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The Creep-damage Model of Salt Rock Based on Fractional Derivative

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Abstract

The use of salt rock for underground radioactive waste disposal facilities requires a comprehensive analysis of creep-damage process in salt rock. A computer-controlled creep setup is employed to carry out a creep test of salt rock lasted as long as 359 days under a constant uniaxial stress. The AE space-time evolution and energy releasing characteristics during creep test are studied in the meantime. A new creep-damage model is proposed on the basis of fractional derivative by combining the AE statistical regularity. It indicates that the AE data in non-decay creep process of salt rock can be divided into three stages. Furthermore, the parameters of new creep-damage model are determined by Quasi-Newton method. The fitting analysis suggests that the creep-damage model based on fractional derivative in this paper provides a precise description of full creep regions in salt rock.

Keywords: salt rock; creep; damage; fractional derivative; acoustic emission

Introduction

Salt rock was widely used in energy storage and radioactive waste disposal as underground engineering facilities, but hard to be predicted its mechanical behavior during long design life[1]. A research on non-decay creep-damage of salt rock is of fundamental importance to avoid the loss of effective storage volume of underground cavities.

The full non-decay creep regions in salt rock can be divided into three stages: the transient creep region (the primary region), the steady-state creep region (the secondary region) and the accelerated creep region (the tertiary region). Many efforts have been expanded on analyzing the creep-damage characteristics through mathematical modeling. Passaris[2] precisely predicted the creep deformation of salt rock by using a three element model composed of one spring and a kelvin system. Hou and Lux[3] proposed a creep constitutive model of salt rock, called Hou/Lux model, which combined theory of strain hardening and recovery, and damage and damage healing. The Hou/Lux model was applied to predict the time-dependent deformation of salt rock in the excavation disturbed zone (EDZ) of a 37-year-old underground cavity[4].

In addition, an enormous amount of effort has been devoted to application of fractional calculus

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to creep constitutive models as it has advantages in explicating the accumulation process of internal stress, reducing the parameters in constitutive model and representing the nonlinear characteristics. Zhou *et al.*[5] proposed a creep constitutive model of salt rock on the basis of time-based fractional derivative by replacing a Newtonian dashpot in the classical Nishihara model with the fractional-derivative Abel dashpot and found that the predicted results are in a good agreement with the experimental data. Wu F. improved the Maxwell creep model and established a constitutive model of salt rock based on variable-order fractional derivatives[6]. By combining ultrasonic testing (UT), Zhou *et al.*[7] introduced a variable-viscosity Abel dashpot in their new creep constitutive model. But ultrasonic testing (UT) can only reflected the two-dimensional (2D) damage information inside of salt rock.

The acoustic emission (AE) test provides a more precise three-dimensional (3D) description of damage during creep process as it records information of any given point inside the salt rock. It is widely used for rock damage testing both in laboratory and situ nowadays.

In this paper, the author completed the uniaxial compression experiment of salt rock which lasted for 359 days and analyzed AE characteristic. It indicated that the AE data in non-decay decay creep process of salt rock can be divided into three stages. Furthermore, the parameters of new creep-damage model were determined by Quasi-Newton method. The fitting analysis suggested that the creep-damage model based on fractional derivative in this paper provide a precise description of full creep regions in salt rock.

Methods

The salt rock specimen used in uniaxial compression experiment was taken from a salt mine in Pingdingshan City, Henan Province, Central China. It was drilled from the PT Well No.1 at depth of 1719m below the ground surface. The main surrounding rock mass had a salt content up to 98.14%-98.71%. The No. 7-27-17 cylindrical salt rock specimen used in uniaxial experiment was processed on a dry lathe and prepared with a required dimension of 80mm in diameter and 160mm in length.

Uniaxial compression test were carried out at Sichuan University using a computer-controlled creep setup, with uniaxial load in the range of 0-600kN. A three-dimensional real-time monitor and display system (model: PCI-2), manufactured by American Physical Acoustics Corporation was used to monitor the AE signals (Fig. 1). The preamplifier gain is 40dB, and the threshold value is settled at 35dB in order to eliminate the background noise. 8 AE sensors were installed symmetrically in the radial direction along the cylinder surface and the distance from the sensor to the nearest end surface is about 1cm (Fig. 1). The system can capture and display the acoustic emissions during the whole rock damage and failure process. To get a better effect in receiving AE signals, the AE sensors are isolated from the specimen by thin Vaseline plates. The experiment lasted from May 10, 2013 to May 4, 2014. Axial load is set to be a constant value of 17Mpa during whole failure process of salt rock and the temperature is kept at 22℃.

