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Solutions to Benjamin-Bona-Mahony equation in two space dimensions by various methods

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Abstract

Four methods in two different families have been constructed to derive the exact solutions to Benjamin-Bona-Mahony equation in two space dimensions. Simply defined hyperbolic tangent, hyperbolic secant and hyperbolic cosecant ansatzes and the expansion method based on the Sine-Gordon equation in two dimensions are directly substituted into the governing ODE reduced from the two dimensional BBM equation. Classical algebraic method is used to find the relations among the target parameters representing the nonzero coefficients in the predicted solutions and the wave transform parameters. Some complex and real solutions have been constructed in explicit forms.

Keywords: 2D RLW Equation; 2D BBM Equation; ansatz; series expansion method based on Sine-Gordon equation; traveling wave solution.
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1 Introduction

The Benjamin-Bona-Mahony Equation (BBME), or well-known Regularized Long Wave Equation (RLWE), in two space dimension of the form

$$u_t + p_1 u_x + p_2 u_y + p_3 u u_x + p_4 u u_y + p_5 u_{xxt} + p_6 u_{yyt} = 0 \tag{1}$$

where $p_i, 1 \leq i \leq 6$ all are non zero constant coefficients, models rossby waves in rotating waters and drifting plasma waves [1]. The collision of cylindrical pulses

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in bell shapes were studied for a particular form ($p_2 = p_4 = 0$) [2]. When the signs of pulses are the same, the collision is elastic. However, it was also reported in the same study that the collision is inelastic that cause deformation in shapes of the pulses when the signs of the pulses are opposite [2]. The similarities of the collision in one dimensional form and conserved quantity based properties for the collision were discussed in details.
Even though the BBME in two dimensions does have significance to understand rossby waves, there are few studies on solutions in the literature. Extended form of the mapping method have been implemented to (1) to reduce it to an equation in elliptic-like form and to derive various solutions covering Jacobi elliptic non degenerative and degenerative solutions [3]. Direct integration technique was also used to set some solutions in traveling solitary wave form [4]. A solution in cnoidal wave form was determined by implementation of the elliptic integral technique [5]. The properties of the uniqueness and the stability of the solution in both space variables were also proved in the same study.
In order to fill some of the gap in the literature related to the solutions of the BBME, we construct some exact solutions in traveling wave forms. We apply two different approach to derive the solutions. The first approach consists of three simple predicted solutions in various hyperbolic function forms. The finite series expansion approach is the second technique that exploits the relation between the trigonometric and hyperbolic function derived from the Sine-Gordon equation in two dimensions.

2 Methods

2.1 Simple Hyperbolic Function Ansatz Methods

Simple hyperbolic function ansatz method has various types. Mostly used forms of the method are based on hyperbolic tangent, secant and cosecant functions.
In the method, the predicted solution is assumed to be one of $A\tanh^B(.)$, $A\,\mathrm{sech}^B(.)$ or $A\,\mathrm{csch}^B(.)$ where B is positive integer and $A \neq 0$. The procedure starts by substitution of one of the predicted solutions into the governing equation. Equating the powers of the hyperbolic functions in the resultant equation is used to determine the positive integer B. The predicted solution with determined positive integer B is substituted into the governing equation again. Rearranging the same powers of the hyperbolic function with known B value, and equating the coefficients of them to zero leads a set of algebraic equations. The solution sets of this resultant algebraic equation system gives the relations among the parameters used in the ansatz, in the wave transform and in the governing equation.

2.2 Sine-Gordon Expansion Method

Consider the Sine-Gordon equation (SGE) in 2 space dimensions:

$$u_{xx} + u_{yy} - u_{tt} = m^2 \sin u, \quad m \text{ is constant} \tag{2}$$

where $u = u(x, y, t)$. The compatible simple classical traveling wave transform

$$\begin{aligned} u(x, y, t) &\to U(\xi) \\ \xi &= ax + by - \nu t \end{aligned} \tag{3}$$

reduces the SGE (2) to

$$U'' = \frac{m^2}{a^2 + b^2 - \nu^2} \sin U \tag{4}$$

where ν is the velocity parameter of the defined traveling wave by the transform [6,7]. Some simple mathematical operations converts the last equation to

$$\left((\frac{U}{2})'' \right)^2 = \frac{m^2}{a^2 + b^2 - \nu^2} \sin^2 U/2 + \tilde{K} \tag{5}$$

where $\tilde{K} = 0$ is constant of integration. Changing variable $w(\xi) = U(\xi)/2$ and assuming $m^2/(a^2 + b^2 - \nu^2) = 1$ yield

$$w' = \sin w \tag{6}$$

Thus, from (6) the following relations can be written:

$$\sin w(\xi) = \left. \frac{2ce^{\xi}}{c^2 e^{2\xi} + 1} \right|_{c=1} = \operatorname{sech} \xi \tag{7}$$

or

$$\cos w(\xi) = \left. \frac{c^2 e^{2\xi} - 1}{c^2 e^{2\xi} + 1} \right|_{c=1} = -\tanh \xi \tag{8}$$

where c is the integral constant .

