

GEOMETRICAL INFORMATION FLOW REGULATED BY TIME LENGTHS

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The article analyzes Bernoulli's binary sequences in representation of empirical events about the distribution of natural resources and population sizes. Considering the event as a nonlinear system, and consisting of two dependent random variables, with memory and probabilities in maximum finite or infinite lengths, constant and equal to $\frac{1}{2}$ for both variables (stationary process). The expressions of the possible trajectories remain constant in sequences that are repeated alternating the presence or absence of one of the variables at each iteration (asymmetric). There are constant oscillations in the event except if the variables X_1 and X_2 are regulated as a function of time Y . It is observed that the variables X_1 and X_2 assume in time $T_k \rightarrow \infty$ specific behaviors (geometric variable) that can be used as management tools for random systems. In this way, the article seeks to know from this analyzes, the maximum entropy of information in the system by a theoretical view and how to model resources distribution or containment in the given problem.

Keywords: probabilistic systems analysis; nonlinear systems; public management; pattern formation; resources distribution, information entropy.

The author declares that he has no relevant or material financial interests that relate to the research described in this paper.

The mathematical and theory data used to support the findings of this study are included within the article and are from previously reported studies and datasets, which have been cited along the text.

1. Introduction

In analyzing the article "Methodology for Analysis of Water Consumption in School Buildings" (Telles, 2017) it can be seen that the flows of oscillations present in expenses, for example, with water in state educational establishments of Secretary of State for education at Paraná, are influenced by different variables which can assume different intensities and natures in the different regions of the State. Therefore, the proposed methodology of knowing in large samples how water consumption in schools occurs, trying to extract universal indicators that can serve as a reference for the whole State, was shown, by article, limited.

Currently, indicators such as school population size are used to estimate urban public provision of services, however, the main objective of the article (Telles, 2017) was to make it visible that the population factor itself can affect the consumption of water in a very smother way, via a continuous growth that accompanies the population, but does not indicate a direct ratio between population and consumption of the resource in a directly proportional order. The intensity in which the school population influences the consumption of water is not proportional in quantitative aspect and thus it is assumed that other variables exert their own internal mechanics in the event and generate modifications in the dynamics of the system. (Yarnold, 2015)

Although the article points out that there is scope for cross-checking of data in order to identify intensities of variables among them, it is pointed out that the combination of several data, subjectively understood in the graph lines, may not be sensitive enough to a management analysis with a view to controlling the system by producing intervention actions on the variables with each other, or in isolation, or estimating what results will be possible with precision when interfering in certain processes of the event. The method is not robust enough to explain events and variables of it in possible expressions. (Bertsimas & Thiele, 2006)

Thus, the following diagram indicates the traditional form of management analysis, in which quantitative indicators, for example, % of population in a school, are used as objective parameters whose purpose is to describe the behavior of dependent variables in a system and in the case analyzed it is stated that the larger the population, the greater the water consumption in a given population.

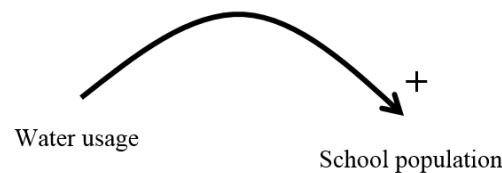


Diagram 1.

In the diagram 1, consumption is directly proportional to population. However, the results of the article (Telles, 2017) indicate that there are other variables that influence the system so that the population factor does not have enough force to produce high vibration movements in the system to generate a growth or reduction of the consumption caused by the same. In this sense, using demographic bases with per capita methodology as management indicators may be an imprecise and controversial criterion since in fact there may be hidden variables that influence the final set of the event and at the same time the analysis variable, in this case the school population. (Aven, 2016)

Thus, it would be more accurate to analyze the variables of the system as static in the sense that approaches on the quantitative aspect of frequency with which the variables interact were not considered in an exact sense (real time for each trial), that is, it is objectively analyzed by variables that reflect the binary mode system as the Bernoulli method of analysis with memory prior to the start of event and constant time length (Lan et al. 2012), so that it is possible to indicate possible sequences of interactions between variables more than the set of intensities in which the variables express themselves. (Gibbins, 1987; Uzi Harush & Baruch Barzel, 2017)

