Massive photons propagation in gravitational field

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Abstract – A single photon inside a gravitational field defined by the accelerates g is found to have a gravitational mass given by $m_g = (\hbar/2c^3)g$, where $\hbar$ is the reduced Planck's constant, and c is the speed of light in vacuum. This force is equivalent to the curvature force introduced by Einstein's general relativity. These photons behave like the radiation emitted by a black hole. A black hole emitting such a radiation develops an entropy that is found to increase linearly with black hole mass, and inversely with the photon mass. Based on this, the entropy of a solar black hole emitting photons of mass $\sim 10^{-33}eV$ amounts to $\sim 10^{77}\,k_B$. The created photons could be seen as resulting from quantum fluctuation during an uncertainty time given by $\Delta t = c/g$. The gravitational force on the photon is that of an entropic nature, and varies inversely with the square of the entropy. The power of the massive photon radiation is found to be analogous to Larmor power of an accelerating charge.

KEYWORDS: Black holes thermodynamics; Entropic force; Electromagnetic-gravity analogy; General Relativity; Massive electrodynamics.

Introduction. – Black holes represent the ultimate state that heavy stars can end their life. The first black hole solution was obtained by Schwarzschild in 1916 employing Einstein's general relativity field equations. Although, even light can't escape from the surface of the black hole, Hawking proposed that a black hole can emit radiation like a black body radiation in thermal equilibrium, due to quantum effects near the black hole event horizon [1, 2]. The emitted radiation is model by the spontaneous pair production process that takes place near the event horizon. If one of the pairs is swallowed by the black hole, the other pair will be free to be dispatched away carrying a positive energy. Hence, we argue that a black hole emits particles and not massless radiation. This effect is reminiscent of the Josephson effect where a current (electrons) passes through the junction even though no voltage is applied ($V = 0$). However, a quantum recipe has been advocated to account for the latter effect [3]. In this work we show that the mass of the emitted particle can be related to the gravitational field in which the particle exists.

Recall that light inside a conductor is governed by a Telegraph equation that reflects an exponential decay in the amplitude of the electromagnetic fields by an amount proportional to the medium conductivity [4]. This behaviour is further shown to be analogous to the propagation of a quantum particle in space [5, 6]. The similarity dictates that the mass acts like an electric conductivity of the medium. Additionally, photons are found to behave like massive particles inside a superconducting medium. They are governed by the quantized Maxwell's equations that are shown to be gauge invariant [7]. It is shown by Acedo and Tung that the

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propagation of light in a conductor under gravity is such that its conductivity is related to the acceleration, g [8].

By combining these two analogies, we deduce the mass and acceleration relationship. Light propagating in conductor is shown to be similar to the particle radiated by a black hole. The space encapsulating the black hole would act as a conducting medium. The Higgs mechanism dictates that massless particles acquire mass when moving in a Higgs field. Consequently, the entropy of the black hole is found to be directly proportional to the black hole mass, and inversely proportional to the photon mass. The number of microstates corresponding to a given thermodynamic macroscopic state is exponentially proportional to the number of Planck's area that a black hole can contain. A single Planck's area represents a single bit of information. Using the Heisenberg uncertainty relation, the radiated power of the black hole by massive photons is $1/30\pi$ of that due to black body radiation of massless photons. The radiated power due to massive photon is found to be analogous to that of the classical Larmor power.

Black hole thermodynamics. – The temperature of the thermal radiation emitted by the black hole is given by [1, 2]

$$T_H = \frac{\hbar g}{2\pi c k_B}\,, \qquad\qquad g = \frac{c^4}{4GM}\,, \tag{1}$$

where g is the surface gravity, $\hbar = h/2\pi$ is the reduced Planck's constant, and k_B is the Boltzmann's constant. Additionally, Unruh found that an accelerating (a) observer in vacuum see a thermal radiation with a black body nature, to be emitted from the vacuum, having a temperature [9]

$$T_U = \frac{\hbar a}{2\pi c k_B}\,, \tag{2}$$

One can associate the conductivity of the medium under gravity by [8]

