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Article

A Theoretically Derived Probability Distribution of Scour

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Abstract: Based on recent contributions regarding the treatment of unsteady hydraulic conditions into the state-of-the-art of scour literature, the theoretically derived probability distribution of bridge scour is introduced. The model is derived assuming a rectangular hydrograph shape with a given duration, and random flood peak following a Gumbel distribution. A model extension for a more complex flood event is also presented, assuming a synthetic exponential hydrograph shape. The mathematical formulation can be extended to any flood-peak probability distribution. The aim of the manuscript is to move forward the current approaches adopted for the bridge design coupling hydrological, hydraulic, and erosional models in a mathematical closed form.

Keywords: Scour; Bridges; Piers; Theoretically Derived Distribution (TDD)

1. Introduction

Scour process is one of the major causes of bridge collapse at the global scale. It is responsible of more than 50% of bridge failures according to the recent studies by Proske [1]. Therefore, understanding the complex interactions between river systems and bridges is critical to advance our ability to prevent such catastrophic events and protect our infrastructures.

It is interesting to mention that actual bridge design manuals recommend the use of the maximum equilibrium scour depth to withstand erosional forces due to extreme flood events (see e.g., [2,3]). Therefore, most of the modern bridges have been designed under such hypotheses assuming a flood peak discharge with a return period of 100-200 years. This should lead to good safety conditions, but scour induced bridge failure occur under considerably scattered flood peaks events with a range of return periods ranging between 1 and more than 1000 years [4]. This clearly highlights some gaps in our understanding of the overall dynamics.

Such an uncertainty on the safety conditions of river bridges may be due to the scale effects involved in estimations of real-life scour using scour formulas derived from laboratory experiments with idealized hydraulic, sedimentological, and geometrical conditions [5]. Unfortunately, the upscale of laboratory models to field case studies is still far to be realized given the complete lack of reliable field scour data. In fact, bridge-scour field monitoring implies a number of technical issues (e.g., turbid currents, sensor damages) that have limited the development of a dedicated technology.

Nevertheless, scientific literature offers a wide spectrum of applications that provide a well-defined formalism to characterize the maximum equilibrium scour depth achieved under clear-water and steady hydraulic conditions [6,7]. However, extreme flood events lead to more complex mechanisms such as the interaction among unsteady hydraulic conditions, short- and long-term riverbed evolution, and refilling of the scour-hole [8,9].

The lack of more realistic hydraulic conditions in laboratory-based scour experiments is attributed to experimental difficulties associated to discharge control in flumes and to the hydraulic pump volume capacity and sediment recirculation systems. Only recently, time-dependent scour formulas without considering a step-hydrograph has been introduced (e.g., [10–13]). Among others

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Link et al. [12] and Pizarro et al. [14] introduced the concept of dimensionless effective flow work that have shown good predictive capabilities under several experimental conditions both in steady and non-steady conditions. The study has stimulated a number of subsequent studies that further explored this index to derive a time-dependent scour formulation (e.g., [13]). Such a mathematical formulation offers the opportunity to export recent results on scour process in the field of probability searching for potential links between flood statistics and scour. This may help to overcome some of the existing limitations in river bridges design.

This paper aims to investigate the theoretically derived probability distribution of the bridge scour process with the aim to motivate and move forward the current approaches adopted for the hydraulic design of bridges. Hydrological, hydraulic, and scour models are coupled in a mathematical closed form. This document is organized as follows: hydraulic assumptions are presented in Section 2 and Section 3 describes the mathematical derivation of the theoretical probability distribution of scour. Conclusions are presented at the end.

2. Hydraulic Assumptions

Interactions between river flow and bridges must be delineated adopting a number of simplifying assumption able to capture the main dynamics occurring in a river. In particular, river flow discharge (Q) can be expressed as the product of two factors, namely mean flow velocity (V) and the associated wetted area of the cross section (Ω):

$$Q = V\Omega. \tag{1}$$

Following Manfreda [15], it is possible to write both terms as a function of the hydraulic water stage (H),

$$\Omega = \mathrm{a}H^b \tag{2}$$

$$V = \mathrm{c}H^d, \tag{3}$$

and therefore, the combination of Eq. (2) and Eq. (3) allows to express the flow discharge as a function of the mean flow velocity:

$$Q = a\,c\,\left(\frac{V}{c}\right)^{\frac{b+d}{d}}. \tag{4}$$

Inverting Eq. (4) allows to derive an estimate of V as a function of Q,

$$V = pQ^{\gamma} \tag{5}$$

where $= \frac{d}{d+b}$, and $p = c\left(\frac{1}{a\,c}\right)^{\frac{d}{d+b}}$.