For all possible sequences of expression between variables in a system there are possible paths in which each variable assumes specific aspects in ordered repeated iterations (Butucea et al, 2015) when it is taken in observation the effect of time lengths within the event, which is observable to the manager in a theoretical and static way, leading him to decide which ways to opt for intervention effects on management risks regarding optimization for resources distribution or containment in a system (Tampieri, 2005). The analysis of this article brings not to trivial ways in calculating by the method of Bernoulli the probabilities of an event in occurrence, but the analysis of static parameters of information inside a system of constant and nonlinear binary sequences, being this last characteristic relative to the number of iterations of the system, since the binary trajectories do not express probabilistic modifications in the time, but the comparison between two binary systems with the same mathematical conditions in itself already reflect oscillations of systems of multivariate and intuitively stochastic mode. (Martin-Löf, 1971).

2. Methodology

In order to problematize the effect of maximum entropy of information (Baran *et al*, 2017) in two systems with equal characteristics, consider the analysis hypothesis in which two schools work in only one shift, in the time of 4 hours. What is the difference between the two schools relating to the size of their population (number of students) as a function of water consumption, every 15 seconds?

We will describe in the items (1), (2) and (3), different analyzes adopting different theoretical models for each one and relating time as the main factor in which iterations (frequency of interaction between variables in quantitative aspect) exert more influence on the behavior of the variables than their probabilities from binary sequences. Later, in results paragraph, each item presented for methodology will be presented accordingly.

School (system) A – 200 population

School (system) B – 1000 population

Number of drinkers in each school: 1

(1) Considering the system as ideal and not possible to have non-observable variables in the system that directly influence consumption behavior, for both systems, school A and B, in the first 15 seconds (Y) there is the possibility of a student consuming the resource (X_1) and the possibility that none will consume (X_2) (without possibility of the same individual consuming again), indicating the consumption of resources by an individual as "1" and not consumption, "0". Therefore, in a given number K of dependent iterations (function of time (short duration)):

$$\begin{array}{l}
 \text{Or,} \\
 Y \cap X_1 \\
 \text{Or as,} \\
 Y \cap X_2 \\
 Y \cap X_1 \\
 Y(K) = x_1(1,0)
 \end{array} \quad (1)$$

After the 15 seconds end, there is the second expression of the system as a potential possibility of interaction between variables. Soon (X_2 = time not determined),

$$\begin{array}{l}
 Y \cap X_2 \\
 Y(K) = x_2(0,1)
 \end{array} \quad (2)$$

Considering the conditions commented on in the article summary, the events of success and failure q , are considered as $p = X_1$ and $q = X_2$, where the variables p and q are dependent and not identically distributed (not *iid*), if we get the following formulation:

$$q = 1 - p$$

And the odds of the event follow as:

$$x_1 - 1 = x_2 \text{ with 50\% chance and } x_2 - 1 = x_1 \text{ with 50\% chance} \quad (3)$$

However, the main concern of this article is not to calculate the probability that the event occurs in time (Y) given the variables and their conditions of analysis, but to verify the behavior of the event from the point of view of the binary sequences as a method that assists also to visualize the event in its information characteristics in maximum information entropy when the probabilities between two systems are identical and not observable in Bernoulli's method regarding the behavior of the two systems among each other in the function Y . Therefore, to conceive the analysis as information entropy (Baran *et al.*, 2017) of the system, the variables considered assume an evolution in time in bits. (Mézard & Montanari, 2009).

As time T_k passes, there is a growth of the variable X_1 and X_2 revealing binary sequences that repeat cumulatively and asymmetrically in time length ($T_k \rightarrow \infty$) according to table 1:

Time	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	...
Variables	X_1	X_2	X_1	X_2	X_1	X_2	X_1	X_2	...
Bits	1	0	1	0	1	0	1	0	...
	0	1	0	1	0	1	0	1	...

Table (1)

The sequences can be described by the next equation:

$$\begin{array}{l}
 \lim_{y \rightarrow \infty} (X_{1t} + X_{2t}) = X_1 + X_2 \\
 {}_n C_r(T_k \rightarrow \infty) = (p) + (p - q) + (p - q + q) + (p - q + p - q) + (p - q + p - q + p)
 \end{array} \quad (4)$$

and so on.

