

# Fluid-Structure Interaction models in pressurized fluid-filled pipes: a review

David Ferràs <sup>1</sup>, Pedro A. Manso<sup>2</sup>, Anton J. Schleiss<sup>2</sup>, and Dídia I.C. Covas<sup>3</sup>

<sup>1</sup>*Department of Environmental Engineering and Water Technology, IHE Delft  
Institute for Water Education (The Netherlands)*

<sup>2</sup>*Laboratory of Hydraulic Constructions (LCH), École Polytechnique Fédérale de  
Lausanne (Switzerland)*

<sup>3</sup>*CERIS, Instituto Superior Técnico, Universidade de Lisboa (Portugal)*

## Abstract

*The present review paper aims at collecting and discussing the research work, numerical and experimental, carried out in the field of Fluid-Structure Interaction (FSI) in one-dimensional (1D) pressurized transient flow in the time-domain approach. Background theory and basic definitions are provided for the proper understanding of the assessed literature. A novel frame of reference is proposed for the classification of FSI models based on pipe degrees-of-freedom. Numerical research is organized according to this classification, while an extensive review on experimental research is presented by institution. Engineering applications of FSI models are described and historical accidents and post-accident analyses documented.*

**Keywords:** hydraulic transients, water-hammer, fluid-structure interaction, degrees-of-freedom, junction coupling, Poisson coupling, friction coupling, Bourdon coupling.

## Notation

$A_f$	fluid cross-sectional area ( $\text{m}^2$ )	$p$	fluid pressure (Pa)
$a_f$	pressure wave speed ( $\text{ms}^{-1}$ )	$r$	radius of the pipe-wall (m)
$A_p$	pipe-wall cross-sectional area ( $\text{m}^2$ )	$R$	rotational velocity ( $\text{rad s}^{-1}$ )
$a_n$	acoustic speed of the $i$ -DOF ( $\text{ms}^{-1}$ )	$R_c$	bend radius of curvature (m)
$D$	pipe inner diameter (m)	$t$	time (s)
$E$	pipe-wall Young's modulus (Pa)	$U$	pipe-wall velocity ( $\text{ms}^{-1}$ )
$e$	pipe-wall thickness (m)	$V$	fluid mean velocity ( $\text{ms}^{-1}$ )
$G$	shear modulus (Pa)	$W$	pipe-wall radial velocity ( $\text{ms}^{-1}$ )
$I$	second moment of area ( $\text{m}^4$ )	$\nu$	Poisson's ratio (–)
$J$	polar second moment of area ( $\text{m}^4$ )	$\rho_f$	fluid density ( $\text{kgm}^{-3}$ )
$K$	bulk modulus of compressibility (Pa)	$\rho_p$	pipe density ( $\text{kgm}^{-3}$ )
$L$	pipe length (m)	$\sigma$	pipe-wall stress (Pa)
$M$	moment (N m)	$\epsilon$	strain (–)

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## 1 Introduction

The first scientific contributions to the field of Fluid-Structure Interaction (FSI) in transient pipe flow took place in the 19<sup>th</sup> century when authors like Korteweg (1878) or Helmholtz (1882) realized about the need of considering both fluid compressibility and pipe-wall distensibility as interacting mechanisms. Classical water-hammer theory is also based on this principle. Since then, many researchers have added their contributions in a step-wise manner, building up and shaping the theory of hydraulic transients in pipe flow.

FSI models deal with the original principle of considering water-hammer waves as a result of the relation between fluid and pipe deformations. Skalak (1955) presented a milestone PhD thesis entitled '*An extension of the theory of water-hammer*'. The basis of one-dimensional (1D) FSI was established, pipe vibration modes were described and the basic formulation for straight pipes was presented. Skalak's work triggered the FSI research on the two-way coupling between fluid dynamics and structural mechanics. Contributions by Wilkinson (1977), Walker & Phillips (1977), Valentin *et al.* (1979), Wiggert *et al.* (1985a), Wiggert (1986), Joung & Shin (1987), Bürmann & Thielen (1988a), Wiggert & Tijsseling (2001) and Tijsseling (2003) developed and completed the theory for all the basic degrees-of-freedom (DOF) of pipe-systems.

Some historical reviews on hydraulic transients in pipe flow are given by Wood (1970), Thorley (1976), Anderson (1976), Tijsseling & Anderson (2007), Tijsseling & Anderson (2008) and Tijsseling & Anderson (2012). The developments in water-hammer research before the 20<sup>th</sup> century are well summarized by Boulanger (1913). Also Lambossy (1950) and Stecki & Davis (1986) presented in-depth reviews that served, at that time, as vision papers. More recently, Ghidaoui *et al.* (2005) presented a complete state-of-the-art review focusing on both historic and most recent research and practice covering most of the water-hammer research topics. Surveys more specific in the field of Fluid-Structure Interaction

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are given by Wiggert (1986), Tijsseling (1996), Wiggert & Tijsseling (2001) and, more recently, by Li *et al.* (2015). The aim of the current review is to report the most significant contributions carried out in water-hammer research related to Fluid-Structure Interaction in 1D hydraulic transients modelling, giving emphasis on the time-domain analyses and focusing on the most recent research. A novel classification of FSI models based on pipe-degrees-of-freedom is presented.

The paper starts with the basic definitions and background theory that frame the research of FSI in water-hammer modelling. Numerical and experimental research is documented following the physically-based classification of pipe degrees-of-freedom. Finally, insights of engineering applications of Fluid-Structure Interaction developments in pipe flow are pointed out.

## 2 Definitions and basic concepts

### 2.1 Fluid-Structure Interaction

In the present review, Fluid-Structure Interaction in pipe systems is defined as the transfer of momentum and forces in both ways, between the pipe-wall and the contained fluid during unsteady flow (Wiggert, 1986). Hence, FSI in pipe flow involves, at least, transient responses of two different physical systems. The interaction arises when the time scales of both system responses are shorter than the time scale of the overall transient event (*i.e.* time lag between the initial and the final steady state). If the disturbance source is shorter than both system responses, then fast fluid and solid transients simultaneously occur. If their interaction is strong enough, then the description of FSI might be worthwhile in water-hammer analyses and interaction mechanisms have to be taken into account.

In a broad sense, Fluid-Structure Interaction embraces any form of energy transfer, one upon another, between the fluid and the structure. In common engineering problems, this transferred energy is typically kinetic and elastic or thermal. The former is termed mechanical Fluid-Structure Interaction and the latter thermal Fluid-Structure Interaction. Heat exchange effects in transient pipe flow are barely significant, processes are assumed adiabatic, and FSI analyses are mainly focused on the momentum exchange between the fluid and the pipe structure.

Two different approaches may be followed to account for the momentum transfer into the structure (Giannopapa, 2004): considering that the structure moves as a rigid solid or by the propagation of a local excitation/deformation of the solid. In the first, no transient event is considered propagating throughout the solid, the structure element moves as a rigid body and its effect on the fluid is analysed. In the second, the modes of vibration of the structure element are excited and their respective transient states are taken into account and coupled with the fluid transient. The present review is focused only on the second form.

FSI analyses may be classified according to the dimensions and the degrees-of-freedom with which the pipe system is allowed to move. Normally, in 1D water-hammer analysis, the classification criterion is based on the modes of vibration of the pipe, which is quite

67 convenient for frequency-domain approaches. However, for time-domain analyses a clas-  
68 sification based on the pipe degrees-of-freedom is more physically intuitive. The latter is  
69 the classification criterion used herein.

## 70 2.2 Degrees-of-freedom in fluid-filled pipes

71 Degrees-of-freedom (DOF) are the number of independent coordinates or parameters that  
72 describe the position or configuration of a mechanical system at any time (Sinha, 2010).  
73 Systems with finite number of degrees-of-freedom are called discrete systems, and those  
74 with infinite degrees-of-freedom are called continuous systems. Pipe systems are contin-  
75 uous systems, however these can be treated as discrete systems for numerical modelling  
76 purposes, with many DOF's depending on the number of nodes.

77 Pipes are slender elements, therefore a 1D approach assuming that the fluid pressure  
78 propagates axially during hydraulic transients is reasonable. However, transient pressures  
79 transmit forces over the pipe wall that make the pipe system move in a 3D space. The  
80 basic degrees-of-freedom for a rigid body in a 3D space are three for translation (*i.e.*  
81 heaving, swaying and surging) and three for rotation (*i.e.* pitching, yawing and rolling).  
82 An infinitesimal control volume of a pipe-segment (like in Fig. 1) will have the referred  
83 six basic degrees-of-freedom. The pipe-wall control-volume is a hollow cylinder, therefore  
84 axisymmetric vibration due to hoop strain must be as well considered, adding another  
85 degree-of-freedom. Additionally, the infinitesimal control volume of the 1D contained  
86 fluid accounts for another degree-of-freedom. Henceforth, in the present 1D FSI analysis,  
87 eight degrees-of-freedom compose the infinitesimal control volume of a pipe.

88 For each degree-of-freedom, momentum and mass conservation laws are applied, giving  
89 as result a set of 16 partial differential equations (*cf.* Eqs. 1 to 16), with time and space  
90 coordinates as independent variables, governing two basic dependent variables related  
91 with the loading and the movement in each degree-of-freedom (*i.e.*, load and deformation  
92 relation). Depending on the pipe geometry, axial, shear, bending and torsional forces and  
93 displacements alternate throughout the pipe. A schematic of such displacements is shown  
94 in Fig. 1.

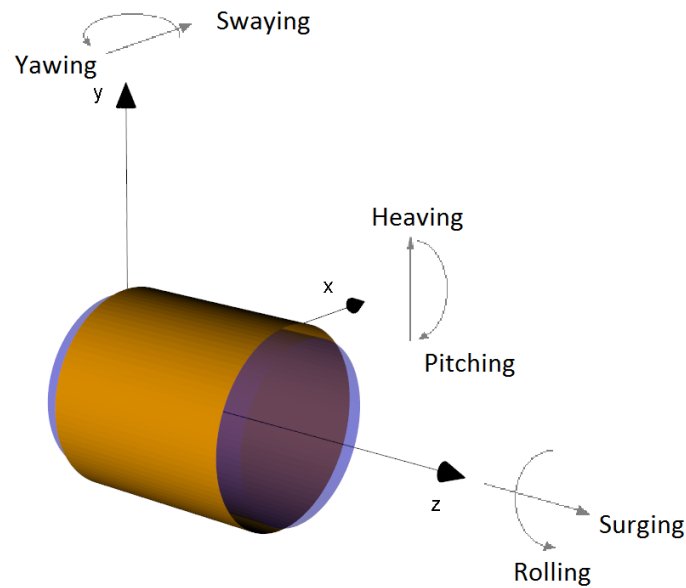


Fig. 1: Spatial reference system and signs convention in a straight pipe element

FSI models in 1D water-hammer analyses can be classified according to the pipe degrees-of-freedom as follows:

- 1-DOF (fluid surging): only the axial fluid transient event is described.
- 2-DOF (breathing): radial inertia of the fluid and the pipe are taken into account.
- 3-DOF (solid surging): refers to the axial movement of the pipe.
- 4-DOF (swaying): includes the effect of horizontal displacement of the pipe.
- 5-DOF (heaving): includes the effect of vertical displacement of the pipe.
- 6-DOF (yawing): includes the rotation of the pipe in the  $\widehat{xz}$  plane.
- 7-DOF (pitching): includes the rotation of the pipe in the  $\widehat{yz}$  plane.
- 8-DOF (rolling): includes the rotation of the pipe on the  $\widehat{xy}$  plane.

