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Modified Kudrayshov Method to solve generalized Kuramoto - Sivashinsky equation

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Abstract

The generalized Kuramoto - Sivashinsky equation is investigated using the modified Kudrayshov method for the exact analytical solution. The modified Kudrayshov method converts the nonlinear partial differential equation to algebraic equations, as a result of various steps, which on solving the so obtained equation systems yields the analytical solution. By this way various exact solutions including complex structures are found and drawn their behaviour in complex plane by Maple to compare the uniqueness of solutions.

Keywords : Generalized Kuramoto - Sivashinsky equation, modified Kudrayshov method, exact solutions, Maple graphs.
Mathematical subject classification : 60H15, 20F70, 83C15.

1 Introduction

In engineering and science, the problems arising from the wave propogation of communication between two (or) more systems such as electromagnetic waves in wireless sensor networks, water flow in dams during earthquake, stability of the output in current electricity, viscous flows in fluid dynamics, magneto hydro dynamics, turbulence in microtides, and other physical phenomenons are described by the non-linear evolution equations (NLEE). The process of solving such NLEE analytically and numerically uses symbolic computation procedures, analytical methods and cardinal functions respectively. In modelling such media continuously takes to the Generalized Kuramoto-Sivashinsky Equation (GKSE) [1] given by the partial differential equation nonlinearly for $u = u(x,t)$ and non-zero α , β , γ constants.

$$u_t + uu_x + \alpha u_{xx} + \beta u_{xxx} + \gamma u_{xxxx} = 0. \tag{1}$$

GKSE and its solutions performs ample roles on flowing in viscous fluids, feedback in the output of self loop controllers, trajectories systems, gas dynamics. While $\alpha = \gamma = 1$ and $\beta = 0$, GKSE Eq (1) leads to Kuramoto-Sivashinsky Equation (KSE). N. A. Kudryashov solved Eq (1) by the method of Weiss-Tabor-Carnevale and obtained exact solutions in [1]. E. J. Parkes *et al* applied tanh method for Eq (1) by taking $\alpha = \beta = 1$ and solved using Mathematica package, they also solved (1) by taking $\alpha = -1$, $\beta = 1$ in [2]. B. Abdel-Hamid in [3] assumed the inital solution as PDE for u and solved exactly for $\alpha = 1$, $\beta = 0$ in Eq (1). D. Baldwin *et al* [4] applied tanh and sech methods to Eq (1) with $\alpha = \gamma = 1$ and solved using mathematica package. C. Li *et al* [5] solved GKSE of the form $u_t + \beta u^{\alpha} u_x + \gamma u^{\tau} u_{xx} + \delta u_{xxxx} = 0$ using Bernoulli equation. By simplest equation method again N. A. Kudrayshov solved by considering $u_x = u^m u_x$ in GKSE Eq (1) and obtained solution for general m with some restrictions in [6]. A. H. Khater *et al* in [7] used Chebyshev polynomials and applied its collocation points to solve approximations of Eq (1). M. G. Porshokouhi *et al* in [8] solved Eq (1) for different values of constants and approximately solved by variational iteration method. In [9] C.M. Khalique reduced Eq (1) by Lie symmetry and solved exactly by simplest

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equation methd with Riccati and Bernoulli equations seperately. D. Feng in [10] solved GKSE using Riccati equation where they taken $\beta = 0$ and $uu_x = \gamma uu_x$ in Eq (1). M. Lakestani *et al* used B-spline approximations function and solved Eq (1) numerically in [11] where they used tanh exact solutions for error estimations. J. Yang *et al* in [12] used sine-cosine method and dynamic bifurcation method to solve more generalized GKSE and its related equations to Eq (1). In [12] J. Rashidinia *et al* solved Eq (1) by Chebyshev wavelets. O.Acan *et al* applied reduced differential transform method to solve Eq (1) by taking $\beta = 0$ in [14].