The time and variables X_1 and X_2 can be represented by the sequences of binary values in the diagram 2:

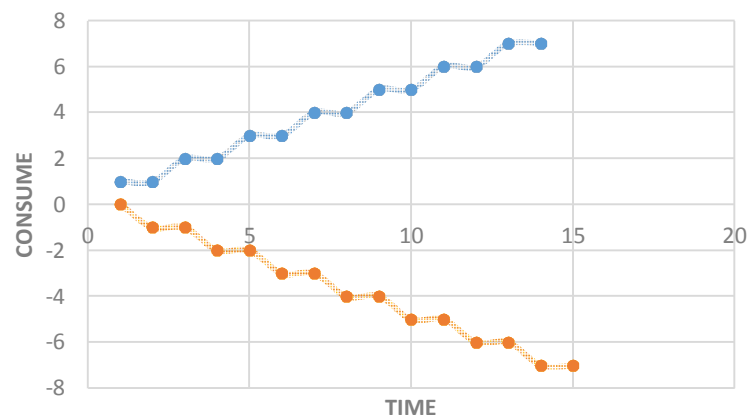
					1
				1	0
			1	0	0
		1	0	0	1
	1	0	0	1	1
1	0	0	1	1	0
0	0	1	1	0	0
	1	1	0	0	1
		0	0	1	1
		1		1	0
				0	0
				1	

Diagram (2)

Proof: The probability of the infinite sequences of iterations set to happen in values 0 and 1 is constant as k becomes infinite and remains always in the given proportion of 50% (Tsitsiklis, 2011). Where,

$$\begin{aligned} & P(Y = 0 \text{ and } 1, \forall Y) \\ & \leq P(T_1 = \dots = Y^k = 0 \text{ and } 1) = P^k \\ & \lim_{k \rightarrow \infty} P^k = 0 \text{ and } 1 \end{aligned} \quad (5)$$

The diagram 2 can be represented by the graph 1:



Graph 1.

The blue line represents the resource consumption variable X_1 and the red line represents the consumption variable X_2 . X_2 is expressed in the same manner as X_1 in the time function Y . The information sequence in which the variable X_1 and X_2 are expressed reveal common periodic oscillations in which regardless of the size of the sequence, the results will always be the same. Thus, the maximum information entropy (Baran *et al.*, 2017) of the system is finitely or infinitely constant but asymmetric in time length to the probability of variables X_1 and X_2 set to happen in which it represents the evolution of iterations (Y) performing a periodic oscillation.

The oscillations present in each system A and B caused by the small and large population do not express irregularities among themselves if compared in a short time. Thus, in real situations in which the maximum time interval in school period for students is 15 minutes, both institutions with large numbers of students and those with small numbers, the frequency of intake of the resource is the same. Therefore, the school population does not affect the consumption of water through ingestion.

Otherwise, the ratio of X_1 to X_2 is shown as increasing but asymmetric in the length of Y function as a geometric variable L :

$$\begin{aligned} X_2(0,1) &\propto X_1(1,0) \\ \forall_{X_1, X_2} \in Y(X_1 R X_2 \rightarrow \neg(X_2 R X_1)) \end{aligned} \quad (6)$$

In this sense it is possible to affirm that both the population of 200 and 1000 will have consumption and idle time defined by Y , in which, if the time is of short duration, the variable X_1 and X_2 will not have different expressions of probability and maximum entropy of information for both populations. Otherwise a result of item (2) is obtained where time affects event influencing number of sequences to happen defined by the following equation of Cauchy (1821):

$$Y = f(X_1 + X_2)$$

Where,

$$\propto \cap Y$$

Produces dynamical properties in the event as:

$$f(Y+\alpha) - f(Y)$$

Resulting in,

$$Y = f(X_1 + X_2) \text{ and } X_1 + X_2 = f(Y) \quad (7)$$

The geometric variable L affects infinitely every time $T_k \rightarrow \infty$, expressing turn shifts between variable X_1 and X_2 . This feature results in the following item (2).

