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Invited Review

New Reentrant Insulating Phases in Strongly Interacting 2D Systems with Low Disorder

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Abstract: The apparent metal-insulator transition (MIT) in two-dimension (2D) was discovered by Kravchenko et al. [1] more than two decades ago in strongly interacting 2D electrons residing in a Si-metal-oxide-semiconductor field-effect transistor (Si-MOSFET). Its origin remains unresolved. Recently, low magnetic field reentrant insulating phases (RIPs), which dwell between the zero-field (B=0) metallic state and the integer quantum Hall (QH) states where the Landau-level filling factor $\upsilon > 1$, have been observed in strongly correlated 2D GaAs hole systems with large interaction parameter r_s (~20-40) and high purity. A new complex phase diagram was proposed, which includes zero field MIT, low magnetic field RIPs, integer QH states, fractional QH states, high field RIPs and insulating phases (HFIPs) with $\upsilon < 1$ in which the insulating phases are explained by the formation of Wigner crystal. Furthermore, evidences of new intermediate phases were reported. All contribute to the further understandings of the puzzle. This review article serves the purpose of summarizing those recent experimental findings and theoretical endeavors, to foster future research efforts.

Keywords: 1; Metal-insulator transitions 2; Electronic transport in interface structures 3; Quantum Hall effects.

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1. Introduction

Tremendous knowledge from more than fifty years of research has been accumulated regarding the transport behaviors of electrons in varieties of 2D materials and systems. It has regained strong interests in the field of physics and material science due to recent discoveries of topological insulator [2] and new 2D materials [3].

With different types and strengths of interactions, many fascinating phenomena can emerge at various temperatures and energy scales, such as Fermi liquid, Wigner crystal [4], integer quantum Hall (QH) effect [5], fractional QH effect [6], etc. The scaling theory of localization for noninteracting 2D systems [7] predicts that the electronic states are localized while temperature T approaches zero. In low density and high mobility Si MOSFET, however, a metal-insulator transition (MIT) was observed by Kravchenko et al. [1]. It implies that the strong correlation effects play a key role in the MIT [8, 9], as the value of r_s (the ratio between Coulomb energy and kinetic energy, $r_s \equiv 1/\left[a_B^*\sqrt{\pi p}\right], a_B^* = \hbar^2\epsilon/m^*e^2$ is the effective Bohr radius) is much greater than one.

In this brief review, we will first introduce and focus on the recently discovered low magnetic field RIPs [10-13], with the new phase diagram which connects the zero field MIT with RIPs, integer QH states, fractional QH states, and high magnetic field insulating phases [14]. Resistivity, capacitance and inductance measurements exploring the RIPs and phase diagram are presented and discussed. Second, experimental observations of possible Wigner crystal melting and new intermediate phases are shown and examined. Finally, relevant theoretical models that try to explain the MIT are briefly discussed.

2. New Reentrant Insulating Phases at Low Magnetic Fields

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Recently, a new reentrant insulating phase at Landau-level filling factor $\upsilon > 1$ was first discovered by Qiu et. al. [10, 11] in ultra clean GaAs quantum well samples. Figure 1 is taken from reference 10, which shows the observed phase diagram at 150, 80 and 50 mK.

