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Modified Kudrayshov Method to solve generalized Kuramoto - Sivashinsky equation

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Abstract

The generalized Kuramoto - Sivashinsky equation is investigated using the modified Kudrayshov equation for the exact analytical solution. The modified Kudrayshov method converts the nonlinear partial differential equation to algebraic equations by results of various steps which on solving the so obtained equation systems yields the analytical solution. By this way various exact including complex solutions are found and drawn their behaviour in complex plane by Maple to compare the uniqueness of various solutions.

Keywords : Kuramoto - Sivashinsky equation, Modified Kudrayshov method, Exact solutions, Maple graphs.
Mathematical subject classification : 60H15, 20F70, 83C15.

1 Introduction

In engineering and science, the problems arising from the wave propogation of communication between two (or) more systems such as 1. Electromagnetic waves in wireless sensor networks, 2. Water flow in dams during earthquake, 3. Stability of the output in current electricity, 4. Viscous flows in fluid dynamics, magneto hydro dynamics, 5. Turbulence in microtides, and other physical phenomenons are described by the non-linear evolution equations (NLEE). The process of solving such NLEE analytically and numerically uses symbolic computation procedures, analytical methods and cardinal functions respectively. In modelling such media continuously takes to the Generalized Kuramoto-Sivashinsky Equation (GKSE) [1] given by the partial differential equation nonlinearly for $u = u(x,t)$ and non-zero α , β , γ constants.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \alpha\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial^3 u}{\partial x^3} + \gamma\frac{\partial^4 u}{\partial x^4} = 0. \tag{1}$$

GKSE and its solutions performs ample roles on flowing in viscous fluids, feedback in the output of self loop controllers, trajectories systems, gas dynamics. While $\alpha = \gamma = 1$ and $\beta = 0$, GKSE (1) leads to Kuramoto-Sivashinsky Equation (KSE). N. A. Kudryashov solved (1) by the method of Weiss-Tabor-Carnevale and obtained exact solutions in [1]. E. J. Parkes *et al* applied tanh method for (1) by taking $\alpha = \beta = 1$ and solved using Mathematica package, they also solved (1) by taking $\alpha = -1$, $\beta = 1$ in [2]. B. Abdel-Hamid in [3] assumed the inital solution as PDE for u and solved exactly for $\alpha = 1$, $\beta = 0$ in (1). D. Baldwin *et al* [4] applied tanh and sech methods to (1) with $\alpha = \gamma = 1$ and solved using mathematica package. C. Li *et al* [5] solved GKSE of the form $u_t + \beta u^{\alpha} u_x + \gamma u^{\tau} u_{xx} + \delta u_{xxxx} = 0$ using Bernoulli equation. By simplest equation method again N. A. Kudrayshov solved by considering $u_x = u^m u_x$ in GKSE (1) and obtained solution for general m with some restrictions in [6]. A. H. Khater *et al* in [7] used Chebyshev polynomials and applied its collocation points to solve approximations of (1). M. G. Porshokouhi *et al* in [8] solved (1) for different values of constants and approximately solved by variational iteration method. In [9] C.M. Khalique reduced (1) by Lie symmetry and solved exactly by simplest equation methd with Riccati and Bernoulli equations seperately. D. Feng in [10] solved GKSE using Riccati equation where they taken $\beta = 0$ and $uu_x = \gamma uu_x$ in (1).

M. Lakestani *et al* used B-spline approximations function solved (1) numerically in [11] and used tanh exact solutions for error estimations. J. Yang *et al* in [12] used sine-cosine method and dynamic bifurcation method to solve more generalized GKSE and its related equations to (1). In [12] J. Rashidinia *et al* solved (1) by Chebyshev wavelets. O.Acan *et al* applied reduced differential transform method to solve (1) by taking $\beta = 0$ in [14].

For solving the nonlinear partial differential equations there as been many schemes applied such as Kudryashov method by M. Foroutan *et al* in [15] and K. K. Ali *et al* in [16]. Modified Kudryashov method by K. Hosseini *et al* in [17,18], D. Kumar *et al* in [19], A. K. Joardar *et al* in [20] and A.R. Seadawy *et al* in [21]. Generalized Kudryashov method by F. Mahmud *et al* in [22], S. T. Demiray *et al* in [23] and S. Bibi *et al* in [24]. Sine-cosine method by K. R. Raslan *et al* in [25]. Sine-Gordon method by H. Bulut *et al* in [26]. Sineh-Gordon equation expansion method by H. M. Baskonus *et al* in [27], Y. Xian-Lin *et al* in [28] and A. Esen *et al* in [29]. Extended trial equation method by K. A. Gepreel in [30],Y. Pandir *et al* in [31] and Y. Gurefe *et al* in [32]. Exponential $-\frac{\phi}{2}$ method by L.K. Ravi *et al* in [33], A. R. Seadawy *et al* in [34] and M. Nur Alam *et al* in [35]. Jacobi elliptic function method by S. Liu *et al* in [36]. F-expansion method by A. Ebaid *et al* in [37]. Extended $\frac{G'}{G}$ method by E. M. E. Zayed, S. Al-Joudi *et al* in [38].