The geometric variable L express no probability functions except if determined by Y , in which this article aims to associate with resources management within a system as an indicator of resources distribution frequencies. This is represented as:

$$L + 1 = \text{geometric } p$$

Where L is equal to,

$$Y = f(X_1 + X_2) + (X_1 + X_2) = f(Y) = 1 \quad (8)$$

And the probability of L_k is equal to the variables X_1 and X_2 in its probabilistic expressions as follows:

$$\begin{aligned} L + 1 = x_1 - 1 = x_2 \text{ with } 50\% \text{ chance and} \\ L - 1 = x_2 - 1 = x_1 \text{ with } 50\% \text{ chance} \end{aligned} \quad (9)$$

And for variable L with geometric distribution of $p = 1/2$, where, $P(L = n) = pq^{n-1}$, $n \in \{1, 2, \dots\}$; the entropy of L in bits is: (Cover & Thomas, 2006)

$$\begin{aligned} H(L) &= - \sum_{n=1}^{\infty} pq^{n-1} \log(pq^{n-1}) \\ &= - \left[\sum_{n=1}^{\infty} pq^n \log p + \sum_{n=0}^{\infty} npq^n \log q \right] \\ &= \frac{-p \log p}{1-q} - \frac{pq \log q}{p^2} \\ &= \frac{-p \log p - q \log q}{p} \\ &= H(p)/p \text{ bits.} \end{aligned} \quad (10)$$

If $p = 1/2$, then $H(L) = 2$ bits.

For these constant features of the system, probability and entropy for X_1 and X_2 and geometric p , other variable influence the event, the number of iterations (Y). This effect, as represented in diagram 3, counts towards amount of resources available and population size. Y can be represented for managerial purposes as a controlling device in

which two types of resources management can be achieved, that is logistical distribution or containment of expenditures.

It means, the 2 bits of information can be controlled by an external variable (time) without changing the maximum output of oscillations due to geometrical property of variables. For other types of entropy information, different behaviors will be observed. 2 bits are easier to regulate with time than, for example, 15 bits due to low length of variables distributions. Entropy information in the example given remains constant as time passes. However, it is possible to control the distribution of information (resources flow among elements) in the given system for arbitrary inputs and outputs (Uzi Harush & Baruch Barzel, 2017). The effects of regulating the event through time can cause specific effects for the phenomena. Therefore, for managerial purposes of this article, the amount of information distribution can be influenced by coupling conditions (time length or other dependent variable) with small or big intensities making possible to obtain low risks and optimal control concerning the flow of resources in a given set of elements, being this flow understood by how variables increase in interactions and interactions frequencies as time of phenomena goes infinite (see figure 1). The effect of it for real systems are observed for the coupling functions regarding bits distribution and real system quantitative aspects.

The next items (2) and (3) and results section of article, discuss the statements previously explained at item (1), analyzing mainly the management of resources and population effects in time.

(1.1) Types of information flow:
Variable *A* = resources
Variable *B* = population

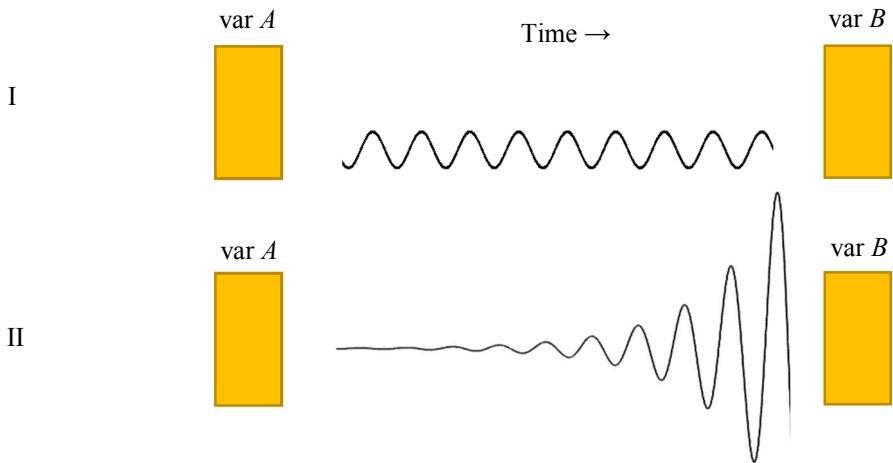


Figure 1. Information flow at geometric variables (I) and not geometric variables (II).