## 2.3 Fundamental formulae

The equations of the system (Eqs. 1 to 16) presented hereby correspond to the basic momentum and continuity conservation equations of a pipe-system with eight degrees-of-freedom, like in the control volume depicted in Fig. 1. Thin-wall assumption is adopted. Eqs. 1 to 6 and their associate characteristic equations can be found in Walker & Phillips (1977); Eqs. 7 to 16 in Wiggert *et al.* (1987). The symbols are declared in the Notation.

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1-DOF (fluid surging):

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f} \frac{\partial p}{\partial z} = 0 \quad (1)$$

$$\frac{1}{K} \frac{\partial p}{\partial t} + \frac{\partial V}{\partial z} = -\frac{2}{r} W \quad (2)$$

2-DOF (breathing):

$$\left( \rho_p r e + \rho_f \frac{r^2}{2} \right) \frac{\partial W}{\partial t} = r p - e \quad \sigma_\theta \quad (3)$$

$$\frac{\partial \sigma_\theta}{\partial t} - E \nu \frac{\partial U_z}{\partial z} = E \frac{W}{r} \quad (4)$$

3-DOF (solid surging):

$$\frac{\partial U_z}{\partial t} - \frac{1}{\rho_p} \frac{\partial \sigma_z}{\partial z} = 0 \quad (5)$$

$$\frac{1}{E} \frac{\partial \sigma_z}{\partial t} - \frac{\partial U_z}{\partial z} = \nu \frac{W}{r} \quad (6)$$

4-DOF (swaying):

$$- \left( \rho_p + \frac{A_f}{A_p} \rho_f \right) \frac{\partial U_x}{\partial t} + \frac{\partial \sigma_x}{\partial z} = 0 \quad (7)$$

$$\frac{\partial \sigma_x}{\partial t} - G \frac{\partial U_x}{\partial z} = -G R_y \quad (8)$$

5-DOF (heaving):

$$- \left( \rho_p + \frac{A_f}{A_p} \rho_f \right) \frac{\partial U_y}{\partial t} + \frac{\partial \sigma_y}{\partial z} = 0 \quad (9)$$

$$\frac{\partial \sigma_y}{\partial t} - G \frac{\partial U_y}{\partial z} = -G R_x \quad (10)$$

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6-DOF (yawing):

$$-\rho_p I_p \frac{\partial R_y}{\partial t} + \frac{\partial M_y}{\partial z} = -\sigma_x A_p \quad (11)$$

$$\frac{\partial M_y}{\partial t} - EI_p \frac{\partial R_y}{\partial z} = 0 \quad (12)$$

7-DOF (pitching):

$$-\rho_p I_p \frac{\partial R_x}{\partial t} + \frac{\partial M_x}{\partial z} = \sigma_y A_p \quad (13)$$

$$\frac{\partial M_x}{\partial t} - EI_p \frac{\partial R_x}{\partial z} = 0 \quad (14)$$

8-DOF (rolling):

$$-\rho_p J \frac{\partial R_z}{\partial t} + \frac{\partial M_z}{\partial z} = 0 \quad (15)$$

$$\frac{\partial M_z}{\partial t} - GJ \frac{\partial R_z}{\partial z} = 0 \quad (16)$$

111 All the degrees-of-freedom are distinguished in the previous system of equations, hence  
 112 the analysis of wave celerities can be reduced to the essential (uncoupled) wave propagating  
 113 speeds in each degree-of-freedom. The following formulae (Eqs. 17 to 21) define the  
 114 uncoupled wave celerities for each wave type considered (note that the sub-index refers to  
 115 the DOF):

$$a_1 = \sqrt{\frac{K}{\rho_f}} \quad (17)$$

$$a_3 = \sqrt{\frac{E}{\rho_p}} \quad (18)$$

$$a_{4,5} = \sqrt{\frac{GA_p}{\rho_p A_p + \rho_f A_f}} \quad (19)$$

$$a_{6,7} = \sqrt{\frac{EI_p}{\rho_p I_p + \rho_f I_f}} \quad (20)$$

$$a_8 = \sqrt{\frac{G}{\rho_p}} \quad (21)$$

120 Notice that, due to the pipe axisymmetry, shear and bending wave celerities are equal  
 121 in both planes (*i.e.*  $a_4 = a_5$  and  $a_6 = a_7$ ). Due to the dispersive nature of a 2-DOF wave

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122 propagating along the pipe-wall, no formula for  $a_2$  is provided (Tijsseling & Anderson,  
123 2012).

124 The advantage of considering the system of Eqs. 1 to 16 is that there is no need of  
125 considering the abstract concept of elastic wave celerity from classic water-hammer theory.  
126 An in-depth critical analysis of the different interpretations of wave speed in both time  
127 and frequency-domains is given by Tijsseling & Vardy (2015).

## 128 2.4 Coupling mechanisms and modelling approaches

129 Pipe systems subjected to water-hammer transients can be regarded as free-damped-  
130 deterministic vibrating systems with multiple modes of vibration, coupled or uncoupled,  
131 according to the degrees-of-freedom of the conduit and exposed to skin friction, dry friction  
132 and structural/hysteretic damping. Although not included in Eqs. 1 to 16, these damping  
133 mechanisms convert hydraulic transients into non-periodic and non-linear phenomena that  
134 are difficult to analyse.

135 The different degrees-of-freedom of a pipe system may interact one upon another.  
136 There are three basic kinds of coupling mechanisms (Tijsseling, 1996): (i) *Poisson coupling*  
137 describes the interaction between the axial motion of the pipe-wall and the pressure in  
138 the fluid occurring by means of the Poisson effect; (ii) *friction coupling* arises from the  
139 shear stress between the pipe-wall and the fluid; (iii) and *junction coupling* results from  
140 unbalanced local forces and by changes in the fluid momentum that occur in pipe bends,  
141 T-junctions or cross-section changes.

142 In time-domain analyses, the Method of Characteristics (MOC), the Finite Element  
143 Method (FEM), the Finite Difference Method (FDM) or the Finite Volume Method (FVM)  
144 are discretization methods used to solve the governing differential equations. Either a  
145 single or combined (hybrid) numerical method can be used for the description of the  
146 different degrees-of-freedom of the pipe. The method of characteristics (MOC) and the  
147 finite-element method (FEM), or a combination of both, are the most common numerical  
148 methods used for solving the one-dimensional basic equations (Tijsseling, 1996). One  
149 single integrating approach, such as MOC-MOC or FEM-FEM, is convenient as all the  
150 information flows into the same numerical scheme (Wiggert & Tijsseling, 2001). Other  
151 combinations are not that common in one-dimensional analyses; FVM is rather used for  
152 3D simulations.

153 A different coupling approach consists of setting up an interaction between two different  
154 computer codes, one specific for the fluid and another for the structure. In each time-step  
155 output information is transferred in both directions. There are contributions proposing  
156 methodologies to carry out this data transfer, such as Ware & Williamson (1982). However,  
157 the main challenge of this approach is the requirement of a considerable computational  
158 effort and data transfer (Belytschko *et al.*, 1986).

159 A-Moneim & Chang (1978) coupled FDM code for the fluid and a FEM for the struc-  
160 ture with the goal to simulate an interesting experimental research carried out at the Stan-  
161 ford Research Institute (SRI). Other authors who tried to simulate the same validating  
162 experiments are Romander *et al.* (1980) and Kulak (1982, 1985) who coupled FEM-FEM



software. Also Erath *et al.* (1998, 1999) used a FDM code for the fluid with a FEM for the structure with the goal to simulate field measurements from a pump shut-down and a closing valve in the nuclear power plant KRB II (Gundremmingen, Germany). Bietenbeck *et al.* (1985) and Mueller (1987) applied a MOC-FEM coupling aiming at describing the response of an experimental facility located at the Karlsruhe Nuclear Research Centre (KfK–Kernforschungszentrum Karlsruhe).

In Casadei *et al.* (2001) FEM and FVM are compared for the fluid domain simulation and coupling techniques are proposed. In Simão *et al.* (2015a,b) the traditional MOC approach for the fluid is compared with a CFD  $k - \epsilon$  model, coupled with a FEM model for the structure.

### 3 Numerical and experimental research

#### 3.1 Introduction

A review of the numerical and experimental research of 1D FSI in the time-domain is presented hereby. Table 1 summarizes and describes the main FSI models according to their DOF's and lists some of the most relevant contributions that enabled the theoretical development, implementation, application and validation of numerical models using adapted versions of the fundamental equations presented in subsection 2.3. Details of these research contributions are provided in the following subsections.

Tab. 1: Summary table of main 1D FSI models in hydraulic transients research.

DOF	Description	Main contributions
1	Only the fluid transient is described. Equations solved: 1, 2	Menabrea (1858); Korteweg (1878); Von Kries (1883); Frizell (1898); Allievi (1902); Joukowsky (1904); Halliwell (1963)
1,3	Solid surging is coupled with the fluid. Equations solved: 1, 2, 5, 6	Schwarz (1978); Wiggert (1983); Kojima & Shinada (1988); Bürmann & Thielen (1988c); Lavooij & Tijsseling (1991); Zhang <i>et al.</i> (1994); Vardy <i>et al.</i> (1996); Li <i>et al.</i> (2003); Tijsseling (2003); Gale & Tiselj (2005); Loh & Tijsseling (2014); Ferrás <i>et al.</i> (2017b)
1,2,3	Fluid, breathing and solid surging interact. Equations solved: 1, 2, 3, 4, 5, 6	Walker & Phillips (1977); Schwarz (1978); Kellner <i>et al.</i> (1983); Gorman <i>et al.</i> (2000); Tijsseling (2007)
1,3 and 4,6 or 5,7	Fluid and solid surging, and either swaying and yawing or heaving and pitching are taken into account. Equations solved: 1, 2, 5, 6 and 7, 8, 11, 12 or 9, 10, 13, 14	Regetz (1960); Wood & Chao (1971); A-Moneim & Chang (1979); Hu & Phillips (1981); Tijsseling <i>et al.</i> (1994, 1996); Vardy <i>et al.</i> (1996); Tijsseling & Heinsbroek (1999); Gale & Tiselj (2006); Simão <i>et al.</i> (2015b)
1,3,4,5,6,7,8	Fluid and solid surging, swaying, heaving, yawing, pitching and rolling are coupled. Equations solved: 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	Weijde (1985); Tijsseling & Lavooij (1990); Lavooij & Tijsseling (1989, 1991); Kruisbrink (1990); Bettinali <i>et al.</i> (1991); Heinsbroek (1997)

181 **3.2 One degree-of-freedom models**

182 The classic water-hammer model (two-equation model) is a sophisticated version of the  
183 basic 1-DOF system (Eqs. 1 and 2), where the right-hand-side term of the continuity  
184 equation is adapted in order to account for the pipe-wall distensibility. Although the  
185 bulk modulus of compressibility and a finite acoustic wave speed are considered in the  
186 fluid, in terms of density variation the fluid is assumed to be incompressible and pressure  
187 changes are related to velocity changes by embedding fluid compressibility and pipe-wall  
188 distensibility into the wave celerity value, which is regarded as a constant parameter and  
189 can be either experimentally or numerically determined. Research works such as Young

(1808), Weber (1866), Resal (1876), Moens (1878), Korteweg (1878), Von Kries (1883) or Halliwell (1963) contributed to the development of wave celerity formulae. The latest presented correcting factors to account for axial FSI.