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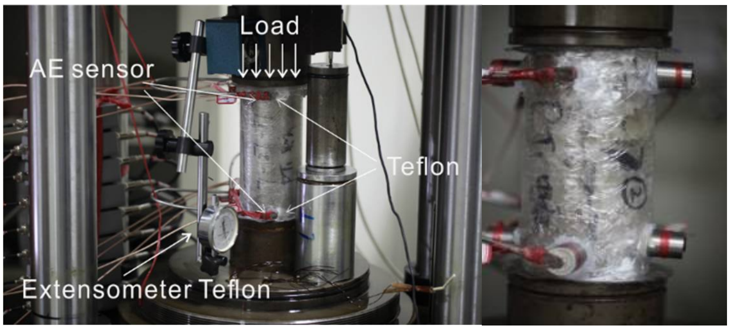

Figure 1. Experimental setup of creep test of salt rock. The applied load along y-axis remain unchanged during creep experiment. The image on right side shows the salt rock sample with AE sensors.

Results

The salt rock appeared expansion failure during creep experiment without confining pressure (Fig. 2). The deformations in salt rock specimen mainly concentrated in the middle part. A mass of penetrative cranny and discrete salt gains can be seen clearly after failure.

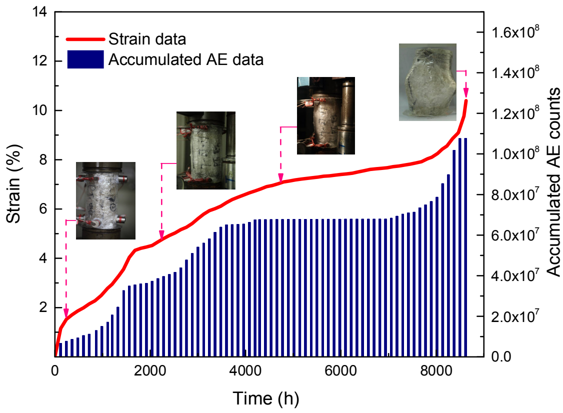

Figure 2. The deformation characters of salt rock during creep process. The red line and dark blue bars represent strain and the accumulated AE counts of salt rock, respectively. The inset images indicated the deformation characters of salt rock in different stages during creep test.

To further investigate relationship between deformation and spatial distribution of AE events during creep process. We analyze the strain rate which calculated from creep data. It highly indicated that the creep process can be divided into three stages (Fig. 3). During stage I, the strain hardening effect is stronger than strain recovery effect at room temperature, so that the strain rate curve showed the downtrend. This stage ends at about 4296 hours after test. Then, the strain rate kept steady at a relatively low level of 10^{-9}/s until 7656 hours, this stage was considered to be the steady stage of creep. After 7656 hours, the axial strain rate sharply increased, which means the creep process entered into the accelerated stage (stage III). The salt rock sample quickly deformed until damage.

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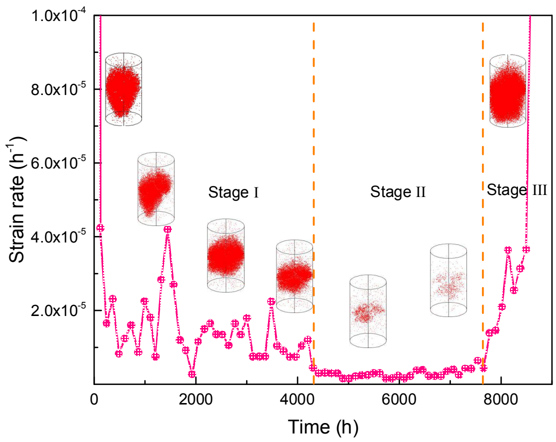

Figure 3. The strain rate and spatial distribution of AE events. The inset images show the spatial distribution characters of AE events in different creep stages.

Figure 3 also illustrated the spatial distribution of AE events in the specimen during full creep process. A set of red points represented the locations of acoustic emission events, and each point corresponds to a fracture surface or volume in the physical space. It revealed that the AE events occur mainly at the middle part of the specimen during loading process. The spatial distribution of AE events was corresponding to the distortion sections of salt rock show in Fig. 2. It also could be concluded from inset images in Fig. 3 that there were obvious differences in spatial distribution of AE events during three creep stages. During stage I, the spatial distribution of AE events was relatively extensive in middle portion and covered comparatively large volume of specimen. Then, the number of AE events markedly decreased and kept stable at a low level for about 90 days. Finally, the AE events rapidly increased until nearly covered almost the whole specimen.