For solving the nonlinear partial differential equations there as been many schemes applied such as Kudryashov method by M. Foroutan *et al* in [15] and K. K. Ali *et al* in [16]. Modified Kudryashov method by K. Hosseini *et al* in [17,18], D. Kumar *et al* in [19], A. K. Joardar *et al* in [20] and A.R. Seadawy *et al* in [21]. Generalized Kudryashov method by F. Mahmud *et al* in [22], S. T. Demiray *et al* in [23] and S. Bibi *et al* in [24]. Sine-cosine method by K. R. Raslan *et al* in [25]. Sine-Gordon method by H. Bulut *et al* in [26]. Sineh-Gordon equation expansion method by H. M. Baskonus *et al* in [27], Y. Xian-Lin *et al* in [28] and A. Esen *et al* in [29]. Extended trial equation method by K. A. Gepreel in [30],Y. Pandir *et al* in [31] and Y. Gurefe *et al* in [32]. Exponential $\left(-\frac{\phi}{2}\right)$ method by L.K. Ravi *et al* in [33], A. R. Seadawy *et al* in [34] and M. Nur Alam *et al* in [35]. Jacobi elliptic function method by S. Liu *et al* in [36]. F-expansion method by A. Ebaid *et al* in [37]. Extended $\left(\frac{G'}{G}\right)$ method by E. M. E. Zayed, S. Al-Joudi *et al* in [38].

2 Analysis of the Modified Kudrayshov Method

Given the nonlinear partial differential equations (NLPDE) which are converted to the ordinary differential equations (ODE) by making the necessary transformation. Then the initially assumed solution is substituted in the ODE, from which the algebraic equations are obtained and solved for unknowns, substituting the obtained unknown values in the assumed solution gives the exact solution of NLPDE. MKM takes the following steps in solving NLPDE [17–21].

Step 1. Consider the given NLPDE of the following form $u = u(x,t)$.

$$P\left(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \cdots\right) = 0. \tag{2}$$

Step 2. Apply the wave transformation $u(x,t) = u(\eta)$ in Eq (2) where

$$\eta = \mu(x - \lambda t). \tag{3}$$

Here μ is the wave variable and λ is the velocity, both are non-zero constants. Hence Eq (2) transforms to the following ODE.

$$O\left(u, u', u'', uu', \cdots\right) = 0. \tag{4}$$

where the prime represents the derivative w. r. t. η.

Step 3. Let the initial solution guess of Eq (4) be,

$$u(\eta) = A_0 + \sum_{i=1}^{N} A_i \left[Q(\eta)\right]^i. \tag{5}$$

where N is non-zero and positive constant calculated by principle of homogeneous balancing of Eq (4), A_i ; $i = 0, 1, 2, \cdots$ are unknowns to be calculated and $Q(\eta)$ is the solution of the following auxilary ODE.

$$\frac{dQ(\eta)}{d\eta} = Q(\eta)\left[Q(\eta) - 1\right]\ln(a)\ ;\ a \neq 1. \tag{6}$$

given by,

$$Q(\eta) = \frac{1}{1 + Da^{\eta}}. \tag{7}$$

where D is the integral constant and assumed $D = 1$.

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Step 4. Substituting Eqs (5) and (6) in Eq (4) leads to the polynomial in $Q(\eta)^i$; $i = 0, 1, 2, \cdots$. As $Q(\eta)^i \neq 0$ and so collecting its coefficients then equating to zero gives the systems of overdetermined algebraic equations, which upon solving gives the unknowns of Eqs (3) and (5).

Step 5. Finally substituting the values of Step 4 in Eq (5) and then in Eq (3) gives the solution $u(x,t)$ of Eq (2).

3 Applications to solve Generalized Kuramoto-Sivashinsky equation

Applying the wave transformation with Eq (3) to the GKSE Eq (1) leads to the ODE and then integrating once the ODE by taking integration constant to zero, transforms to the following ODE.

$$-\lambda u + \frac{u^2}{2} + \alpha\mu u^{(1)} + \beta\mu^2 u^{(2)} + \gamma\mu^3 u^{(3)} = 0. \tag{8}$$

where $u = u(\eta)$ and the superscripts (.) represents the derivatives w. r. t. η. By homogeneous balancing of Eq (8) gives $N = 3$ and hence the initial guess solution of Eq (8) from Eq (5) is given by,

$$u(\eta) = A_0 + A_1 Q(\eta) + A_2 (Q(\eta))^2 + A_3 (Q(\eta))^3. \tag{9}$$

Substituting Eqs (9) and (6) in Eq (8) results in the $6-th$ order polynomial of $Q(\eta)$. Collecting the coeffecients of $(Q(\eta))^i$; $i = 1, 2, \cdots, 7$ and equating each coeffecients to zero gives the systems of algebraic equations which upon solving by Maple gives the unknowns in Eqs (9) and (3). The resulting values are substituted in Eq (9) along with Eqs (7) and (3) gives the exact solution of GKSE Eq (1) for specific values of constants α and β. Substituting the α and β values in Eq (1) and the unknowns A_i ; $i = 0, 1, 2, 3$ in Eq (9) where $Q(\eta)$ taken from Eq (7) gives the following exact solutions. Let $\delta_1 = \gamma\mu\ln(a)$, $\delta_2 = \gamma\mu^2\ln(a)^2$, $\delta_3 = \gamma\mu^3\ln(a)^3$ and $i^2 = -1$ in the following cases.