This formalism allows to describe the mathematical relationship between mean flow velocity and discharge in a given cross-section. Considering that the probability distribution of flow peak discharge is a well described process in hydrology, it is possible to exploit this knowledge to derive the probability distribution of mean flow velocity and consequently of scour. The details of such an idea are better addressed in the following section.

3. The Probability distribution of scour

3.1. Simplified rectangular flood hydrograph

A simplified hydrograph shape (with constant discharge Q and duration k) is assumed to derive the probability distribution of scour. This leads to a mathematical description of the dynamic of the scour process over their entire possible range of flood frequencies.

On one hand, the probability distribution of floods is well known and parameters of the flood distribution can be calibrated using local or regional approaches. In the present case, it is started from the hypothesis of a Gumbel distribution,

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$$P(Q) = \frac{1}{\alpha} e^{-\frac{Q-b1}{\alpha} - e^{-\frac{Q-b1}{\alpha}}}, \tag{6}$$

where $P(Q)$ is the probability distribution of floods, and α and $b1$ are Gumbel parameters.

On the other hand, the relationship between a given flood event and the produced scour can be interpreted by different methods. The authors have recently proposed the dimensionless effective flow work (W^*) which is computed as a function of V [14]:

$$W^* = \int_0^k \frac{1}{t_R} \left(\frac{V(t) - u_{cs}}{u_R} \right)^4 dt = \int_0^k \frac{1}{t_R} \left(\frac{pQ(t)^\gamma - u_{cs}}{u_R} \right)^4 dt \tag{7}$$

where u_{cs} is the critical velocity for the incipient scour, $u_R = \sqrt{\rho' g d_s}$ is a reference velocity, $t_R = z_R/u_R = (D^2/2d_s)/u_R$ is a reference time, D is the pier-diameter, d_s is the sediment grain-size, $\rho' = (\rho_s - \rho_w)/\rho_w$ is the relative density with subscripts s and w referring to sediment and water respectively, g is the gravitational acceleration, and z_R is a reference length.

It is true that V does not necessarily represent properly local dynamics, but it is an indispensable approximation to build a probabilistic model of scour. This formulation gives the great advantage to properly interpret the scour process on time under any hydrograph. Using the assumption of rectangular hydrograph makes to solution of the integral tractable for the subsequent steps. It is possible to assume a more complex hydrograph shape, but this makes more complex the mathematical tractability of the function. Therefore, the authors decided to keep the model as simple as possible to identify an analytical solution. Nevertheless, additional complexity can be introduced in the presented modelling scheme by means of the use of numerical integration in the subsequent steps.

Inverting Eq. (7), it is possible to obtain an expression of Q in function of W^*,

$$Q = \left[\frac{\left(\frac{W^* tr\, u_r^4}{k} \right)^{1/4} + u_{cs}}{p} \right]^{1/\gamma}. \tag{8}$$

Furthermore, given W^*, the scour depth can be estimated using the BRISENT model recently introduced by Pizarro et al. [13]:

$$Z^* = \frac{1}{\lambda} \ln\left\{1 + \frac{W^*}{W^*_{max}} [exp(S) - 1]\right\}, \tag{9}$$

where $Z^* = z/z_R$ is the normalized scour depth, z is the scour depth, $S = \lambda Z^*_{max}$ is the entropic-scour parameter, λ is a fitting coefficient, and $Z^*_{max} = \max(Z^*)$ is the maximum relative scour depth associated to the maximum dimensionless, effective flow work, $W^*_{max} = \max(W^*)$. Inverting Eq. (9), it is possible to express W^* in function of Z^*,

