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Accounting Games: Using Matrix Algebra in Creating the Accounting Models

Abstract

The aim of this paper is to show how the mathematical basis is used for a precise treatment of double-entry bookkeeping, which was firstly developed in the nineteenth century by Sir William Rowan Hamilton. This is done by using basic notions of matrix algebra with the roots to the idea of using ordered pairs. We also reveal how the complex numbers and the rationals (fractions) developed in a mainstream of accounting science and became a leading platform for the ongoing processes within the Industry 4.0.

The paper concludes with the examples on how accounting operations can be represented by matrix equations with a result of the essential report generating. The author represents a mathematical model of accounting which is independent of the specific existential forms but is capable to undertake the form of any of them and thus has the prospect of being understood and accepted by specialists globally.

Keywords: Matrix Algebra, Accounting, Industry 4.0.

Introduction

Sangster and Scataglinibelghitar (2010) in their research address the evidence of accounting mathematical origins. Earlier, the review of the literature on matrix accounting was provided by Leech (1986). Described publications gave understanding on how accounting matrices transformed typical approaches to accounting practice. Most named publications described the convenience of using mathematics in accounting practice.

Later, Stoner and Vysotskaya (2012) described practice of teaching accounting via a form of matrix mathematics at Southern Federal University,

Russia. By giving an alternative explanation and understanding of the main principles of accounting this method provided useful insights for accounting educators.

This article is structured as follows. The next section provides an overall literature review on the matrix accounting field, which is followed by sections describing the matrix modeling approaches, and the game based example. The paper concludes on the usefulness of the suggested approach.

Literature Review

Matrix algebra approach to accounting starts with a matrix framework for accounting introduced by Augustus De Morgan in 1846. Further, this approach was developed along with the development of PC in relevant software which allowed changing accounting practice making it faster and more reliable.

Through the decades, the authors were making attempts to apply different mathematical approaches to accounting modeling (see, for example, De Morgan A. (1846), Churchill N. (1964), Demski J., FitzGerald S. (2008), Didier Leclère et Jean-Guy Degos (1990, 1995), Gardner M. Jones. (1965), Sorter G.H. (1969), Schroderheim G. (1964), Williams T. (1964), Mattessich R. (1994), Mattessich R., Galassi G.. (2000), Mephram M.J. (1988)). Most of the papers are devoted to the matrix algebra applications to accounting and linked accounting practice with the matrix algebra approach.

Situational-matrix approach served as an alternative language for accounting during the process of it technological transition (Mephram M.J., 1988, Kolvakh O., 1996; Kolvakh O., 1999; Kolvakh O., 2000, Vysotskaya A., 2011, Vysotskaya A. and Aleshin V., 2013, Vysotskaya A., 2014, Sbitneva S., 2013).

In his papers (see, for example: Kolvakh, 1996, 1999, 2000) O. Kolvakh examines the group of accounting matrix models, calling them "situational-matrix accounting". This approach appears to be able to combine situational (or event driven) nature of accounting with the means of matrix algebra, and, consequently, serve as a prototype of a mathematical model of accounting.

Research conducted by Vysotskaya A. (2011, 2014) eliminates the requirement of the modern accounting practice to be suitable for operation using information technologies (IT) and suggests using this approach in planning practice, e.g. tax planning, etc.

Finally, Stoner and Vysotskaya (2012) examined the relevance of the approach to accounting teaching practice suggesting that alternative mathematical methods of the transaction and report representation be useful to students in providing them understanding of accounting basics. Later research by Vysotskaya, Kolvakh and Stoner (2016) represented the mathematical accounting game used in teaching basic accounting course in Russia. This technique is useful for teaching students how to understand basics of recording of business transactions through the explanation with mathematics based algorithmic methods. Such kind of competence is important and was outlined by Smirnova, Sokolov and Emmanuel (1995), Sangster (2010), Vysotskaya, Kolvakh and Stoner (2016).

There is certain recognition of accounting teaching cases based on games theory since American Institute of CPAs supports teachers in their activities in engaging, entertaining and educating high school students¹ by launching different successful methods used in classroom.

The development of Information technologies (IT) influence on accounting started the debate on the importance of the use of modeling in accounting. In their report (2010), the leadership programme of the ICAEW Financial Reporting Faculty argued on the assumption that one way of relating the theory of the firm and accounting measurement is via firms' business models. Along with stating that the business model in non-financial reporting, the report sheds light on the potential importance of business models in both financial and non-financial reporting and define this as a topic of emerging interest.