In this communication work we apply the Modified Kudrayshov Method (MKM) used in [17–21] to solve GKSE (1) and gotten the new exact solutions. Description about the methodology of MKM is given in section 2, Applications of MKM to GKSE given in the section 3, Numerical study of GKSE solutions through graphs given in the section 4, comparative study to the previous solution given in the section 5 and processing GKSE by other methods and their limitations are given in the section 6 followed by conclusions at the end.

2 Analysis of MKM

Given nonlinear partial differential equations (NLPDE) are converted to ordinary differential equations (ODE) by making necessary transform. Then the initially assumed solution is applied in ODE from which the algebraic equations are obtained and solved, when substituting in assumed solution gives the NLPDE solution. MKM takes the following steps in solving NLPDE [17–21] as described in Algorithm 1.

Remark 1. When $\ln(a) = 1$ in (6), Algorithm 1 gives the Kudrayshov method [15,16]. Next When $\ln(a) = 1$ in (6) and $u(\eta) = \frac{\sum_{i=0}^{N} A_i Q(\eta)^i}{\sum_{j=0}^{M} B_i Q(\eta)^j}$ in (5), Algorithm 1 gives the generalized Kudrayshov method [22–24].

3 Applications to solve GKSE

Applying the wave transformation with (3) to the GKSE (1) and integrating once by taking integration constant zero, transforms to the following ODE.

$$-\lambda u + \frac{u^2}{2} + \alpha\mu u^{(1)} + \beta\mu^2 u^{(2)} + \gamma\mu^3 u^{(3)} = 0. \tag{8}$$

where $u = u(\eta)$ and the superscripts $(.)$ represents the derivatives w. r. t. η. By homogeneous balancing of (8) gives $N = 3$ and hence the initial guess solution of (8) from (5) is given by,

$$u(\eta) = A_0 + A_1 Q(\eta) + A_2 (Q(\eta))^2 + A_3 (Q(\eta))^3 . \tag{9}$$

Substituting (9) and (6) in (8) results in the polynomial of $Q(\eta)$ order 6. Collecting the coeffecients of $(Q(\eta))^i$; $i = 1, 2, \cdots, 7$ and equating to zero gives the systems of algebraic equations which upon solving by Maple gives the unknowns of (9) and (3). The obtained values are applied in (9) along with (7) and (3) gives the exact solution of GKSE (1) for specific values of constants α and β. Substituting the α and β values found the from set of algebraic equations in (1) and the unknowns A_i ; $i = 0, 1, 2, 3$ in (9) where $Q(\eta)$ taken from (7) gives the following exact solutions. Let $\delta_1 = \gamma\mu \ln(a)$, $\delta_2 = \gamma\mu^2 \ln(a)^2$ and $\delta_3 = \gamma\mu^3 \ln(a)^3$ in the following cases.

Solution 1 **:**

$$A_0 = A_1 = A_2 = 0 \ , \ A_3 = -120\delta_3 \ , \ \lambda = -60\delta_3 \ , \ \mu = \mu \ , \ \alpha = 47\delta_2 \ , \ \beta = 12\delta_1 .$$

Algorithm 1 Modified Kudrayshov method

Step 1. Consider the given NLPDE of the following form $u = u(x,t)$.

$$P\left(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \cdots\right) = 0. \tag{2}$$

Step 2. Apply the wave transformation $u(x,t) = u(\eta)$ in (2) where

$$\eta = \mu(x - \lambda t). \tag{3}$$

Here μ is the wave variable and λ is the velocity, both are non-zero constants. Hence (2) transforms to ODE.

$$O\left(u, u^{'}, u^{''}, uu^{'}, \cdots\right) = 0. \tag{4}$$

where the prime represents the derivative w. r. t. η.