3 Solutions

The two dimensional traveling wave transform $u(x, y, t) \to U(\xi), \xi = ax + by - \nu t$ reduces the BBME (1) to

$$(ap_3 + bp_4)\, U'U + (ap_1 + bp_2 - \nu)\, U' + \left(-a^2 p_5 - b^2 p_6\right) \nu\, U''' = 0 \tag{9}$$

where $'$ denotes the classical derivative w.r.t the traveling wave variable ξ.

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3.1 $\operatorname{sech}^B(.)$ solution

Consider the ODE form (9) of the BBME (1) has a solution of the form

$$U(\xi) = A \operatorname{sech}^B \xi \tag{10}$$

where A nonzero constant and B positive integer. Substitution of the predicted solution (10) into (9) and rearranging the resultant algebraic expression gives

$$\begin{aligned}
&(p_5\nu\, Aa^2B^3 + \nu\, Ap_6b^2B^3 - p_1 AaB - AB\, bp_2 + \nu\, AB) \operatorname{sech}^B \xi \\
&+ (-A^2ap_3B - p_4A^2bB) \operatorname{sech}^{2B} \xi \\
&+ (-p_5\nu\, Aa^2B^3 - \nu\, Ap_6b^2B^3 - 3\, p_5\nu\, Aa^2B^2 - 3\, \nu\, Ap_6b^2B^2 - 2\, p_5\nu\, Aa^2B - 2\, \nu\, Ap_6b^2B) \operatorname{sech}^{B+2} \xi = 0
\end{aligned} \tag{11}$$

The relation $2B = B + 2$ gives $B = 2$. Thus, the predicted solution (10) takes the form

$$U(\xi) = A \operatorname{sech}^2 \xi \tag{12}$$

Putting this solution into (9) gives

$$\begin{aligned}
&(-24\, p_5\nu\, Aa^2 - 24\, \nu\, Ap_6b^2 - 2\, A^2ap_3 - 2\, p_4A^2b) \operatorname{sech}^4 \xi \\
&+ (8\, p_5\nu\, Aa^2 + 8\, \nu\, Ap_6b^2 - 2\, p_1 Aa - 2\, Abp_2 + 2\, \nu\, A) \operatorname{sech}^2 \xi = 0
\end{aligned} \tag{13}$$

Since the predicted solution is nonzero, the coefficients of $\operatorname{sech}^4 \xi$ and $\operatorname{sech}^2 \xi$ both have to be zero. Thus, solving the algebraic system set by equating the coefficients of $\operatorname{sech}^4 \xi$ and $\operatorname{sech}^2 \xi$ to zero for A and ν gives

$$\begin{aligned}
A &= -12\, \frac{a^3p_1p_5 + a^2bp_2p_5 + ab^2p_1p_6 + b^3p_2p_6}{4\, a^3p_3p_5 + 4\, a^2bp_4p_5 + 4\, ab^2p_3p_6 + 4\, b^3p_4p_6 + ap_3 + bp_4} \\
\nu &= \frac{ap_1 + bp_2}{4\, a^2p_5 + 4\, b^2p_6 + 1}
\end{aligned} \tag{14}$$

for arbitrary a and b used in the traveling wave transform. Using these relations, the solution to the BBME (1) can be written as

$$u(x, y, t) = -12\, \frac{a^3p_1p_5 + a^2bp_2p_5 + ab^2p_1p_6 + b^3p_2p_6}{4\, a^3p_3p_5 + 4\, a^2bp_4p_5 + 4\, ab^2p_3p_6 + 4\, b^3p_4p_6 + ap_3 + bp_4} \operatorname{sech}^2 \left(ax + by - \frac{ap_1 + bp_2}{4\, a^2p_5 + 4\, b^2p_6 + 1} t \right) \tag{15}$$