Case 1. For $\alpha = \delta_2$ and $\beta = 4\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = A_1 = 0\ ,\ A_2 = 120\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 6\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_1(x,t) = \frac{120\delta_3 a^{\mu x - 6\delta_3\mu t}}{\left(1 + a^{\mu x - 6\delta_3\mu t}\right)^3}. \tag{10}$$

Case 2. For $\alpha = \delta_2$ and $\beta = 4\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = -12\delta_3\ ,\ A_1 = 0\ ,\ A_2 = 120\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -6\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_2(x,t) = -\frac{12\delta_3\left(1 + a^{3\mu x}e^{3(6\delta_3\mu\ln(a)t)} + 3a^{2\mu x}e^{2(6\delta_3\mu\ln(a)t)} - 7a^{\mu x}e^{6\delta_3\mu\ln(a)t}\right)}{\left(1 + a^{\mu x}e^{6\delta_3\mu\ln(a)t}\right)^3}. \tag{11}$$

Case 3. For $\alpha = \delta_2$ and $\beta = -4\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 0\ ,\ A_1 = -120\delta_3\ ,\ A_2 = 240\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -6\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_3(x,t) = -\frac{120\delta_3 a^{2(\mu x + 6\delta_3\mu t)}}{\left(1 + a^{\mu x + 6\delta_3\mu t}\right)^3}. \tag{12}$$

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Case 4. For $\alpha = \delta_2$ and $\beta = -4\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 12\delta_3\ ,\ A_1 = -120\delta_3\ ,\ A_2 = 240\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 6\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_4(x,t) = \frac{12\delta_3\left(a^{3(\mu x-6\delta_3\mu t)} - 7a^{2(\mu x-6\delta_3\mu t)} + 3a^{\mu x-6\delta_3\mu t} + 1\right)}{\left(1+a^{\mu x-6\delta_3\mu t}\right)^3}. \tag{13}$$

Case 5. For $\alpha = -19\delta_2$ and $\beta = 0$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = -60\delta_3\ ,\ A_1 = 0\ ,\ A_2 = 180\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -30\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_5(x,t) = -\frac{60\delta_3 e^{2(30\delta_3\mu\ln(a)t)}\left(a^{3\mu x}e^{30\delta_3\mu\ln(a)t} + 3a^{2\mu x}\right)}{\left(1+a^{\mu x}e^{30\delta_3\mu\ln(a)t}\right)^3}. \tag{14}$$

Case 6. For $\alpha = -19\delta_2$ and $\beta = 0$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = A_1 = 0\ ,\ A_2 = 180\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 30\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_6(x,t) = \frac{60\delta_3\left(1+3a^{\mu x-30\delta_3\mu t}\right)}{\left(1+a^{\mu x-30\delta_3\mu t}\right)^3}. \tag{15}$$

Case 7. For $\alpha = 47\delta_2$ and $\beta = 12\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = A_1 = A_2 = 0\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -60\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_7(x,t) = \frac{120\delta_3}{\left(1+a^{\mu x+60\delta_3\mu t}\right)^3}. \tag{16}$$

Case 8. For $\alpha = 47\delta_2$ and $\beta = 12\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 120\delta_3\ ,\ A_1 = A_2 = 0\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 60\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_8(x,t) = \frac{120\delta_3\left(3a^{\mu x}e^{2(60\delta_3\mu\ln(a)t)} + 3a^{2\mu x}e^{60\delta_3\mu\ln(a)t} + a^{3\mu x}\right)}{\left(a^{\mu x}+e^{60\delta_3\mu\ln(a)t}\right)^3}. \tag{17}$$

Case 9. For $\alpha = 47\delta_2$ and $\beta = -12\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 0\ ,\ A_1 = -360\delta_3\ ,\ A_2 = 360\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -60\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_9(x,t) = -\frac{120\delta_3\left(3a^{2\mu x}e^{2(60\delta_3\mu\ln(a)t)} + 3a^{\mu x}e^{60\delta_3\mu\ln(a)t} + 1\right)}{\left(1+a^{\mu x}e^{60\delta_3\mu\ln(a)t}\right)^3}. \tag{18}$$