Step 3. Let the initial solution guess of (4) be,

$$u(\eta) = A_0 + \sum_{i=1}^{N} A_i \left[Q(\eta)\right]^i. \tag{5}$$

where N is non-zero and positive constant calculated by principle of homogeneous balancing of (4), A_i ; $i = 0, 1, 2, \cdots$ are unknowns to be calculated and $Q(\eta)$ is the solution of following auxilary ODE.

$$\frac{dQ(\eta)}{d\eta} = Q(\eta)\left[Q(\eta) - 1\right]\ln(a) \; ; \; a \neq 1. \tag{6}$$

given by,

$$Q(\eta) = \frac{1}{1 + Da^{\eta}}, \tag{7}$$

where D is the integral constant and assumed $D = 1$.

Step 4. Substituting (5) and (6) in (4) leads to the polynomial in $Q(\eta)^i$; $i = 0, 1, 2, \cdots$. As $Q(\eta)^i \neq 0$ and so collecting its coefficients then equating to zero gives systems of overdetermined algebraic equations, upon solving gives the unknowns of (3) and (5).

Step 5. Finally substituting the values of Step 4 in (5) and then in (3) gives the solution $u(x,t)$ of (2).

$$u_1(x,t) = \frac{120\delta_3}{(1+a^{\eta})^3}. \tag{10}$$

where $\eta = \mu x + 60\delta_3\mu t$.

Solution 2 **:**

$$A_0 = A_1 = 0 \ , \ A_2 = 120\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = 6\delta_3 \ , \ \mu = \mu \ , \ \alpha = \delta_2 \ , \ \beta = 4\delta_1.$$

$$u_2(x,t) = \frac{120\delta_3 a^{\eta}}{(1+a^{\eta})^3}. \tag{11}$$

where $\eta = \mu x - 6\delta_3\mu t$.

Solution 3 **:**

$$A_0 = A_1 = 0 \ , \ A_2 = 180\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = 30\delta_3 \ , \ \mu = \mu \ , \ \alpha = -19\delta_2 \ , \ \beta = 0.$$

$$u_3(x,t) = \frac{60\delta_3\,(1+3a^{\eta})}{(1+a^{\eta})^3}. \tag{12}$$

where $\eta = \mu x - 30\delta_3\mu t$.

Solution 4 **:**

$$A_0 = 0 \ , \ A_1 = -\frac{720}{11}\delta_3 \ , \ A_2 = 180\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = -\frac{30}{11}\delta_3 \ , \ \mu = \mu \ , \ \alpha = \frac{19}{11}\delta_2 \ , \ \beta = 0.$$

$$u_4(x,t) = -\frac{60\delta_3}{11}\left[\frac{1-9a^{\eta}+12a^{2\eta}}{(1+a^{\eta})^3}\right]. \tag{13}$$

where $\eta = \mu x + \frac{30}{11}\delta_3\mu t$.

Solution 5 **:**

$$A_0 = 0 \ , \ A_1 = -120\delta_3 \ , \ A_2 = 240\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = -6\delta_3 \ , \ \mu = \mu \ , \ \alpha = \delta_2 \ , \ \beta = -4\delta_1.$$

$$u_5(x,t) = -\frac{120\delta_3 a^{2\eta}}{(1+a^{\eta})^3}. \tag{14}$$

where $\eta = \mu x + 6\delta_3\mu t$.

Solution 6 **:**

$$A_0 = 8i\delta_3 \ , \ A_1 = -60\mu^3\ln(a)^3(\gamma+i\gamma) \ , \ A_2 = 180\delta_3+60i\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = 4i\delta_3 \ , \ \mu = \mu \ , \ \alpha = -\delta_2 \ , \ \beta = -4i\delta_1.$$

$$u_6(x,t) = \frac{8\delta_3}{(1+a^{\eta})^3}\left[i+ia^{3\eta}-\left(\frac{15+9i}{2}\right)a^{2\eta}+\left(\frac{15-9i}{2}\right)a^{\eta}\right]. \tag{15}$$

where $\eta = \mu x - 4i\delta_3\mu t$ and $i^2 = -1$.

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Solution 7 **:**

$A_0 = 0$, $A_1 = -60\mu^3 \ln(a)^3(\gamma + i\gamma)$, $A_2 = 180\delta_3 + 60i\delta_3$, $A_3 = -120\delta_3$, $\lambda = -4i\delta_3$, $\mu = \mu$, $\alpha = -\delta_2$, $\beta = -4i\delta_1$.

$$u_7(x,t) = -\frac{60\delta_3 a^\eta \left(i - 1 + (i+1)a^\eta\right)}{\left(1 + a^\eta\right)^3}. \tag{16}$$

where $\eta = \mu x + 4i\delta_3\mu t$ and $i^2 = -1$.