(1.2) Coupling effects of variables *X₁* and *X₂* towards variables *A* and *B*, regulated by time *Y* and with geometrical properties:

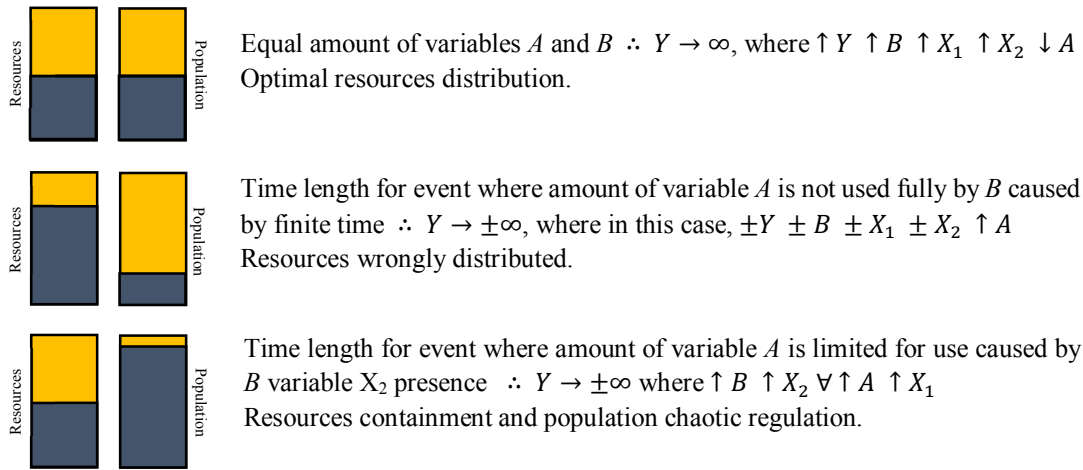
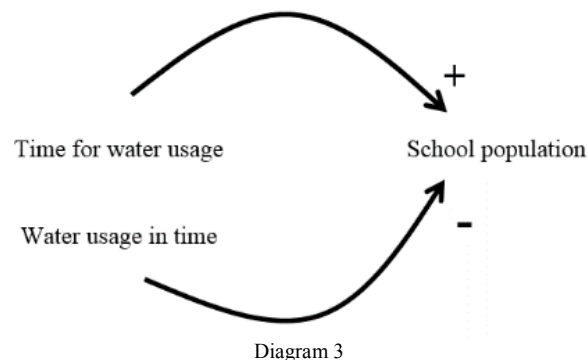


Figure 2. Couplings quantitative aspects due to information distribution regulated by time lengths.

(2) In contrast to item (1), if the time is of long duration (Y , number of iterations), it is possible to affirm that the larger the school population, the lower the water consumption in time (figure 2), in an effect of increasing the frequency with which the variable X_2 will be present in the system. Larger populations generate more void spaces (variable X_2) and soon extend resource consumption over time when compared to smaller populations. In the problem in question, a variable X_3 , defined as the number of drinkers in the place will affect the dynamics of the event, however, although there were 2 or 10 drinkers in the place, still the system has its behavior as already described. The difference in the increase in the number of drinkers is in the rate of frequency with which the water resource is consumed and the increase of frequency with which the variable X_2 of the system also expresses itself. For management purposes, it is possible to reduce the number of drinking troughs to reduce resource consumption or increase as needed. It is to be considered that a large number of drinking fountains are inefficient, as idleness in the system is a constant and very large quantities of drinking fountains would be required to provide water for an entire population of, say, 1000 students in a short number of iterations.

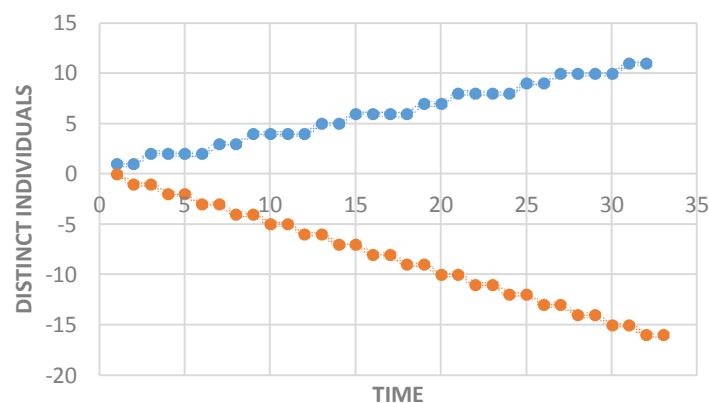
The main objective of the manager is to work with the risks and uncertainty of the system in order to analyze how the system expresses itself and to have the best decision making (Aven, 2016). The example described in this article illustrates situations that are in large numbers present in the public management of the administrative and financial scope of public institutions (Sapra, 2016; Telles *et al.* 2018). In this way it is possible to use the results of analysis in maximum entropies of information in identical systems as an indicator of how to operate the system variables.