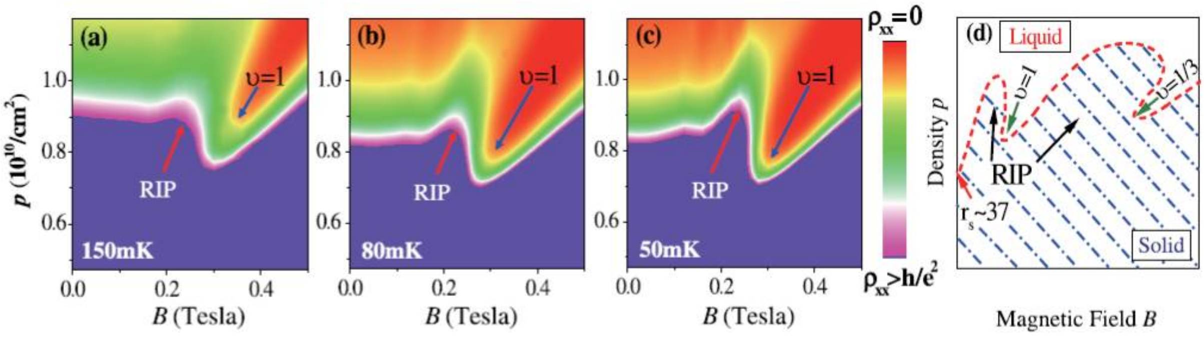

Figure 1. Longitudinal resistivity map plotted in the density (p) – magnetic field (B) plane at T = 150 (a), 80 (b) and 50 (c) mK for dilute 2D holes in a 10nm wide high mobility GaAs quantum well. The RIP phase ($\rho_{xx} > h/e^2$) becomes more prominent at lower temperature. (d) The proposed phase diagram includes MIT, RIP, integer and fractional QH states. Figure is adapted from reference 10.

The formations of WC were widely believed to be the origin of the HFIP and RIPs where the Landau-level filling factor $\upsilon < 1$ [14]. Experiment evidences were given from transport, thermodynamic compressibility, and microwave transmission measurements [14]. Most recent study [15] observed tunneling resonance feature that was attributed to the vibrations of WC. However, theoretically, liquid-WC transition was predicted to appear both at $B = 0$ and $\upsilon > 1$ around $r_s \sim$ 30-40 [16, 17], which had not been shown in experimental measurements until the study in Reference 10. Therefore, the observation of the new RIP in reference 10 provided a consistent phase diagram with theory [17], which further suggests that the zero-field MIT is a liquid-WC transition.

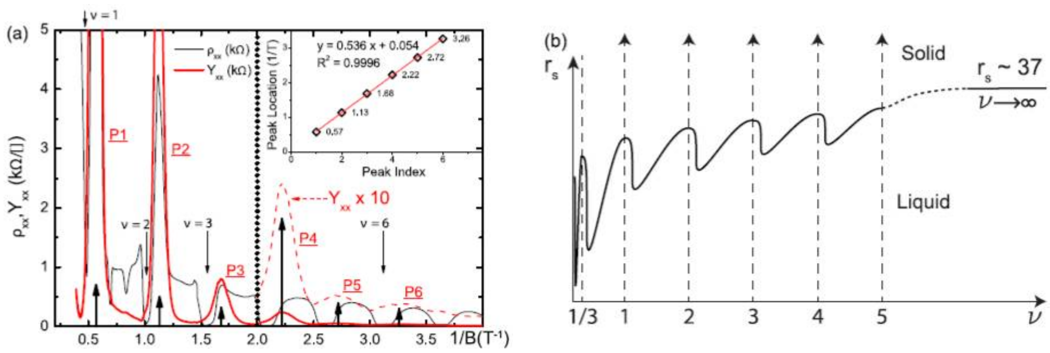

Figure 2. (a) Observation of multiple RIPs between Landau-level filling factors 1, 2, 3 and 4 in a dilute 2D hole system in GaAs with r_s~20. (b) The modified phase diagram with multiple RIPs. Figure is taken from reference 12.

Later, the existence of low field RIPs ($\upsilon > 1$) was confirmed by Knighton *et al*. [12, 13], see Figure 2 adopted from reference 12. Moreover, three RIPs were observed, which are seen between υ = 1, 2, 3 and 4. This observation implies a phase diagram that the 2D WC or low field RIP can alternate with integer QH states to take lower energy down to low fields where ν >1.

The reported resistance, capacitance and inductance characteristics of the RIPs are discussed below.