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Case 10. For $\alpha = 47\delta_2$ and $\beta = -12\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 120\delta_3\ ,\ A_1 = -360\delta_3\ ,\ A_2 = 360\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 60\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_{10}(x,t) = \frac{120\delta_3 a^{3(\mu x - 60\delta_3\mu t)}}{\left(1 + a^{\mu x - 60\delta_3\mu t}\right)^3}. \tag{19}$$

Case 11. For $\alpha = 73\delta_2$ and $\beta = 16\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 180\delta_3\ ,\ A_1 = 0\ ,\ A_2 = -60\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 90\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_{11}(x,t) = \frac{60\delta_3\left(8a^{\mu x}e^{2(90\delta_3\mu\ln(a)t)} + 9a^{2\mu x}e^{90\delta_3\mu\ln(a)t} + 3a^{3\mu x}\right)}{\left(e^{90\delta_3\mu\ln(a)t} + a^{\mu x}\right)^3}. \tag{20}$$

Case 12. For $\alpha = 73\delta_2$ and $\beta = 16\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = A_1 = 0\ ,\ A_2 = -60\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -90\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_{12}(x,t) = -\frac{60\delta_3\left(3 + a^{\mu x + 90\delta_3\mu t}\right)}{\left(1 + a^{\mu x + 90\delta_3\mu t}\right)^3}. \tag{21}$$

Case 13. For $\alpha = 73\delta_2$ and $\beta = -16\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 180\delta_3\ ,\ A_1 = -480\delta_3\ ,\ A_2 = 420\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 90\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_{13}(x,t) = \frac{60\delta_3\left(a^{2\mu x}e^{90\delta_3\mu\ln(a)t} + 3a^{3\mu x}\right)}{\left(e^{90\delta_3\mu\ln(a)t} + a^{\mu x}\right)^3}. \tag{22}$$

Case 14. For $\alpha = 73\delta_2$ and $\beta = -16\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 0\ ,\ A_1 = -480\delta_3\ ,\ A_2 = 420\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -90\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_{14}(x,t) = -\frac{60\delta_3\left(8a^{2\mu x}e^{2(90\delta_3\mu\ln(a)t)} + 9a^{\mu x}e^{90\delta_3\mu\ln(a)t} + 3\right)}{\left(1 + a^{\mu x}e^{90\delta_3\mu\ln(a)t}\right)^3}. \tag{23}$$

Case 15. For $\alpha = \frac{19}{11}\delta_2$ and $\beta = 0$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = \frac{60}{11}\delta_3\ ,\ A_1 = -\frac{720}{11}\delta_3\ ,\ A_2 = 180\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = \frac{30}{11}\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_{15}(x,t) = \frac{60\delta_3 a^{\left(\mu x - \frac{30}{11}\delta_3\mu t\right)}\left(a^{2\left(\mu x - \frac{30}{11}\delta_3\mu t\right)} - 9a^{\left(\mu x - \frac{30}{11}\delta_3\mu t\right)} + 12\right)}{11\left(1 + a^{\left(\mu x - \frac{30}{11}\delta_3\mu t\right)}\right)^3}. \tag{24}$$

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Case 16. For $\alpha = \frac{19}{11}\delta_2$ and $\beta = 0$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 0\ ,\ A_1 = -\frac{720}{11}\delta_3\ ,\ A_2 = 180\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -\frac{30}{11}\delta_3.$$

Therefore the exact solution of Eq (1) is given by,

$$u_{16}(x,t) = -\frac{60\delta_3\left(1 - 9a^{\left(\mu x + \frac{30}{11}\delta_3\mu t\right)} + 12a^{2\left(\mu x + \frac{30}{11}\delta_3\mu t\right)}\right)}{11\left(1 + a^{\left(\mu x + \frac{30}{11}\delta_3\mu t\right)}\right)^3}. \quad (25)$$

Case 17. For $\alpha = -\delta_2$ and $\beta = 4i\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 0\ ,\ A_1 = -60\mu^3\ln(a)^3(\gamma - i\gamma)\ ,\ A_2 = 60(3-i)\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 4i\delta_3.$$

Therefore the exact complex solution of Eq (1) is given by,

$$u_{17}(x,t) = \frac{60\delta_3 a^{\mu x - 4i\delta_3\mu t}\left(i + 1 + (i-1)a^{\mu x - 4i\delta_3\mu t}\right)}{\left(1 + a^{\mu x - 4i\delta_3\mu t}\right)^3}. \quad (26)$$

The 3D complex graph of real and imaginary parts of $u_{17}(x,t)$ for $a = 7$ and $\mu = \gamma = 1.5$ are drawn in Figure 1.

