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Towards Energy Discretization in Quantum Cosmology

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Abstract

In this article we presented an application of the quantum cosmological model in teleparallel gravity. Working with a vacuum solution, the gravitational energy density is quantized with the Weyl procedure and we obtain a discrete expression for the gravitational energy. As an immediate consequence the empty space exhibits an expansion for an early universe.

Keywords: Teleparallel gravity; Quantum Cosmology, Inflation.

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I. INTRODUCTION

Since the seminal papers of Einstein [1] and Hubble [2] our understanding of the universe has grown enormously. Even though the pillars of modern Physics, General Relativity (GR) and quantum physics, have had their paths developed over the years separately, now a crucial challenge is to develop a quantum theory of gravitation. As early as 1930 Rosenfeld was the first to verify that there were problems quantizing gravity [3–5] and others attempts have been made until 1959 when Arnowitt, Deser and Misner proposed the Hamiltonian formulation of GR [6]. Thus the quantization of gravitation could have been achieved for instance with the help of Dirac method [7] but the concept of gravitational energy is controversy which imposes severe difficulties when to use the Hamiltonian formulation to achieve such a goal. In 1967 DeWitt, in dialogue with Wheeler, made an Einstein-Schrödinger equation [8] which later became known as the Wheeler-DeWitt equation. Throughout these 50 years this theory has had extensive study in the community, despite its intrinsic limitations, such as the so called problem of time [9–11]. Despite all this a Quantum Cosmology (QC) should be a theory that applies the concepts of quantum physics and gravity throughout in the whole universe since its birth. It should contain an expression of total energy, with fields of matter and gravity, and a quantization procedure that is clear and unambiguous.

The Teleparalelism Equivalent to General Relativity (TEGR) is an alternative gravitational theory which is dynamically equivalent to GR and allows a well-behaved definition of gravitational energy-momentum and angular momentum. Einstein himself in 1930, while investigating the unification between gravitation and electromagnetism, introduced the ideas of teleparallel gravity [12]. Møller [13], Pellegrini along with Plebanski [14] have made important contributions to the development of TEGR, among others, until in 1994 Maluf presented the Hamiltonian formulation of TEGR [15]. The important feature of TEGR concerning the definition of gravitational energy makes such a theory a natural candidate to evolve to a quantum theory of gravitation once a quantization method is used. The Weyl's prescription [16] is an interesting method to create operators from classical functions, for instance this method was used to quantize the Schwarzschild black hole [17]. Of course such a theory has to have implications on a QC. In this sense a first step was to use the gravitational energy of Friedman-Lemaitre-Robertson-Walker (FLRW) metric together with the Weyl quantization procedure to obtain an analogous Wheeler-DeWitt equation [18].

The Einstein field equations when used with the FLRW metric lead to the Friedman equations. These equations describe the evolution of the universe where a perfect fluid represent its content.

Thus a state equation for this fluid is settled to express the dominance of a specific kind of matter in the evolution of the universe. An immediately problematic situation arises, i.e., the initial conditions are undefined. Another problem is related to the flatness of universe, it means that the curvature of the universe with its content is not dominating today hence in the past it was very insignificant [19, 20]. In addition the problem of horizon means that the different regions of the universe which are not casually connected exhibit a surprisingly homogenity [20]. In order to deal with such problems the theory of inflation was proposed [21–25]. Supposedly a scalar field took place at a given moment in the beginning of the universe driving the inflation of space. There are several models that use inflation to try to answer some of the problems of the early universe, such as Natural inflation, Higgs inflation, among others [26]. Most of the inflationary models require a scalar field and several adjustable parameters to explain the anisotropy of the CMB. That is, the available inflation models do not explain very well the quantum fluctuations that are observed in the CMB [26].

The QC we have established in reference [18] has some undesirable features such as the necessity to choose the matter fields of Einstein equation before the quantization procedure, otherwise a perfect fluid solution would be undistinguishable from a vacuum solution. Hence in this article the gravitational energy-momentum tensor is quantized separately, thus it is possible to obtain a purely vacuum cosmological quantum equation which implies an expanding universe. Then such a quantum expansion is associated to the process of inflation due to purely gravitational interaction in the beginning of the universe.