2.1. Resistivity

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Both Qiu *et al*. [10-11] and Knighton *et al*. [12-13] reported clearly insulating behavior ($d\,\rho_{xx}/d\,T < 0$) of the RIPs that are far stronger than the SdH oscillation amplitude. The resistivity values of the RIP between ν=1 and 2 are all above the quantum resistivity h/e^2. However, there are some discrepancies in the amplitude and temperature dependency, which could be caused by the differences in sample structures and qualities. Figure 3 presents the result from reference 10, which used thermal activation model ($R_{RIP} \propto exp(\Delta_{RIP}/2T)$) to fit the temperature dependence of the RIP peak resistance. On the other hand, Knighton *et al*. [12-13] fitted the data to Efros-Shklovskii variable-range-hopping model $\rho_{xx} \propto exp\left(\sqrt{T^*/T}\right)$. Figure 4 shows their data and fitting results. It is worth to point out that the resistivity value of the RIPs at ν > 2 in Knighton *et al*.'s work was much lower than h/e^2 (Figure 4a) and the data do not have high dynamic range to warrant a reliable fitting to the model. Therefore, the mechanism (thermal activation vs variable-range-hopping etc.) of temperature dependent resistivity in the RIPs remains to be seen.

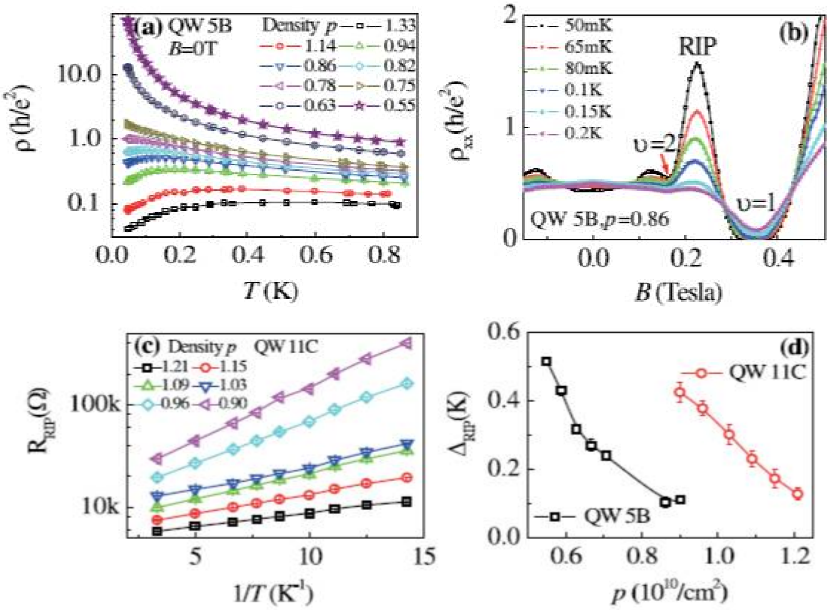

Figure 3. (a) 2D MIT at zero magnetic field in a dilute 2D hole system in 10 nm wide GaAs quantum well, $p_c \sim 0.8 \times 10^{10}/cm^2$ (b) $\rho_{xx}(B)$ with $p = 0.86 \times 10^{10}/cm^2$ (c) Arrhenius plot of the RIP peak resistance at various hole densities. (d) Fitted thermal activation gap. From Reference 10.

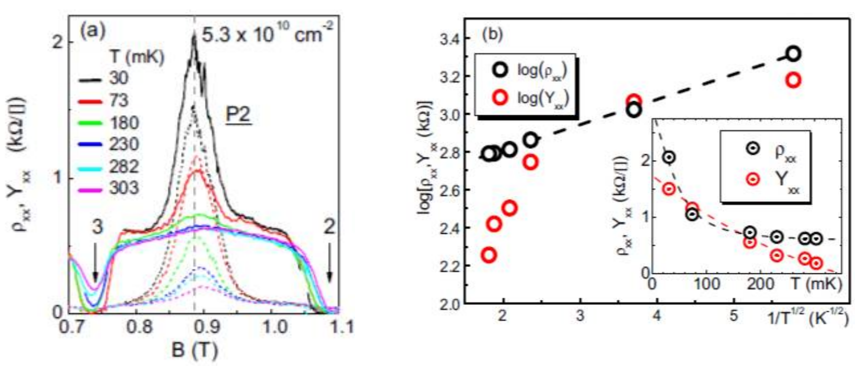

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Figure 4. (a) Real (solid) and imaginary (dashed) parts of the magnetoresistance for RIP between υ = 2 and 3 for 2D holes in a 20nm wide GaAs quantum well. (b) Real and imaginary components of the magnetoresistance plotted in a semi-log scale. From the reference 12.