$$\sigma = \frac{\varepsilon_0}{c}\, g\,, \tag{3}$$

where g is the acceleration due to gravity (gravitational field). The above conductivity can be seen as the conductivity of the gravitational field (space-time) that behaves like a conducting medium. Owing to Eq.(3), the Earth gravitational field acts like a conductor with an electric conductivity, $\sigma_E = 2.894 \times 10^{-19}\,\Omega^{-1} m^{-1}$, that is exceeding low. It is however found that the photon inside a conducting medium behaves like a particle with mass given by [5, 6]

$$\sigma = \frac{2m}{\mu_0 \hbar}\,. \tag{4}$$

where σ is the electric conductivity. The massive photon is found to be governed by the quantized Maxwell's equations that are gauge invariant [7]. Therefore, Eqs.(3) and (4) define the mass of the photon in a gravitational field as

$$m_g = \frac{\hbar}{2c^3}\, g\,. \tag{5}$$

We call the mass in Eq.(5) the *gravitational mass* of the photon. It is interesting that this mass has a quantum nature. Therefore, the stronger the gravitational field, the bigger the photon mass. The photon traveling in a gravitational field due to a massive object M with radius R will acquire a mass of

$$m_g = \frac{\hbar GM}{2c^3 R^2}\,, \tag{6}$$

where $g = GM/R^2$. Using Eqs.(1) and (3), the electric conductivity of the space (medium) surrounding the black hole is

$$\sigma = \frac{c}{4\mu_0}\,\frac{1}{GM}\,.$$

Therefore, a black hole has electromagnetic properties besides its thermodynamic ones. The space around a solar black hole has a conductivity of $4.496 \times 10^{-7} S/m$. A photon with mass in a gravitational field would contribute to the overall mass of the universe. It also contributes to the dark matter that astronomers deem to exist.

The gravitational mass of the photon under Earth gravity is thus $1.916 \times 10^{-59} kg$, that is exceedingly low. The gravitational force acted on the above mass will be a non-linear gravitational force given by

$$F_g = \frac{\hbar}{2c^3}\, g^2 \,. \tag{7}$$

Using Eq.(1), the force on the massive photon near a black hole will be

$$F_g = \frac{c^5 \hbar}{32 G^2 M^2} \,.$$

The potential energy derived from general relativity is given by [10]

$$V_f = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{mc^2 r^3} \,.$$

A conservative force can then be obtained as

$$F_f = -\frac{GMm}{r^2} + \frac{L^2}{mr^3} - \frac{3GML^2}{mc^2 r^4} \,. \tag{7a}$$

The last term in the above force is attributed to the curvature of the space. Now defining the acceleration due to gravity by $g = -GM/r^2$, the above equation can be written as

$$F_f = mg + \frac{3L^2}{GMmc^2}\, g^2 + \frac{L^2}{mr^3} \,. \tag{7b}$$

The third term in the above equation can be compared with that in Eq.(7) to obtain

$$L^2 = \frac{GMm}{6c}\, \hbar \,. \tag{7c}$$

Hence, one can attribute the last term in general relativity to the photon mass. Hence, owing to general relativity, the force in Eq.(7) is a curvature force.

Equation (5) can be rewritten to give the photon mass, near a gravitating mass whose surface area A, as

$$m_g = \left(\frac{A_P}{A}\right) M \,, \qquad A_p = \frac{Gh}{c^3} \,, \tag{8}$$

where A_P is the Planck's area. The above equation suggests a quantization of the mass, M, in terms of the mass, m, *viz.*, $M = nm_g$, where $n = A/A_P$. The photon mass in the gravitational field due to the black hole, using (1) and Eq.(5), is

$$m_g = \frac{\hbar c}{8GM} \,, \qquad m_g = \frac{\hbar}{4cR_S} \,, \tag{9}$$

where $R_S = 2GM/c^2$ is the Schwarzschild radius. Hence, the photon mass near a solar black hole surface is $m_g = 1.67 \times 10^{-11} eV/c^2$. The mass M in Eq.(9) represents the maximum mass of a non interacting bosonic star, where m stands for the boson mass and $\hbar c/G = M_P^2$ [11–13]. This can be compared with a fermionic star having the Chandrasekhar mass $M_{Ch} \approx M_P^3/m_p^2$, where m_p is the mass of the proton [14]. It is interesting that if one employs the Heisenberg uncertainty relation, $\Delta x\, \Delta p > \hbar/2$, with $\Delta x = R_S$ and $\Delta p = mc$, one finds the relation $m = c\hbar/2GM$. This result can be compared with the exact result in Eq.(9).