The article is divided as follows. In section II the ideas of TEGR are presented and the field equations are obtained in the natural unities system $c = G = 1$. In section III we introduce the Weyl quantization and apply this to the vacuum gravitational energy establishing a cosmological quantum equation for the beginning of the universe. Finally in the last section we present our concluding remarks.

II. ON TELEPARALLELISM EQUIVALENT TO GENERAL RELATIVITY

Telepararallelism Equivalent to General Relativity is a gravitational theory dynamically equivalent to GR. In such a theory the torsion tensor is responsible for gravitation instead of curvature as in GR and it is formulated in terms of the tetrad field $e^a{}_\mu$, rather than in terms of the usual metric tensor. The tetrad field connects tensors under Lorentz symmetry and tensors under

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coordinates transformations. Thus the latin indices $a = (0), (i)$ stand for $SO(3,1)$ group while the greek indices represent the diffeomorphic group. Therefore there should be two kinds of connections one changing the derivatives of Lorentz vectors, called spin connection $\omega_{\mu ab}$, and another one that affects greek indices, $\Gamma^{\lambda}{}_{\mu\nu}$. Then by the condition $\nabla_{\mu} e^{a}{}_{\nu} = 0$ which leads to $\partial_{\mu} e^{a}{}_{\nu} - \Gamma^{\lambda}{}_{\mu\nu} e^{a}{}_{\lambda} + \omega_{\mu}{}^{a}{}_{b} e^{b}{}_{\nu} = 0$, we get $\Gamma^{\lambda}{}_{\mu\nu} = e^{a\lambda} e^{b}{}_{\nu} \omega_{\mu ab} + e^{a\lambda} \partial_{\mu} e_{a\nu}$ and as a consequence the usual definition of curvature tensor,

$$R^{\lambda}{}_{\gamma\mu\nu}(\Gamma) = \partial_{\mu} \Gamma^{\lambda}{}_{\gamma\mu} - \partial_{\nu} \Gamma^{\lambda}{}_{\gamma\mu} + \Gamma^{c}{}_{\gamma\nu} \Gamma^{\lambda}{}_{c\mu} - \Gamma^{c}{}_{\gamma\mu} \Gamma^{\lambda}{}_{c\nu},$$

yields $R^{\lambda}{}_{\gamma\mu\nu}(e,\omega) = e_{a}{}^{\lambda} e^{b}{}_{\gamma} (\partial_{\mu} \omega_{\nu ab} - \partial_{\nu} \omega_{\mu ab} + \omega_{\mu ac} \omega_{\nu cb} - \omega_{\nu ac} \omega_{\mu cb})$ [15, 18].

The torsion tensor which is defined by $T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}$ reads $T^{a}{}_{\mu\nu}(e,\omega) = \partial_{\mu} e^{a}{}_{\nu} - \partial_{\nu} e^{a}{}_{\mu} + \omega_{\mu a\nu} - \omega_{\nu a\mu}$. The spin connection $\omega_{\mu a\nu}$, the Levi-Civita connection ${}^{0}\omega_{\mu ab}$ and the contorsion tensor $K_{\mu ab}$ are related identically by $\omega_{\mu ab} = {}^{0}\omega_{\mu ab} + K_{\mu ab}$, where ${}^{0}\omega_{\mu ab} = -\frac{1}{2} e^{c}{}_{\mu} (\Omega_{abc} - \Omega_{bac} - \Omega_{cab})$ and $K_{\mu ab} = -\frac{1}{2} e_{a}{}^{\lambda} e_{b}{}^{\nu} (T_{\lambda\mu\nu} + T_{\nu\lambda\mu} + T_{\mu\lambda\nu})$ with $\Omega_{abc} = e_{a\nu} (e_{b}{}^{\mu} \partial_{\mu} e_{c}{}^{\nu} - e_{c}{}^{\mu} \partial_{\mu} e_{b}{}^{\nu})$. In order to specify this geometry into something equivalent to Riemannian geometry it is necessary to settle $\omega_{\mu ab} = 0$ which is known as the teleparallel condition. Hence it follows that $e R(e,\omega) = e R(e) + e \left(\frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - T^{a}{}_{a}\right) - 2 \partial_{\mu} (e T^{\mu})$. Consequently, the Hilbert-Einstein Lagrangian density reads $e R(e) \equiv -e \left(\frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - T^{a}{}_{a}\right) + 2 \partial_{\mu} (e T^{\mu})$, where $T^{a}{}_{a} = T^{b}{}_{b}{}^{a}$ and $T^{a}{}_{\mu\nu}(e) = \partial_{\mu} e^{a}{}_{\nu} - \partial_{\nu} e^{a}{}_{\mu}$. Then, after drooping the total divergence in the above expression, the teleparallel Lagrangian density $\mathscr{L}$ is described by