The fundamental equations of classic water-hammer theory (*i.e.* mass and momentum conservation) can be derived from Navier-Stokes equations (Ghidaoui, 2004) or by directly applying the Reynolds Transport Theorem (Chaudhry, 2014) to a control volume of the pipe. From an FSI standpoint, these fundamental equations can be also reached from the system of equations presented in Section 2, as the classical theory considers a combination of the first two degrees-of-freedom. The fundamental momentum conservation equation is directly the one presented in 1-DOF (Eq. 1). For mass conservation (continuity equation), the cross-sectional area of the control volume is assumed to vary and this variation is related to the fluid inner pressure by applying a quasi-static assumption in the 2-DOF. This derivation is described in Appendix B.

The system of PDE's (Eq. 22 and Eq. 23) represents the fundamental conservation equations of classic frictionless water-hammer theory.

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f} \frac{\partial p}{\partial z} = 0 \quad (22)$$

$$\frac{\partial V}{\partial z} + \frac{1}{\rho_f a_h} \frac{\partial p}{\partial t} = 0 \quad (23)$$

Usually the system of mass conservation and momentum equations is solved by means of the Method of Characteristics (MOC), which is the most popular and extensively used method by researchers and engineers thanks to its easy programming, computational efficiency and accuracy of the results (Vardy & Tijsseling, 2015). Over all methods MOC stays the closest to the physics of the problem.

### 3.3 Two degree-of-freedom models

The historical development of four-equation models can be traced back from Korteweg (1878) who already pointed out the need of considering axial stress waves. Gromeka (1883) and Lamb (1898), qualitatively, took into account pipe axial inertia and Poisson coupling in their analyses. Skalak (1955), who extended Lamb's work, presented the four basic fundamental equations and introduced the concept of precursor waves. Thorley (1969) was the first to experimentally observe precursor waves, which are, at the same time, the evidence of the Poisson coupling effect. Bürmann (1979), Thielen & Bürmann (1980) and Bürmann & Thielen (1988b) used the simplified version of Skalak's equations which represent the well-known four-equation system for axial FSI. Skalak's work was revisited and analysed in Tijsseling *et al.* (2008).

For the description of pressure waves in pipe systems, two or four-equation models are sufficient (Tijsseling, 1996). Four-equation models consider the combination of classic theory with the 3-DOF equations. Hence, four fundamental equations, two for the fluid and two for the pipe axial movement, are to be solved. The right-hand-side terms of the continuity equations of the 1-DOF and 3-DOF systems must be adapted in order to

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227 describe the Poisson coupling in terms of the dependent variables of the four-equation  
 228 model (*i.e.*, respectively, axial stress of the pipe-wall and fluid pressure). This derivation  
 229 is explained in Appendix C from which Eqs. 25 and 27 are obtained.

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f} \frac{\partial p}{\partial z} = 0 \quad (24)$$

$$\frac{\partial V}{\partial z} + \frac{1}{\rho_f a_h^2} \frac{\partial p}{\partial t} = \frac{2\nu}{E} \frac{\partial \sigma_z}{\partial t} \quad (25)$$

$$\frac{\partial U_z}{\partial t} - \frac{1}{\rho_p} \frac{\partial \sigma_z}{\partial z} = 0 \quad (26)$$

$$\frac{\partial U_z}{\partial z} - \frac{1}{\rho_p a_3^2} \frac{\partial \sigma_z}{\partial t} = -\frac{r\nu}{eE} \frac{\partial p}{\partial t} \quad (27)$$

233 E

234 Several numerical methods can be used to solve the above system of equations, either  
 235 integrating both the fluid and the structure in the same numerical scheme (*e.g.*, MOC-  
 236 MOC) or by a combination between different schemes (*e.g.*, MOC-FEM).

237 In Bürmann (1979) and Bürmann & Thielen (1988b) the four-equation system was  
 238 solved using MOC procedure for the first time. Bürmann (1975, 1979) and Bürmann &  
 239 Thielen (1988c) presented a series of tests carried out on a vertical pipe line located in  
 240 a subterranean salt cavern. In Bürmann *et al.* (1985, 1986b, 1987) measurements were  
 241 shown from a water-main bridge, and in Bürmann *et al.* (1986a) and Bürmann & Thielen  
 242 (1988a) from a loading line between tanks and ships. These measurements were used to  
 243 develop and validate the four-equation model and to understand FSI mechanisms.

244 In Vardy & Alsarraj (1989) the Method of Characteristics for both the fluid and  
 245 the structure (*i.e.*, MOC-MOC) was shown to have useful advantages. This approach  
 246 was supported by experimental evidence from Vardy & Fan (1986, 1987, 1989) and Fan  
 247 (1989), who carried out measurements in which FSI effects were particularly well isolated  
 248 by means of suspended pipe rigs that were excited by the impact of a solid rod. In  
 249 combination with their numerical developments, they showed how FSI coupling changes  
 250 the natural vibrating frequencies, which cannot be predicted by uncoupled approaches.

251 Schwarz (1978) used a FDM scheme in his four-equation model as a simplified version  
 252 of a six-equation model which was solved by MOC. Ellis (1980) modelled fluid and axial  
 253 stress waves in conduits by means of MOC, taking into account only junction coupling  
 254 (ignoring Poisson coupling). Kojima & Shinada (1988) also used a FDM approach which  
 255 was validated by tests on a thin-walled straight pipe for Poisson coupling as well as junction  
 256 coupling at a closed-free pipe end. Ferràs *et al.* (2016a) derived an explicit Joukowsky-like  
 257 expression from the four-equation system aiming at estimating maximum pressures during  
 258 water-hammer with FSI.

259 Wiggert *et al.* (1985a), Elansary & Contractor (1990, 1994), Elansary *et al.* (1994) and  
 260 Budny *et al.* (1991) explained how to solve the four-equation system considering Poisson  
 261 coupling. They presented the characteristic equations after MOC transformation and how  
 262 to integrate them within the same characteristic grid using time-line interpolations as

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explained by [Goldberg \(1983\)](#). The MOC transformation that allows hyperbolic partial differential equation systems to be converted to a set of ordinary differential equations was based on [Forsythe \*et al.\* \(1960\)](#). [Zhang \*et al.\* \(1994\)](#) used a FEM scheme for both the fluid and the structure. In [Bouabdallah & Massouh \(1997\)](#) and [Ghodhbani & Hadj-Taïeb \(2013\)](#), time interpolation and wave adjustment methods are compared for MOC-MOC solutions. [Wiggert \(1983\)](#) used a hybrid MOC-FEM approach, MOC for the fluid and FEM for the structure, and experimental data was used for model verification. A FVM approach was presented in [Gale & Tiselj \(2005\)](#) to solve the four-equation model, which was successfully verified using the Delft Hydraulics Benchmark Problem A ([Tijsseling & Lavooij, 1990](#); [Lavooij & Tijsseling, 1991](#)). In [Lavooij & Tijsseling \(1991\)](#) both approaches MOC-MOC and MOC-FEM are compared, concluding that for straight pipe problems the MOC procedure is more accurate and efficient. [Ferràs \*et al.\* \(2017a\)](#) used MOC-MOC coupling to simulate a kind of FSI which was experimentally observed in pipe coils by [Ferràs \*et al.\* \(2014, 2016a\)](#).

The Delft Hydraulics Benchmark Problem A (20 m long, steel pipe, 0.4 m diameter) is a good test case for the verification of four-equation numerical codes (*v.i.* Fig. 2). In [Li \*et al.\* \(2003\)](#), [Tijsseling \(2003\)](#) and [Tijsseling \(2009\)](#), a theoretical development of an exact solution of the four-equation system by means of a recursion was presented. The drawback of the method is its exponential computational effort for longer simulation periods. Recently, in [Loh & Tijsseling \(2014\)](#), the computation for the exact solution was upgraded in order to increase computational efficiency and applicability. The analysis suggested to keep the scope of exact solutions to generate test cases and to benchmark solutions for more conventional numerical methods. Also, in [Xu & Jiao \(2017\)](#), the efficiency is improved by using a hybrid cubic time-line interpolation scheme.

It is important to highlight that numerical outputs, such as the one presented in *v.i.* Fig. 2, show how FSI phenomena can cause pressure surges higher than the ones expected from classical theory. [Tijsseling \(1997\)](#) has demonstrated the Poisson coupling beat, which is a phenomenon that arises from resonance between 1-DOF and 3-DOF. Poisson coupling beat was already numerically observed by [Wiggert \(1986\)](#). So far, there is no experimental evidence about it, as damping mechanisms tend to hide the oscillating resonance between the pipe-wall and the fluid vibrations.

More recently, [Ferràs \*et al.\* \(2017b\)](#) numerically observed the Liebau effect in pipe flow using a four-equation model which was adapted to describe the inertia of thrust blocks. The Liebau effect is rather an object of study in the field of physiological flows and is defined as the occurrence of valveless pumping through the application of a periodic force at a place which lies asymmetric with respect to the system configuration ([Borzi & Propst, 2003](#)). [Ferràs \*et al.\* \(2017b\)](#) claimed that the Liebau effect in pipe flow may be induced by Poisson coupling and should be object of further research (*v.i.* Fig. 3).

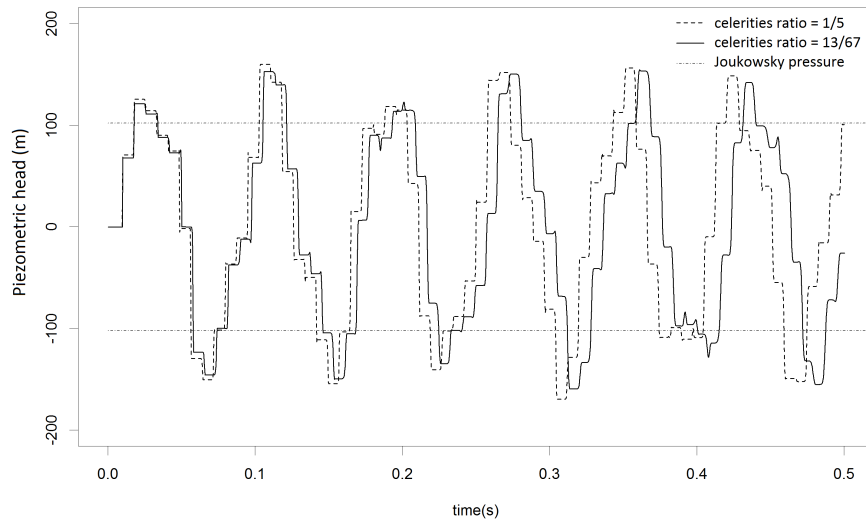


Fig. 2: Four-equation code verified by means of the Delft Hydraulics Benchmark Problem A (Ferràs, 2016).

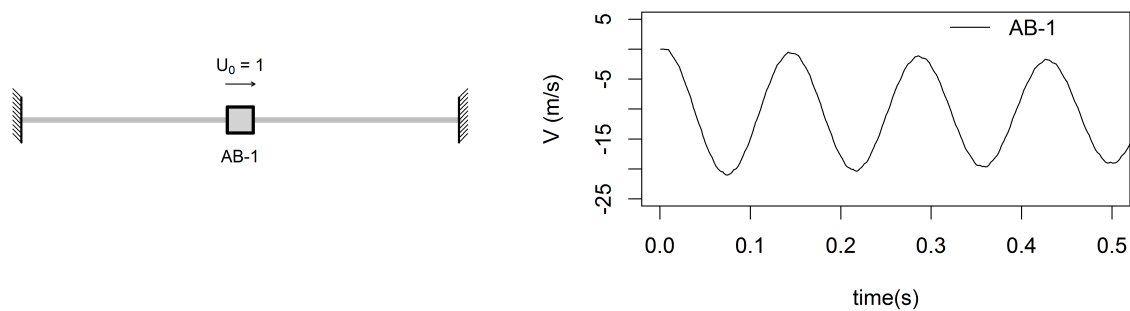


Fig. 3: Numerical evidence of Liebau effect depicted in Ferràs *et al.* (2017b).