Case 18. For $\alpha = -\delta_2$ and $\beta = 4i\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = -8i\delta_3\ ,\ A_1 = -60\mu^3\ln(a)^3(\gamma - i\gamma)\ ,\ A_2 = 60(3-i)\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -4i\delta_3.$$

Therefore the exact complex solution of Eq (1) is given by,

$$u_{18}(x,t) = -\frac{8\delta_3}{\left(1 + a^{\mu x + 4i\delta_3\mu t}\right)^3}\left[i(1 + a^{3(\mu x + 4i\delta_3\mu t)}) + \left(\frac{15-9i}{2}\right)a^{2(\mu x + 4i\delta_3\mu t)} - \left(\frac{15+9i}{2}\right)a^{\mu x + 4i\delta_3\mu t}\right]. \quad (27)$$

The 3D complex graph of real and imaginary parts of $u_{18}(x,t)$ for $a = 7$ and $\mu = \gamma = 1.5$ are drawn in Figure 2.

Case 19. For $\alpha = -\delta_2$ and $\beta = -4i\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 0\ ,\ A_1 = -60\mu^3\ln(a)^3(\gamma + i\gamma)\ ,\ A_2 = 60(3+i)\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -4i\delta_3.$$

Therefore the exact complex solution of Eq (1) is given by,

$$u_{19}(x,t) = -\frac{60\delta_3 a^{\mu x + 4i\delta_3\mu t}\left(i - 1 + (i+1)a^{\mu x + 4i\delta_3\mu t}\right)}{\left(1 + a^{\mu x + 4i\delta_3\mu t}\right)^3}. \quad (28)$$

The 3D complex graph of real and imaginary parts of $u_{19}(x,t)$ for $a = 7$ and $\mu = \gamma = 1.5$ are drawn in Figure 3.

Case 20. For $\alpha = -\delta_2$ and $\beta = -4i\delta_1$ in Eq (1), the unknown coeffecients are given by,

$$A_0 = 8i\delta_3\ ,\ A_1 = -60\mu^3\ln(a)^3(\gamma + i\gamma)\ ,\ A_2 = 60(3+i)\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 4i\delta_3.$$

Therefore the exact complex solution of Eq (1) is given by,

$$u_{20}(x,t) = \frac{8\delta_3}{\left(1 + a^{\mu x - 4i\delta_3\mu t}\right)^3}\left[i(1 + a^{3(\mu x - 4i\delta_3\mu t)}) - \left(\frac{15+9i}{2}\right)a^{2(\mu x - 4i\delta_3\mu t)} + \left(\frac{15-9i}{2}\right)a^{\mu x - 4i\delta_3\mu t}\right]. \quad (29)$$

The 3D complex graph of real and imaginary parts of $u_{20}(x,t)$ for $a = 7$ and $\mu = \gamma = 1.5$ are drawn in Figure 4.

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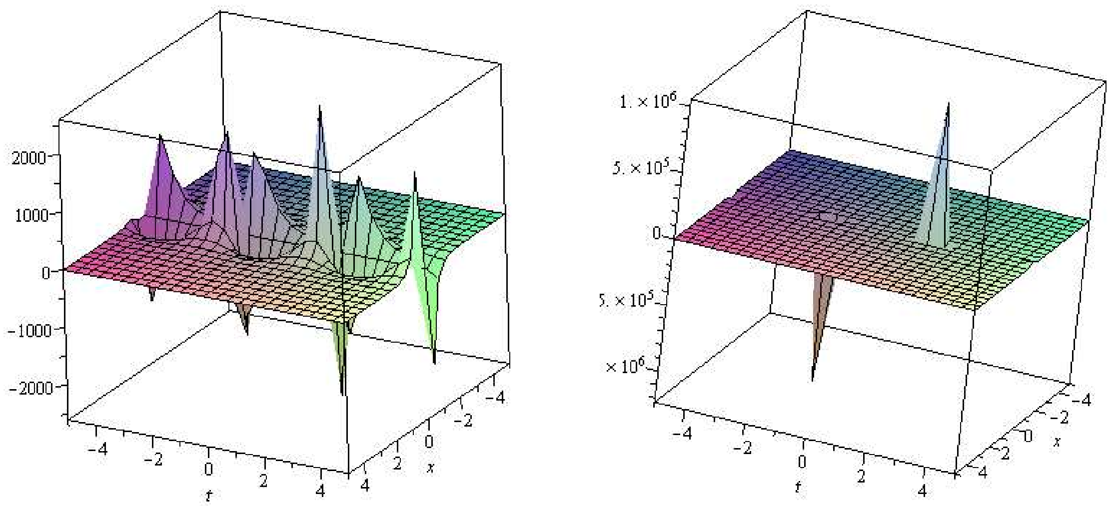