The creep curves of full regions and their corresponding AE events could be observed in Fig. 4. During the initial stage of salt rock creep process (stage I), the AE events number and energy of salt rock sample was at a relatively high level and fluctuated greatly during the primary region (Fig. 4a and b). At 1536h, the AE counts rate reached the peak of 8.34×10^4/h, in the meantime, the released energy also reached the peak of 3.16×10^{-6} J. The irregular fluctuation of AE rate in this stage was mainly caused by inhomogeneous distribution of different components in nature salt rock.

The AE counts and released energy of salt rock in steady stage (stage II) was rather small contrast with the initial stage. It appeared that the fracture growth retardation so that only a few AE signals could be generated and received. During the accelerated stage (stage III), the AE events number and energy rapidly increased to the maximum value of whole creep process. The sample kept damage as strain energy released and strain rate accelerated until failure. The ultimate deformation value of salt rock was 10.396mm. Further, it could be found that AE counts and released energy of salt rock has the same trend through a comparison of the two in Fig. 4.

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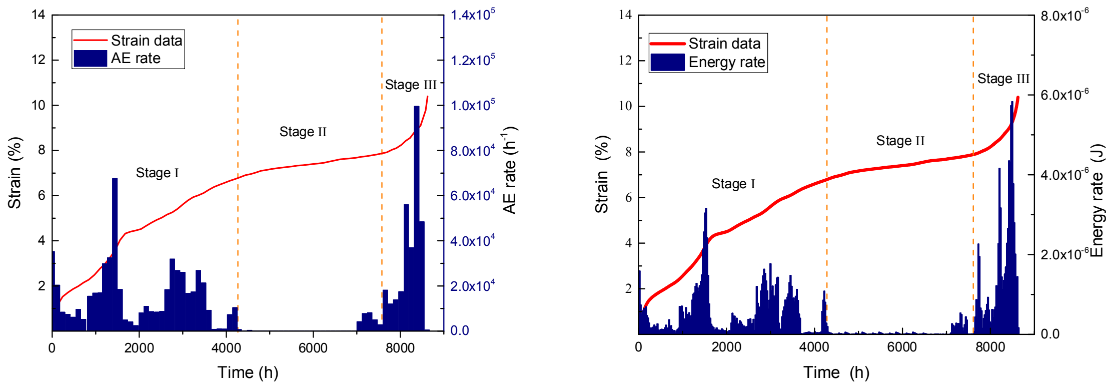

(a) (b)

Figure 4. The curves for the AE counts rate & released energy.
(a) AE counts rate vs. strain-time curve. (b) Released energy rate vs. strain-time curve. The red curves in two figures both represent the creep strain in full region. The blue columns in two figures represent AE rate and Energy rate, respectively. The creep process was divided into three stages by two orange dashed lines.

Creep-damage model of salt rock based on fractional derivative

The creep behavior of salt rock can be divided into decay creep process and non-decay creep process. The former usually happened when external load is under long-time strength of salt rock, the creep rate gradually decreased until approaching to small constant close to zero under ideal conditions. The non-decay creep process happens when external load greater than or equal to long-time strength of salt rock. The typical non-decay creep process is usually divided into three stages which are named initial transient creep period, the steady-state period and the tertiary stage (an unsteady state period). The constitutive model in this paper is aimed at study the non-decay creep process of salt rock.

The Maxwell model

The Maxwell model is the simplest viscoelasticity model, the setup of Maxwell model is by putting a spring and a dashpot together in a series. The Maxwell constitutive model is given as

$$\varepsilon(t)=\frac{\sigma_0}{E}+\frac{\sigma_0}{\eta}t \tag{1}$$

where $\sigma_0 = const$; E is the elasticity modulus, η is the viscosity coefficient, the initial condition is $\varepsilon_0 = \sigma_0 / E$ when $t = 0$, ε_0 represent the instantaneous strain of salt rock. It can only describe the ideal fluid characters of salt rock.

However, creep behavior of salt rock is the interactions among elastic, viscoelastic, viscoplastic behavior. So, the Maxwell model should be improved in order to get a better explanation on creep behavior of nature salt rocks.

The Abel dashpot: a fractional derivative element

A typical application of fractional calculus is the Abel dashpot, which is a fractional derivative description of the Newtonian dashpot. The constitutive relation of the Abel dashpot is given by[8]

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$$\sigma = \eta^{\beta} D^{\beta}[\varepsilon(t)] \quad (0 \le \beta \le 1) \tag{2}$$

where η^{β} is the viscosity coefficient and D^{β} indicates fractional differentiation. The Abel dashpot in equation (2) can be used to describe both the Newtonian dashpot in a special case of $\beta = 1$, representing an ideal fluid, and a spring in the special case of $\beta = 0$, representing an ideal solid. The Abel dashpot exhibits characteristics of both a spring and the Newtonian dashpot and eliminates the limitation of an element being solely either a spring or the Newtonian dashpot.