Solution 8 **:**

$$A_0 = 180\delta_3 \ , \ A_1 = 0 \ , \ A_2 = -60\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = 90\delta_3 \ , \ \mu = \mu \ , \ \alpha = 73\delta_2 \ , \ \beta = 16\delta_1.$$

$$u_8(x,t) = \frac{60\delta_3 \left(8a^{\mu x}e^{2\eta} + 9a^{2\mu x}e^\eta + 3a^{3\mu x}\right)}{\left(e^\eta + a^{\mu x}\right)^3}. \tag{17}$$

where $\eta = 90\delta_3\mu \ln(a)t$.

Solution 9 **:**

$$A_0 = -12\delta_3 \ , \ A_1 = 0 \ , \ A_2 = 120\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = -6\delta_3 \ , \ \mu = \mu \ , \ \alpha = \delta_2 \ , \ \beta = 4\delta_1.$$

$$u_9(x,t) = -\frac{12\delta_3 \left(1 + a^{3\mu x}e^{3\eta} + 3a^{2\mu x}e^{2\eta} - 7a^{\mu x}e^\eta\right)}{\left(1 + a^{\mu x}e^\eta\right)^3}. \tag{18}$$

where $\eta = 6\delta_3\mu \ln(a)t$.

Solution 10 **:**

$$A_0 = 180\delta_3 \ , \ A_1 = -480\delta_3 \ , \ A_2 = 420\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = 90\delta_3 \ , \ \mu = \mu \ , \ \alpha = 73\delta_2 \ , \ \beta = -16\delta_1.$$

$$u_{10}(x,t) = \frac{60\delta_3 \left(a^{2\mu x}e^\eta + 3a^{3\mu x}\right)}{\left(e^\eta + a^{\mu x}\right)^3}. \tag{19}$$

where $\eta = 90\delta_3\mu \ln(a)t$.

Solution 11 **:**

$$A_0 = -60\delta_3 \ , \ A_1 = 0 \ , \ A_2 = 180\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = -30\delta_3 \ , \ \mu = \mu \ , \ \alpha = -19\delta_2 \ , \ \beta = 0.$$

$$u_{11}(x,t) = -\frac{60\delta_3 e^{2\eta} \left(a^{3\mu x}e^\eta + 3a^{2\mu x}\right)}{\left(1 + a^{\mu x}e^\eta\right)^3}. \tag{20}$$

where $\eta = 30\delta_3\mu \ln(a)t$.

Solution 12 **:**

$$A_0 = \frac{60}{11}\delta_3 \ , \ A_1 = -\frac{720}{11}\delta_3 \ , \ A_2 = 180\delta_3 \ , \ A_3 = -120\delta_3 \ , \ \lambda = \frac{30}{11}\delta_3 \ , \ \mu = \mu \ , \ \alpha = \frac{19}{11}\delta_2 \ , \ \beta = 0.$$

$$u_{12}(x,t) = \frac{60\delta_3 a^\eta \left(a^{2\eta} - 9a^\eta + 12\right)}{11\left(1 + a^\eta\right)^3}. \tag{21}$$

where $\eta = \mu x - \frac{30}{11}\delta_3\mu t$.

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Solution 13 **:**

$$A_0 = 120\delta_3\ ,\ A_1 = A_2 = 0\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 60\delta_3\ ,\ \mu = \mu\ ,\ \alpha = 47\delta_2\ ,\ \beta = 12\delta_1.$$

$$u_{13}(x,t) = \frac{120\delta_3\left(3a^{\mu x}e^{2\eta} + 3a^{2\mu x}e^{\eta} + a^{3\mu x}\right)}{\left(a^{\mu x} + e^{\eta}\right)^3}. \tag{22}$$

where $\eta = 60\delta_3\mu\ln(a)t$.

Solution 14 **:**

$$A_0 = 0\ ,\ A_1 = -360\delta_3\ ,\ A_2 = 360\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -60\delta_3\ ,\ \mu = \mu\ ,\ \alpha = 47\delta_2\ ,\ \beta = -12\delta_1.$$

$$u_{14}(x,t) = -\frac{120\delta_3\left(3a^{2\mu x}e^{2\eta} + 3a^{\mu x}e^{\eta} + 1\right)}{\left(1 + a^{\mu x}e^{\eta}\right)^3}. \tag{23}$$

where $\eta = 60\delta_3\mu\ln(a)t$.