$$W^* = W^*_{max} \left(\frac{e^{\lambda Z^*} - 1}{e^S - 1} \right). \tag{10}$$

In consequence, the mathematical relation between Q and Z^* can be written as,

$$Q = \left(\left(\left(\frac{W^*_{max}(e^{\lambda Z^*} - 1) tr\, u_r^4}{k\,(e^S - 1)} \right)^{\frac{1}{4}} + u_{cs} \right) \Big/ p \right)^{\frac{1}{\gamma}} = \left(\left(\left(\theta (e^{\lambda Z^*} - 1) \right)^{1/4} + u_{cs} \right) / p \right)^{1/\gamma}, \tag{11}$$

where $\theta = \frac{W^*_{max} tr\, u_r^4}{k\,(e^S - 1)}$.

Using Eq. (11), it can derive the probability distribution of scour, $P(Z^*)$, based on the probability distribution of floods (Eq. (6)). The mathematical formulation becomes:

$$P(Z^*) = \frac{\theta \lambda e^{\lambda Z^*}}{4\,\gamma\,\alpha\,p} \left(\theta (e^{\lambda Z^*} - 1) \right)^{-\frac{3}{4}} \left(\left(\left(\theta (e^{\lambda Z^*} - 1) \right)^{\frac{1}{4}} + u_{cs} \right) \Big/ p \right)^{\frac{1}{\gamma} - 1} \tag{12}$$

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$$e^{-\frac{\left(\left(\left(\theta\left(e^{\lambda Z^{*}}-1\right)\right)^{1/4}+u_{cs}\right)/p\right)^{1/\gamma}-b1}{\alpha}-e^{-\frac{\left(\left(\left(\theta\left(e^{\lambda Z^{*}}-1\right)\right)^{1/4}+u_{cs}\right)/p\right)^{1/\gamma}-b1}{\alpha}}}.$$

Note that Eq. (12) is a function defined between 0 and Z^{*}_{max}, and therefore it has two mass probabilities at these scour depths. Consequently, to properly describe the distribution, it is necessary to estimate the probability that the scour depth is equal zero for all the possible realizations of floods. This value can be estimated integrating the derived probability distribution of mean flow velocities $P(V)$,

$$P(V) = \frac{(V/p)^{\frac{1}{\gamma}-1}}{\alpha\, p\, \gamma} e^{-\frac{\left(\frac{V}{p}\right)^{\frac{1}{\gamma}}-b1}{\alpha}-e^{-\frac{(V/p)^{1/\gamma}-b1}{\alpha}}}. \tag{13}$$

The probability of zero scour can be thus defined as:

$$P(0) = \int_{0}^{u_{cs}} \frac{(V/p)^{\frac{1}{\gamma}-1}}{\alpha\, p\, \gamma} e^{-\frac{(V/p)^{1/\gamma}-b1}{\alpha}-e^{-\frac{(V/p)^{1/\gamma}-b1}{\alpha}}} dV. \tag{14}$$

3.2. Exponential flood hydrograph

The proposed theoretical formulation can be extended to a case where the hydrograph assumes a complex or more realistic shape. Given the experience from previous studies, it is known that the total scour is function of the total flow work. Therefore, it is possible to redefine the formulation just assigning the correct parametrization to the mathematical formalism proposed. Figure 1 shows a conceptual diagram to link the hydrograph shape and scour depth.

