non-linearity employing travelling wave hypothesis. Veljkovic et al. [6] studied super-sech soliton dynamics in MMs by collective variable approach. Triki et al. [7] investigated the MMs with Kerr law nonlinearity, and derived dipole soliton solutions by adopting the complex amplitude ansatz. Biswas et al. [8] obtained bright and dark solitons for MMs. Ebadi et al. [9] demonstrated the existence of solitons in MMs with Kerr law nonlinearity using F-expansion approach. In this perspective, many methods for obtaining exact solutions of NLSE was investigate, such as Tan-sech method [10,11], Jacobi elliptic function expansion method [12], the sine-cosine method [13,14], the modified simple equation method [15], and so on.
In this paper, we aim is to obtain the exact solutions and soliton solutions of Gerdjikov Ivanov equation.The paper is organized as follows, section 2 presents the GI equation. The section 3 is devoted to the derivation of soliton-like solutions of GI and the last section gives the summary and remarks.

2 Governing equation

The following governing equation,is the Gerdjikov-Ivanov model as in [16,17]

$$iq_t + aq_{xx} + ibq^2q_x^* + c|q|^4q = 0. \tag{1}$$

where $q_(x,t)$ is the complex component amplitude and q^* the correspondent complex conjugate. Therefore a,b and c are respectively a group velocity dispersion, coefficients of nonlinearity and term of nonlinear dispersion. The parameters a, b and c are all real-valued constants. To extract nonlinear chirp soliton-like solutions and traveling wave solutions to Eq.(1), we use the following transformation

$$q(x,t) = \rho(x,t)e^{i(\chi\xi(x,t)-\Omega_0 t)}, \tag{2}$$

where $\rho(x,t)$ is the real amplitude function of $\xi(x,t) = x - vt$. Therefore v is the wave velocity and Ω_0 the correspondent frequency of the wave oscillation. Substituting Eq.(2) into Eq. (1), the imaginary parts gives:

$$-v\rho_\xi + 2a\chi_\xi\rho_\xi + a\chi_{\xi\xi}\rho + b\rho^2\rho_\xi = 0, \tag{3}$$

and the real part gives

$$\Omega_0\rho + v\chi_\xi\rho + a\rho_{\xi\xi} - a\chi_\xi^2\rho + b\chi_\xi\rho^3 + c\rho^5 = 0, \tag{4}$$

Considering that the associated chirp can be written in the following form

$$\delta\omega = -\frac{\partial}{\partial x}[\chi(\xi) - \Omega_0 t] = -\chi_\xi, \tag{5}$$

where $\chi_\xi = \frac{\partial\chi}{\partial\xi}$
We assume that the corresponding chirp takes the following expression

$$-\chi_\xi = -(A\rho^2 + B), \tag{6}$$

Inserting Eq.(6) into Eq.(3) leads to

$$A = -\frac{b}{4a} \tag{7}$$

$$B = \frac{v}{2a} \tag{8}$$

thence Eq.(4) becomes

$$a_0 - a_1\rho^6 - a_2\rho^4 - a_3\rho^2 - \rho_\xi^2 = 0, \tag{9}$$

$$a_0 = Cst, \tag{10}$$

$$a_1 = -\frac{1}{3a}(c - \frac{5b^2}{16a}), \tag{11}$$

$$a_2 = \frac{-vb}{16a^2}, \tag{12}$$

$$a_3 = \frac{(16\omega_0 a + 3v^2)}{16a^2} \tag{13}$$

2.1 Chirped Soliton-like solutions

Eq.(5) is an elliptic equation well known to solve partial differential equations[18,19]. As in [18,19],we gain the following soliton solutions
1.**i**:$a_0 = 0, a_3 > 0, a_2 < 0$ and $a_2^2 - 4a_3a_1 > 0$,

$$\delta\omega_1 = -\frac{2Aa_3 sech(\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} - (\sqrt{a_2^2-4a_3a_1}+a_2)sech(\sqrt{a_3}(x-vt))^2} - \frac{v}{2a}, \quad (14)$$

$$\delta\omega_2 = -\frac{2Aa_3 csch(\pm\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} + (\sqrt{a_2^2-4a_3a_1}+a_2)csch(\pm\sqrt{a_3}(x-vt))^2} - \frac{v}{2a}, \quad (15)$$

$$q_1(x,t) = \sqrt{\frac{2a_3 sech(\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} - (\sqrt{a_2^2-4a_3a_1}+a_2)sech(\sqrt{a_3}(x-vt))^2}}\, e^{i(-kx+\omega t+\theta_0)}, \quad (16)$$

$$q_2(x,t) = \sqrt{\frac{2a_3 csch(\pm\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} + (\sqrt{a_2^2-4a_3a_1}+a_2)csch(\pm\sqrt{a_3}(x-vt))^2}}\, e^{i(-kx+\omega t+\theta_0)}, \quad (17)$$