2.2. Capacitance Measurement

Qiu *et al*. [10-11] also reported the thermodynamic compressibility study through capacitance measurement in the RIP, shown in Figure 5. It is found that the RIP tends to be incompressible, like the zero-field insulating phase and the HFIP. The phase diagram from capacitance measurement matches well with that from the transport measurement. The observation suggests the possibility of the same origin for them, i.e. the liquid-WC transition. We also point out that in the capacitance measurement of a 2D heterostructure with a single-gate configuration, the geometric gate capacitance usually dominates over the 2D system's quantum capacitance which is related to the compressibility. It is only when the 2D system becomes very incompressible and the 2D system's quantum capacitance is greatly reduced, the measured gate capacitance starts to show observable deviation from the geometric capacitance. More intricate methods such as the penetration field measurements in 2D structures with both top and bottom gate [18] will be desirable to further study the compressibility of RIPs and their connection or competition with the zero field MIT or integer QHs to a better precision.

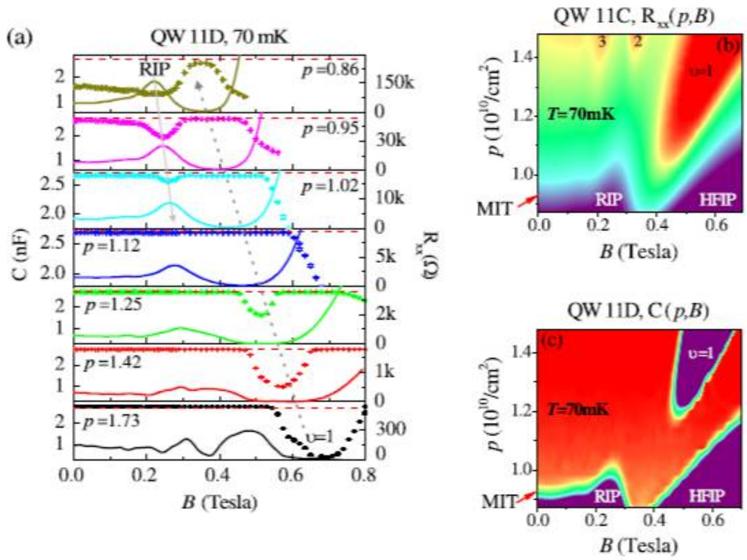

Figure 5. (a) Capacitance (symbol) and resistance (line) Vs. perpendicular magnetic field at several hole densities in a 10nm wide GaAs quantum well system. (b) The phase diagram viewed in the longitudinal resistance map at 70mK. (c) The phase diagram viewed in the capacitance map at 70mK. From Reference 10.

2.3. Inductance

Inductance of 2D systems is a rarely studied topic. Knighton *et al*. [12-13] reported the inherent inductive behavior of the RIPs, illustrated in Figure 6. The inductance behaviors are different between the RIPs at low magnetic field with $\upsilon > 1$ and the RIPs at high magnetic field where $\upsilon < 1$. There has not been much (or any) theoretical studies on the inductive behavior of correlated 2D electron systems. Thus the understanding of these anomalous inductance observations is limited and awaits further theoretical investigation.