Considering $\sigma(t) = \sigma = const$ in equation (3), taking the fractional integral calculation of equation (2) on the basis of the Riemann–Liouville operator, we obtain

$$\varepsilon(t) = \frac{\sigma}{\eta^{\beta}} \frac{t^{\beta}}{\Gamma(1+\beta)} \quad (0 \le \beta \le 1) \tag{3}$$

where equation (3) denotes the creep strain characterized by Abel dashpot. More details on the Abel dashpot can be referred to Zhou et al.[5,7].

Maxwell model based on fractional derivative

The creep-damage behavior of salt rock has been studied through indoor creep experiment which lasted for 359 days. By replacing the Newtonian dashpot in Maxwell model with the Abel dashpot, the new constitutive relation of Maxwell creep model (Fig. 5) based on fractional derivative is given by

$$\varepsilon(t) = \frac{\sigma}{E} + \frac{\sigma}{\eta^{\beta}} \frac{t^{\beta}}{\Gamma(1+\beta)} \tag{4}$$

Equation (4) denotes the creep strain characterized by the Abel dashpot. Substituting $\sigma = 20\text{MPa}$ and $\eta^{\beta} = 8\text{GPa}\cdot\text{h}$ into equation (4), one finds a series of creep curves under the cases of different derivative orders β (Fig. 5). It can be easily observed that Maxwell creep model based on fractional derivative has advantages in describing the characters of decay creep process.

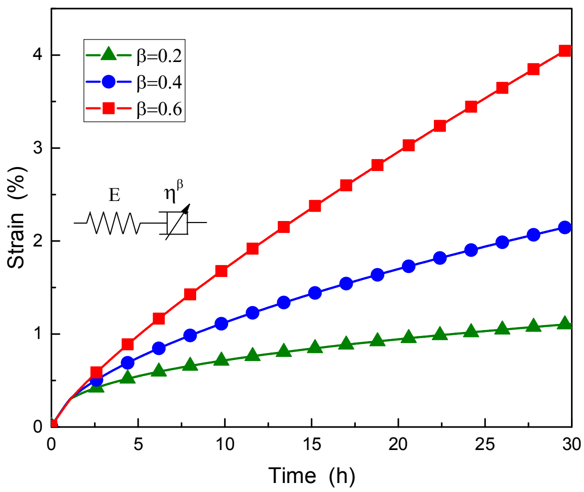

Figure 5. Creep strain of Maxwell model based on fractional derivative by: $\sigma = 20\text{MPa}$ **and** $\eta^{\beta} = 8\text{GPa}\cdot\text{h}$**.** The inset image is the Maxwell model based on fractional derivative, it composed of spring and Abel dashpot. The lines represent modeling results.

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The creep-damage model based on fractional derivative

Many researchers considered when ignored the accelerating creep stage which is caused by damaged accumulation, the creep of salt rock can be seen as decay creep process. So they usually ignored the damage in first two creep stages when established the constitutive model. But according to the discontinuity in nature salt rock, the damage is actually existed since beginning of the experiment, it directly affected the whole creep process. So when considered damage during full creep region, the total creep deformation can be concluded as the combination of two parts: primary and steady creep strains ε_{t+ss} (the decay creep process without damage) and the strain induced by damage ε_d. So the total creep deformation is given as

$$\varepsilon_c = \varepsilon_{t+ss} + \varepsilon_d \tag{5}$$

where ε_c is the total strain of creep.

By using Maxwell creep model based on fractional derivative to define the primary and steady creep strain ε_{t+ss}, the constitutive relation of ε_{t+ss} and ε_d is given as[9]

$$\varepsilon_{t+ss} = \frac{\sigma}{E} + \frac{\sigma}{\eta^\beta}\frac{t^\beta}{\Gamma(1+\beta)} \tag{6}$$

$$\varepsilon_d = A(\frac{\sigma}{1-D})^n t \tag{7}$$

where E is the elasticity modulus, η^β is the viscosity coefficient, A, n is material coefficients, D is the damage factor, $0 \le D \le 1$.

The accumulated energy of AE events was used to evaluate the damage. Considering the data from 5th sensor, the damage variable d can be defined as

$$d = \int_0^{\varepsilon_t} p_{i\varepsilon} d_\varepsilon / \int_0^{\varepsilon_c} p_{i\varepsilon} d_\varepsilon \tag{8}$$

where ε_t is the strain of the specimen at time t, ε_c is the total strain, and $p_{i\varepsilon}$ is the energy density of the 5th sensor at the loading level of ε.