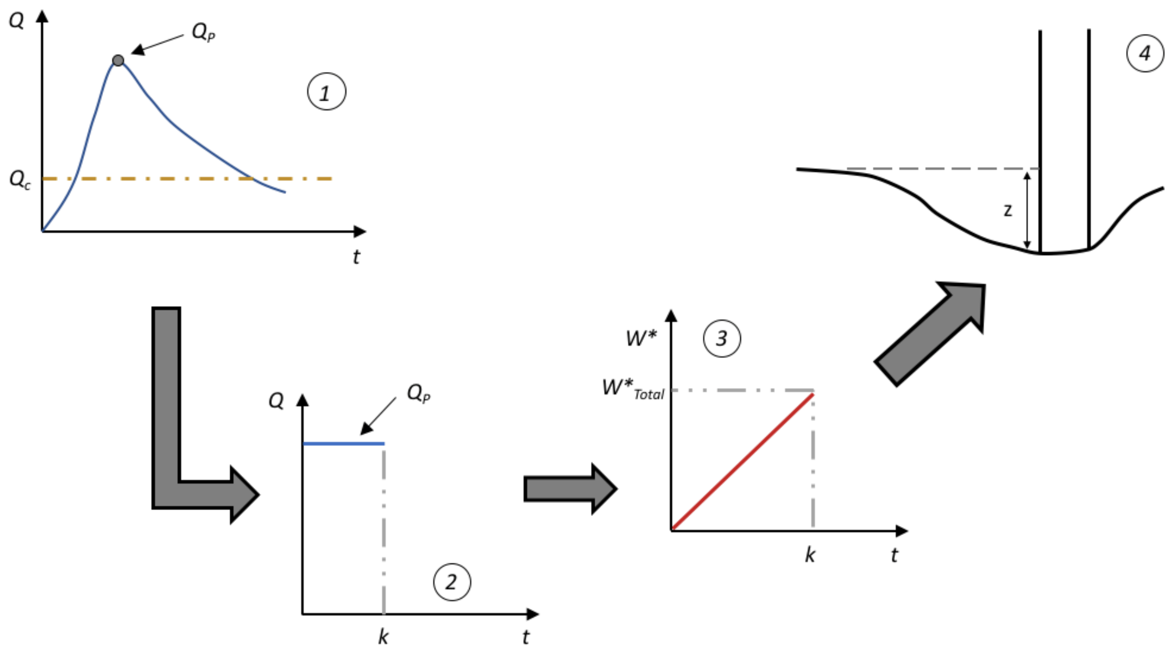

Figure 1. Conceptual diagram to link flood hydrograph and scour depth. (1) Natural flood shape. (2) Equivalent hydrograph in terms of W^*. (3) W^* on time. (4) Scour depth produced by W^*_{Total}.

In particular, one limitation that constraints the proposed approach is the fact that no hydrograph in nature is rectangular (a natural hydrograph is characterized by a raising limb and a recession phase). Therefore, the equivalent hydrograph of duration k producing the same total work W^* as the synthetic scour hydrograph is obtained imposing:

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$$\frac{k}{tr}\left(\frac{V-u_{cs}}{u_r}\right)^4 = \int_0^{t_{end}} \frac{1}{tr}\left(\frac{V(t)-u_{cs}}{u_r}\right)^4 dt. \tag{15}$$

Assuming that the flow hydrograph is represented by the NERC approximation [16],

$$Q(t) = Q_{max}\, e^{-t/\omega}, \tag{16}$$

the mean flow velocity is derived as,

$$V(t) = pQ_{max}{}^{\gamma}\, e^{-\gamma\frac{t}{\omega}}. \tag{17}$$

Using the above formulation, the total work W^* is defined as long as $V > u_{cs}$. Therefore, the upper limit of the integral is

$$t_{end} = -\frac{\omega}{\gamma}\ln\left(\frac{u_{cs}}{pQ_{max}^{\gamma}}\right). \tag{18}$$

The total W^* associated to a random flood event with a synthetic exponential hydrograph becomes

$$W^* = \int_0^{t_{end}} \frac{1}{tr}\left(\frac{V(t)-u_{cs}}{u_r}\right)^4 dt = \frac{\omega\left(3p^4Q^{4\gamma}-16p^3u_cQ^{3\gamma}+36p^2u_c{}^2Q^{2\gamma}-48p\,u_c{}^3Q^{\gamma}+25u_c{}^4-12\,u_c{}^4 ln\left(\frac{u_c}{pQ^{\gamma}}\right)\right)}{12\,\gamma\,tr\,u_r{}^4}, \tag{19}$$

and imposing Eq. (7) equal to Eq. (19), the parameter k can be estimated as

$$k = \frac{\omega\left(3p^4Q^{4\gamma}-16p^3u_cQ^{3\gamma}+36p^2u_c{}^2Q^{2\gamma}-48p\,u_c{}^3Q^{\gamma}+25u_c{}^4-12\,u_c{}^4 ln\left(\frac{u_c}{pQ^{\gamma}}\right)\right)}{12\,\gamma\,(pQ^{\gamma}-u_c)^4}. \tag{20}$$

An example of application of the proposed model is given in Figure 2, where the derived probability distributions of the mean flow velocities associated to a specific configuration (panel A) and the associated probability distribution of scour depth obtained assuming different equivalent durations k are depicted (panel B).