Matrix accounting modelling using blocks of matrices

¹ <https://www.aicpa.org/press/pressreleases/2014/aicpa-creates-interactive-online-accounting-game-for-educators.html>

The methods of accounting modeling (from recording transactions to financial reporting) basically repeat the steps implemented in practice, by using certain numerical examples. Still the visible variety of accounting techniques complicates accounting system conceivability. The main assumption here relates to the evidence that one of the effective tools of creating accounting models can be mathematical modeling, particularly, the one which uses basic notions and operations of matrix algebra.

By using matrix algebra we found it possible to:

1. Record transactions and to present the Ledger on their basis in the form of equivalent matrices.
2. Transform the initial data into Trial Balances corresponding to their equivalents in the system of matrix algebra transactions.
3. Relate the opening and closing account balances by means of the basic accounting equation in a matrix form.
4. Determine the formulae to receive the trial Balance.
5. Determine matrix model as a system of equivalents of the data presented by corresponding Trial Balances.

The foundations laid into the system considered below are the well-known categories of accounting such as correspondence of accounts and accounting entry. In this paper they are not defined in conventional terms but are represented as mathematical concepts and operations of matrix algebra (Vysotskaya and Aleshin, 2013). We provide these definitions below.

Definition 1. The correspondence matrix is a square matrix $E(X,Y)$ with the dimensions $m \times m$, where the point of intersection of the debit account X and the credit account Y is equal to 1, and other elements of the matrix are equal to zero. The correspondence matrix itself is indicated as $E(X,Y)$, and its non-zero element is always equal to 1 ($E_{X,Y}=1$).

Definition 2. The entry matrix represents its connection with the sum of transaction by a correspondence matrix, as follows:

$$\mathbf{M}(X,Y) = S_{x,y} \cdot \mathbf{E}(X,Y), \quad (1)$$

The represented definitions of the correspondence matrix and entry matrix are here axioms which provide the foundations of further conceptual theory of matrix accounting. Here we use them as a basis for mathematical constructions with the elements of matrix algebra. Further, we obtain formulae to receive the Ledger Matrix (**LM**), as follows:

$$LM = \sum_{i=1}^n S_i \cdot \mathbf{E}(X_i, Y_i), \quad (2)$$

Where i – is the number of the entry in the journal, S_i – is the sum of transactions corresponding to entry i ; $\mathbf{E}(X_i, Y_i)$ – a correspondence matrix referring to the entry i .

The formula of the Matrix of Debit Turnovers (**MDT**) - is obtained from **TM**, as follows:

$$\mathbf{MDT} = \sum_{X=c_1}^{c_m} \sum_{Y=c_1}^{c_m} S_{X,Y} \cdot \mathbf{E}(X, Y), \quad (3)$$

Here $S_{X,Y} = \sum_{i_{XY}=1}^{n_{X,Y}} S_{i_{XY}}$ - is the total sum of the transaction referred to the given

correspondence of accounts X, Y . In this case $\sum_{X=c_1}^{c_m} \sum_{Y=c_1}^{c_m} n_{XY} = n$, where n represents the total number of the entries in the Ledger (the total of n_{XY} of similar XY). The correspondence of accounts is equal to the total number of entries n in the Ledger.

The formula of the Matrix of Credit Turnovers (**MCT**) is a transposed matrix **MDT**, as follows:

$$\mathbf{MCT} = \mathbf{MDT}' = \sum_{Y=c_1}^{c_m} \sum_{X=c_1}^{c_m} S_{X,Y} \cdot \mathbf{E}(Y, X)$$

Here $X, Y := c_1, c_2, \dots, c_m$ are accounting symbols used to represent the matrix model of an accounting system, where $S_{X,Y} = \sum_{i_{XY}=1}^{n_{X,Y}} S_{i_{XY}}$ is the total sum of the transaction referred to the accounts X, Y . In this case $\sum_{X=c_1}^{c_m} \sum_{Y=c_1}^{c_m} n_{XY} = n$, where n – is the total number of entries in the Ledger.

Here an example for the trial balance in the form of "Assets-Liabilities-Capital"- grouping (ALC blocks) is represented as an (A) matrix and is structured as a block of matrices, which consists of nine types of the blocks of matrices, as follows:

AA - the matrix of "assets-assets" transactions;

AL – the matrix of "assets-liabilities" transactions;

LA – the matrix of "liabilities- assets" transactions;

LL - the matrix of "liabilities-liabilities" transactions;

CA - the matrix of "capital-assets" transactions;

CC – the matrix of "capital-capital" transactions.