Solution 15 **:**

$$A_0 = A_1 = 0\ ,\ A_2 = -60\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -90\delta_3\ ,\ \mu = \mu\ ,\ \alpha = 73\delta_2\ ,\ \beta = 16\delta_1.$$

$$u_{15}(x,t) = -\frac{60\delta_3\left(3 + a^{\eta}\right)}{\left(1 + a^{\eta}\right)^3}. \tag{24}$$

where $\eta = \mu x + 90\delta_3\mu t$.

Solution 16 **:**

$$A_0 = 0\ ,\ A_1 = -480\delta_3\ ,\ A_2 = 420\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -90\delta_3\ ,\ \mu = \mu\ ,\ \alpha = 73\delta_2\ ,\ \beta = -16\delta_1.$$

$$u_{16}(x,t) = -\frac{60\delta_3\left(8a^{2\mu x}e^{2\eta} + 9a^{\mu x}e^{\eta} + 3\right)}{\left(1 + a^{\mu x}e^{\eta}\right)^3}. \tag{25}$$

where $\eta = 90\delta_3\mu t$.

Solution 17 **:**

$$A_0 = 0\ ,\ A_1 = -60\mu^3\ln(a)^3\left(\gamma - i\gamma\right)\ ,\ A_2 = 180\delta_3 - 60i\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 4i\delta_3\ ,\ \mu = \mu\ ,\ \alpha = -\delta_2\ ,\ \beta = 4i\delta_1.$$

$$u_{17}(x,t) = \frac{60\delta_3 a^{\eta}}{\left(1 + a^{\eta}\right)^3}\left[i + 1 + (i-1)\,a^{\eta}\right]. \tag{26}$$

where $\eta = \mu x - 4i\delta_3\mu t$.

Solution 18 **:**

$$A_0 = 12\delta_3\ ,\ A_1 = -120\delta_3\ ,\ A_2 = 240\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 6\delta_3\ ,\ \mu = \mu\ ,\ \alpha = \delta_2\ ,\ \beta = -4\delta_1.$$

$$u_{18}(x,t) = \frac{12\delta_3\left(a^{3\eta} - 7a^{2\eta} + 3a^{\eta} + 1\right)}{\left(1 + a^{\eta}\right)^3}. \tag{27}$$

where $\eta = \mu x - 6\delta_3\mu t$.

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Solution 19 **:**

$$A_0 = 120\delta_3\ ,\ A_1 = -360\delta_3\ ,\ A_2 = 360\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = 60\delta_3\ ,\ \mu = \mu\ ,\ \alpha = 47\delta_2\ ,\ \beta = -12\delta_1.$$

$$u_{19}(x,t) = \frac{120\delta_3 a^{3\eta}}{(1+a^{\eta})^3}. \tag{28}$$

where $\eta = \mu x - 60\delta_3\mu t$.

Solution 20 **:**

$$A_0 = -8i\delta_3\ ,\ A_1 = -60\mu^3\ln(a)^3(\gamma - i\gamma)\ ,\ A_2 = 180\delta_3 - 60i\delta_3\ ,\ A_3 = -120\delta_3\ ,\ \lambda = -4i\delta_3\ ,\ \mu = \mu\ ,\ \alpha = -\delta_2\ ,\ \beta = 4i\delta_1.$$

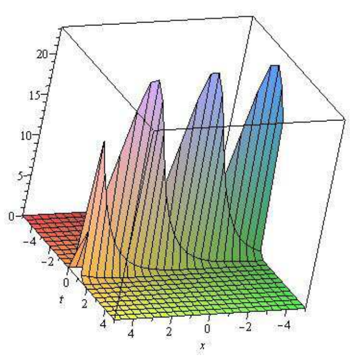
$$u_{20}(x,t) = -\frac{8\delta_3}{(1+a^{\eta})^3}\left[i(1+a^{3\eta}) + \left(\frac{15-9i}{2}\right)a^{2\eta} - \left(\frac{15+9i}{2}\right)a^{\eta}\right]. \tag{29}$$

where $\eta = \mu x + 4i\delta_3\mu t$.