### 3.4 Three degree-of-freedom models

Six-equation models aim at describing the 1,2,3-DOF's. As in the four-equation model, similar numerical schemes can be used for solving the six-equation system. However, the right-hand-sides of the three continuity equations are not expressed in differential terms. A first or second-order approximation can be applied for integrating these equations.

Walker & Phillips (1977) were the first proposing and solving by MOC the six-equation model. These authors have compared results from the frequency and time domains and carried out their validation using experimental data collected from a water-filled copper pipe excited by hammering the pipe-end.

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310 With a similar MOC numerical scheme Schwarz (1978) solved the equations and com-  
 311 pared them to a four-equation model solved by FDM; the effect of Poisson coupling in  
 312 each case was also analysed. Kellner *et al.* (1983) extended the work of Walker & Phillips  
 313 (1977) by proposing an added fluid mass term and solving the equations by a MOC-FEM  
 314 approach. Gorman *et al.* (2000) used a MOC-FDM scheme in their numerical analysis,  
 315 the effect of initial axial tensional stress was included in their derivation.

316 From the six-equation system, Tijsseling (2007) derived a four-equation model which  
 317 included correction terms and factors accounting for the pipe-wall thickness (*v.i.* Fig. 4).  
 318 The model was validated with exact solutions in the time-domain (Li *et al.*, 2003; Tijssel-  
 319 ing, 2003). The authors concluded that, in the low-frequency range, a transient description  
 320 of the 2-DOF is only important for very thick pipes ( $r/e < 2$ ).

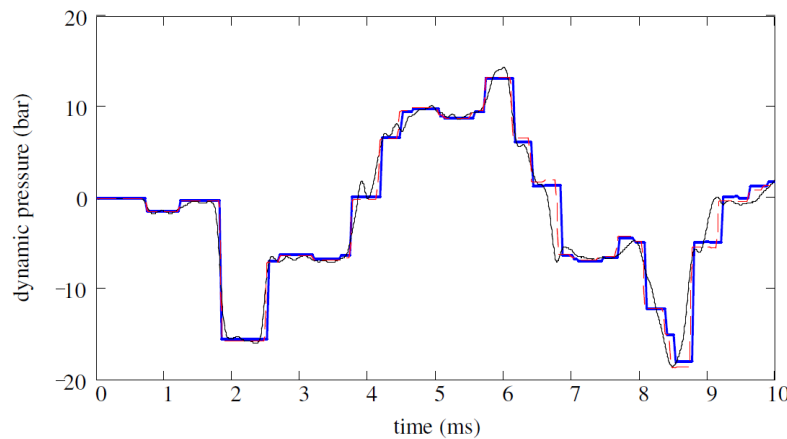


Fig. 4: Comparison of transient pressures considering thick-wall theory (thick solid blue line), thin-wall theory (thin broken red line) and experimental data (thin solid black line) (Tijsseling, 2007),  $e/r = 0.15$ .

### 3.5 Four degree-of-freedom models

322 According to the classification proposed in Subsection 2.2, eight-equation models solve the  
 323 system of equations for either 1,3,4,6-DOF's or 1,3,5,7-DOF's. These kind of models are  
 324 used to describe in-plane axial, torsional and flexural pipe displacements, respectively, in  
 325 the  $\widehat{xz}$  or  $\widehat{yz}$  planes. Radial deformation is nested in the celerity of the 1-DOF as in the  
 326 classic water-hammer theory. Poisson coupling may be included such that the system of  
 327 equations to be solved becomes composed of Eqs. 24, 25, 26 and 27 (*i.e.* the four-equation  
 328 model) together with Eqs. 7, 8, 11 and 12 or Eqs. 9, 10, 13 and 14. The 4,5,6,7-DOF's  
 329 are only coupled by means of junction coupling.

330 Pipe systems like the one depicted in Fig. 5 can be described by 4-DOF models. In  
 331 this pipe scheme, a water-hammer wave generated by the valve manoeuvre would induce  
 332 not only transient pressures but axial stress, shear stress and bending waves in the pipe  
 333 wall, hence exciting the 1,3,4,6-DOF's.



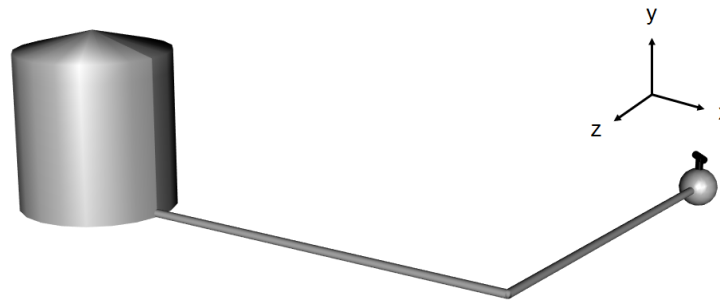


Fig. 5: Reservoir-pipe-valve system with a 90° elbow at the mid-length pipe section.

334 [Valentin \*et al.\* \(1979\)](#) presented the eight-equation model for curved pipes for 1,3,4,6-  
 335 DOF's. [Hu & Phillips \(1981\)](#) solved the same equations using MOC and validated their  
 336 results against new experimental data. Radial inertia was included by [Joung & Shin](#)  
 337 [\(1987\)](#) who solved a nine-equation model. [Tijsseling \*et al.\* \(1994, 1996\)](#) and [Tijsseling &](#)  
 338 [Heinsbroek \(1999\)](#) used a MOC-MOC scheme in combination with cavitation, which was  
 339 modelled by means of a lumped parameter model. In [Gale & Tiselj \(2006\)](#) a FVM method  
 340 was used to solve the eight-equation model, which was tested for different set-ups (*v.i.*  
 341 Fig. 6). In this analysis, [Gale & Tiselj \(2005\)](#) highlighted that a two-phase flow model is  
 342 needed for simulations of more universal FSI problems occurring in pipelines.

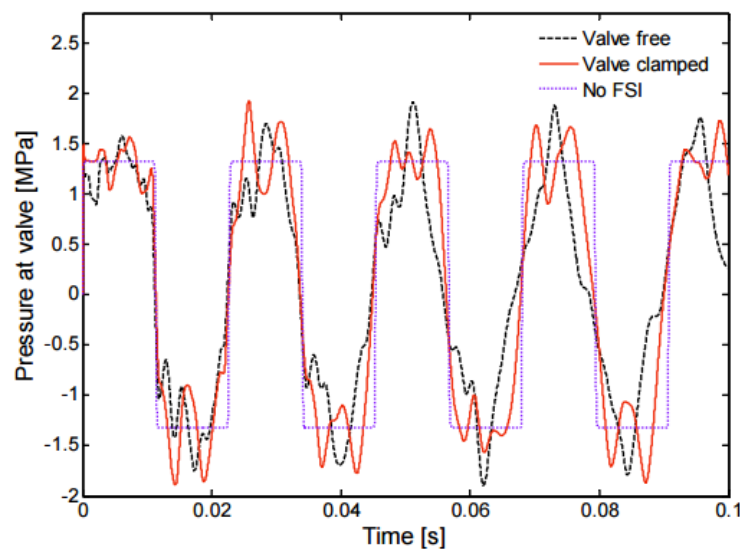


Fig. 6: Numerical output from [Gale & Tiselj \(2006\)](#) considering a free moving valve (black dashed line), anchored (red solid line) and compared with the classic water-hammer model output (purple dotted line).

343 A compilation of sixteen experiments dedicated to systems with a single elbow (L-



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344 pipes) was presented in Tijsseling (2016), eight experiments focused on the frequency-  
 345 domain approach and eight on the time-domain. The experiments based on the time-  
 346 domain approach are presented in Table 2.

Tab. 2: Main time-domain experiments carried out for single-elbow pipes.

Reference	Experimental setup	Transient test
Swaffield (1968–1969)	45° – 180°, hor. mitre, hor. curved bends 0.85 < $R_c/D$ < 5.0, rigid (2 jacks)	valve closure: 2 - 5 ms initial flow vel.: 0.6 - 2.4 m/s
Wood & Chao (1971)	30° – 150°, hor. mitre, rigid and free	valve closure: 2 ms initial flow vel.: 2 - 3 m/s
A-Moneim & Chang (1979)	hor. 114.3 mm, $D = 70.6$ mm, $R_c/D = 1.6$ , rigid	gun: 150 bar pulse 3
Hu & Phillips (1981)	$R_c/D = 6$	pellet impact 0.2 m/s
Otwell (1984) Wiggert <i>et al.</i> (1985b)	hor. $R_c/D = 0.8$	valve closure: 4 ms initial flow vel.: 1.2 m/s
Tijsseling <i>et al.</i> (1996) Tijsseling & Vaugrante (2001)	hor. 0.88 kg	rod impact 0.15 m/s
Steens & Pan (2008)	hor. $R_c/D = 2.2$	impact hammer pulse 1 - 2 ms
Altstadt <i>et al.</i> (2008)	vert. elbow $R_c/D = 1.5$	valve opening: 20 - 200 ms initial flow vel.: 2 - 17 m/s

347 Swaffield (1968–1969) tried to experimentally prove that a pressure wave reflects par-  
 348 tially when passing through a rigidly supported elbow. This work generated in-depth  
 349 discussion pointing out the importance of considering FSI even for rigid supports as-  
 350 suming that the movement of anchorages is nearly impossible to avoid. This idea was  
 351 supported by Wood & Chao (1971) who stated that pipelines are never anchored suffi-  
 352 ciently to eliminate motion due to a water-hammer surge. In A-Moneim & Chang (1979)  
 353 a complete pipe rig was used (*v.i.* 7), nonetheless they experimented difficulties in getting  
 354 rid off undesired FSI effects, emphasizing the importance of properly testing experimental  
 355 setups preventing such phenomena. Finally, Wiggert *et al.* (1985b) verified that there is  
 356 no pressure wave reflection from an immobile elbow but that there is due to the elbow  
 357 movement. In Altstadt *et al.* (2008) these findings were confirmed once more.

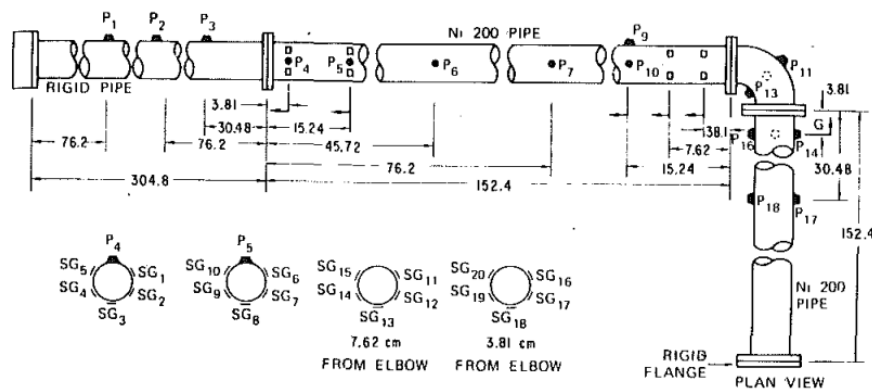


Fig. 7: Experimental pipe rig used in A-Moneim & Chang (1979).