3.2 $\tanh^B(.)$ solution

Consider the ODE form (9) of the BBME (1) has a solution of the form

$$U(\xi) = A \tanh^B \xi \tag{16}$$

where A nonzero constant and B positive integer. Substitution of the predicted solution (16) into (9) and rearranging the resultant algebraic expression gives

$$\begin{aligned}&\left(-AB^3a^2\nu\,p_5-\nu\,p_6B^3Ab^2+3\,AB^2a^2\nu\,p_5+3\,\nu\,p_6B^2Ab^2-2\,ABa^2\nu\,p_5-2\,\nu\,p_6BAb^2\right)\tanh^{B-3}\xi\\&+\left(3\,AB^3a^2\nu\,p_5+3\,\nu\,p_6B^3Ab^2-3\,AB^2a^2\nu\,p_5-3\,\nu\,p_6B^2Ab^2+2\,ABa^2\nu\,p_5+2\,\nu\,p_6BAb^2+p_1BAa+ABp_2b-\nu\,BA\right)\tanh^{B-1}\xi\\&+\left(-3\,AB^3a^2\nu\,p_5-3\,\nu\,p_6B^3Ab^2-3\,AB^2a^2\nu\,p_5-3\,\nu\,p_6B^2Ab^2-2\,ABa^2\nu\,p_5-2\,\nu\,p_6BAb^2-p_1BAa-ABp_2b+\nu\,BA\right)\tanh^{B+1}\xi\\&+\left(AB^3a^2\nu\,p_5+\nu\,p_6B^3Ab^2+3\,AB^2a^2\nu\,p_5+3\,\nu\,p_6B^2Ab^2+2\,ABa^2\nu\,p_5+2\,\nu\,p_6BAb^2\right)\tanh^{B+3}\xi\\&+\left(A^2Bap_3+p_4BA^2b\right)\tanh^{2B-1}\xi\\&+\left(-A^2Bap_3-p_4BA^2b\right)\tanh^{2B+1}\xi=0\end{aligned}\tag{17}$$

Choosing $2B-1=B+1$ gives $B=2$. Thus, the predicted solution takes the form

$$U(\xi)=A\tanh^2\xi\tag{18}$$

Substituting the power determined predicted solution (18) into the (9) gives

$$\begin{aligned}&\left(24\,Aa^2\nu\,p_5+24\,Ab^2\nu\,p_6-2\,A^2ap_3-2\,A^2bp_4\right)\tanh^5\xi\\&+\left(-40\,Aa^2\nu\,p_5-40\,Ab^2\nu\,p_6+2\,A^2ap_3+2\,A^2bp_4-2\,Aap_1-2\,Abp_2+2\,A\nu\right)\tanh^3\xi\\&+\left(16\,Aa^2\nu\,p_5+16\,Ab^2\nu\,p_6+2\,Aap_1+2\,Abp_2-2\,A\nu\right)\tanh\xi=0\end{aligned}\tag{19}$$

Since the predicted solution is assumed non-trivial, then the coefficients of powers of tanh(.) should be zero by the polynomial equality. Solving the resultant algebraic system of equations for A and ν gives

$$\begin{aligned}A&=-12\,\frac{a^3p_1p_5+a^2bp_2p_5+ab^2p_1p_6+b^3p_2p_6}{8\,a^3p_3p_5+8\,a^2bp_4p_5+8\,ab^2p_3p_6+8\,b^3p_4p_6-ap_3-bp_4}\\\nu&=-\frac{ap_1+bp_2}{8\,a^2p_5+8\,b^2p_6-1}\end{aligned}\tag{20}$$

Thus the solution to the BBME(1) is expressed as

$$u(x,y,t)=-12\,\frac{a^3p_1p_5+a^2bp_2p_5+ab^2p_1p_6+b^3p_2p_6}{8\,a^3p_3p_5+8\,a^2bp_4p_5+8\,ab^2p_3p_6+8\,b^3p_4p_6-ap_3-bp_4}\tanh^2\left(ax+by+\frac{ap_1+bp_2}{8\,a^2p_5+8\,b^2p_6-1}t\right)\tag{21}$$