(a) Real part of $u(x,t)$ in Eq (26)

(b) Imaginary part of $u(x,t)$ in Eq (26)

Figure 1: Real and Imaginary part of solution in the case 17 of Eq (1) **respectively from left to right for** $a = 7$ **and** $\mu = \gamma = 1.5$ **in** $x \in [-5,5]$ **and** $t \in [-5,5]$

4 Conclusion

In this work the generalized Kuramoto-Sivashinsky equation is solved for exact solutions. The said GKSE have exact solutions for the different α and β values, which we obtained by the application of modified Kudrayshov method and found 10 classes of $(\alpha\ ,\ \beta)$ pairs and their corresponding two distinct exact solutions for each class of GKSE Eq (1) from cases 1 through 20. All the solutions are validated in Maple computer algebra. The three dimensional simulations of solutions shows their behavioural pattern. We reckon all the solutions obtained through this communication will help further study of GKSE in the physical field.

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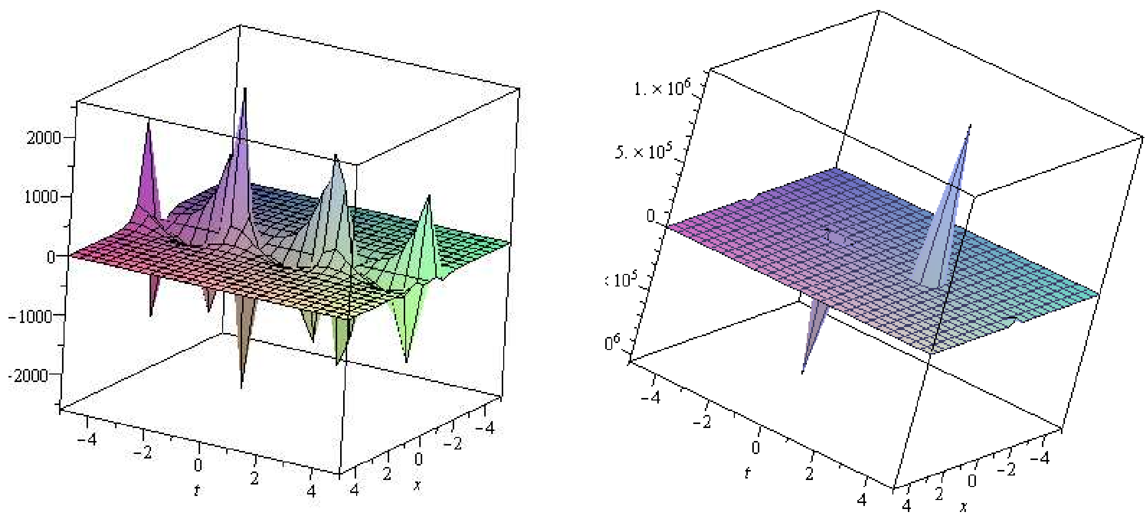

(a) Real part of $u(x,t)$ in Eq (27)

(b) Imaginary part of $u(x,t)$ in Eq (27)

Figure 2: Real and Imaginary part of solution in the case 18 of Eq (1) **respectively from left to right for** $a = 7$ **and** $\mu = \gamma = 1.5$ **in** $x \in [-5,5]$ **and** $t \in [-5,5]$