$$\begin{aligned} \mathscr{L}(e_{a\mu}) &= -k e \left(\frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - T^{a} T_{a} \right) - \mathscr{L}_{M} \\ &\equiv -k e \Sigma^{abc} T_{abc} - \mathscr{L}_{M} \end{aligned}$$

with $k = \frac{1}{16\pi}$, $\mathscr{L}_{M}$ being the lagrangian density of matter fields and Σ^{abc} defined as follows $\Sigma^{abc} = \frac{1}{4}\left(T^{abc} + T^{bac} - T^{cab}\right) + \frac{1}{2}\left(\eta^{ac} T^{b} - \eta^{ab} T^{b}\right)$ [15, 18].

Performing a functional derivative in the Lagrangian density $\mathscr{L}$ with respect to the tetrads $e^{a\mu}$, the field equations read

$$\partial_{\lambda} (e \Sigma^{a\mu\lambda}) - e \left(\Sigma^{b\lambda\mu} T_{b\lambda}{}^{a} - \frac{1}{4} e^{a\mu} T_{bcd} \Sigma^{bcd} \right) = \frac{1}{4k} e T^{a\mu}, \tag{1}$$

where $T^{a\mu}$ is the energy-momentum tensor of the matter fields. It is possible to show that these field equations can be expressed as

$$\begin{aligned} \partial_{\lambda} (e \Sigma^{a\mu\lambda}) - e \left(\Sigma^{b\lambda\mu} T_{b\lambda}{}^{a} - \frac{1}{4} e^{a\mu} T_{bcd} \Sigma^{bcd} \right) &= \frac{1}{2} e \left[R^{a\mu}(e) - \frac{1}{2} e^{a\mu} R(e) \right], \\ G_{a\mu} = R_{a\mu}(e) - \frac{1}{2} e_{a\mu} R(e) &= \frac{1}{2k} T_{a\mu}, \end{aligned}$$

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where $G_{a\mu} = e_a{}^{\nu} G_{\nu\mu}$ is the projected Einstein tensor, that is, the equivalence between GR and teleparallel gravity becomes clear. If equation (1) is rewritten then it follows $\partial_\nu \left(e\Sigma^{a\lambda\nu} \right) = \frac{1}{4} e\, e^a{}_\mu \left(t^\lambda{}_\mu + T^{\lambda\mu} \right)$, where $t^{\lambda\mu} = k\left(4\Sigma^{bc\lambda} T_{bc}{}^{\mu} - g^{\lambda\mu}\Sigma^{bcd} T_{bcd}\right)$. This tensor is interpreted as the energy-momentum tensor of the gravitational field and it is a true tensor under coordinate transformations although it is not symmetric. The tensor $\Sigma^{a\mu\nu}$ is skew-symmetric $\Sigma^{a\mu\nu} = -\Sigma^{a\nu\mu}$ which implies $\partial_\mu \partial_\nu \left(e\Sigma^{a\mu\nu} \equiv 0 \right)$, then $\partial_\lambda \left[e\, e^a{}_\mu (t^\lambda{}_\mu + T^{\lambda\mu}) \right] = 0$. This is a conservation equitation for gravitational and matter fields separately. It should be noticed that there is no such an expression in the framework of GR, although its dynamical equivalence to teleparallel gravity. Then the total energy-momentum vector is given by $P^a = \int_V d^3x\, e\, e^a{}_\mu (t^{0\mu} + T^{0\mu})$ which can be rewritten as

$$P^a = -\int_V d^3x\, \partial_j \Pi^{aj} = -\oint_S dS_j \Pi^{aj} \tag{2}$$

with $\Pi^{aj} = -4ke\Sigma^{a0j}$. This expression is invariant under coordinate transfomations but it changes depending on the reference frame since it is a vector under Lorentz symmetry [15].