3.3 $\operatorname{csch}^B(.)$ solution

Consider the ODE form (9) of the BBME (1) has a solution of the form

$$U(\xi)=A\operatorname{csch}^B\xi\tag{22}$$

where A nonzero constant and B positive integer. Substitution of the predicted solution (22) into (9) and rearranging the resultant algebraic expression gives

$$\begin{aligned}&\left(AB^3a^2\nu\,p_5+AB^3b^2\nu\,p_6-AB\,ap_1-AB\,bp_2+AB\,\nu\right)\operatorname{csch}^B\xi\\&+\left(-A^2B\,ap_3-A^2B\,bp_4\right)\operatorname{csch}^{2B}\xi\\&+\left(AB^3a^2\nu\,p_5+AB^3b^2\nu\,p_6+3\,AB^2a^2\nu\,p_5+3\,AB^2b^2\nu\,p_6+2\,AB\,a^2\nu\,p_5+2\,AB\,b^2\nu\,p_6\right)\operatorname{csch}^{B+2}\xi=0\end{aligned}\tag{23}$$

It can be deduce easily that $2B = B + 2$ gives $B = 2$. Thus, the predicted solution (22) takes the form

$$U(\xi) = A\,\mathrm{csch}^2\,\xi \tag{24}$$

Substitution of the power determined predicted solution (24) into (9) gives

$$\begin{aligned}&\left(24\,Aa^2\nu\,p_5 + 24\,Ab^2\nu\,p_6 - 2\,A^2ap_3 - 2\,A^2bp_4\right)\mathrm{csch}^4\xi\\&+\left(8\,Aa^2\nu\,p_5 + 8\,Ab^2\nu\,p_6 - 2\,Aap_1 - 2\,Abp_2 + 2\,\nu\,A\right)\mathrm{csch}^2\xi = 0\end{aligned} \tag{25}$$

Since the predicted solution is assumed non-trivial, the coefficients of powers of csch(.) should be zero. Thus, The resultant system of equations is solved algebraically to give

$$\begin{aligned}A &= 12\,\frac{a^3p_1p_5 + a^2bp_2p_5 + ab^2p_1p_6 + b^3p_2p_6}{4\,a^3p_3p_5 + 4\,a^2bp_4p_5 + 4\,ab^2p_3p_6 + 4\,b^3p_4p_6 + ap_3 + bp_4}\\ \nu &= \frac{ap_1 + bp_2}{4\,a^2p_5 + 4\,b^2p_6 + 1}\end{aligned} \tag{26}$$

The relations among A, ν and other parameters used in the governing equation and in the traveling wave transform allows to write the solution to (1) as

$$u(x,y,t) = 12\,\frac{a^3p_1p_5 + a^2bp_2p_5 + ab^2p_1p_6 + b^3p_2p_6}{4\,a^3p_3p_5 + 4\,a^2bp_4p_5 + 4\,ab^2p_3p_6 + 4\,b^3p_4p_6 + ap_3 + bp_4}\,\mathrm{csch}^2\left(ax + by - \frac{ap_1 + bp_2}{4\,a^2p_5 + 4\,b^2p_6 + 1}t\right) \tag{27}$$

3.4 Solution set derived by Sine-Gordon expansion method

Consider the ODE reduced form (9) of the BBME (1) has a solution of the form

$$U(w) = A_0 + \sum_{i=1}^{N} \cos^{i-1}(w)\left(B_i \sin w + A_i \cos w\right) \tag{28}$$

where A_0 and $A_i, B_i, 1 \leq i \leq N$ are the coefficients to be determined with the condition that at least A_N or B_N is nonzero. The homogeneous balance between UU' and U''' leads $N = 2$. Thus, the predicted solution (28) takes the form

$$U(w) = A_0 + \sum_{i=1}^{2} \cos^{i-1}(w)\left(B_i \sin w + A_i \cos w\right) \tag{29}$$