The basic equation of the trial balance (MTB) using the ALC-grouping² is represented as follows:

$$\text{MTB} = \begin{pmatrix} \Delta \text{AA}_0 & \Delta \text{AL}_0 & \Delta \text{AC}_0 \\ \Delta \text{LA}_0 & \Delta \text{LL}_0 & \Delta \text{LC}_0 \\ \Delta \text{CA}_0 & \Delta \text{CL}_0 & \Delta \text{CC}_0 \end{pmatrix} + \begin{pmatrix} \text{AA} & \text{AL} & \text{AC} \\ \text{LA} & \text{LL} & \text{LC} \\ \text{CA} & \text{CL} & \text{CC} \end{pmatrix} - \begin{pmatrix} \text{AA}' & \text{LA}' & \text{CA}' \\ \text{AL}' & \text{LL}' & \text{CL}' \\ \text{AC}' & \text{LC}' & \text{CC}' \end{pmatrix} = \begin{pmatrix} \Delta \text{AA}_1 & \Delta \text{AL}_1 & \Delta \text{AC}_1 \\ \Delta \text{LA}_1 & \Delta \text{LL}_1 & \Delta \text{LC}_1 \\ \Delta \text{KA}_1 & \Delta \text{KO}_1 & \Delta \text{CC}_1 \end{pmatrix} \quad (\text{A})$$

By multiplying MTB on the corresponding block vector \mathbf{e} , the equation of the General Ledger appears:

$$\begin{pmatrix} \Delta \mathbf{a}_0 \\ \Delta \mathbf{l}_0 \\ \Delta \mathbf{c}_0 \end{pmatrix} + \begin{pmatrix} \text{AA} & \text{AL} & \text{AC} \\ \text{LA} & \text{LL} & \text{LC} \\ \text{CA} & \text{CL} & \text{CC} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{e}_A \\ \mathbf{e}_L \\ \mathbf{e}_C \end{pmatrix} - \begin{pmatrix} \text{AA}' & \text{LA}' & \text{CA}' \\ \text{AL}' & \text{LL}' & \text{CL}' \\ \text{AC}' & \text{LC}' & \text{CC}' \end{pmatrix} \cdot \begin{pmatrix} \mathbf{e}_A \\ \mathbf{e}_L \\ \mathbf{e}_C \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{a}_1 \\ \Delta \mathbf{l}_1 \\ \Delta \mathbf{c}_1 \end{pmatrix} \quad (\text{A1});$$

Further, we provide the Trial Balance equation, as follows:

$$\begin{pmatrix} \Delta \mathbf{a}_0 \\ \Delta \mathbf{l}_0 \\ \Delta \mathbf{c}_0 \end{pmatrix} + \begin{pmatrix} \mathbf{aa} + \mathbf{al} + \mathbf{ac} \\ \mathbf{la} + \mathbf{ll} + \mathbf{lc} \\ \mathbf{ca} + \mathbf{cl} + \mathbf{cc} \end{pmatrix} - \begin{pmatrix} \mathbf{aa}' + \mathbf{la}' + \mathbf{ca}' \\ \mathbf{al}' + \mathbf{ll}' + \mathbf{cl}' \\ \mathbf{ac}' + \mathbf{lc}' + \mathbf{cc}' \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{a}_1 \\ \Delta \mathbf{l}_1 \\ \Delta \mathbf{c}_1 \end{pmatrix} \quad (\text{B});$$

² Here the «0» subscript sign means the beginning of the period $t-1=0$, the «1» sign means the end of the period $t=1$.

From (B) we can receive the Trial Balance matrix with a certain structural change, as follows:
$$\begin{pmatrix} \overline{\mathbf{aa}} \\ \overline{\mathbf{ll}} \\ \overline{\mathbf{cc}} \end{pmatrix} - \begin{pmatrix} \overline{\mathbf{aa}} \\ \overline{\mathbf{ll}} \\ \overline{\mathbf{cc}} \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{a}_{aa} \\ \Delta \mathbf{l}_{ll} \\ \Delta \mathbf{c}_{cc} \end{pmatrix} \quad (\text{B1});$$

Thus, the modification of the equation (B2) appears which is connected with the closing liabilities, assets and capital accounts:
$$\begin{pmatrix} \overline{\mathbf{al}} \\ \overline{\mathbf{la}} \\ \overline{\mathbf{cl}} \end{pmatrix} - \begin{pmatrix} \overline{\mathbf{la}} \\ \overline{\mathbf{al}} \\ \overline{\mathbf{lc}} \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{a}_{al} \\ \Delta \mathbf{l}_{la} \\ \Delta \mathbf{c}_{cl} \end{pmatrix} \quad (\text{B2});$$