III. ON QUANTUM COSMOLOGY WITH INFLATION

In this section a cosmological quantum approach is constructed for a vacuum solution which allows one to interpret the obtained expansion of the early universe as a mechanism of inflation. In this sense the gravitational energy-momentum should be quantized to achieve such a goal.

A. On Weyl quantization

The Weyl quantization is a procedure that can be used in any function including those geometrical objects described early. It is a relatively simple method that requires at least two independent parameters. Let us consider a classical system described by a function f of n variables z_l, such as the Hamiltonian, then the respective quantum operator is obtained by the Weyl transformation $W[f(z_l)]$ which is given by

$$W[f(z_l)] := \frac{1}{(2\pi\omega)^n} \int d^n k\, d^n z\, f(z_l)\, exp\left(\frac{i}{\omega} \sum_{l=1}^{n} k_l (z_l - \hat{z}_l) \right). \tag{3}$$

Here $\hat{z}_l$ are operators associated to the classical variables that obey following commutation relation $[\hat{z}_a, \hat{z}_b] = i\omega_{ab}$ where ω_{ab} is a anti-symmetric quantity. For instance the Weyl prescription is

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superior to the canonical quantization because there is no need to impose symmetrization out of the scope of the transformation. In addition when using the Weyl transformation there is no need to implement that on phase space as it is the case of canonical quantization. Thus it is the natural procedure to be used on a quantum theory of gravitation. It should be pointed out that the immediate function with physical meaning that arises for such a procedure is the very gravitational energy-momentum in teleparallel gravity [17, 18].

B. An expanding early universe

The cosmological principle asserts that the universe is homogeneous and isotropic which is realized by the FLRW metric $ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-\kappa r^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$. A possible tetrad field obtained from this metric tensor and adapted to a stationary observer is

$$e^a{}_\mu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{a}{\sqrt{1-kr^2}} & 0 & 0 \\ 0 & 0 & ar & 0 \\ 0 & 0 & 0 & ar\sin\theta \end{bmatrix}.$$

Thus the gravitational energy density can be calculated using equation (2), it yields

$$et^{0(0)} = \left(\frac{2}{16\pi}\right)\frac{(-a\sin\theta)}{\sqrt{1-kr^2}}\left[(3\dot{a}^2 - k)\,r^2 + 1\right] = \frac{(-a\sin\theta)}{8\pi}\left[\frac{(3\dot{a}^2 r^2)}{\sqrt{1-kr^2}} + \sqrt{1-kr^2}\right]$$

The zero component of P^a from equation (2) is the total energy of the system and it is denoted by

$$E = E_g + E_M\,,$$

where E_g and E_M are

$$E_g = \int d^3x\, e t^{0(0)}\,, \qquad E_M = \int d^3x\, e\, T^{(0)0}\,. \tag{4}$$

It is well known that the FLRW metric has a dynamical horizon given by [27]

$$R = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} = \frac{a}{\sqrt{\dot{a}^2 + k}}$$

where $H \equiv \frac{\dot{a}}{a}$ is Hubble parameter. Regarding the observable universe as a sphere of radius R, and integrating E_g from equation (4) is given by

$$E_g = -\frac{a}{2}\left[3\dot{a}^2\int_0^R \frac{r^2}{\sqrt{1-kr^2}}\,dr + \int_0^R \sqrt{1-kr^2}\,dr\right].$$

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Then we get straightforwardly

$$E_g = \frac{a}{4}\left[\left(\frac{3\dot{a}^2}{k}-1\right)R\sqrt{1-kR^2}-\left(\frac{3\dot{a}^2}{k}-1\right)\frac{1}{\sqrt{k}}\arctan\left(\frac{\sqrt{k}R}{\sqrt{1-kR^2}}\right)\right],$$

and using the definition of R it yields

$$k\,E_g = \frac{a^2(3\dot{a}^2-k)\sqrt{\dot{a}^2+k(1-a^2)}}{4(\dot{a}^2+k)} - \frac{a(3\dot{a}^2+k)}{4\sqrt{k}}\arctan\left[\frac{\sqrt{k}a}{\sqrt{\dot{a}^2+k(1-a^2)}}\right].$$

It is noteworthly that this expression is identically satisfied for $k=0$. Thus let us restrict our attention to $k=\pm 1$.