The diagram 3 represents the item (2) regarding the behavior of variables X_1 and X_2 as a function of Y and not as a function of probabilities or entropy of event variables:



The diagram indicates that the larger the population, the longer the time for water consumption and the Y function, considering two variables X_1 and X_2 , the lower the water consumption when there are two identical systems and the same probability of occurrence of the event.

(3) Consider the use of the resource by individuals, now with the possibility of repetition, in other words, it is possible that an individual will ingest water again. Thus, it is concluded that the higher the school population, the lower the water consumption (item (2)), in an effect of increasing the frequency with which the variable X_2 will be present in the system in a certain time of analysis (item (2)) and in reason not proportional to the number of different individuals who will have access to the resource (blue line of the graph 2).



Graph 2.

The analysis of information by means of static parameters (maximum information input and output) in binary Bernoulli sequences in this sense exposed, reveals that the ratio between variable X_1 and X_2 increases as time passes, so that time is not determined (not shown in the graph) for the variable X_2 . However, this does not invalidate the analysis because it is concluded that there is no directly proportional water consumption between the number of students and the time available for water consumption since the non-consumption idle time exists and is expressed indefinitely in the system removing from the final result of the system possibilities of prediction on the previously treated question that the population directly affects the consumption of water in its quantitative aspect only.

Much more than the quantitative aspect of the population, it is the frequency (length of event) with which the variables interact that generate distortions, that is for management purposes the main objective of this research. When it is defined that oscillations can be caused by the frequency with which the variables in a system interact, it means that in addition to the above example there are other more complex interactions that promote great fluctuation in the continuous expenses of the institution, if, for example, the amount of water consumed be interpreted as a final expenditure. Other analyzes as performed previously are of great importance for the manager to have the indicators that can define the effects generated by a variable or another in the treatment of the management of a large number of administrative units in public sector.

3. Results

Example of indicators for management

The results were discussed according to the presentation of items (1), (2) and (3) in the methodology. So they are listed as item (a), (b) and (c) accordingly and are represented generally by figure 2.

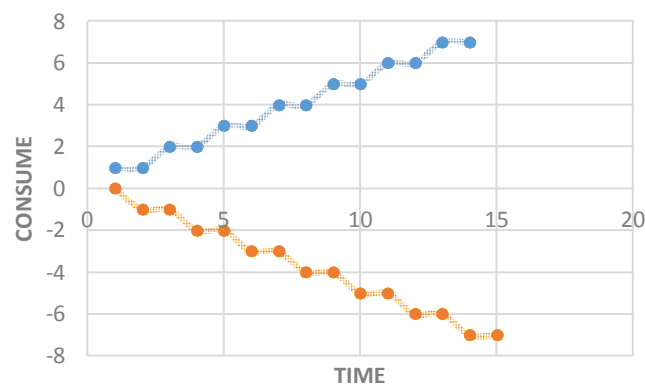
(a) At short intervals, the number of students in a school has no influence whatsoever on the amount of water consumed by ingestion.

Thus, the binary sequences were shown to be constant for both populations not reflecting possible oscillations between the two systems. In consideration of a real approximation of the event, in which there are different values in the time of water consumption and idle time, the results will be the same, since the range of variations, although diverse, will not determine in the event a sufficient force intensity so that there is a specific internal mechanics in the event which redirects the same to another logical conclusion other than that obtained in item a) of the methodology.

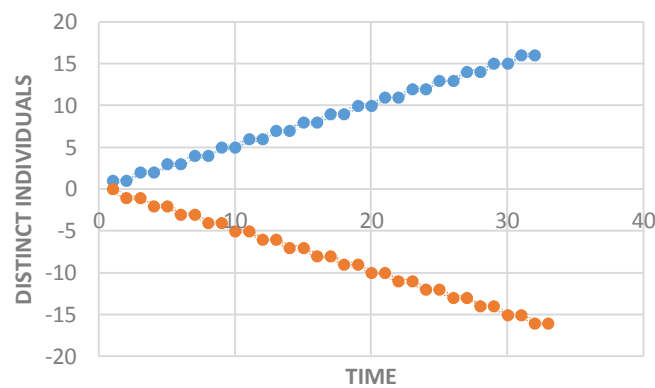
For management purposes, the system of item a) should not be modified, as the balance of the same does not reflect the possibility of major changes if an intervention is made.