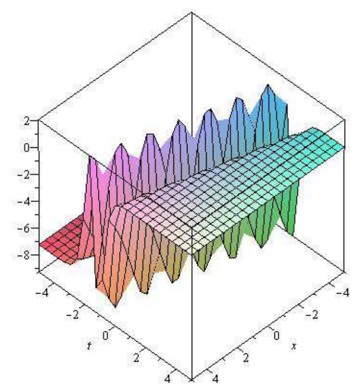
Remark 2. All the solutions (10) through (29) are verified in Maple by substituting them in origianl equation (1) with respective values of α and β which satisfies the GKSE. As far as the references we collected, we trust all the solutions in this communication are new. Also we believe the complex valued solutions (15), (16), (26) and (29) are appearing for the first time in this work.

4 Graphical study of solutions

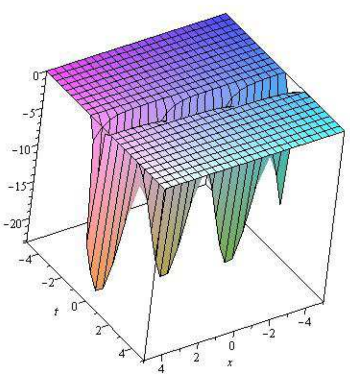
The three dimensional graph of solutions (11), (13), (14) for $a = 3$ and $\mu = \gamma = 1$ are drawn in Figure 1. The real and imaginary part of solutions (15) and (16) for $a = 5$ and $\mu = \gamma = 1$ are drawn in Figure 2 and Figure 3 respectively. The three dimensional graph of solutions (18) for $a = 3$, $\mu = \gamma = 1$, (21) for $a = 5$, $\mu = \gamma = 0.5$ and (24) for $a = 3$, $\mu = \gamma = 0.5$ are drawn in Figure 4. All the complex graphs are plotted in the domain $[-5,5]$ for x and t. Also the graphical simulations shows the solutions are unique.



(a) $u(x,t)$ in Eq (11)



(b) $u(x,t)$ in Eq (13)



(c) $u(x,t)$ in Eq (14)

Figure 1: Solutions 2, 4 and 5 of (1) **respectively from left to right for** $a = 3$ **and** $\mu = \gamma = 1$ **in** $x \in [-5,5]$ **and** $t \in [-5,5]$

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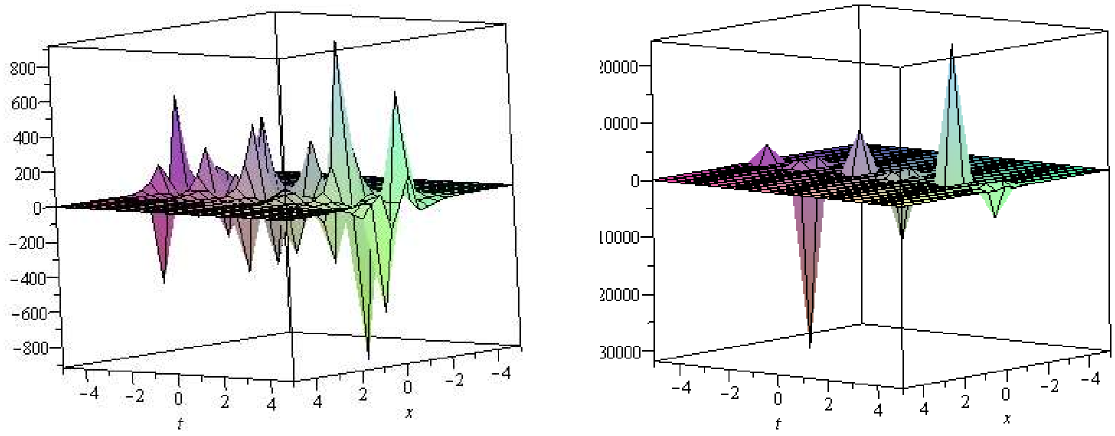

(a) Real part of $u(x,t)$ in Eq (15)

(b) Imaginary part of $u(x,t)$ in Eq (15)

Figure 2: Real and Imaginary part of Solution 6 of (1) **respectively from left to right for** $a = 5$ **and** $\mu = \gamma = 1$ **in** $x \in [-5,5]$ **and** $t \in [-5,5]$

5 Comparision of results from the literature

N.A. Kudryashov in [6] for the exact solution of (1) based on the homogeneous balancing taken the following initial solution.

$$u(\eta) = A_0 + A_1 g(\eta) + A_2 g(\eta)^2 + A_3 g(\eta)^3.$$

where $g(\eta)$ is the solution of $\frac{dg(\eta)}{d\eta} = b - g(\eta)^2$ and obtained the following values.