We suppose a quantum system governed by a Schrödinger-like equation $(k\hat{H}_g + k\hat{H}_M)\Psi = E\,k\Psi$, with $W[k\,E_g] = k\hat{H}_g$ and $W[E_M] = \hat{H}_M$. The first attempt is to consider a vacuum solution $E_M = 0$, which means a purely quantum gravitational system. Therefore we need to calculate $\hat{H}_g$, in this sense we create two auxiliary quantities which read

$$\varepsilon_g^1 = \frac{1}{4}a^2\frac{3\dot{a}^2-k}{\dot{a}^2+k}\sqrt{\dot{a}^2+k(1-a^2)}$$

and

$$\varepsilon_g^2 = -\frac{a(3\dot{a}^2+k)}{4\sqrt{k}}\arctan\left[\frac{\sqrt{k}a}{\sqrt{\dot{a}^2+k(1-a^2)}}\right].$$

If we choose the representation $W[\dot{a}] = \hat{\dot{a}} = -i\omega\frac{\partial}{\partial a}$ and $W[a] = a$, then using

$$\hat{\dot{a}}\,a^6 = -i\omega\frac{\partial}{\partial a}(a^6) = -i\omega 6a^5 - i\omega a^6\frac{\partial}{\partial a}$$

$$\hat{\dot{a}}^2 a^6 = -\omega^2\left[30a^4 + 6a^5\frac{\partial}{\partial a} + 6a^5\frac{\partial}{\partial a} + a^6\frac{\partial^2}{\partial a^2}\right] = \omega^2\left(30a^4 + 12a^5\frac{\partial}{\partial a}\right) + a^6\frac{\partial^2}{\partial a^2}$$

$$\hat{\dot{a}}^2 a^4 = -\omega^2\left(12a^2 + 8a^3\frac{\partial}{\partial a} + a^4\frac{\partial^2}{\partial a^2}\right),$$

the expression $W[\varepsilon_g^1] = \widehat{\varepsilon_g^1}$ by use the Weyl quantization 3 is given by

$$\widehat{\varepsilon_g^1} = -\frac{\sqrt{k}a^2\sqrt{1-a^2}}{4} + \frac{\omega^2}{8\sqrt{k}\sqrt{1-a^2}}\left[(42+120a^2) + (28+48a^2)a\frac{\partial}{\partial a} + (7+4a^2)a^2\frac{\partial^2}{\partial a^2}\right]. \quad (5)$$

It should be pointed out that this quantity was obtained for small values of the scale factor which corresponds to very beginning of the universe, thus the arctan function was expanded using such a condition. A similar procedure can be used to obtain $W[\varepsilon_g^2] = \widehat{\varepsilon_g^2}$ which explicitly is given by

$$\widehat{\varepsilon_g^2} = -\frac{\sqrt{k}a^2}{4} + \frac{3\omega^2}{4\sqrt{k}}\left(1 + 2a\frac{\partial}{\partial a} + a^2\frac{\partial^2}{\partial a^2} + \right). \quad (6)$$

If we consider a stationary Schrödinger-like equation in the absence of matter fields for an early universe then we have $\hat{H}\,\psi = E\psi$ and $(\widehat{\varepsilon_g^1} + \widehat{\varepsilon_g^2})\,\psi = k\,E\,\psi$, that yields after using expressions (5) and (6) to

$$\left[15\,a^2 + \frac{66}{8} + \left(6\,a^2 + \frac{76}{8}\right) a\,\frac{d}{da} + \left(\frac{a^2}{2} + \frac{31}{8}\right) a^2\,\frac{d^2}{da^2}\right] \psi = \varepsilon\,\psi\,, \tag{7}$$

with

$$\varepsilon = \frac{k^{\frac{3}{2}}}{\omega^2}\,E$$

which is a dimensionless quantity. Defining $B^2 = 992\,\varepsilon - 6159$ the solution of equation (7) is given by

$$\psi\,(a) = a^{-(45\pm B)/62}\,{}_2F_1\left(\frac{327\pm B}{124}, \frac{265\pm B}{124}; 1\pm\frac{B}{62}; -\frac{4\,a^2}{31}\right),$$