Using Eq.(9), the above gravity force can be expressed as [2]

$$F_g = \frac{2c^3 m_g^2}{\hbar} = \frac{Gm_g^2}{r_P^2} \,, \qquad r_P = \sqrt{\frac{G\hbar}{2c^3}} \,.$$

[2] If we assume an electric force like F_g, where $g = (q/m)E$, also exists, then $F_q = \frac{\hbar q^2}{2c^3 m^2}\, E^2$. For a Schwinger electric field ($E = m^2 c^3/q\hbar$), this force becomes, $F_q = \frac{c^3 m^2}{2\hbar}$, which is not different from that due to a black force gravity, F_g. Hence, Schwinger electric field is like the black hole gravitational field.

Therefore, massive photons exist near a black hole as pairs at Planck's distance bound by gravity.

Using Eq.(9), the Compton wavelength of the emitted massive photon from a black hole is

$$\lambda_b = \frac{\hbar}{m_g c} = 4R_S\,.$$

Equation (9) states that the mass of the photon in the gravitational field of the black hole is inversely proportional to the mass of the black hole.

It is proposed that our universe today is filled with a massive radiation so that the space accelerates with an acceleration of $a_H \sim 10^{-10} m/s^2$. Such an acceleration can be connected with an emitted photons having mass of $10^{-33} eV/c^2$. This value is recently assumed to represent the mass of the graviton [15]. The mass in Eq.(5) coincides with the mass having Compton wavelength of the radius of our present universe ($m = h/Rc$).

For light climbing a gravitational field for a small distance, d, the loss in its potential energy is given by $\Delta U_g = h\Delta\nu$, where $\Delta U_g = F_g\, d$. Using Eqs.(5) with $m_g c^2 = h\nu$, one finds

$$\frac{\Delta\nu}{\nu} = \frac{gd}{c^2}\,. \tag{10}$$

Applying Eq.(5) in Eq.(10) yields the photon mass

$$m_g = \frac{\hbar}{2cd}\,\frac{\Delta\,\nu}{\nu}\,.$$

Thus, the mass energy of the photon under gravity will be

$$E_g = \frac{\hbar}{2c}\,g = \hbar\,\omega\,, \qquad\qquad \omega = \frac{g}{2c}\,, \tag{11}$$

where ω is the angular frequency of light (massive photon) in the gravitational field. This formula can be compared with the Unruh and Hawking formulae. Equations (1), (2) and (9) imply that $m_g c^2 = \pi k_B T$. Hence, the thermal radiation is now equal to particle radiation. Therefore, owing to Eq.(5), the two effects would imply that the gravitational field acts like an electrically conducting medium, where light has a non-zero effective mass. Hence, our theory says that the radiation from Unruh and Hawking is a particle radiation with mass that can be determined from Eq.(5). The emitted radiation follows a wave propagated down a transmission line that is governed by the Telegraph equation instead of the Klein-Gordon.

For the present cosmic acceleration of $a_H \sim 10^{-10}\, m/s^2$, Eq.(3) shows that the vacuum acts like a conductor with electric conductivity and resistivity as,

$$\sigma \sim 3\times 10^{-30}\,\Omega^{-1} m^{-1}\,, \qquad\qquad \rho \sim 3\times 10^{29}\,\Omega\, m\,. \tag{12}$$

These values would imply that the vacuum is a super dielectric. Moreover, the self - inductance associated with this dielectric is found to be $L = 2/(\sigma c) = 8\mu_0 GM/c^2 \sim 6\times 10^{21} H$ [16]. Hence, the electromagnetic energy embedded in this dielectric would be so huge. More over it is also shown that the diffusivity of the conductor is given by $D = 1/(\mu_0\sigma) = c^3/g$ [17]. The diffusion coefficient of the space around a black hole, using Eq.(1), will be

$$D = \frac{4GM}{c}\,.$$

The vacuum thus endows with very special electromagnetic characteristics.