1.

$$A_0 = -\frac{\beta^3}{576\gamma^2}\,,\; A_1 = \frac{5\beta^2}{4\gamma}\,,\; A_2 = -15\beta\,,\; A_3 = 120\gamma\,,\; \alpha = \frac{47\beta^2}{144\gamma}\,,\; b = \frac{\beta^2}{576\gamma^2}\,,\; C_0 = -\frac{5\beta^3}{144\gamma^2}.$$

2.

$$A_0 = \frac{30\beta^3}{128\gamma^2}\,,\; A_1 = -\frac{30\beta^2}{16\gamma}\,,\; A_2 = -30\beta\,,\; A_3 = 120\gamma\,,\; \alpha = \frac{\beta^2}{16\gamma}\,,\; b = \frac{\beta^2}{64\gamma^2}\,,\; C_0 = \frac{3\beta^3}{32\gamma^2}.$$

N.A. Kudryashov in [6] solved (1) with the auxiliary equations $\left(\frac{dg(z)}{dz}\right)^2 + 4g(z)^3 - ag(z)^2 - 2bg(z) + d = 0$ and $\frac{d^2g(z)}{dz^2} + 6g(z)^2 - ag(z) - b = 0$ and obtained other values for unknowns.

C.M. Khalique in [9] solved (1) by Bernoulli equation $\frac{dh(\eta)}{d\eta} = ah(\eta) + bh(\eta)^2$ and Riccati equation $\frac{dh(\eta)}{d\eta} = ah(\eta)^2 + bh(\eta) + c$ and obtained the following values respectively for each auxiliary equations with $a = 1$, $b = 3$, $c = 1$ for Riccati equation.

1.

$$A_0 = \nu - 6a^3\gamma\,,\; A_1 = -120a^2b\gamma\,,\; A_2 = 240ab^2\gamma\,,\; A_3 = -120b^3\gamma\,,\; \alpha = a^2\gamma\,,\; \beta = 4a\gamma.$$

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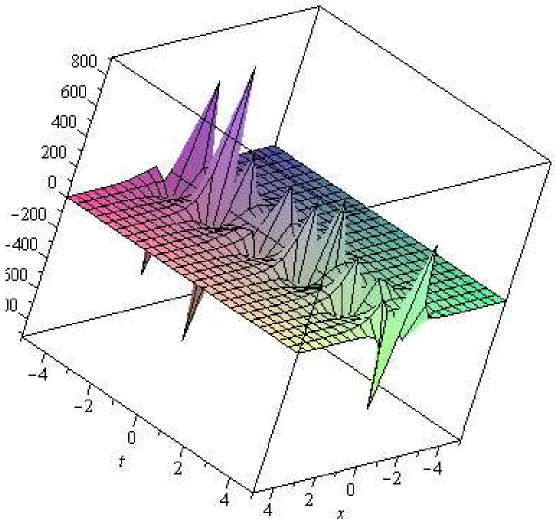
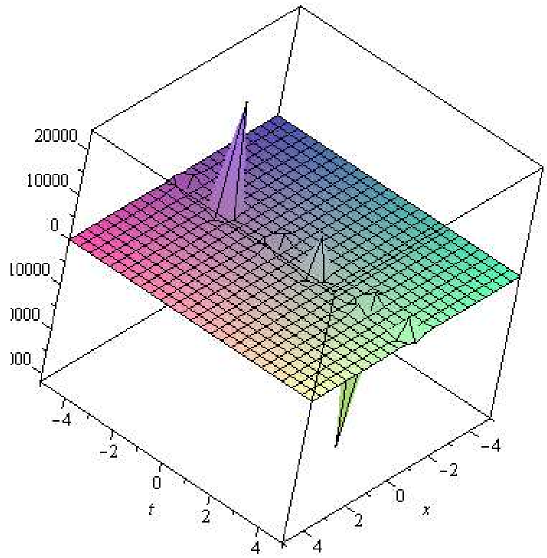


(a) Real part of $u(x,t)$ in Eq (16)



(b) Imaginary part of $u(x,t)$ in Eq (16)

Figure 3: Real and Imaginary part of Solution 7 of (1) **respectively from left to right for** $a = 5$ **and** $\mu = \gamma = 1$ **in** $x \in [-5,5]$ **and** $t \in [-5,5]$

2.

$$A_0 = -990\gamma + 60\gamma k + \nu \ , \ A_1 = 60\gamma + 180\gamma k \ , \ A_2 = 60\gamma k \ , \ A_3 = -120\gamma \ , \ \alpha = 365\gamma \ , \ \beta = -36\gamma - 4\gamma k.$$

When comparing the above values from [6] and [9] our solutions of GKSE in this work are new to the literature as far as we reviewed.