where ${}_2F_1$ is the hypergeometric function. Thus in order to get a well behaved solution the parameter of the hypergeometric function should be a negative integer

$$327 - B = -124\,n; \Rightarrow B_n = 124\,n + 327 \Rightarrow B_n^2 = 124^2\left[n^2 + 2\,\frac{327\,n}{124} + \left(\frac{327}{124}\right)^2\right]$$

as a consequence

$$992\,\varepsilon_n - 6159 = 124^2\left[n^2 + 2\,\frac{327\,n}{124} + \left(\frac{327}{124}\right)^2\right]$$

and for $n = 0 \Rightarrow 992\,\varepsilon_0 = 327^2 + 6159$ we have $\varepsilon_0 \sim 114$. This ε_0 represents the lowest level of energy in the beginning of the universe. We stress out the fact that our result is valid only in the early universe. Therefore the fundamental energy level of the universe at this stage is dependent on the magnitude of ω. Therefore it worths to analyze how such a constant could be experimentally determined. Since there is commutation relation between the operators $\hat{a}$ and $\hat{\dot{a}}$ which is $[\hat{a}, \hat{\dot{a}}] = i\,\omega$, there is also an uncertain relation between the respective measurement of the observables. Thus the present value of the scale factor establishes the limiting value for ω as the error in the determination of Hubble's constant. Thus following the value of $H_0 = 73.24 \pm 1.74\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [28], we estimate the present ω parameter as

$$\omega \sim 2\cdot 10^{-20}\,\mathrm{s}^{-1}\,.$$

It is interesting to note that the eigenvalue of $\hat{\dot{a}}$ is different from zero, that means an expanding early universe. In addition the value of k should be one in order to get a real eigenvalue of such an operator. As a consequence the quantum cosmology of the beginning of the universe predicts an expanding universe with positive curvature in the absence of matter fields. This scenario is what one should expect for the mechanism of inflation.

IV. FINAL CONSIDERATIONS

In this article we analyzed the consequences of an alternative equation for quantum cosmology which was established in the context of TEGR. The standard way to introduce QC is by means the Wheeler-DeWitt equation. However after almost half a century such an equation remains with serious limitations. We doubt that any quantization method would work in GR since the theory is invariant under Lorentz transformations. On the other hand TEGR exhibits a break in the local Lorentz symmetry. We proposed a quantization of the gravitational energy-momentum tensor which shows an expansion for an empty early universe. Such a mechanism is possibly related to inflation. For instance A. Guth assume a scalar field to explain such a conjecture [21, 22]. Others attempts to explain inflation, involving scalar fields, had been proposed such as a massive field with $|m^2| \ll H^2$ and more models [23–25, 29, 30]. We propose a quantum theory of gravitation that is enough to deal with an early expansion in the universe without any matter field. This is equivalent to an inflationary field.

For future perspectives we see the possibility to analyze the features of the graviton by interpreting the gravitational energy of the beginning of the universe as an gas of such particles. Of course we need to obtain a quantum cosmological equation for a perfect fluid and for another range of the scale factor in order to describe others stages of the universe.

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[1] A. Einstein. Die Grundlage der allgemeinen Relativitätstheorie. *Ann. Phys.*, 354(7):769–822, 1916.

[2] E. Hubble. A relation between distance and radial velocity among extra-galactic nebulae. *Proc. Natl. Acad. Sci.*, 15(3):168–173, mar 1929.

[3] L. Rosenfeld. Zur Quantelung der Wellenfelder. *Ann. Phys.*, 397(1):113–152, 1930.

[4] L. Rosenfeld. Über die Gravitationswirkungen des Lichtes. *Zeitschrift für Phys.*, 65(9-10):589–599, sep 1930.

[5] A. Rocci. On first attempts to reconcile quantum principles with gravity. *J. Phys. Conf. Ser.*, 470(1):012004, dec 2013.

[6] R. Arnowitt, S. Deser, and C. W. Misner. Dynamical Structure and Definition of Energy in General Relativity. *Phys. Rev.*, 116(5):1322–1330, dec 1959.