By using acoustic emission data of uniaxial compression test, the energy density $p_{i\varepsilon}$ relative to the strain curve of the specimen is plotted in Fig. 6. The damage variable d at any creep strain can be calculated by integration methods. According to the least square method, the relation between damage variable d and strain at time t can be defined as

$$d = a \cdot [\varepsilon(t)]^b \tag{9}$$

The value of a and b from the fitted curve of damage variable are 0.0161 and 1.7572 respectively. It can be seen from the global evolution of damage that the damage increases slowly and steadily at beginning of the loading procedure and then increases rapidly towards the end.

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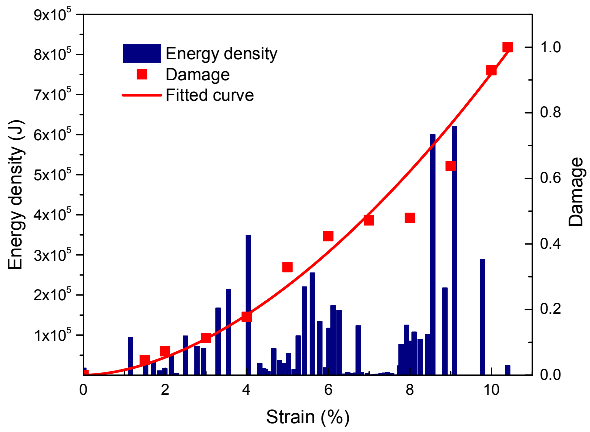

Figure 6. Curves of energy density and damage relative to strain of the salt rock sample

Synthetically, the creep-damage model based on fractional derivative is given by

$$\varepsilon(t)=\frac{\sigma}{E}+\frac{\sigma}{\eta^{\beta}}\frac{t^{\beta}}{\Gamma(1+\beta)}+A(\frac{\sigma}{1-a\cdot[\varepsilon(t)]^{b}})^{n}t \tag{10}$$

Parameter determination by fitting analysis

The efficacy of the creep-damage model based on fractional derivative is dependent on its ability to adequately fit experimental data. Using the experimental data of salt rock creep under uniaxial compression, the parameters of the creep-damage model in equation (10) can be determined by Quasi-Newton method (Fig. 7, Table 1). It is indicated that the creep-damage model based on fractional derivative proposed in this paper can adequately represent the creep deformation of salt rock and in better agreement with the experimental data than the results estimated by the Maxwell model.

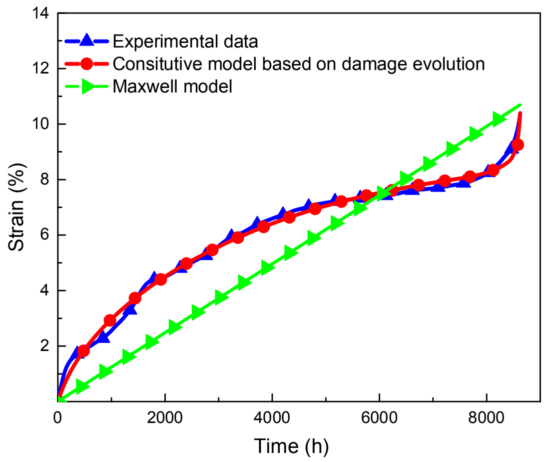

Figure 7. Experimental data and the fitting curves.

Table 1. Parameters determined by fitting analysis based on creep tests of salt rock

Model	E (GPa)	η^{β} (GPa)	β	A	n	correlation coefficient
creep-damage model	1.102	0.178	0.486	2.263×10^{-13}	0.998	0.989
Maxwell model	4.533	13.702	/	/	/	0.627

Discussion

By replacing the Newtonian dashpot in the classical Maxwell model with the variable-viscosity Abel dashpot, a new creep-damage constitutive model is proposed with AE statistical regularity. The parameters of new creep-damage model are determined by Quasi-Newton method.

The results show that the creep deformation reflected the three stages characters of non-decay creep process. Meanwhile, the AE signals can be also divided into three obvious different stages. The distribution of acoustic emission signal agreed well with the location where the deformation or growing fractures appeared in the salt rock. Further, the cumulated AE events and released energy present the same trend.

By considered damage factor during the whole process of creep, a new mathematical model which named creep-damage model based on fractional derivative was proposed. In addition, the fitting analysis suggests that the creep-damage model based on fractional derivative in this paper provides a precise description of full creep regions in salt rock.

Data availability statement:

Data generated during the study are subject to a data sharing mandate and available in supplemental appendix.

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Additional Information

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