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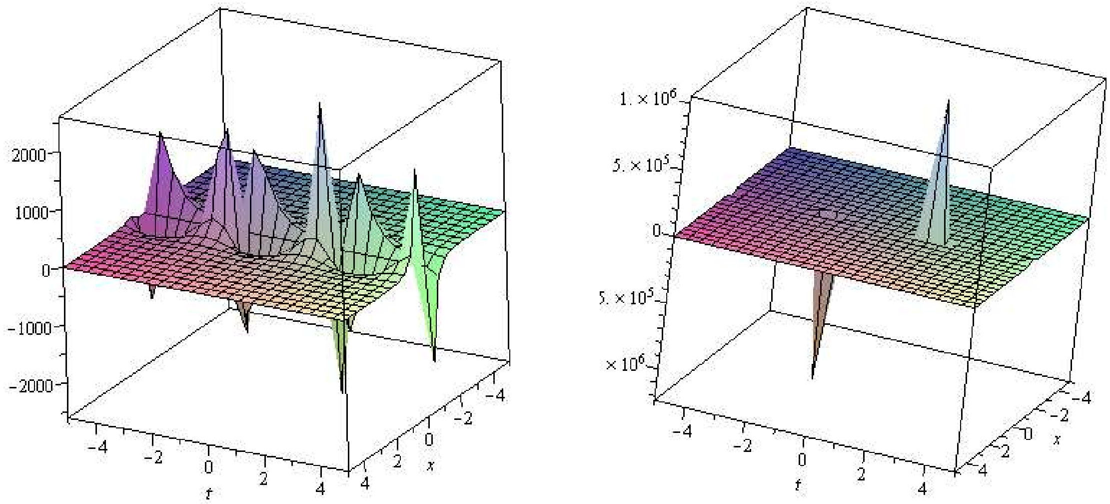

(a) Real part of $u(x,t)$ in Eq (28)

(b) Imaginary part of $u(x,t)$ in Eq (28)

Figure 3: Real and Imaginary part of solution in the case 19 of Eq (1) **respectively from left to right for** $a = 7$ **and** $\mu = \gamma = 1.5$ **in** $x \in [-5,5]$ **and** $t \in [-5,5]$

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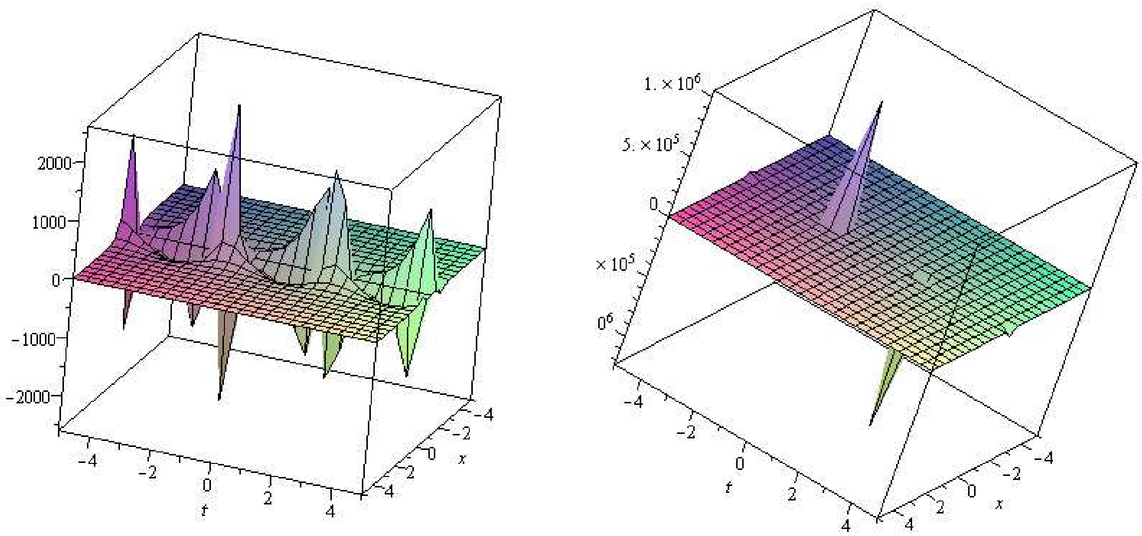

(a) Real part of $u(x,t)$ in Eq (29)

(b) Imaginary part of $u(x,t)$ in Eq (29)

Figure 4: Real and Imaginary part of solution in the case 20 of Eq (1) **respectively from left to right for** $a=7$ **and** $\mu=\gamma=1.5$ **in** $x\in[-5,5]$ **and** $t\in[-5,5]$

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Appendix A GKSE in the previous studies

N.A. Kudryashov in [6] solved for the exact solution of Eq (1). Based on the homogeneous balancing taken the following initial solution.

$$u(\eta) = A_0 + A_1 g(\eta) + A_2 g(\eta)^2 + A_3 g(\eta)^3.$$

where $g(\eta)$ is the solution of $\frac{dg(\eta)}{d\eta} = b - g(\eta)^2$ and obtained the following values.