(b) In events with a large time interval (determined), the larger the number of participants, the smaller the amount of resources consumed by them in time (it means higher time length is required to complete the distribution).

Relatively, the binary sequences of each group of school population express variations between them occurring in time length. However real situations in this sense in a school environment do not occur except for events that mobilize the school population for a number of hours. In the case of events as described in the previous paragraph, there are two theoretical situations for both schools A 200 students and B 1000 students. This situation can be seen in the graph 3 and 4. Note that graph 3 represents school A and graph 4 represents school B.



Graph 3.



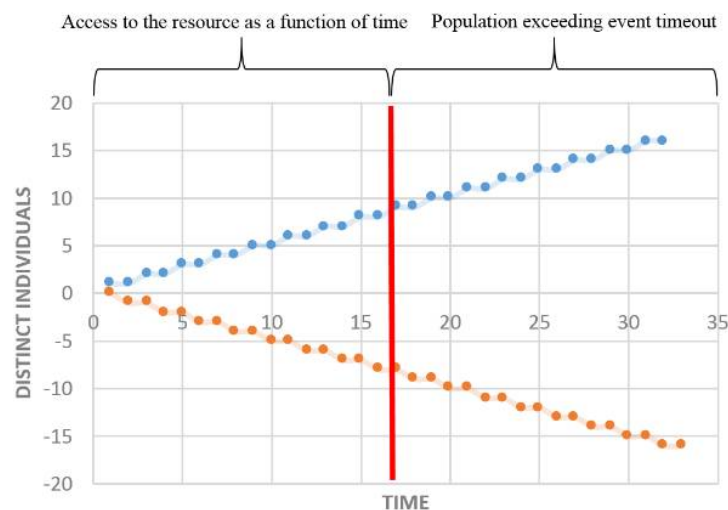
Graph 4.

Equal events in different schools (school population) necessarily consume unequal resources. School B consumes more than school A.

1 - Derivation of item b): Consider for this the supply of resource + amount of participants expected + consumption of product + time length. Following item b), the provision of a given resource to the population will have lower quality than expected if the objective is to provide a resource for the largest number of participants. The assertion can be understood by the aspect of the analysis of the binary sequences of the event, in which, as there is frequency of resource consumption, there is also the frequency of idleness for the consumption, finally generating unequal times for both variables, which are added together and generate the impossibility that in short spaces of time, large populations can consume a certain resource. It is attributed to the external variable, the time influencing X_1 and X_2 .

To correct this effect, it is necessary to extend the maximum stay time of the participants or to increase the speed of the variables that participate in the logistics of distribution and supply of the resource to the participants.

The next graph 5 represents the limiting situation above, in which at a given time (vertical red line), the population consumes a certain resource and part of the population will not have access to the resource due to lack of time or variables that influence the logistics of the system.

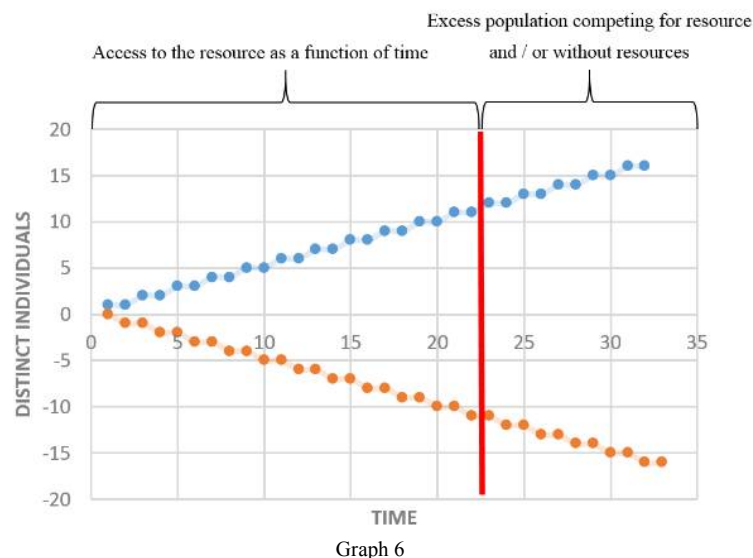


Graph 5

Consider in the above graph a maximum time of permanence in the place (red line vertical) in which the population (lines in projection of consumption of resources) demands more time so that all the individuals can have the same access to the resource.