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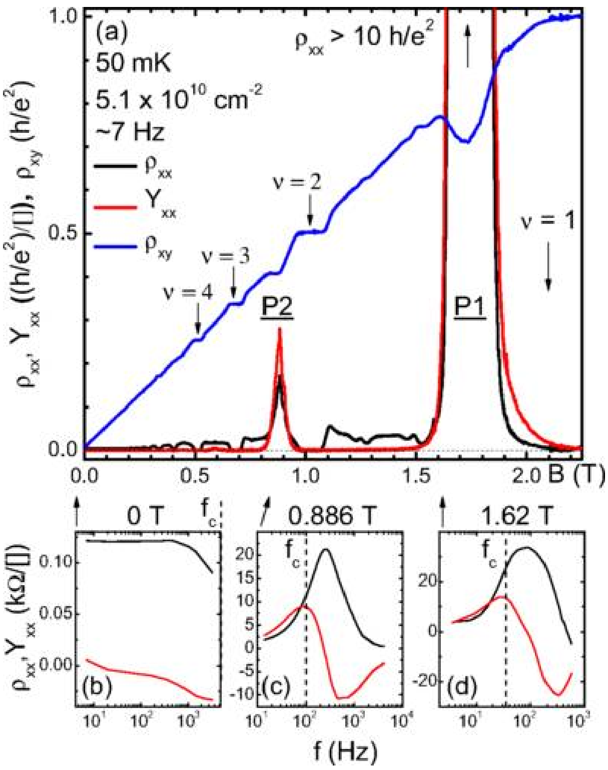

Figure 6. (a) Magnetoresistance of 2D holes in GaAs showing two RIPs at ν>1 (labeled as P1 and P2). (b-d) The longitudinal resistivity ρ_{xx} and inductive signal Y_{xx} vs. frequency at various magnetic fields, showing the clear inductive effect in the RIPs. Figure taken from Reference 12.

In summary, all the measured properties of the RIPs indicate that the RIPs share the same origin with the zero-field insulating phase and the RIPs at higher magnetic field $\upsilon < 1$. The phase diagram of very high mobility 2D p-GaAs systems with high r_s in the density-perpendicular magnetic field plane is consistent with the liquid-WC transition phase diagram in *clean* 2D systems.

3. Possible transport evidences for intermediate phases and Wigner crystal melting

Since the discovery of the RIPs at lower magnetic field $\upsilon > 1$ implies a liquid to Wigner crystal transition in clean 2D systems with high r_s, then an important question arises: what type of phase transition is the experimentally observed 2D metallic liquid to insulator (whereas the insulator is either the zero field insulator or the RIPs at low magnetic fields). Given the well-known theoretical results that there is no long range order in 2D solid and the possible existence of various phases intermediate between 2D WC and liquid [19-32], are there any experimental evidences for intermediate phases when the WC melts into liquid?

Evidences of intermediate phases have been presented by several groups [33-35]. Qiu *et al.* studied how the low field RIP is suppressed by either increasing the temperature or carrier density and found evidences for the 2D holes in p-GaAs transforming into a mixture state of incipient RIP and metallic liquid [36]. Other researchers took a different approach and used an increasing voltage (or electric field) applied to the sample to probe the breakdown of a presumed 2D WC deep in the insulating state where the resistivity is very high. In such state where the WC is presumably well formed, Brussarski *et al*. [34] and Knighton *et al*. [35] found non-linear I-V curves at low temperatures, shown in Figure 7 and Figure 8. Different threshold voltages were seen, suggesting two-stage phase transition instead of direct phase transition, although different explanations were given by the two groups. More experimental data and theoretical models are needed to further elucidate the situation.