2.**i**:$a_0 = 0, a_3 < 0, a_2 \geq 0, a_1 < 0$ and $a_2^2 - 4a_3a_1 > 0$,

$$\delta\omega_3 = -\frac{2Aa_3 \csc(\pm\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} - (\sqrt{a_2^2-4a_3a_1}+a_2)\csc(\pm\sqrt{a_3}(x-vt))^2} - \frac{v}{2a}, \quad (18)$$

$$\delta\omega_4 = \frac{2Aa_3 \sec(\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} - (\sqrt{a_2^2-4a_3a_1}+a_2)\sec(\sqrt{a_3}(x-vt))^2} - \frac{v}{2a}, \quad (19)$$

and the exact soliton solution

$$q_3(x,t) = \sqrt{\frac{2a_3 \csc(\pm\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} - (\sqrt{a_2^2-4a_3a_1}+a_2)\csc(\pm\sqrt{a_3}(x-vt))^2}}\, e^{i(-kx+\omega t+\theta_0)}, \quad (20)$$

$$q_4(x,t) = \sqrt{\frac{-2a_3 \sec(\sqrt{a_3}(x-vt))^2}{2\sqrt{a_2^2-4a_3a_1} - (\sqrt{a_2^2-4a_3a_1}+a_2)\sec(\sqrt{a_3}(x-vt))^2}}\, e^{i(-kx+\omega t+\theta_0)}, \quad (21)$$

3.**i**:$a_0 = 0, a_3 > 0, a_2 < 0, a_1 < 0$ and $a_2^2 = 4a_3a_1 > 0$,

$$\delta\omega_5 = A\frac{a_3}{a_2}(1+\tanh(\pm\sqrt{a_3}(x-vt))) - \frac{v}{2a}, \quad (22)$$

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$$\delta\omega_6 = A\frac{a_3}{a_2}(1+\coth(\sqrt{a_3}(x-vt))) - \frac{v}{2a}, \tag{23}$$

$$q_5(x,t) = \sqrt{\frac{-a_3}{a_2}(1+\tanh(\pm\sqrt{a_3}(x-vt)))}e^{i(-kx+\omega t+\theta_0)}, \tag{24}$$

$$q_6(x,t) = \sqrt{\frac{-a_3}{a_2}(1+\coth(\sqrt{a_3}(x-vt)))}e^{i(-kx+\omega t+\theta_0)}, \tag{25}$$

4.i:$a_0 = 0, a_3 > 0, a_2 < 0, a_1 < 0$ and $a_2^2 = 4a_3a_1 > 0$, the corresponding solutions are

$$\delta\omega_7 = A\frac{a_2a_3sech(\sqrt{a_3}(x-vt))^2}{a_2^2 - a_3a_1(1+\tanh(\sqrt{a_3})(x-vt))^2} - \frac{v}{2a}, \tag{26}$$

$$\delta\omega_8 = -A\frac{a_2a_3csch(\sqrt{a_3}(x-vt))^2}{a_2^2 - a_3a_1(1+\coth(\sqrt{a_3})(x-vt))^2} - \frac{v}{2a}, \tag{27}$$

$$\delta\omega_9 = -4A\frac{a_3e^{2\sqrt{a_3}(x-vt)}}{(e^{2\sqrt{a_3}(x-vt)} - 4a_2)^2 - 64a_3a_1} - \frac{v}{2a}, \tag{28}$$

$$q_7(x,t) = \sqrt{\frac{-a_2a_3sech(\sqrt{a_3}(x-vt))^2}{a_2^2 - a_3a_1(1+\tanh(\sqrt{a_3})(x-vt))^2}}e^{i(-kx+\omega t+\theta_0)}, \tag{29}$$

$$q_8(x,t) = \sqrt{\frac{a_2a_3csch(\sqrt{a_3}(x-vt))^2}{a_2^2 - a_3a_1(1+\coth(\sqrt{a_3})(x-vt))^2}}e^{i(-kx+\omega t+\theta_0)}, \tag{30}$$

$$q_9(x,t) = \sqrt{4\frac{a_3e^{2\sqrt{a_3}(x-vt)}}{(e^{2\sqrt{a_3}(x-vt)} - 4a_2)^2 - 64a_3a_1}}e^{i(-kx+\omega t+\theta_0)}, \tag{31}$$