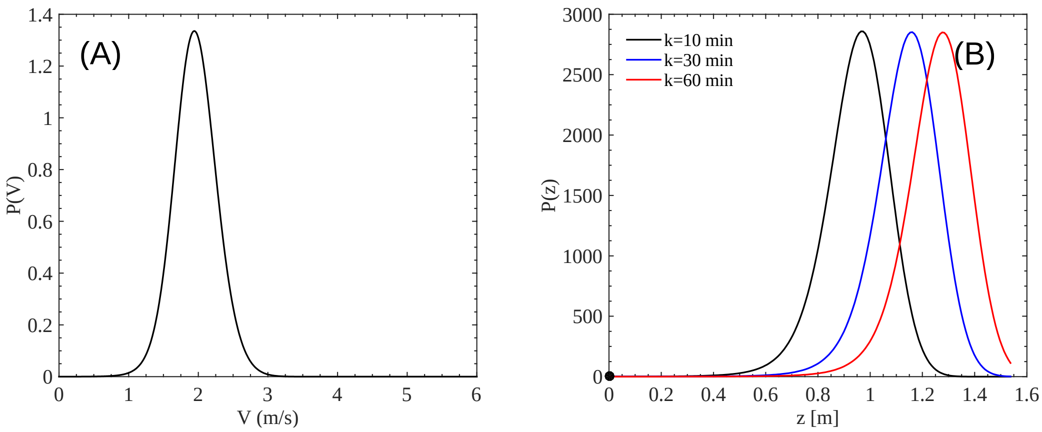

Figure 2. Derived probability density function of the mean flow velocities associated to a given river basin (A) and the corresponding PDFs of scour depth (B) obtained assuming hydrographs with variable equivalent duration k, ranging from 10 minutes up to 1 hour. Other parameters are: D = 1.07m; d_{50} = 0.0007 m; ρ_s=2.65 t/m^3; u_c=0.319 m/s; λ = 4.7252e+03; W^*_{max} = 1.8613e+05; S = 8.9026; α= 67.8627 m^3/s; b_1 = 143.643; γ =0.33; p=0.36 1/m^2.

This graph allows to underline the relative impact of hydrograph duration on the expected scour of a given cross-section. Complementary information respect to those just described are provided in Figure 3, where a description of the relative influence of the hydraulic parameters γ and p on the

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probability distribution of scour is given using a similar set of parameters adopted in the previous figure.