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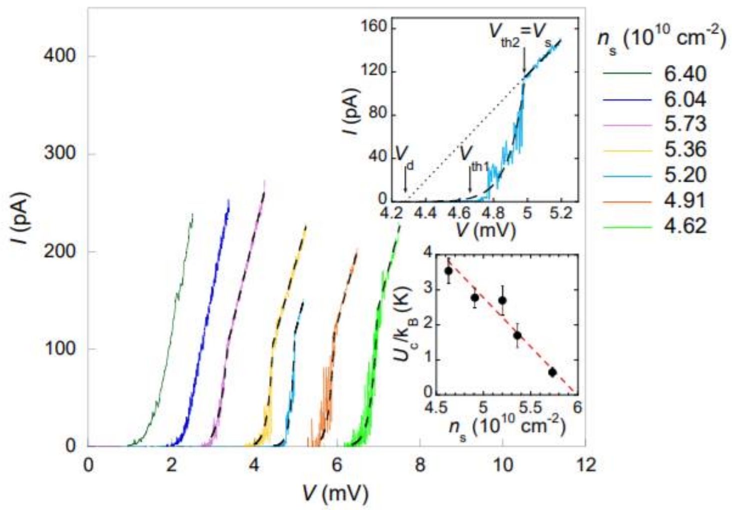

Figure 7. Voltage-current characteristics of 2D electrons in Si-MOSFET in a possible WC state. V-I curves of different electron densities are shown to show the depinning of WC. Figure taken from Reference 34.

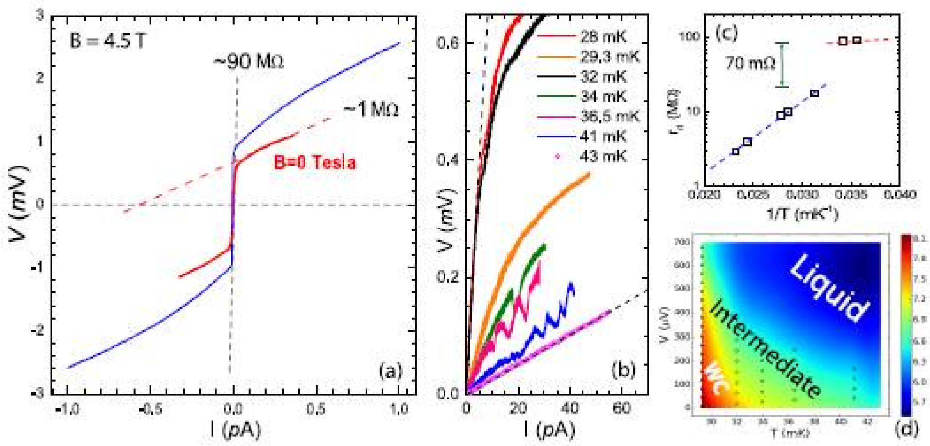

Figure 8. Voltage-current characteristics of 2D holes in GaAs in a possible WC state to show the two-stage melting of WC. (a) dc *IV* at 28 mK. (b) *IVs* at different temperatures. (c) Temperature dependence of $r_{d(T)}|_{V\to 0}$ (d) Suggested phase diagram. From Reference 35.

4. Discussion and Outlook

Although various recent transport experiments and findings point to the existence of 2D Wigner crystal and relevance of liquid-WC transition in the observed RIPs and peculiar voltage-current characteristics in strongly correlated 2D carrier systems, due to the limitations of transport data and various challenging aspects of the experiments (ultra-low temperatures, low noise-low level measurements, and stringent sample quality requirements), information about how the 2D Wigner crystal transforms into metallic liquid transition and the nature of intermediate phase are still limited and further experiments are required to obtain more in-depth understandings.

On the theoretical side, from the beginning of the discovery of 2D MIT, a number of weak-interaction based models were proposed to explain the metal-insulator transition or cross-over behavior in the electrical resistivity such as screening, potential fluctuation, percolation [36-38]. In these theories, conventional Anderson localization (weak-localization in the metal-side and strong localization in the insulator side of the MIT) is still relevant. From extensive prior studies on the Anderson insulator to 2D QH transition, it is expected the zero field insulator would directly transition to an integer QH state upon the application of perpendicular magnetic fields [39]. This is