In 1973 Bekenstein found that the entropy associated with a black hole of surface area A is given by [18]

$$S = \frac{c^3 A k_B}{4G\hbar}\,. \tag{13}$$

It is interesting that Eq.(13) combines the thermodynamic, relativity, quantum mechanics and gravity fundamental constants. The Bekenstein entropy can be related to the Hawking temperature by the relation

$$TdS_H = c^2 dM\,.$$

Equation (13) now reads

$$S_H=\frac{4\pi GM^2}{\hbar c}\,k_B\,,\qquad S_H=\left(\frac{M}{M_H}\right)^2k_B\,,\qquad M_H=\sqrt{\frac{\hbar c}{4\pi G}}\,,\tag{14}$$

where $A=4\pi R_s^2$, and M_H is of the order of Planck's mass. However, if the black hole radiation is that of the one in Eq.(5) (*e.g.*, massive photon), then Eqs.(8) and (13) yield

$$S_A=\frac{\pi}{2}\left(\frac{M}{m_g}\right)k_B\,,\qquad S_A=\frac{hck_B}{32Gm_g^2}\,,\tag{15}$$

where $N=\frac{M}{m_g}$ is the number of massive (photon) particles the mass M embodies. The entropy in Eq.(15) is reminiscent of the entropy of the string that is directly proportional to its mass [19]. Hence, our entropy brings a unified picture between the black hole and string paradigms. In string theory, people relate a black hole to a string with a high degree of excitation but with zero momentum. Equation (15) shows that the entropy is inversely proportional to the photon mass squared, $\propto m_g^{-2}$. Alternatively, one can say the entropy measures the number of massive photons that the mass M consists of, as evident from Eq.(15). However, the entropy in Eq.(14) counts the number of Planck's mass a given black hole consists of.

Upon using Eq.(7), Eq.(15) becomes

$$S_A=\frac{\pi}{2}\left(\frac{A}{A_P}\right)k_B\,.\tag{16}$$

We would assume that a single Planck's area contains a single bit of information. Therefore, the entropy represents the number of these information. The entropy defined in Eq.(16) counts the number of bits (information) contained in the black hole area. Therefore, the surface area of the black hole holds all the information that black hole contains.

For instance, if the mass of our Sun consists mainly of protons, then the entropy of our sun is $10^{57}\,k_B$, which measures the number of protons in our Sun ($M=2\times10^{30}kg$).

If we now assume that our universe consists of protons, then the entropy of the universe, owing to Eq.(15), will be $\sim 10^{80}k_B$. The entropy of our universe can also be estimated if one uses the photon mass $m\sim10^{-68}kg$, and $M_U\sim10^{53}kg$, in Eq.(15), which yields a value of $S_U\sim10^{121}k_B$. Note that our universe satisfy the condition for a black hole where its present mass and radius obey the Schwarzschild radius, $R_S=2GM/c^2$. Interestingly, Eq.(14) gives the same estimate as well. This agrees with the estimate found in [20, 21].

While the entropy of two systems described by Eq.(15) is additive, the one defined in Eq.(14) is not. However, with the Hawking picture, the entropy is $S_H\propto M^2$. While the black hole radiation in Hawking paradigm is a thermal radiation, the radiation in our case is particle radiation.

The internal number of microstates Ω, that a given thermodynamic state of the system has is related to the entropy by the Boltzmann formula

$$S=k_B\ln\Omega\,.\tag{17}$$

Therefore, Eqs.(15) and (16) now read

$$\Omega=e^{N\pi/2}\,,\tag{18}$$

where $N=n$.

Now one can express Eq.(5) as

$$m_gc^2\left(\frac{c}{g}\right)=\frac{\hbar}{2}\,,\qquad \Delta E\,\Delta t=\frac{\hbar}{2}\,,\tag{19}$$

that could express a Heisenberg's uncertainty relation with

$$\Delta E=m_gc^2\,,\qquad \Delta t=\frac{c}{g}\,.\tag{20}$$

The time Δt expresses the time during which the mass m_g is created. Inasmuch as g for ordinary stars is not very big, it is apparent that the creation rate is so slow, and the created radiation can hardly be detected. For instance, near the Earth surface, one finds, $\Delta t = 2.058\times 10^7 s$, and the mass energy of the emitted radiation is $1.077\times 10^{-23} eV$. For a solar black hole, one has $\Delta t = 2.949\times 10^{-5} s$, and the emitted mass energy is $3.35\times 10^{-11} eV$.