5.i:$a_0 = 0$, $a_3 > 0$ and $a_2^2 - 4a_3a_1 > 0$, the solution obtained are

$$\delta\omega_{10} = -\frac{2Aa_3}{\sqrt{a_2^2 - 4a_3a_1}\cosh(2\sqrt{a_3}(x-vt)) - a_2} - \frac{v}{2a}, \tag{32}$$

$$q_{10}(x,t) = \sqrt{\frac{2a_3}{\sqrt{a_2^2 - 4a_3a_1}\cosh(2\sqrt{a_3}(x-vt)) - a_2}}e^{i(-kx+\omega t+\theta_0)}, \tag{33}$$

6.i: $a_0 = 0$, $a_3 > 0$ and $a_2^2 - 4a_3a_1 < 0$, the associated chirp and singular solution as

$$\delta\omega_{11} = -\frac{2Aa_3}{\sqrt{-a_2^2 + 4a_3a_1}\sinh(2\sqrt{a_3}(x-vt)) - a_2} - \frac{v}{2a}, \tag{34}$$

$$q_{11}(x,t) = \sqrt{\frac{2a_3}{\sqrt{-a_2^2+4a_3a_1}\sinh(2\sqrt{a_3}(x-vt))-a_2}}e^{i(-kx+\omega t+\theta_0)}, \tag{35}$$

7.i:$a_0 = 0$, $a_3 > 0$ and $a_2^2 - 4a_3a_1 < 0$, we gain

$$\delta\omega_{12} = -\frac{2Aa_3}{\sqrt{a_2^2+4a_3a_1}\cos(2\sqrt{-a_3}(x-vt))-a_2} - \frac{v}{2a}, \tag{36}$$

$$\delta\omega_{13} = -\frac{2Aa_3}{\sqrt{a_2^2+4a_3a_1}\sin(2\sqrt{-a_3}(x-vt))-a_2} - \frac{v}{2a}, \tag{37}$$

$$q_{12}(x,t) = \sqrt{\frac{2a_3}{\sqrt{a_2^2+4a_3a_1}\cos(2\sqrt{-a_3}(x-vt))-a_2}}e^{i(-kx+\omega t+\theta_0)}, \tag{38}$$

$$q_{13}(x,t) = \sqrt{\frac{2a_3}{\sqrt{a_2^2+4a_3a_1}\sin(2\sqrt{-a_3}(x-vt))-a_2}}e^{i(-kx+\omega t+\theta_0)}, \tag{39}$$

8.i:$a_0 = 0$, $a_3 > 0$ and $a_1 > 0$, we obtained

$$\delta\omega_{14} = A\frac{a_3 sech(\sqrt{a_3}(x-vt))^2}{a_2+2\sqrt{a_1a_2}\tanh(\sqrt{a_3}(x-vt))} - \frac{v}{2a}, \tag{40}$$

$$\delta\omega_{15} = -A\frac{a_3 csch(\sqrt{a_3}(x-vt))^2}{a_2+2\sqrt{a_1a_2}\coth(\sqrt{a_3}(x-vt))} - \frac{v}{2a}, \tag{41}$$

$$q_{14}(x,t) = \sqrt{\frac{-a_3 sech(\sqrt{a_3}(x-vt))^2}{a_2+2\sqrt{a_1a_2}\tanh(\sqrt{a_3}(x-vt))}}e^{i(-kx+\omega t+\theta_0)}, \tag{42}$$

$$q_{15}(x,t) = \sqrt{\frac{a_3 csch(\sqrt{a_3}(x-vt))^2}{a_2+2\sqrt{a_1a_2}\coth(\sqrt{a_3}(x-vt))}}e^{i(-kx+\omega t+\theta_0)}, \tag{43}$$