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in stark contrast to the observation of low field RIPs between the zero field insulator and integer QH state [10-12]. It appears that it is difficult to use weak-interaction based models to reconcile the observations of various RIP, QH and HFIP states and their connection to the liquid-WC transition when the behavior of strongly interacting high r_s 2D systems is examined beyond the zero magnetic field to finite perpendicular magnetic fields. In contrast to weakly interaction theories, many other theories emphasize the importance of strong correlations and relevance of WC physics in the systems showing 2D MIT [9, 25, 26, 32] and therefore may be further compared with the experimental results. These theories are based on a number of different approaches: analytical mean-field models [25, 26], quantum Monte Carlo simulations [29], or dynamic mean field theory (DMFT) [31, 32]. In the mean-field theories by Kivelson and Spivak [25, 26], the various spectacular transport behavior in correlated 2D systems showing MIT are attributed to the Fermi liquid to WC transition where intermediate states are unavoidable [30]. In the intermediate 'micro-emulsion' states (e.g. WC bubbles in a Fermi liquid background), it is the interplay or transformation between Fermi liquid and WC components tuned by temperature or magnetic field that dictates the transport behavior and gives rise to the resistivity change of the system. It seems that the most relevant micro-emulsion phase to the experimentally observed RIPs and intermediate phases is the scenario where WC bubbles co-exists in a Fermi liquid. Whether other micro-emulsion states (Fermi liquid bubbles in WC, 1D ordered stripes) exist in experiments requires further research and more theoretical developments are desired to establish more quantitative predictions on the experimental systems. New theoretical approaches based on hydrodynamics seem quite promising and are currently being developed [40, 41]. In addition to the mean-field models by Kivelson and Spivak, strong interaction and Wigner-Mott transition based theoretical studies led by Dobrosavljevic and collaborators [31, 32] may also be relevant to the experimental findings. Modern DMFT was applied to study the MIT in 2D carriers with high r_s and the early approaches of Wigner and Mott were reconciled. Based on this 'Wigner-Mott' transition scenario, DMFT calculations are able to explain many detailed behavior in the electrical transport and charge ordered intermediate phases similar to charge density wave (CDW) are predicted to form before the system enters WC. It is worth to note that in the DMFT theory, both metallic CDW and insulating CDW are found [32]. It will be very interesting to see whether such CDW states exist in experiments.

On the experimental side, besides the previously mentioned compressibility and inductance measurements that require further advancements and understanding, other striking effects found in the zero field 2D MIT are very worth to explore in relation to the RIPs. One particular case is the effect of an in-plane magnetic field. In the zero field MIT, it was established that an in-plane magnetic field causes large positive magneto-resistance and drives the system toward the insulating phase [8, 9, 42, 43]. Recent in-plane field magneto-transport experiments revealed that the resistivity of dilute 2D electrons in Si-MOSFET in the insulating state are the same for zero field and in the presence of an in-plane field that polarizes the spins [43], a behavior different from the metallic phase of the 2D MIT [8, 9, 42]. It would be thus very insightful to study the effect in-plane magnetic field induced spin-polarization effect on the RIPs and examine how the in-plane field affects the transition between the RIPs and metallic liquid. In addition to spin-polarization effect, understanding the physics of correlated 2D systems at 'high temperatures' comparable with the Fermi temperature is also an interesting topic. In this 'semi-quantum' regime, the correlated electron fluid is expected to show unique behavior in viscosity [26]. While there were many experiments done at low temperatures, transport studies in this 'semi-quantum' regime where $T \sim T_F$ is limited and worth exploring further [45]. In addition to transport, new techniques such as thermopower measurement are strongly desired to shed new light on the RIPs.

In summary, the recent observations of RIPs in low magnetic fields where $\nu > 1$ and the connections between low field RIPs and high field RIP and HFIP with $\nu < 1$ point to the formation of WC as the origin of the low field RIPs and the 2D MIT being driven by the liquid-WC transition. There are also transport evidences for possible new intermediate phases when the insulating phase (either the insulator in zero field or the RIPs at low magnetic fields) is destructed by an increasing voltage, temperature or carrier density. Further experimental and theoretical progresses in these

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fronts are needed and expected to yield many more exciting new insights on the long standing problems of 2D WC-liquid transition and MIT.

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Conflicts of Interest: The authors declare no conflict of interest.

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