Using Eqs.(5) and (20), the radiated power of the massive photons radiation can be expressed as

$$P_\gamma = \frac{\Delta E}{\Delta t} = \frac{\hbar g^2}{2c^2}\,, \tag{21}$$

which is independent of the mass of the photon. The quantum power in Eq.(21) can be compared with the classical Larmor power due to an accelerating charge, q, that is $P_L = \frac{\mu_0 q^2}{6\pi c}\, a^2$. Hence, owing to the Equivalence Principle, one can assume that photons inside a conductor emits radiation whose power (quantum) is given by

$$P_A = \frac{\hbar a^2}{2c^2}\,, \tag{22}$$

where a is the massive photon acceleration. The above power can be called the quantum Larmor power of photons. If the massive photons radiate as Larmor power, then we equate the Larmor power with the power in Eq.(22) to obtain the charge of the massive photon, as

$$q_\gamma = \sqrt{\frac{3h}{2\mu_0 c}}\,. \tag{23}$$

It is interesting that the charge of the massive photon is a pure quantum number. This is equal to $\sqrt{3}/2$ of the Planck's charge [22]. It would be intriguing if a photon acquires mass when moves in gravitational field depending on the strength of the field, and a constant charge.

The radiation (massive photons) power of the black hole, can be obtained from Eq.(21) using Eq.(1) as

$$P_b = \frac{c^6\hbar}{32\, G^2 M^2}\,. \tag{24}$$

Therefore, for a solar black hole one has $P_b = 1.364\times 10^{-25} W$.

Using Eq.(24), one can obtain the intensity of the black hole radiation, as massive photon, as

$$I_b = \frac{c^{10}\hbar}{512\,\pi\, G^4 M^4}\,, \qquad I_b = \frac{\hbar}{2\pi c^6}\, g^4\,, \qquad g = \frac{c^4}{4GM}\,. \tag{25}$$

Using Eq.(1), Eq.(25) can be expressed as $I_b \sim \sigma\, T^4$.

The radiation power for a black hole, under the assumption of a pure photon emission, is however, given by [23]

$$P_0 = \frac{c^6\hbar}{960\,\pi\, G^2 M^2}\,, \tag{26}$$

which is equals to $1/30\pi$ less than that in Eq.(24).

Entropic force. It is recently argued that gravity can be explained as an entropic force caused by changes in the information associated with the positions of material bodies [24]. To manifest such a situation with our present formalism, we combine Eqs.(6) and (15) to obtain

$$F_g = \frac{\pi^3 c}{h}\left[\frac{Mc}{S/k_B}\right]^2\,, \qquad F_g = \frac{\pi^3 c^3 k_B^2}{h}\,\frac{M^2}{S^2}\,. \tag{27}$$

Equation (27) suggests that the force F_g can be associated with an entropic potential that is

$$F_g = -\frac{d\phi_S}{dS}\,, \qquad \phi_S = \frac{G_S M^2}{S}\,, \qquad G_S = \frac{\pi^3 c^3 k_B^2}{h}\,, \tag{28}$$

where we call G_S the entropic gravitational's constant. Using Eq.(5), one can argue that through a distance (from the horizon) $\Delta x = \frac{\hbar}{mc}$, the entropy in Eq.(15) changes by

$$\frac{\Delta S}{\Delta x} = \frac{\pi^2 c k_B M}{h} \,. \tag{29}$$

Hence, the information contained in the black hole decreases as one goes away from the black hole. Or equivalently, the information increases as one approaches the black hole, and the maximum information is contained on its surface.