2 - Derivation of item b): The economy of the resources offered must be maintained. Following the previous analysis, the difference lies in the logical proposition that instead of being necessary for the participants to consume the resources, objective non-consumption. Thus, the larger the population in a given location, the less time

available for everyone to consume the resource equally, considering for this the non-modification in the variables that provide the logistics of distribution and supply of the resource in the system. This example can be seen in graph 6 where the situation was simulated for observation.



(c) The analysis results for item c) are the same as in item b), differing the proportion with which larger populations will not have time to consume a resource and the management purposes of the item are analogous to item b) concomitantly.

The main results obtained for management purposes regarding the objective of this article that is understand distribution of resources for a public institution, are resumed as:

a) In a given event in which manager needs to make proper distribution of resources for all elements in a set, the proportion of resources need to be equal to the number of elements of the set, however, if time of event is relatively short to the number of elements expressions, the resources are not going to be equally distributed among all elements. Incorrect management of resources distribution will probably be reported.

b) In a given event in which manager needs to make a cut in the resources distribution for all elements in a set, the proportion of resources can be lower than the number of elements of the set, however, it is needed to adequate time of event in a way all elements and their expressions are not going to be able to supply themselves in the given time. Competition or lack of supplies for system' elements will probably be reported.

4. Discussion

The scope of the article relies not only about resources consumption, managerial or risk assessment for administrative or finance aspects in public administration but other fields in which analysis of information is set by the conditions exposed in the methodology section. For this analysis, the main approach deals with the flow of information in a nonlinear model system, intended to be used for real systems. As the number of bits remains static it is possible to adjust the entropy through time lengths resulting in many possibilities of control and assessment of a sample towards physical, chemical and biological dimensions, considering it as a broader suggestion of analysis for the specific conditions stated for solving article issue problem.

The main proposition of considering time as a tool to regulate entropy follows the outer dimensional aspect of the sample as specific variables of analysis might interact with other independent dimensional variables, as the example given in this article, the consume of water, time and outer dimensions regulated by time, population and amount of resources. The aspect of oscillation showed by samples analyzed reveals how entropy flows within the system and how evolution of process can be forwarded to the containment or distribution rules desired to be achieved as well as what phenomena of duality can be sustained by binary based events. The intervention at binary based phenomena is obtained as far as its expressions can be time regulated and dependent of outer independent dimensional variables interacting with the system. It means by axiomatic reasons, the binary nature of variable can be attributed only with the presence of independent outer dimensional variables, making possible to dismantle the binary valued orientation existence.

The main objectives for future researches in sequence to this article founds should be focused at duality phenomena control and behavior observations.

5. Conclusion

The analysis of information by binary sequences of Bernoulli allows to predict behavior of nonlinear systems in a continuous way, allowing the manager to visualize the events in their particularities of behavior. The analysis does not take place as an exhaustive methodology for understanding those type of events. However, logical propositions arising from such analyzes can be useful for planning, monitoring and controlling complex systems in order to reduce the estimation risks for these events.

The main objective of the analyzes is to allow to understand diverse variables that are located in a chain of events, which, probabilistic functions can be identical and in this way there is no visibility on the behavior of the event in differing in the time. It is considered as an unobservable factor, the analysis of the conditions in which the system is, when compared to another of the same type, and in both systems, it is verified that a geometric variable, in the case of this article, the number of iterations (time) affects the expression of possible outcomes of the event without its internal factors changing in its probabilities or entropies.

Probabilistic system analysis with geometrical properties regulated by external factor (time) is the main result of this article when analyzing an event from the perspective of binary sequences using static parameters (maximum information input and output) that constitute the event itself. Through the results achieved the manager can intuit or simulate dynamics of nature that express themselves by promoting oscillations in their final result. These resultants promote together with other variables, increasing margins of possible outcomes in a complex chain of events.

4. References

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