Substituting the predicted solution with determined N (29) into (9) yields

$$
\begin{aligned}
&\left(-a^2\nu\, B_2p_5 - b^2\nu\, B_2p_6\right)\sin(w(\xi))(\cos(w(\xi)))^4 \\
&+\left(8\,a^2\nu\, A_2p_5 + 8\,b^2\nu\, A_2p_6 - 2\,aA_2{}^2p_3 + aB_2{}^2p_3 - 2\,bA_2{}^2p_4 + bB_2{}^2p_4\right)(\sin(w(\xi)))^2(\cos(w(\xi)))^3 \\
&+\left(-a^2\nu\, B_1p_5 - b^2\nu\, B_1p_6 + aA_1B_2p_3 + aA_2B_1p_3 + bA_1B_2p_4 + bA_2B_1p_4\right)\sin(w(\xi))(\cos(w(\xi)))^3 \\
&+\left(18\,a^2\nu\, B_2p_5 + 18\,b^2\nu\, B_2p_6 - 4\,aA_2B_2p_3 - 4\,bA_2B_2p_4\right)(\sin(w(\xi)))^3(\cos(w(\xi)))^2 \\
&+\left(4\,a^2\nu\, A_1p_5 + 4\,b^2\nu\, A_1p_6 - 3\,aA_1A_2p_3 + 2\,aB_1B_2p_3 - 3\,bA_1A_2p_4 + 2\,bB_1B_2p_4\right)(\sin(w(\xi)))^2(\cos(w(\xi)))^2 \\
&+(aA_0B_2p_3 + aA_1B_1p_3 + bA_0B_2p_4 + bA_1B_1p_4 + aB_2p_1 + bB_2p_2 - \nu\, B_2)\sin(w(\xi))(\cos(w(\xi)))^2 \\
&+\left(-16\,a^2\nu\, A_2p_5 - 16\,b^2\nu\, A_2p_6 - aB_2{}^2p_3 - bB_2{}^2p_4\right)(\sin(w(\xi)))^4\cos(w(\xi)) \\
&+\left(5\,a^2\nu\, B_1p_5 + 5\,b^2\nu\, B_1p_6 - 2\,aA_1B_2p_3 - 2\,aA_2B_1p_3 - 2\,bA_1B_2p_4 - 2\,bA_2B_1p_4\right)(\sin(w(\xi)))^3\cos(w(\xi)) \\
&+\left(-2\,aA_0A_2p_3 - aA_1{}^2p_3 + aB_1{}^2p_3 - 2\,bA_0A_2p_4 - bA_1{}^2p_4 + bB_1{}^2p_4 - 2\,aA_2p_1 - 2\,bA_2p_2 + 2\,\nu\, A_2\right)(\sin(w(\xi)))^2\cos(w(\xi)) \\
&+(aA_0B_1p_3 + bA_0B_1p_4 + aB_1p_1 + bB_1p_2 - \nu\, B_1)\sin(w(\xi))\cos(w(\xi)) + \left(-5\,a^2\nu\, B_2p_5 - 5\,b^2\nu\, B_2p_6\right)(\sin(w(\xi)))^5 \\
&+\left(-2\,a^2\nu\, A_1p_5 - 2\,b^2\nu\, A_1p_6 - aB_1B_2p_3 - bB_1B_2p_4\right)(\sin(w(\xi)))^4 \\
&+(-aA_0B_2p_3 - aA_1B_1p_3 - aA_2B_2p_3 - bA_0B_2p_4 - bA_1B_1p_4 - bA_2B_2p_4 - aB_2p_1 - bB_2p_2 + \nu\, B_2)(\sin(w(\xi)))^3 \\
&+(-aA_0A_1p_3 - bA_0A_1p_4 - aA_1p_1 - bA_1p_2 + \nu\, A_1)(\sin(w(\xi)))^2 + (aA_2B_2p_3 + bA_2B_2p_4)\sin(w(\xi)) = 0
\end{aligned}
\tag{30}
$$

Equation the coefficients of all trigonometric terms to zero gives a system of algebraic equations. The solution of this resultant system gives

$$
\begin{aligned}
&\nu = \frac{A_2\,(ap_3 + bp_4)}{12\,a^2p_5 + 12\,b^2p_6}, \\
&A_0 = -\frac{8\,a^3A_2p_3p_5 + 8\,a^2bA_2p_4p_5 + 8\,ab^2A_2p_3p_6 + 8\,b^3A_2p_4p_6 + 12\,a^3p_1p_5 + 12\,a^2bp_2p_5 + 12\,ab^2p_1p_6 + 12\,b^3p_2p_6 - aA_2p_3 - bA_2p_4}{12\,(a^2p_5 + b^2p_6)\,(ap_3 + bp_4)}, \\
&A_1 = 0,\ A_2 = A_2,\ B_1 = 0,\ B_2 = 0
\end{aligned}
\tag{31}
$$

where A_2 is arbitrary. The solution determined from the relations given above is set as

$$
\begin{aligned}
u(x,y,t) = &-\frac{8\,a^3A_2p_3p_5 + 8\,a^2bA_2p_4p_5 + 8\,ab^2A_2p_3p_6 + 8\,b^3A_2p_4p_6 + 12\,a^3p_1p_5 + 12\,a^2bp_2p_5 + 12\,ab^2p_1p_6 + 12\,b^3p_2p_6 - aA_2p_3 - bA_2p_4}{12\,(a^2p_5 + b^2p_6)\,(ap_3 + bp_4)} \\
&+ A_2\tanh^2\left(ax + by - \frac{A_2\,(ap_3 + bp_4)}{12\,a^2p_5 + 12\,b^2p_6}t\right)
\end{aligned}
\tag{32}
$$