A pipe system with multiple elbows, bends and junctions can be described by eight-equation models if these are located in the very same plane. This is the case for the experiment carried out in the University of Guanajuato, Mexico, in collaboration with IST, Portugal. Simão *et al.* (2015c,b) collected data from a pipe rig assembled by concentric elbows of 90°. The apparatus was equipped with pressure transducers and accelerometers. Water-hammer events were generated by a downstream valve manoeuvre. The aim of the experimental data collection was the validation of a numerical model which coupled CFD software for the fluid with FEM software for the structure. The model was compared also with a modified MOC approach which included damping coefficients to account for structural damping. The work highlighted the importance of integrated analyses including the description of both fluid and structure behaviours.

### 3.6 Seven degree-of-freedom models

The fourteen-equation model includes all the degrees-of-freedom presented in Section 2 except the 2-DOF corresponding to the radial inertia of the pipe-wall, which is nested in the celerity of the 1-DOF like in the classic water-hammer theory. Hence, the system to be solved is composed of Eqs. 24, 25, 26 and 27 (*i.e.* the four-equation model) together with Eqs. 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16. Pipe systems like the one depicted in Fig. 8 can be described by 7-DOF models, where all the related DOF's would be excited by a water-hammer wave generated at the downstream valve.

Wilkinson (1977) introduced the fourteen-equation model in the time-domain, which was finally implemented by Wiggert *et al.* (1985a, 1987, 1985b) and Wiggert (1986) with MOC approach, both in the fluid and in the structure. Experimental measurements from Wiggert *et al.* (1987), corresponding to a similar set-up as the one depicted in Fig. 8, are shown in Fig. 9. A good fitting with measurements was obtained but the analysis concluded that further model developments were necessary. Lesmez *et al.* (1990) extended the work using an experimental set-up consisting of a copper pipe containing a U-bend free to move in an in-plane fashion. This method was used also by Obradović (1990), who simulated an accident.

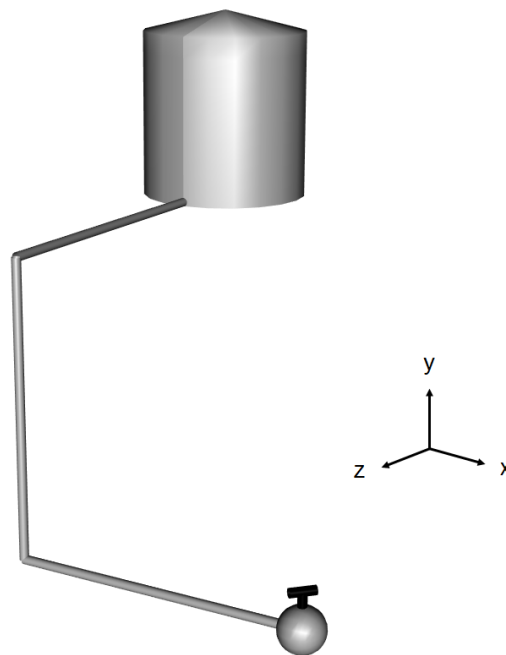


Fig. 8: Reservoir-pipe-valve system with two out-of-plane 90° elbows.

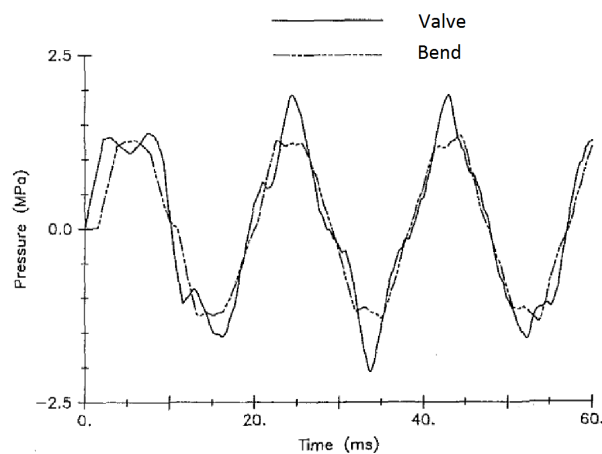


Fig. 9: Experimental pressure measurements next to the downstream valve and at a bend Wiggert *et al.* (1987).

386 Weijde (1985) carried out experiments in a PVC pipe containing a U-shaped section  
 387 at the laboratory of Delft Hydraulics, The Netherlands. He concluded that classic water-  
 388 hammer theory was not accurate enough to describe the behaviour of the pipe-rig and,  
 389 consequently, the FLUSTRIN project was launched. A complex and large-scale apparatus  
 390 (Fig. 10) held by suspension wires and specially designed for FSI tests was assembled at

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391 Delft Hydraulics laboratory and used for the development and verification of the FLUS-  
 392 TRIN code, which is based on a MOC-FEM approach (Tijsseling & Lavooij, 1990; Lavooij  
 393 & Tijsseling, 1989). In this framework Kruisbrink & Heinsbroek (1992) and Heinsbroek &  
 394 Kruisbrink (1993) carried out a series of numerical benchmark tests. Coupled and uncoupled  
 395 Poisson effect solutions were compared for the Delft Hydraulics Benchmark Problem  
 396 F (Lavooij, 1987), which is a good approach for verifying fourteen-equation model imple-  
 397 mentations (*v.i.* Fig. 11). Experimental measurements were used in this comparison and  
 398 a guideline was provided suggesting when FSI is important. The same computer code was  
 399 used by Kruisbrink (1990), Lavooij & Tijsseling (1991) and Heinsbroek (1997) with similar  
 400 purposes of comparing with other modelling assumptions and using experimental tests for  
 401 validation. Heinsbroek (1997) suggested that for four-equation modelling an MOC-MOC  
 402 approach is more convenient, while for higher degrees-of-freedom an MOC-FEM scheme is  
 403 preferable as higher grid resolution is required. Bettinali *et al.* (1991) presented a similar  
 404 MOC-FEM code with differences in the implementation of the Poisson coupling mecha-  
 405 nism. Kochupillai *et al.* (2005) developed a model using a velocity based FEM formulation  
 406 which was validated with benchmark problems. Time-domain solutions can be also ob-  
 407 tained from frequency-domain analyses, however, Hatfield & Wiggert (1983) concluded  
 408 that the time-domain solutions derived from frequency-domain results are difficult and  
 409 impractical.

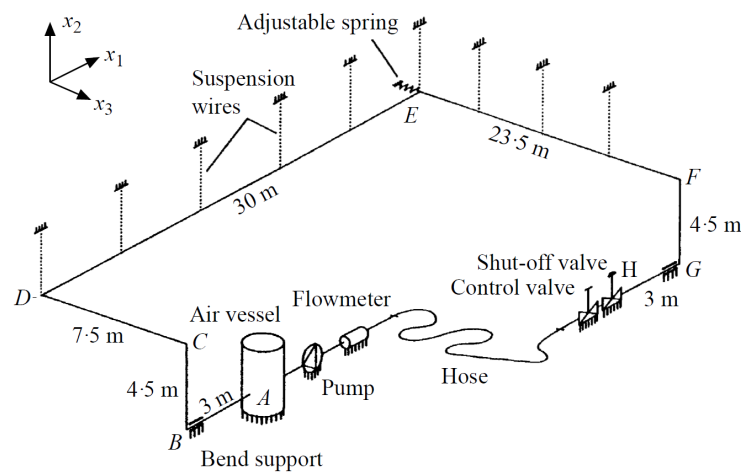


Fig. 10: FSI experimental set-up at Delft Hydraulics (Kruisbrink & Heinsbroek, 1992)

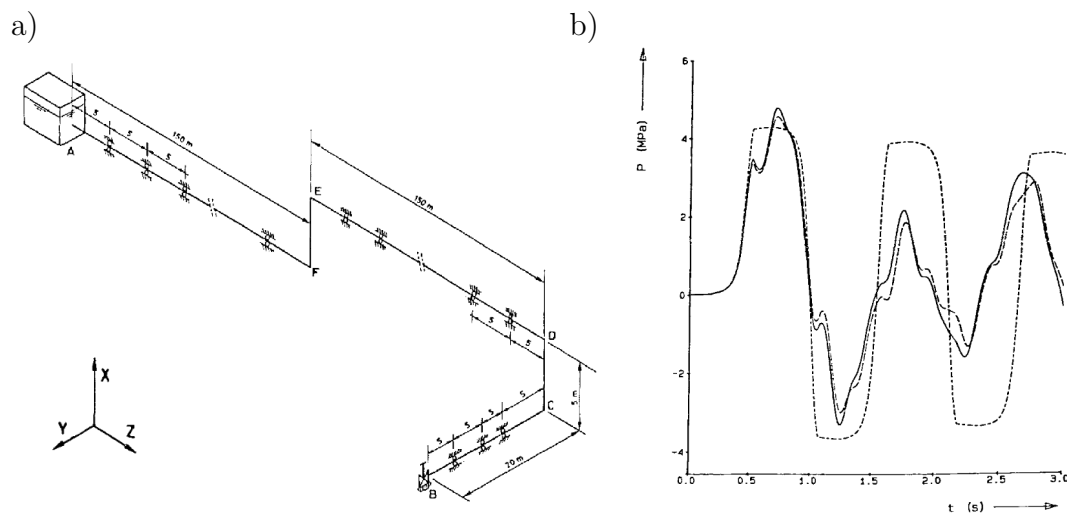


Fig. 11: Set-up of the Delft Hydraulics Benchmark Problem F (a); and numerical output (b) for: Poisson and junction coupling (solid line), only junction coupling (dashed line) and for classic water-hammer model (dash-dotted line) (Tijsseling & Lavooij, 1990).

### 3.7 Other FSI mechanisms

In curved pipes of non-circular cross-section, an additional coupling mechanism, called Bourdon coupling, affects the pipe behaviour. This mechanism consists of the change of ovality of the pipe cross-section in function of the internal pressure loading. In Davidson & Samsury (1969, 1972) Fluid-Structure Interaction was analysed, respectively, in straight and curved pipes. In Clark & Reissner (1950) and Reissner *et al.* (1952) the Bourdon tube deformation mechanism is explained and a methodology based on the Boltzmann superposition principle to describe stress-strain states is presented. Bathe & Almeida (1980, 1982) studied Bourdon phenomena by means of a FEM approach. The Bourdon effect was first dynamically coupled with the fluid response in Tentarelli (1990). The work was extended in Brown & Tentarelli (2001) and Tentarelli & Brown (2001), where experimental measurements were used for validation of the numerical output in the frequency domain (*v.i.* Fig. 12). Budny *et al.* (1990) and Fan (1989) gave as well experimental evidence of Bourdon coupling.