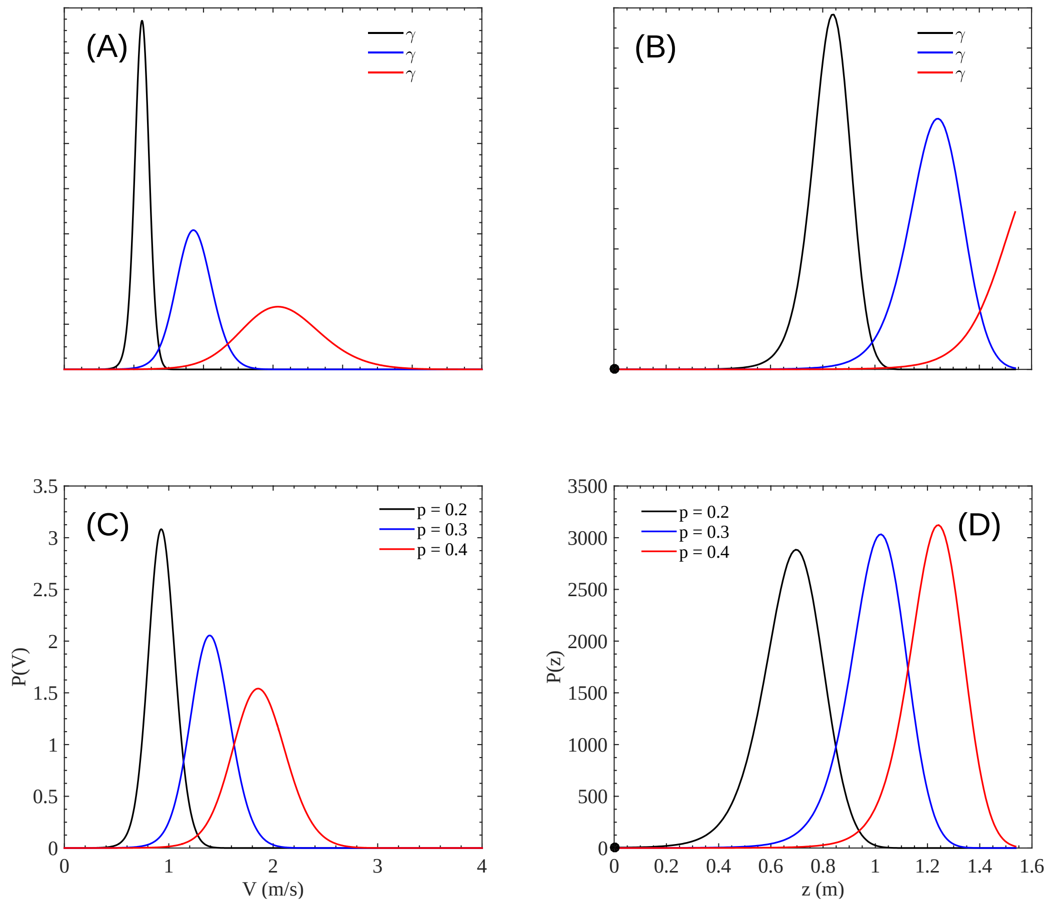

Figure 3. Derived probability density functions of the mean flow velocities associated to a given river basin (A and C) and the corresponding PDFs of scour depths obtained modifying the shape parameter of the cross-section γ, with a fixed value of p = 0.4 and k = 60 min (A and B) or modifying the scale parameter p with fixed values of γ = 0.3 and k = 60 min (C and D). Other parameters are: D = 1.07m; d_{50} = 0.0007 m; ρ_s=2.65 t/m^3; u_c=0.319 m/s; λ = 4.7252e+03; W^*_{max} = 1.8613e+05; S = 8.9026; α= 67.8627 m^3/s; b_1 = 143.643; γ =0.33; p=0.36 1/m^2.

The duration of a rectangular hydrograph producing an equivalent scour as a real-shape hydrograph is clearly a function of Q. Eq. (20) is depicted in Figure 4, where one can clearly see how the parameter k/ ω rapidly reaches an asymptotic value increasing Q. The upper limit of the parameter k is

$$k \approx \lim_{Q\to\infty} k(Q) = \frac{\omega}{4\,\gamma}. \tag{21}$$

Therefore, a possible approximation for the probability distribution of scour is represented by Eq. (12), where the parameter k can be derived using Eq. (20). In this way, the total W^* associated to each flood event is representative of an equivalent synthetic exponential hydrograph. It is worthy to underline that the proposed expression for k tend to overestimate the total duration of the equivalent rectangular hydrograph and it will produce a scour estimate slightly conservative.

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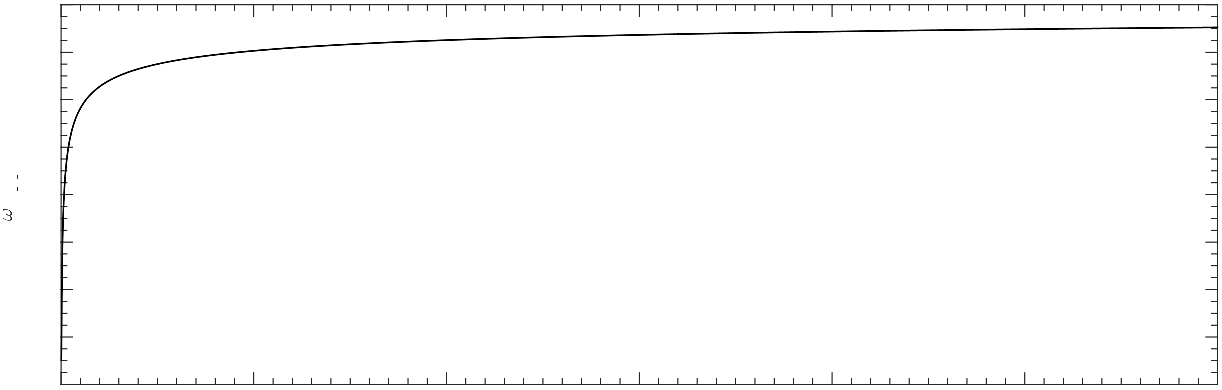

Figure 4. Ratio between equivalent duration k and ω as a function of the discharge. It can be notice that the ratio k/ω tends to stabilize when Q tends to increase.