for arbitrary nonzero A_2. Another solution set of algebraic system of equations can be found as

$$
\begin{aligned}
&\nu = \frac{iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6)}, \\
&A_0 = -\frac{5\,iB_2a^3p_3p_5 + 5\,iB_2a^2bp_4p_5 + 5\,iB_2ab^2p_3p_6 + 5\,iB_2b^3p_4p_6 + 6\,a^3p_1p_5 + 6\,a^2bp_2p_5 + 6\,ab^2p_1p_6 + 6\,b^3p_2p_6 - iB_2ap_3 - iB_2bp_4}{6\,(a^2p_5 + b^2p_6)\,(ap_3 + bp_4)}, \\
&A_1 = 0,\ A_2 = B_2i,\ B_1 = 0,\ B_2 = B_2
\end{aligned}
\tag{33}
$$

where $i = \sqrt{-1}$ and B_2 is arbitrarily chosen nonzero constant. These relation among the parameters gives the solution

$$
\begin{aligned}
u(x,y,t) = &-\frac{5\,iB_2a^3p_3p_5 + 5\,iB_2a^2bp_4p_5 + 5\,iB_2ab^2p_3p_6 + 5\,iB_2b^3p_4p_6 + 6\,a^3p_1p_5 + 6\,a^2bp_2p_5 + 6\,ab^2p_1p_6 + 6\,b^3p_2p_6 - iB_2ap_3 - iB_2bp_4}{6\,(a^2p_5 + b^2p_6)\,(ap_3 + bp_4)} \\
&- B_2\,\mathrm{sech}\left(ax + by - \frac{iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6})t\right)\tanh\left(ax + by - \frac{iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6})t\right) \\
&+ iB_2\tanh^2\left(ax + by - \frac{iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6})t\right)
\end{aligned}
\tag{34}
$$

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where B_2 is arbitrarily chosen nonzero constant. The last solution derived from the solution of the algebraic system of equations as

$$\nu = \frac{-iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6)}$$
$$A_0 = -\frac{-5\,iB_2a^3p_3p_5 - 5\,iB_2a^2bp_4p_5 - 5\,iB_2ab^2p_3p_6 - 5\,iB_2b^3p_4p_6 + 6\,a^3p_1p_5 + 6\,a^2bp_2p_5 + 6\,ab^2p_1p_6 + 6\,b^3p_2p_6 + iB_2ap_3 + iB_2bp_4}{6\,(a^2p_5 + b^2p_6)\,(ap_3 + bp_4)}$$
$$A_1 = 0,\; A_2 = -B_2i,\; B_1 = 0,\; B_2 = B_2 \tag{35}$$

Thus, the last solution to the BBME (1) is constructed as

$$\begin{aligned} u(x,y,t) = &-\frac{-5\,iB_2a^3p_3p_5 - 5\,iB_2a^2bp_4p_5 - 5\,iB_2ab^2p_3p_6 - 5\,iB_2b^3p_4p_6 + 6\,a^3p_1p_5 + 6\,a^2bp_2p_5 + 6\,ab^2p_1p_6 + 6\,b^3p_2p_6 + iB_2ap_3 + iB_2bp_4}{6\,(a^2p_5 + b^2p_6)\,(ap_3 + bp_4)} \\ &- B_2\tanh\left(ax + by - \frac{-iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6)}t\right)\operatorname{sech}\left(ax + by - \frac{-iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6)}t\right) \\ &- B_2 i\tanh^2\left(ax + by - \frac{-iB_2\,(ap_3 + bp_4)}{6(a^2p_5 + b^2p_6)}t\right) \end{aligned} \tag{36}$$

for arbitrarily chosen B_2.

4 Conclusion

Solutions to the BBME equation defined in two space dimensions are determined by implementation of two different approaches. Simple traveling wave transform in two dimensions reduces the BBME to some nonlinear ODE with integer ordered derivatives. In the first approach, simple hyperbolic function ansatzes are used to derive the solutions. Following some simple algorithms, some solutions in forms of powers of tanh (.), sech (.) and csch (.) functions are constructed. In the second approach, the predicted solutions are assumed to be a finite series of multiplications some hyperbolic functions. The relation between the hyperbolic and trigonometric functions obtained by the Sine-Gordon equation in two space dimension is the significant key to set the solutions. It should be emphasized that some solutions derived by the Sine-Gordon expansion method are in complex traveling wave forms.

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