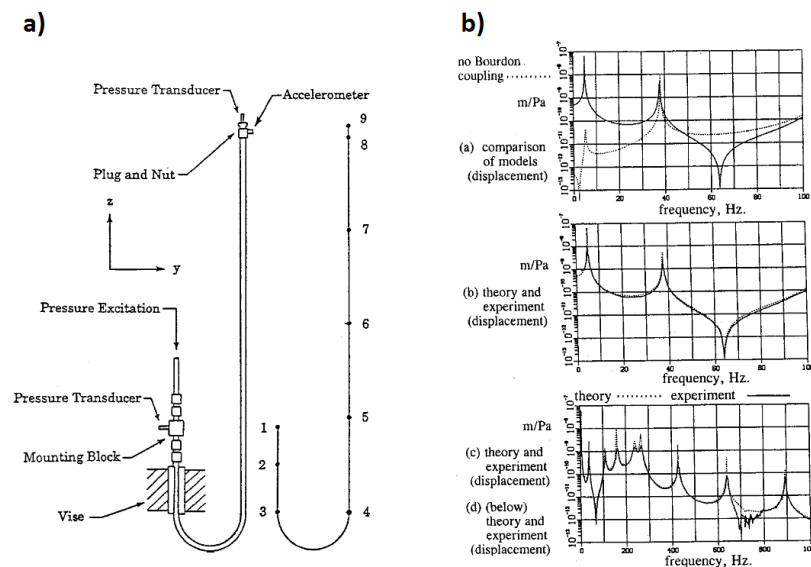


Fig. 12: Experimental set-up (a) and output in the frequency-domain (b) for Bourdon coupling analysis (Brown & Tentarelli, 2001).

Other FSI mechanisms, not that common in regular engineering practices, are the buckling and flutter induced by centrifugal and Coriolis forces. Authors that have contributed on this matter are: Housner (1952), Gregory & Païdoussis (1966), Païdoussis & Issid (1974) and Païdoussis & Laithier (1976). Experimental research focused on describing the buckling and flutter effects in pipe systems was conducted in Gregory & Païdoussis (1966) and Jendrzejczyk & Chen (1985). Païdoussis (2016) gives an encyclopaedic treatment of the subject.

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### 4.1 FSI consideration in codes and standards

Table 3 refers to the Codes and Standards belonging to those engineering fields that frequently require water-hammer analyses. Other Standards and Guidelines have been reviewed by Leslie & Vardy (2001). However, none of the Standards directly consider any kind of FSI coupling. Several industrial cases of FSI generated by internal flows are analysed in Moussou *et al.* (2004). The paper highlights the complexity of FSI problems and the need for guidelines and rules in international Codes and Standards.

Tab. 3: Codes and Standards in industries where water-hammer analyses are frequent.

Industry	Application	International standards
Hydropower energy	penstocks	ASME-B31.3 DIN-19704-1 ASCE MOP 79 CECT-1979
Nuclear/Thermal energy	cooling systems	ASME-BPV NS-G-1.9
Oil/Gas transportation	oil/gas mains	ASME-B31.2 ASME-B31.4 ISO-13628
Water distribution	water pipes	ANSI/ASSE-1010 PDI-WH 201
Aerospace	fuel pipes	ISO/FDIS-8575 NASA-STD-8719

## 4.2 Anchor and support forces

Fluid-Structure Interaction and specially the behaviour of pipe supports have a direct applicability in above-ground or non-buried pipe systems, such as hydropower systems, long oil and gas pipes, cooling systems of nuclear, thermal plants or any fluid distribution system in industrial compounds. However, only few authors investigated anchor and support behaviour in the context of water-hammer theory. Frequently, studies are based on qualitative discussions focused on post-accident analyses and mitigation measures case-by-case oriented. An example is Almeida & Pinto (1986) where recommendations for design criteria, operating rules and post-accident analyses were given. Also Hamilton & Taylor (1996a,b) and Locher *et al.* (2000) presented qualitative discussions of the performance of different industrial piping systems, giving insights of pipe supports behaviour. The latter highlighted the case-by-case dependency of Fluid-Structure Interaction and the high computational demand of including anchor analyses, stating that the scope of such studies should be justifiable only for very critical systems, such as in nuclear power plants.

Bürmann & Thielen (1988b) collected data from a firewater facility pipeline and carried out numerical analyses by means of MOC. Heinsbroek & Tijsseling (1994) studied the effect of support rigidity of pipe systems and discussed for what rigidity of the supports FSI becomes a dominant effect. In their analysis they applied both classic water-hammer theory and a MOC-FEM approach by means of the FLUSTRIN code (Lavooij & Tijsseling, 1989; Kruisbrink & Heinsbroek, 1992). The simulated facility corresponded to the one from Delft Hydraulics laboratory.

Tijsseling & Vardy (1996a) studied the effect of a pipe-rack considering the dry friction occurring between the rack and the pipe-wall. Recommendations were given in order to assess when dry friction must be considered. Following this line Ferras *et al.* (2017b); Ferràs *et al.* (2016a,b) carried out experimental and numerical work based on a straight

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copper pipe which allowed a broad variety of anchoring configurations. In Ferras *et al.* (2017b) a robust and accurate MOC-MOC code to simulate anchoring blocks taking into account their inertia and dry friction is presented. The blocks were nested in the numerical scheme as internal conditions and junction coupling was considered. Fig. 13 depicts the model output *vs.* experimental measurements for different anchoring set-ups.

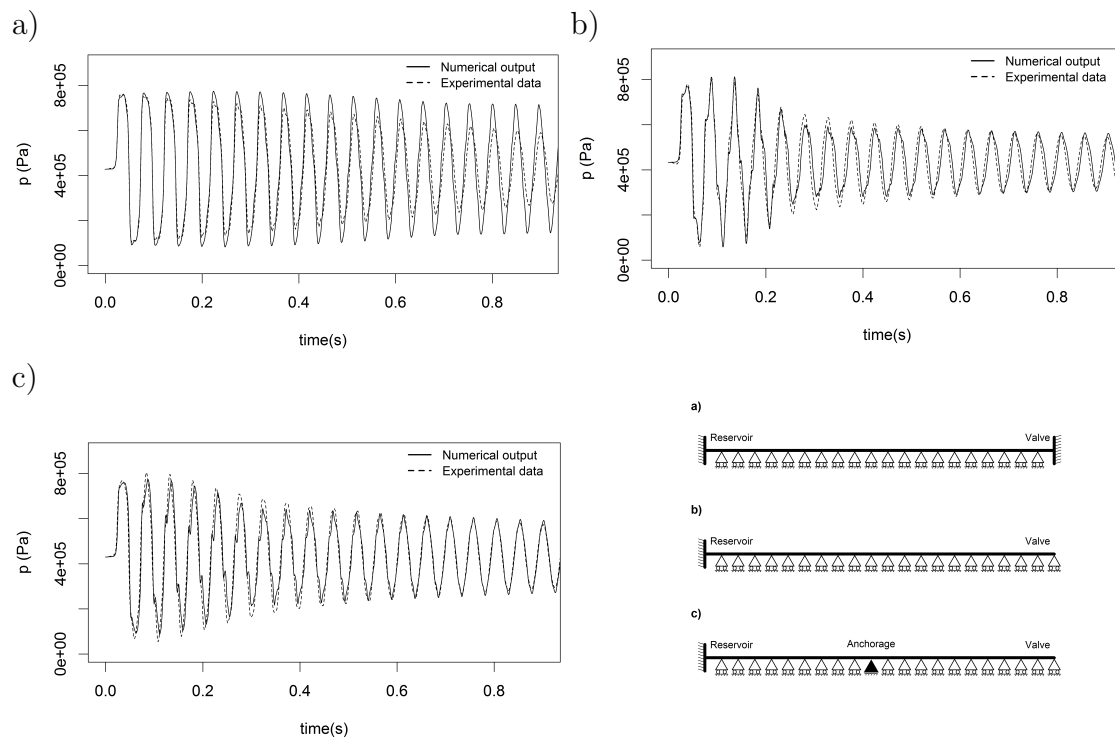


Fig. 13: Validation of the numerical model developed in Ferras *et al.* (2017b) for: anchored pipe ends (a); released downstream end (b); and released downstream end but anchored midstream (c).

Different anchoring conditions were assessed in Simão *et al.* (2015c) using CFD software, which was validated by means of experimental data. The analysis pointed out the need of CFD simulations for the proper description of pipe supports behaviour. In Zanganeh *et al.* (2015) the aim was the simulation of hydraulic transients in a straight pipe anchored with axial supports using a MOC-FEM approach. Both pipe-wall and supports had a viscoelastic behaviour. The study concluded that the viscoelastic supports significantly reduce displacements and stresses in the pipe and eliminate the high frequency fluctuations produced due to FSI. In Wu & Shih (2001) and Yang *et al.* (2004) a multi-span pipe system, with middle rigid constraints was analysed in the frequency-domain using the transfer matrix method, concluding that the middle rigid constraints have a much larger effect than the Poisson coupling. These types of multi-span pipes with middle rigid constraints set-ups are common in engineering practices and, so far, only a limited num-



ber of investigations has been carried out addressing this issue, especially in time-domain analyses.

### 4.3 Vibration damping and noise reduction

Pipe vibration may induce audible noise and FSI analyses are required for the assessment of such noise. Moser *et al.* (1986) investigated the vibrating modes that produce sound. Kwong & Edge (1996, 1998) carried out experimental analyses and developed a technique to reduce noise generation by the specific positioning of pipe clamps. De Jong (1994) suggested that for the full description of sound generation in pipe-systems, seven degrees-of-freedom are required. This statement was verified in Janssens *et al.* (1999). In Chen (2012) a pump-induced fluid-borne noise investigation is carried out by means of a distributed-parameter transfer-matrix model in the frequency-domain. It was claimed that the method could be used as well for structure-born noise as long as Fluid-Structure Interaction was taken into account.

Tijsseling & Vardy (1996b) carried out experimental water-hammer tests on a steel pipe containing a short segment of ABS. MOC was successfully used to reproduce the experiments and they concluded that the vibration could be adapted and modified in function of the segment material and geometry. Hachem & Schleiss (2012) reached a similar conclusion in an aluminium pipe set-up with a short segment of PVC. The analysis was carried out in the frequency-domain. Related with the previous subsection, Koo & Park (1998) proposed a methodology to reduce vibrations by the installation of intermediate supports.

### 4.4 Earthquake engineering

Water-hammer waves can be produced by earthquake excitation on a pipe system. Fluid-Structure Interaction or soil-pipe interaction may be one of the potential damaging factors during earthquakes, specially for relatively low pressure and large diameter pipelines (Young & Hunter, 1979). The Fukushima Daiichi nuclear disaster in Japan is a prominent example of this (Lo Frano & Forasassi, 2012; Mitsume *et al.*, 2014). Some authors have studied this kind of transients coupled with FSI. Hara (1988) analysed a Z-shaped piping system subjected to a one-directional seismic excitation. A numerical analysis of a 3D pipe system was carried out in Hatfield & Wiggert (1990). It was found that assuming the piping to be rigid produced an upper-bound estimate of pressure, but assuming the liquid to be incompressible resulted in underestimating the displacement of the piping. Coupled and uncoupled analyses applied to a single straight pipe were compared in Bettinali *et al.* (1991), who also concluded that coupled analyses accurately predicted lower wave amplitudes.

## 4.5 Aerospace engineering

Strong fluid transients occur in the filling up process of propulsion feedlines of satellites and launchers. In the experimental works of [Regetz \(1960\)](#), [Blade \*et al.\* \(1962\)](#), [A-Moneim & Chang \(1978\)](#) and [A-Moneim & Chang \(1979\)](#) different configurations of rocket fuel-filled pipe rigs were tested. An overview of the main concerns experienced in the aerospace community with respect to fluid-hammer is reported by [Steelant \(2015\)](#). The study remarks the need of detailed investigation of Fluid-Structure Interaction in combination with thermal heat transfer during fluid-hammer waves in satellites or launchers. [Bombardieri \*et al.\* \(2014\)](#) also highlight the importance of FSI in the filling of pipelines during the start up of the propulsion systems of spacecrafts, claiming that more experimental research should be focused on this line.