[7] P. A. M. Dirac. Generalized Hamiltonian Dynamics. *Proc. R. Soc. A Math. Phys. Eng. Sci.*, 246(1246):326–332, aug 1958.

[8] Bryce S. DeWitt. Quantum Theory of Gravity. I. The Canonical Theory. *Phys. Rev.*, 160(5):1113–1148, aug 1967.

[9] Asher Peres. Critique of the Wheeler-DeWitt Equation. In *Einstein's Path*, pages 367–379. Springer New York, New York, NY, 1999.

[10] E. Anderson. Problem of time in quantum gravity. *Ann. Phys.*, 524(12):757–786, dec 2012.

[11] Tatyana P. Shestakova. Is the Wheeler-DeWitt equation more fundamental than the Schrödinger equation? *Int. J. Mod. Phys. D*, 27:1841004, dec 2017.

[12] A. Einstein. Auf die Riemann-Metrik und den Fern-Parallelismus gegründete einheitliche Feldtheorie. *Math. Ann.*, 102(1):685–697, dec 1930.

[13] C Møller. Further remarks on the localization of the energy in the general theory of relativity. *Ann. Phys. (N. Y).*, 12(1):118–133, jan 1961.

[14] C. Pellegrini and J. Plebanski. Tetrad fields and gravitational fields. *Mat. Fys. Skr. Dan. Vid. Selsk.*, 2(4), 1963.

[15] José W. Maluf. The teleparallel equivalent of general relativity. *Ann. Phys.*, 525(5):339–357, may 2013.

[16] Hermann Weyl. *The theory of groups and quantum mechanics*. Dover Publications, New York, 1931.

[17] S. C. Ulhoa and R. G. G. Amorim. On Teleparallel Quantum Gravity in Schwarzschild Space-Time. *Adv. High Energy Phys.*, 2014:1–6, may 2014.

[18] A. S. Fernandes, S. C. Ulhoa, and R. G. G. Amorim. On Quantum Cosmology in Teleparallel Gravity. *J. Phys. Conf. Ser.*, 965:012014, feb 2018.

[19] Planck Collaboration. Planck 2015 results. XIII. Cosmological parameters. *Astron. Astrophys.*, 594:A13, oct 2016.

[20] Leonardo Senatore. TASI 2012 Lectures on Inflation. In *Search. New Phys. Small Large Scales*, pages 221–302. World Scientific, nov 2013.

[21] Alan H. Guth. Inflationary universe: A possible solution to the horizon and flatness problems. *Phys. Rev. D*, 23(2):347–356, jan 1981.

[22] Alan H. Guth and So-Young Pi. Fluctuations in the New Inflationary Universe. *Phys. Rev. Lett.*, 49(15):1110–1113, oct 1982.

[23] A.D. Linde. Scalar field fluctuations in the expanding universe and the new inflationary universe scenario. *Phys. Lett. B*, 116(5):335–339, oct 1982.

[24] A.D. D. Linde. A new inflationary universe scenario: A possible solution of the horizon, flatness,

homogeneity, isotropy and primordial monopole problems. *Phys. Lett. B*, 108(6):389–393, feb 1982.

[25] A. D. Linde. Chaotic inflation. *Phys. Lett. B*, 129(3-4):177–181, 1983.

[26] K.A. Olive. Review of Particle Physics. *Chinese Phys. C*, 40(10):100001, oct 2016.

[27] Valerio Faraoni. Cosmological apparent and trapping horizons. *Phys. Rev. D*, 84(2):024003, jul 2011.

[28] Adam G. Riess, Lucas M. Macri, Samantha L. Hoffmann, Dan Scolnic, Stefano Casertano, Alexei V. Filippenko, Brad E. Tucker, Mark J. Reid, David O. Jones, Jeffrey M. Silverman, Ryan Chornock, Peter Challis, Wenlong Yuan, Peter J. Brown, and Ryan J. Foley. A 2.4% determination of the local value of the Hubble Constant. *Astrophys. J.*, 826(1):56, jul 2016.

[29] John D. Barrow. The deflationary universe: An instability of the de Sitter universe. *Phys. Lett. B*, 180(4):335–339, nov 1986.

[30] John McDonald. Reheating temperature and inflaton mass bounds from thermalization after inflation. *Phys. Rev. D*, 61(8):083513, mar 2000.