Applying Eq.(29) in Eq.(27) yields

$$F_g = \frac{ch}{\pi}\frac{1}{x^2} \,. \tag{30}$$

The force per area ($4\pi x^2$) of a spherical shell around the black hole, can be expressed as

$$\frac{F_g}{A} = \frac{\hbar c}{2\pi^2 x^4} \,.$$

It is interesting that, with the condition in Eq.(29), the entropic force in Eq.(28) reduces to a quantum force. Equation (29) can be expressed as

$$F_g = -\frac{d\phi_x}{dx} \,, \qquad \phi_x = \frac{hc}{\pi\, x} \,. \tag{31}$$

The entropic force due to the entropy increase is defined as [24]

$$F_S = T\left(\frac{\Delta S}{\Delta x}\right) = \frac{mc^2}{\pi k_B}\frac{\Delta S}{\Delta x} = \frac{Mg}{4\pi} \tag{32}$$

upon using Eqs.(5) and (29), where we use the relation that $\pi k_B T = m_g c^2$. Employing Eq.(5), Eq.(32) yields

$$F_S = m_g g_c \,, \qquad g_c = \frac{Mc^3}{h} \,, \tag{33}$$

where g_c is some characteristic quantum acceleration induced by the mass M. It is the black hole gravitational force that a massive photon will experience. This force is by all measures a huge force. One prefers to call this force, the entropic force. Recall that the entropy, is in some sense, amounts to the resistance "inertia" that a given thermal system respond when subject to heat. Therefore, entropy in thermodynamics is like mass in mechanics.

For a black hole, one finds

$$F_S = \frac{c^4}{16\pi G} \,. \tag{34}$$

The induced gravitational pressure due to massive photon can be obtained as [25]

$$P_g = T\left(\frac{\partial S}{\partial V}\right)_T = \frac{m_g\, c^2}{\pi k_B}\frac{\partial S}{\partial x}\frac{\partial x}{\partial V} \,, \tag{35}$$

which upon using Eqs.(29) and (5) yields

$$P_g = \frac{M\, g}{16\pi^2 x^2} \,, \tag{36}$$

where we have considered a spherical shell of thickness Δx. Employing Eq.(5), Eq.(36) yields

$$P_g = \frac{m_g g_c}{4\pi x^2} \,. \tag{37}$$

The heat capacity of the black hole emitting massive photon is given by

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \,, \tag{38}$$

which upon using Eq.(15), and the fact that $m_gc^2 = \pi k_BT$, becomes

$$C_V = -\frac{\pi}{2}\left(\frac{M}{m_g}\right)k_B\,, \qquad C_V = -\frac{c^3Ak_B}{4\hbar G} = -S\,, \tag{39}$$

upon using Eq.(8). A negative heat capacity is a property of all astronomical objects, since their temperature will increase as they lose energy. The standard black hole heat capacity is twice this vale, assuming massless radiation. The same formulae would have agreed had we used the black hole energy $E_b = \frac{Mc^2}{2}$ instead of $E_b = Mc^2$.

For a black hole emitting massive photons, Eq.(37) yields

$$P_g = \frac{c^4}{64\pi^2G\,x^2}\,, \tag{40}$$

which is an inexorably huge pressure. Here x is the distance of the point from the black hole horizon. Therefore, an infinite pressure will forbid any particle from falling toward the event horizon of the black hole.

The intensity of the radiation (in terms of massive photons) emitted by any mass of radius R can be obtained using Eq.(5) as

$$I = \frac{\hbar G^2M^2}{8\pi c^2R^6}\,. \tag{41}$$

Hence, our Earth emits massive photons at an intensity of $1.1\times 10^{-64}W/m^2$.

Concluding remarks. – The photon (light) placed under gravity behaves like a particle with mass that depends on the gravitational acceleration. Such a force could be related to the curvature force of the Einstein's general relativity. Unlike the standard entropy for a black hole that is directly proportional to M^2, the entropy of a black hole emitting such a radiation is found to be directly proportional to its mass, M. Thus, its entropy is additive like other thermodynamic systems. The created particle near the black hole horizon takes too long time to be emitted. A single photon needs about one year to be created near the Earth surface with energy of $\sim 10^{-23}\,eV$. The gravitational force on the massive photon is found to be of a quantum character. The radiated power by massive photon is analogous to that of the classical Larmor power. Using the Heisenberg uncertainty relation, the power radiated by a black hole as massive photon is greater than that of a massless photon, due to the black body radiation, by a factor of $1/30\pi$.

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