## 4.6 Biomechanics

The disciplines of hydraulic transients and physiological flows share a good basis of the classic water-hammer theory as long as the assumptions of liquids with relatively low compressibility contained in thin-walled elastic cylindrical tubes are considered. Studies such as [Lambossy \(1950\)](#), [McDonald \(1974\)](#), [Nakoryakov \*et al.\* \(1976\)](#), [Anderson & Johnson \(1990\)](#), [Sherwin \*et al.\* \(2003\)](#), [Van de Vosse & Stergiopulos \(2011\)](#), [Nichols \*et al.\* \(2011\)](#) and [Alastruey \*et al.\* \(2012\)](#) focused on adapting classic water-hammer to the main factors that affect physiological flows. For instance, in [Anderson & Johnson \(1990\)](#), the Korteweg formula for wave celerity computation was reviewed in order to include pipe cross-section ovality effects. The study concluded that even for a low ovality of the pipe cross-section there may be significant reductions of the wave velocity due to bending-induced changes in the tube cross-section. The analysis carried out by [Anderson & Johnson \(1990\)](#) serves also in the field of hydraulic transients for pipe bends and coils where the pipe cross-section becomes elliptic.

Nowadays, computational-fluid-dynamics (CFD) tools are used to model the complexity of haemodynamics. Not just the pipe-wall viscoelasticity and the elliptic pipe cross-section, but the inner fluid defies as well classic water-hammer theory assumptions as blood is a non-Newtonian fluid, presenting shear-thinning, viscoelasticity and thixotropy. [Wathen \*et al.\* \(2009\)](#) presents a review of modern modelling approaches for haemodynamical flows. In [Janela \*et al.\* \(2010\)](#) a comparison of different physiological assumptions is carried out by means of a FEM-FEM approach. Newtonian and non-Newtonian assumptions are considered with Fluid-Structure Interaction, highlighting their differences and the importance of good modelling criteria. More specific to blood flow diseases diagnoses, [Simão \*et al.\* \(2016a\)](#) also used CFD tools, including FSI, for modelling a vein blockage induced by a deep venous thrombosis and the occurrence of reverse flow in human veins.

## 4.7 Accidents and post-accident analyses

FSI may generate overpressures higher than that predicted by Joukowsky's formula and not only caused by water-hammer waves, but also by turbulence-induced vibrations,

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cavitation-induced vibrations or vortex shedding with lock-in. These phenomena are poorly understood (Moussou *et al.*, 2004) and are rarely taken explicitly into consideration in engineering designs, leading to accidents and service disruptions of important infrastructure with large social relevance (*e.g.* industrial compounds, water and wastewater treatment plants, thermal plants, nuclear power plants, hydropower plants).

Jaeger *et al.* (1948) reviewed a number of the most serious accidents due to water-hammer in pressure conduits until WWII. Many of the failures described were related to vibration, resonance and auto-oscillation (Bergant *et al.*, 2006). Table 4 summarizes a selection of accidents caused by strong hydraulic transients found in the literature, noting that the majority of incidents and accidents remains ‘unpublished’.

Normally, accidents in hydraulic facilities are associated not only to a single phenomenon but to a sequence of events that make the system collapse. Although not all the accidents listed in Table 4 were caused directly by FSI, in many cases FSI is involved in this sequence of events and its understanding is crucial in post-accident analyses, such as reported in Almeida & Pinto (1986), Wang *et al.* (1989), Obradović (1990) and Simão *et al.* (2016b). Leishear (2017) investigated water-hammer related accidents in nuclear power plants, where water-hammer waves compress flammable gasses to their autoignition temperatures in piping systems. In this paper several examples of incidents and accidents are analysed enhancing the understanding of nuclear power plant explosions.

## 4 Engineering applications

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Tab. 4: Selection of historical accidents in pressurized pipe systems mentioned in the literature.

Location	Facility	Description and citations
Oigawa, Japan	Penstock	A water-hammer wave, caused by a fast valve-closure, split the penstock open and produced the pipe collapse upstream. <a href="#">Bonin (1960)</a> .
Big Creek, U.S.A.	Penstock	Burst turbine inlet valve caused by a fast closure. <a href="#">Trenkle (1979)</a> .
Azambuja, Portugal	Pump station	Collapse of water column separation causing the burst of the pump casing. <a href="#">Chaudhry (2014)</a>
Lütschinen, Switzerland	Penstock	Penstock failure during draining due to the buckling produced by a frozen vent at the upstream end. <a href="#">Chaudhry (2014)</a> .
Arequipa, Peru	Penstock	The clogging of the control system of a valve resulted in buckling and the failure of the welding seams of the penstock due to fatigue. <a href="#">Chaudhry (2014)</a>
Ok, Papua New Guinea	Power house	The draft tube access doors were damaged and the power house flooded due to column separation in the system. <a href="#">Chaudhry (2014)</a> .
Lisbon, Portugal	Water main	Rupture of concrete support blocks during the slow closure of an isolation valve installed in a large suction pipe. <a href="#">Almeida &amp; Ramos (2010)</a> ; <a href="#">Simão et al. (2016b)</a> .
New York, U.S.A.	Steam pipe	Condensation-induced water-hammer caused the rupture of the steam pipe. <a href="#">Vecchio et al. (2015)</a>
Lapino, Poland	Penstock	Burst of the penstock caused by a rapid cut-off and low quality of the facility. <a href="#">Adamkowski (2001)</a> .
Chernobyl, Ukraine	Nuclear reactor	Fuel pin failure, fuel-coolant interaction and Fluid-Structure Interaction were involved in the failure of the nuclear reactor. <a href="#">Wang et al. (1989)</a> .
New York, U.S.A	Nuclear reactor	Circumferential weld failure in one of the feedwater lines due to a steam generator water-hammer. <a href="#">Meserve (1987)</a> .

## 5 Conclusions

Not considering pipe-wall movement during water-hammer events is going against the essence of water-hammer research. As shown in Appendix B, the classic water-hammer equations assume a quasi-steady circumferential deformation of the pipe-wall. The information of this quasi-steady behaviour of the piping structure affecting the pressure wave is, in the classic approach, enclosed in the water-hammer wave celerity, which may be eventually affected, as well, if other pipe degree-of-freedom are considered. Jumping from this quasi-steady assumption of the pipe structure to an unsteady one is what makes the trade between the fluid and the structure dynamic; Fluid-Structure Interaction arises and the classic water-hammer theory becomes invalid. Even in very well controlled conditions of hydraulic laboratories, undesired FSI phenomena are frequent. An important challenge of experimental research in FSI is the setting up of the right design to fit the research purpose. Validation of the test rig itself is, therefore, crucial.

Fluid-Structure Interaction is a case-dependent problem; there is no general solution or numerical model capable of describing and simulating any pipe setup. The technical challenge in the scope of 1D FSI is not resolving the fundamental equations, but assuming the appropriate coupling between the different pipe degrees-of-freedom without ending up in expensive computations. This case-dependency feature and the lack of user-friendly tools is what makes FSI problems difficult to tackle in engineering practice. Additionally, there is a general consuetudinary thinking that classical approaches remain on the conservative side. Though, in this review, it has been shown how authors demonstrated, both numerically and experimentally, that FSI may generate overpressures higher than ones estimated by the classical solutions. Moreover, there is no engineering code or standard specifying when FSI has to be considered. All these factors pinpoint that the physics of FSI phenomena are not fully understood in common engineering practices and this involves the potential risk of underrated designs.

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**A Appendix: Summary tables of experimental and numerical research**

Table 5 summarizes some of the most relevant contributions that enabled the theoretical development, implementation and application of numerical models using adapted versions of the fundamental equations presented in Subsection 2.3.

Tab. 5: Summary table of relevant numerical research in 1D FSI.

DOF	Method	References
1 & 3	MOC-MOC	Ellis (1980), Vardy & Alsarraj (1989, 1991), Lavooij & Tijsseling (1991), Bouabdallah & Massouh (1997), Ghodhbani & Hadj-Taïeb (2013), Ferras <i>et al.</i> (2017a,b); Ferràs <i>et al.</i> (2016b).
1 & 3	FDM-FDM	Schwarz (1978), Kojima & Shinada (1988).
1 & 3	FEM-FEM	Zhang <i>et al.</i> (1994).
1 & 3	MOC-FEM	Wiggert (1983), Lavooij & Tijsseling (1991).
1 & 3	FVM-FVM	Gale & Tiselj (2005).
1 & 3	Analytical solution	Li <i>et al.</i> (2003), Tijsseling (2003, 2009), Loh & Tijsseling (2014).
1, 2 & 3	MOC-MOC	Walker & Phillips (1977), Schwarz (1978).
1, 2 & 3	MOC-FEM	Kellner <i>et al.</i> (1983).
1, 2 & 3	MOC-FDM	Gorman <i>et al.</i> (2000).
1, 3 & 4, 6 or 5, 7	MOC-MOC	Hu & Phillips (1981), Tijsseling <i>et al.</i> (1994, 1996), Tijsseling & Heinsbroek (1999).
1, 3 & 4, 6 or 5, 7	FVM-FVM	Gale & Tiselj (2006).
1, 3, 4, 5, 6, 7 & 8	MOC-MOC	Wiggert <i>et al.</i> (1985a, 1987), Wiggert (1986), Obradović (1990).
1, 3, 4, 5, 6, 7 & 8	MOC-FEM	Tijsseling & Lavooij (1990), Lavooij & Tijsseling (1991, 1989), Kruisbrink (1990), Bettinali <i>et al.</i> (1991), Heinsbroek (1997).

In Table 6 a summary of the main experimental research work related with FSI in pipe transient flow is depicted, organized by research institutes, authors and dates. Details of these research contributions are provided in the following subsections.

## B Appendix: Two-equation model

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Tab. 6: Summary table of relevant experimental work in 1D FSI.