9.i:$a_0 = 0$, $a_3 < 0$ and $a_1 > 0$, we obtained the following combined solutions

$$\delta\omega_{16} = A\frac{a_3\sec(\sqrt{-a_3}(x-vt))^2}{a_2+2\sqrt{-a_1a_2}\tanh(\sqrt{-a_3}(x-vt))} - \frac{v}{2a}, \tag{44}$$

$$\delta\omega_{17} = A\frac{a_3\csc(\sqrt{-a_3}(x-vt))^2}{a_2+2\sqrt{-a_1a_2}\cot(\sqrt{-a_3}(x-vt))} - \frac{v}{2a}, \tag{45}$$

$$q_{16}(x,t) = \sqrt{\frac{-a_3\sec(\sqrt{-a_3}(x-vt))^2}{a_2+2\sqrt{-a_1a_2}\tanh(\sqrt{-a_3}(x-vt))}}e^{i(-kx+\omega t+\theta_0)}, \tag{46}$$

$$q_{17}(x,t) = \sqrt{\frac{-a_3\csc(\sqrt{-a_3}(x-vt))^2}{a_2+2\sqrt{-a_1a_2}\cot(\sqrt{-a_3}(x-vt))}}e^{i(-kx+\omega t+\theta_0)}, \tag{47}$$

10.**i**:$a_0 = 0$, $a_3 > 0$ and $a_2 = 0$, we gain

$$\delta\omega_{18} = \pm A\frac{a_3e^{2\sqrt{a_3}(x-vt)}}{1-64a_3a_1e^{4\sqrt{a_3}(x-vt)}} - \frac{v}{2a}, \tag{48}$$

$$q_{18}(x,t) = \sqrt{\pm\frac{a_3e^{2\sqrt{a_3}(x-vt)}}{1-64a_3a_1e^{4\sqrt{a_3}(x-vt)}}}e^{i(-kx+\omega t+\theta_0)}, \tag{49}$$

11.**i**:$a_0 = \frac{8a_3^2}{27a_1}$, $a_1\frac{a_2^2}{4a_3}$ and $a_3 > 0$ and $a_2 > 0$, we gain

$$\delta\omega_{19} = A\frac{8Ab\coth(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\coth(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))} - \frac{v}{2a}, \tag{50}$$

$$\delta\omega_{20} = \frac{8Ab\tanh(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\tanh(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))} - \frac{v}{2a}, \tag{51}$$

$$q_{19}(x,t) = \sqrt{\frac{-8b\coth(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\coth(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))}}e^{i(-kx+\omega t+\theta_0)}, \tag{52}$$

$$q_{20}(x,t) = \sqrt{\frac{-8b\tanh(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\tanh(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))}}e^{i(-kx+\omega t+\theta_0)}, \tag{53}$$

12.**i**:$a_0 = \frac{8a_3^2}{27a_1}$, $a_1\frac{a_2^2}{4a_3}$ and $a_3 < 0$ and $a_2 > 0$, we gain

$$\delta\omega_{21} = -\frac{8Ab\cot(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\cot(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))} - \frac{v}{2a}, \tag{54}$$

$$\delta\omega_{22} = -\frac{8Ab\tan(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\tan(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))} - \frac{v}{2a}, \tag{55}$$

$$q_{21}(x,t) = \sqrt{\frac{8b\cot(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\cot(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))}}e^{i(-kx+\omega t+\theta_0)}, \tag{56}$$

$$q_{22}(x,t) = \sqrt{\frac{8b\tan(\pm\sqrt{\frac{-a_3}{3}}(x-vt))^2}{3a_2(3+\tan(\pm\sqrt{\frac{-a_3}{3}}(x-vt)^2))}}e^{i(-kx+\omega t+\theta_0)}, \tag{57}$$

3 Summary

This paper secured the chirped soliton-like solutions to the Gerdjikov Ivanov equation. Our model have been studied by authors [1,12,13]. The existence criteria have been presented as constraint conditions. The chirped soliton-like solutions obtained are well known in the fiber optic and nonlinear electrical transmission lines, which are the telecommunication tools. Our results, will certainly help to amplify or compress signal in telecommunication. We also note that, the model becomes close to the nonlinear cubic equation obtained in the left-handed transmission lines where the coefficient of nonlinear dispersion is not consider ($c = 0$).

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