1.

$$A_0 = -\frac{\beta^3}{576\gamma^2}\ ,\ A_1 = \frac{5\beta^2}{4\gamma}\ ,\ A_2 = -15\beta\ ,\ A_3 = 120\gamma\ ,\ \alpha = \frac{47\beta^2}{144\gamma}\ ,\ b = \frac{\beta^2}{576\gamma^2}\ ,\ C_0 = -\frac{5\beta^3}{144\gamma^2}.$$

2.

$$A_0 = \frac{30\beta^3}{128\gamma^2}\ ,\ A_1 = -\frac{30\beta^2}{16\gamma}\ ,\ A_2 = -30\beta\ ,\ A_3 = 120\gamma\ ,\ \alpha = \frac{\beta^2}{16\gamma}\ ,\ b = \frac{\beta^2}{64\gamma^2}\ ,\ C_0 = \frac{3\beta^3}{32\gamma^2}.$$

In the same work he solved Eq (1) with the auxiliary equations $\left(\frac{dg(z)}{dz}\right)^2 + 4g(z)^3 - ag(z)^2 - 2bg(z) + d = 0$ and $\frac{d^2g(z)}{dz^2} + 6g(z)^2 - ag(z) - b = 0$ and obtained other values for unknowns.

C.M. Khalique in [9] solved Eq (1) by taking the Bernoulli equation $\frac{dh(\eta)}{d\eta} = ah(\eta) + bh(\eta)^2$ and Riccati equation $\frac{dh(\eta)}{d\eta} = ah(\eta)^2 + bh(\eta) + c$ as the auxillary ODE, and obtained the following values respectively by using each ODE. For both the auxiliary equation $a = 1$, $b = 3$ and $c = 1$.

1.

$$A_0 = \nu - 6a^3\gamma\ ,\ A_1 = -120a^2b\gamma\ ,\ A_2 = 240ab^2\gamma\ ,\ A_3 = -120b^3\gamma\ ,\ \alpha = a^2\gamma\ ,\ \beta = 4a\gamma.$$

2.

$$A_0 = -990\gamma + 60\gamma k + \nu\ ,\ A_1 = 60\gamma + 180\gamma k\ ,\ A_2 = 60\gamma k\ ,\ A_3 = -120\gamma\ ,\ \alpha = 365\gamma\ ,\ \beta = -36\gamma - 4\gamma k.$$

While comparing the above values, our solutions of GKSE Eq (1) in this work are new to the literature surveyed.

Appendix B Studying GKSE by GKM and SGEEM

1. For solving the GKSE Eq (1) by the generalized Kudrayshov method [22–24], the homogeneous balancing of Eq (8) gives $N = M + 3$ which is having infinite solutions. For the value $M = 1$, gives $N = 4$. So,

$$u(\eta) = \frac{A_0 + A_1 Q(\eta) + A_2 (Q(\eta))^2 + A_3 (Q(\eta))^3 + A_4 (Q(\eta))^4}{B_0 + B_1 Q(\eta)}.$$

where $Q(\eta)$ is the solution of $\frac{dQ(\eta)}{d\eta} = Q(\eta)(Q(\eta) - 1)$, Applying these equations in Eq (8) leads to the polynomial in $Q(\eta)$ and its powers. Collecting the coefficients of $(Q(\eta))^i$; $i = 0, 1, 2, \cdots$ and attempting to solve the overdetermined equations results in the continuous execution of Maple. Therefore we reckon GKSE cannot be solved by generalized Kudrayshov method.

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2. Next for solving the GKSE Eq (1) by the Sine-Gordon equation expansion method [26], the homogeneous balancing is same as MKM given by $N = 3$. Thus,

$$u(\eta) = A_0 + A_1 \tanh(\eta) + B_1 \operatorname{sech}(\eta) + A_2 \tanh^2(\eta) + B_2 \tanh(\eta)\operatorname{sech}(\eta) + A_3 \tanh^3(\eta) + B_3 \tanh^2(\eta)\operatorname{sech}(\eta).$$

Substituting the above equation $u(\eta)$ in Eq (8) and following the steps in [26] leads to the polynomials in $\sin(w)$, $\cos(w)$, their products, powers. Collecting the coefficients, equating them to zero and solving in Maple results in the continuous execution. Hence we reckon GKSE cannot be solved by the Sine-Gordon equation expansion method as well.