Location	Description and references
City University London, U.K.	Aluminium alloy straight pipe. Experimental evidence of precursor waves is depicted. <a href="#">Thorley (1969)</a> .
University of Dundee, U.K.	Suspended pipe rigs excited by the impact of a solid rod aiming at isolating FSI effects. <a href="#">Vardy &amp; Fan (1986, 1987, 1989)</a> ; <a href="#">Vardy et al. (1996)</a> , <a href="#">Fan (1989)</a> ; <a href="#">Fan &amp; Vardy (1994)</a>
University of Karlsruhe, Germany	Physical data from diverse case-studies: subterranean salt cavern, water-main bridge and tank-ship loading line. The aim was the development and validation of a four-equation model. <a href="#">Bürmann (1975, 1979)</a> ; <a href="#">Bürmann &amp; Thielen (1988c,a)</a> , <a href="#">Bürmann et al. (1985, 1986b, 1987, 1986a)</a>
Delft Hydraulics, The Netherlands	Complex apparatus held by suspension wires and specially designed for FSI tests. Used for the development and verification of the FLUSTRIN code. <a href="#">Weijde (1985)</a> , <a href="#">Kruisbrink &amp; Heinsbroek (1992)</a> , <a href="#">Heinsbroek &amp; Kruisbrink (1993)</a> .
Michigan State University, U.S.A.	U-bend and multi-plane copper pipe aiming at validating a fourteen-equation model. <a href="#">Wiggert (1983)</a> , <a href="#">Wiggert et al. (1985b, 1987)</a> , <a href="#">Lesmez et al. (1990)</a>
Stanford Research Institute, U.S.A.	Straight pipe extensively equipped with pressure and strain gauges in order to analyse pipe flexure during the transient events generated by a pulse gun. <a href="#">Regetz (1960)</a> , <a href="#">Blade et al. (1962)</a> <a href="#">A-Moneim &amp; Chang (1978, 1979)</a>
University of Berkeley, U.S.A.	Conduit excited by firing steel spheres onto the pipe ends with the goal to study axial stress waves. <a href="#">Krause et al. (1977)</a> , <a href="#">Barez et al. (1979)</a>
University of Kentucky, U.S.A.	Rigidly supported straight pipe terminated by a spring-mass device. <a href="#">Wood (1968, 1969)</a>
IST, University of Lisbon, Portugal; and EPFL, Switzerland	Straight copper pipe rig, copper coil and polyethylene coil. <a href="#">Ferràs et al. (2014)</a> ; <a href="#">Ferràs et al. (2017a,b)</a> ; <a href="#">Ferràs et al. (2016a,b)</a>
University of Guanajuato, Mexico; and IST University of Lisbon, Portugal	Pipe-rig assembled by concentric elbows aiming at validation of a CFD model. <a href="#">Simão et al. (2015b,c)</a>

## B Appendix: Two-equation model

In the classic water-hammer theory, only 1-DOF is described and the distensibility of the pipe in the radial direction is taken into account neglecting the radial inertia of the pipe-



## B Appendix: Two-equation model

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1139 wall and the fluid, and assuming a quasi-steady linear-elastic circumferential deformation  
1140 of the pipe-wall.

1141 On the one side, if inertial terms (proportional to  $\partial W/\partial t$ ) are neglected in the mo-  
1142 mentum equation of the 2-DOF, Eq. 3 becomes the well-known hoop stress formula:

$$\sigma_{\theta} = \frac{rp}{e} \quad . \quad (28)$$

1143 Applying time partial derivative to both sides of Eq. 28 and expanding differential  
1144 terms, one obtains:

$$\frac{\partial \sigma_{\theta}}{\partial t} = \frac{p}{e} \frac{\partial r}{\partial t} + \frac{r}{e} \frac{\partial p}{\partial t} \quad . \quad (29)$$

1145 The left-hand-side of Eq. 29 can be written in terms of circumferential strain:

$$E \frac{\partial \epsilon_{\theta}}{\partial t} = \frac{p}{e} \frac{\partial r}{\partial t} + \frac{r}{e} \frac{\partial p}{\partial t} \quad , \quad (30)$$

1146 and knowing that  $\epsilon_{\theta} = \partial r/r$ , one gets:

$$E \frac{\partial \epsilon_{\theta}}{\partial t} = \frac{pr}{e} \frac{\partial \epsilon_{\theta}}{\partial t} + \frac{r}{e} \frac{\partial p}{\partial t} \quad . \quad (31)$$

1147 Rearranging Eq. 31 and assuming  $\frac{pr}{e} \ll E$ , one obtains:

$$\frac{\partial \epsilon_{\theta}}{\partial t} = \frac{r}{eE} \frac{\partial p}{\partial t} \quad . \quad (32)$$

1148 On the other side, the classic water-hammer theory does not consider any axial move-  
1149 ment of the pipe. Hence, in Eq. 4,  $\partial U_z/\partial z = 0$  and becomes:

$$\frac{\partial \sigma_{\theta}}{\partial t} = E \frac{W}{r} \quad , \quad (33)$$

1150 which in terms of circumferential strain is:

$$\frac{\partial \epsilon_{\theta}}{\partial t} = \frac{W}{r} \quad . \quad (34)$$

1151 Combining Eq. 32 with Eq. 34 an expression for the radial velocity of the pipe-wall,  
1152 in function of the inner pressure, is obtained:

$$W = \frac{r^2}{eE} \frac{\partial p}{\partial t} \quad . \quad (35)$$

1153 Substituting Eq. 35 into the right-hand-side of the continuity equation of the 1-DOF:

$$\frac{1}{K} \frac{\partial p}{\partial t} + \frac{\partial V}{\partial z} = -\frac{2r}{eE} \frac{\partial p}{\partial t} \quad (36)$$

1154 rearranging Eq. 36:

$$\frac{\partial V}{\partial z} + \left( \frac{1}{K} + \frac{D}{eE} \right) \frac{\partial p}{\partial t} = 0 \quad . \quad (37)$$

## C Appendix: Four-equation model

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1155 Finally, defining the elastic wave celerity as:

$$a_h = \sqrt{\frac{K}{\rho_f \left(1 + \frac{DK}{eE}\right)}} \quad , \quad (38)$$

1156 the continuity equation Eq. 39 for classic water-hammer theory is obtained:

$$\frac{\partial V}{\partial z} + \frac{1}{\rho_f a_h^2} \frac{\partial p}{\partial t} = 0 \quad . \quad (39)$$

1157 The fundamental system of equations of the classic water-hammer theory, neglecting  
1158 damping mechanisms, is therefore composed by Eqs. 1 and 39, forming the following  
1159 system of equations 40:

$$\text{two-equation model} \left\{ \begin{array}{l} \frac{\partial V}{\partial t} + \frac{1}{\rho_f} \frac{\partial p}{\partial z} = 0 \\ \frac{\partial V}{\partial z} + \frac{1}{\rho_f a_h^2} \frac{\partial p}{\partial t} = 0 \end{array} \right. \quad (40)$$

1160 If the acoustic wave celerity in the unconfined fluid is considered ( $a_1 = \sqrt{\frac{K}{\rho_f}}$ ), Eq. 1  
1161 and Eq. 39 are equivalent. Hence, the only difference between 1-DOF wave propagation  
1162 and classic water-hammer theory is determined by how the elastic wave celerity is defined.  
1163 The first, assumes an entirely rigid pipe, while the second takes into account the hoop  
1164 distensibility of the pipe-wall.

## 1165 C Appendix: Four-equation model

1166 The four-equation model describes the 1-DOF (fluid surging) and 3DOF (solid surging)  
1167 of the pipe system and takes into account the 2-DOF (breathing) in a similar manner as  
1168 the classic water-hammer theory.

### 1169 C.1 Continuity in 1-DOF

1170 Subtracting from the 2-DOF continuity equation Poisson ratio times the 3-DOF continuity  
1171 equation (*i.e.* Eq. 4 –  $\nu$  Eq. 6) the following expression is obtained:

$$\frac{\partial \sigma_\theta}{\partial t} - \nu \frac{\partial \sigma_z}{\partial t} = (1 - \nu^2) E \frac{W}{r} \quad . \quad (41)$$

1172 Notice that, dividing both sides of Eq. 41 by  $E$ , the left-hand-side is actually the local  
1173 time rate of change of the circumferential strain. Hoop stress can be written in terms of  
1174 pressure according to Eq. 28, which is also valid in the present derivation. Thus Eq. 42 is  
1175 obtained:

## C Appendix: Four-equation model

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$$\frac{1}{e} \frac{\partial (pr)}{\partial t} - \nu \frac{\partial \sigma_z}{\partial t} = (1 - \nu^2) E \frac{W}{r} \quad , \quad (42)$$

1176 expanding the differential term, as indicated below:

$$\frac{p}{e} \frac{\partial r}{\partial t} + \frac{r}{e} \frac{\partial p}{\partial t} - \nu \frac{\partial \sigma_z}{\partial t} = (1 - \nu^2) E \frac{W}{r} \quad , \quad (43)$$

1177 and by considering that  $\frac{p}{e} \frac{\partial r}{\partial t}$  is negligible for low frequencies compared to other terms  
 1178 and by rearranging Eq. 43, one obtains:

$$W = \frac{\frac{r^2}{e} \frac{\partial p}{\partial t} - r \nu \frac{\partial \sigma_z}{\partial t}}{(1 - \nu^2) E} \quad . \quad (44)$$

1179 Substituting Eq. 44 into Eq. 2 leads to:

$$\frac{1}{K} \frac{\partial p}{\partial t} + \frac{\partial V}{\partial z} = - \frac{D}{e(1 - \nu^2) E} \frac{\partial p}{\partial t} + \frac{2\nu}{(1 - \nu^2) E} \frac{\partial \sigma_z}{\partial t} \quad , \quad (45)$$

1180 neglecting second-order Poisson-ratio terms and rearranging Eq. 45, one gets:

$$\frac{\partial V}{\partial z} + \left( \frac{1}{K} + \frac{D}{eE} \right) \frac{\partial p}{\partial t} = \frac{2\nu}{E} \frac{\partial \sigma_z}{\partial t} \quad . \quad (46)$$

1181 Finally, applying the definition of elastic wave celerity from Eq. 38, the continuity  
 1182 equation (Eq. 47) for the 1-DOF of a four-equation model is obtained:

$$\frac{\partial V}{\partial z} + \frac{1}{\rho_f a_h^2} \frac{\partial p}{\partial t} = \frac{2\nu}{E} \frac{\partial \sigma_z}{\partial t} \quad . \quad (47)$$

## 1183 C.2 Continuity in 3-DOF

1184 Substituting Eq. 44 into Eq. 6:

$$\frac{\partial \sigma_z}{\partial t} - E \frac{\partial U_z}{\partial z} = \frac{\frac{\nu r}{e} \frac{\partial p}{\partial t} - \nu^2 \frac{\partial \sigma_z}{\partial t}}{(1 - \nu^2)} \quad , \quad (48)$$

1185 neglecting second order Poisson ratio terms and rearranging the continuity equation (Eq. 48),  
 1186 one gets:

$$\frac{\partial U_z}{\partial z} - \frac{1}{E} \frac{\partial \sigma_z}{\partial t} = - \frac{\nu r}{eE} \frac{\partial p}{\partial t} \quad . \quad (49)$$

1187 Finally, defining the acoustic wave speed in the pipe-wall as:

$$a_3 = \sqrt{\frac{E}{\rho_p}} \quad , \quad (50)$$

## C Appendix: Four-equation model

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1188 and substituting Eq. 50 into Eq. 49, the continuity equation of the pipe-wall (Eq. 51) for  
 1189 the four-equation model is obtained:

$$\frac{\partial U_z}{\partial z} - \frac{1}{\rho_p a_3^2} \frac{\sigma_z}{\partial t} = -\frac{\nu r}{eE} \frac{\partial p}{\partial t} \quad (51)$$

1190 The four fundamental equations of a four-equation model are composed, therefore, of  
 1191 Eq. 1, 47, 5 and 51. Forming the following system of equations 52:

$$\text{four-equation model} \left\{ \begin{array}{l} \left. \begin{array}{l} \frac{\partial V}{\partial t} + \frac{1}{\rho_f} \frac{\partial p}{\partial z} = 0 \\ \frac{\partial V}{\partial z} + \frac{1}{\rho_f a_h^2} \frac{\partial p}{\partial t} = \frac{2\nu}{E} \frac{\partial \sigma_z}{\partial t} \end{array} \right\} \text{1-DOF} \\ \left. \begin{array}{l} \frac{\partial U_z}{\partial t} - \frac{1}{\rho_p} \frac{\partial \sigma_z}{\partial z} = 0 \\ \frac{\partial U_z}{\partial z} - \frac{1}{\rho_p a_3^2} \frac{\sigma_z}{\partial t} = -\frac{\nu r}{eE} \frac{\partial p}{\partial t} \end{array} \right\} \text{3-DOF} \end{array} \right. \quad (52)$$