6 Constraints of solving GKSE by other methods

1. For solving GKSE (1) by generalized Kudrayshov method [22–24] mentioned in Remark 1, homogeneous balancing of (8) gives $N = M + 3$ having infinite solutions. For the least value $M = 1$ takes $N = 4$. So,

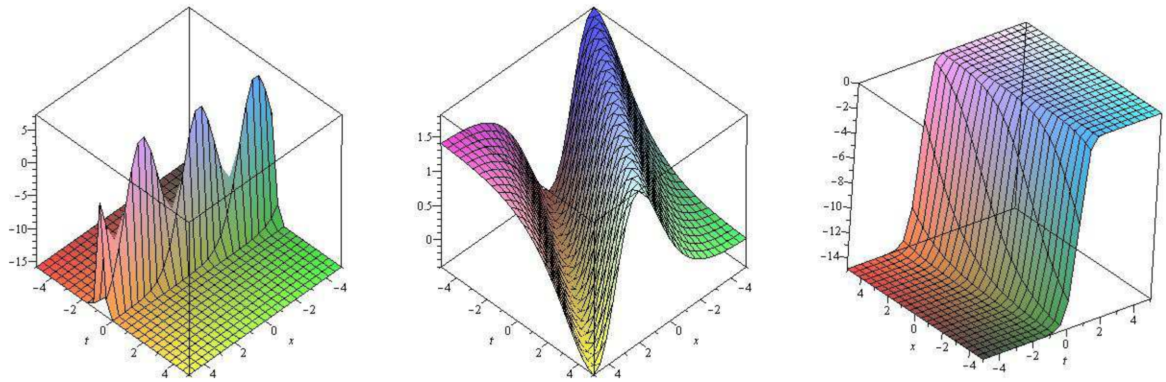
$$u(\eta) = \frac{A_0 + A_1 Q(\eta) + A_2 (Q(\eta))^2 + A_3 (Q(\eta))^3}{B_0 + B_1 Q(\eta)}.$$

where $\frac{dQ(\eta)}{d\eta} = Q(\eta)(Q(\eta) - 1)$, Applying these equations in (8) leads to the polynomial in $Q(\eta)$ and its powers. Collecting the coefficients of $Q(\eta)$ attempting to solve the equations results in continuous execution of Maple, which does not solves the overdetermined equations. However for $M = 0$ takes $N = 3$ which may solve the (1) but if we consider only the non-zero solutions of $N = M + 3$ the second case is also ignored. Therefore we believe GKSE cannot be solved by generalized Kudrayshov method.

2. Next for solving GKSE (1) by Sine-Gordon equation expansion method [26], the homogeneous balancing is same as MKM calculated $N = 3$ Thus,

$$u(\eta) = A_0 + A_1 \tanh(\eta) + B_1 \text{sech}(\eta) + A_2 \tanh^2(\eta) + B_2 \tanh(\eta)\text{sech}(\eta) + A_3 \tanh^3(\eta) + B_3 \tanh^2(\eta)\text{sech}(\eta).$$

Substituting the above $u(\eta)$ in (8) and following the steps in [26] leads to the polynomials in $\sin(w)$, $\cos(w)$, their products, powers. Collecting the coefficients and equating to zero then solving in Maple executes continulosuly. Hence we believe GKSE cannot be solved by Sine-Gordon equation expansion method.



(a) $u(x,t)$ in Eq (18) (b) $u(x,t)$ in Eq (21) (c) $u(x,t)$ in Eq (24)

Figure 4: Solution 9 for $a = 3$ **and** $\mu = \gamma = 1$**, solution 12 for** $a = 5$ **and** $\mu = \gamma = 0.5$**, solution 15 for** $a = 3$ **and** $\mu = \gamma = 0.5$ **of** (1) **respectively from left to right in** $x \in [-5,5]$ **and** $t \in [-5,5]$

7 Conclusion

The MKM is implemented for solving GKSE analytically and twenty cases of unique exact solutions are gotten for the nonlinear partial differential equation. For six set of solutions complex plots are drawn and shown their uniqueness in Figures 1 and 4. Complex solutions structures are obtained for the first time in this work and shown their simulations of real and imaginary parts in Figures 2 and 3. The constraints of other methods for solving GKSE are analysed and proved the effectiveness of MKM.

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