4. Conclusions

The recent contributions on the treatment on bridge scour processes in unsteady hydraulic conditions are a cornerstone to reduce uncertainty within the hydraulic design of bridges, bridge scour assessment, and future applications. These results have been exploited to provide a new theoretically derived probability distribution of scour (TDDS) that allows to link within a mathematical framework the main variables involved in the process, such as: river basin hydrology, hydraulic characteristics of the river and the cross-section, the sediment and the pier characteristics (e.g., pier diameter). The proposed framework allows a better understanding of the impact of each of those components on scour dynamics.

The proposed TDDS represents the first attempt to formalize the scour statistics in an event-based modeling scheme. It is a simple and suitable approach to estimate flood-induced scour probability or conversely to estimate the risk of collapse (i.e. failure risk associated to the probability of exceeding a given scour depth) of a given bridge under specific conditions.

The model represents a preliminary attempt to introduce a new formalism to support the bridge design. All model parameters can potentially be linked to physical features as displayed in the manuscript by *Pizarro et al.* (2017a). For this reason, it would have been extremely interesting to test the proposed framework on a real study case, but the total absence of monitored data over an extended temporal window prevented such an attempt. It may potentially be tested on long term numerical simulations and this task is currently under investigation.

In conclusion, we should also remind that the proposed framework can be easily expanded using a different probability distribution of floods or eventually introducing the relative dependence between the flood peak discharge and the relative duration of the flood hydrograph.

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Conflicts of Interest: The authors declare no conflict of interest.

Notation

a [m]	=	Coefficient of the power law cross-section area function of H;
b [-]	=	Exponent of the power law cross-section area function of H;
c [1/s]	=	Coefficient of the power law mean flow velocity function of H;
d [-]	=	Exponent of the power law mean flow velocity function of H;
p [1/m²]	=	Coefficient of the power law mean flow velocity function of Q;
γ [-]	=	Exponent of the power law mean flow velocity function of Q;
α [-]	=	location parameter of the Gumbel distribution;
b_1 [m³/s]	=	scale parameter of the Gumbel distribution;
D [m]	=	Pier diameter;
d_s [m]	=	Sediment grain-size;
g [m/s]	=	Acceleration of gravity;
H [m]	=	Flow depth;
k [sec]	=	Equivalent time;
λ [-]	=	BRISENT coefficient;
ω [sec]	=	Characteristic time;
Ω [m²]	=	Cross-section area;
P(Q) [-]	=	Probability distribution of floods;
P(V) [-]	=	Probability distribution of velocities;
P(Z*) [-]	=	Probability distribution of scour;
Q [m³/s]	=	River discharge;
Q_{max} [m³/s]	=	Maximum river discharge in a flood event;
ρ' [kg/m³]	=	Relative density;
ρ_s [kg/m³]	=	Sediment density;
ρ_w [kg/m³]	=	Water density;
S [-]	=	Entropic-scour parameter;
t [sec]	=	Time;
t_{end} [sec]	=	Time in which a hydrograph is able to make work;
t_R [sec]	=	Reference time;
u_c [m/s]	=	Critical velocity for the initiation of sediment motion;
u_{cs} [m/s]	=	Critical velocity for the incipient scour;
u_R [m/s]	=	Reference velocity;
V [m/s]	=	Cross-section-averaged velocity;
W^* [-]	=	Dimensionless, effective flow work;
W^*_{max} [-]	=	Maximum possible W*, according to BRISENT formulation;
Z^* [-]	=	Normalized scour depth;
Z^*_{max} [-]	=	Maximum possible Z*, according to BRISENT formulation;
z_R [m]	=	Reference scour depth;

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