The modification (B3) of the represented equation appears with closing the capital flow, assets and liabilities accounts:
$$\begin{pmatrix} \overline{\mathbf{ac}} \\ \overline{\mathbf{lc}} \\ \overline{\mathbf{ca}} \end{pmatrix} - \begin{pmatrix} \overline{\mathbf{ac}} \\ \overline{\mathbf{cl}} \\ \overline{\mathbf{ac}} \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{a}_{ac} \\ \Delta \mathbf{l}_{lc} \\ \Delta \mathbf{c}_{ca} \end{pmatrix} \quad (\text{B3}).$$

These blocks are built on the nine possible types of transactions. By providing (B1), (B2), (B3) equations we represent the formulae for the quantitative evaluation of the dynamics of the Trial Balance.

The structure of the transposed matrix is formed in accordance with the rules of block matrices transposing:

$$\text{MCT} = \text{MDT}' = \begin{pmatrix} \overline{\mathbf{AA}} & \overline{\mathbf{AL}} & \overline{\mathbf{AC}} \\ \overline{\mathbf{LA}} & \overline{\mathbf{LL}} & \overline{\mathbf{LC}} \\ \overline{\mathbf{CA}} & \overline{\mathbf{CL}} & \overline{\mathbf{LL}} \end{pmatrix}' = \begin{pmatrix} \overline{\mathbf{AA}'} & \overline{\mathbf{LA}'} & \overline{\mathbf{CA}'} \\ \overline{\mathbf{AL}'} & \overline{\mathbf{LL}'} & \overline{\mathbf{CL}'} \\ \overline{\mathbf{AC}'} & \overline{\mathbf{LC}'} & \overline{\mathbf{CC}'} \end{pmatrix}$$

The transposed matrix (MCT) contains the data from the transactions with inverted correspondences of the accounts used.

Accounting game by means of matrix modelling

This part of the paper provides a case statement and its solution for the accounting game played during accounting classes at Southern Federal University, Russia and was described in 2016 by Vysotskaya, Kolvakh and Stoner in the accounting education research paper. This game is based on the matrix accounting modelling system described earlier.

We assume that three players A, B, and C play some game the result of which will be some money reward (or loss). The game is played in pairs and can be run between any pairs of the players: A and B, A and C, B and C.

We also assume that as a result of the game, all players owe each other certain sums of money and there is somebody (e.g. an accountant) who registers emerging liabilities during a certain period of time (for example, during a month) in the order of their occurring during the period considered, for example, in the chronological order.

The fact that liabilities occur can be registered as a description, for instance, in the next form (resembling a diary):

1. September, 4 – player A won the sum of money of 10 c.u. from the player B.
2. On the same day player C won 4 c.u. from the player B.
3. On September, 7 player B won 3 c.u. from the player A, etc.

We provide an evidence that to register these events by means of a chronological register is more convenient – The Ledger.

Table 1 – The Journal of operations of players A, B, C (September)

№	Date	Liabilities		Total, c.u.
		To Receive	To Pay	
1	4.09	A	B	10
2	4.09	C	A	4
3	7.09	B	A	3
4	15.09	A	C	7
5	17.09	B	C	8
6	24.09	C	B	6
7	28.09	A	B	9
8	30.09	C	A	3
Total:				50

Such Ledger refers to the group of the chronological accounting registers as it contains information on the entries in the chronological order. It should be, however, noted, that the date of the event can't be uniquely identified since in the same interval several events can take place. Therefore, the identification of the events is done by the consecutive number or record number.

Such indicating of players settlement by notional names (accounts) in the Ledger (e.g., A – to receive, B – to pay), is usually called correspondence of the accounts in accounting. And the transaction itself will have the next interpretation:

Liabilities		Sum, c.u.
To receive	To pay	
A	B	10

The Solution

By calculating the total sums of the liabilities (“rewards – losses”) within the following equation $S(X,Y) = S_{x,y}$, the liabilities in their general form will be represented as follows: X – to receive, Y – to pay, on the left a value of these liabilities comes.

In our example the sum of liabilities “A – to receive, B – to pay” will be equal to the next: $S(A,B) = S_1(A,B) + S_7(A,B) = 10 + 9 = 19$ c.u..

Accordingly, $S(C, A) = S_2(C,A) + S_8(C, A) = 4 + 3 = 7$ c.u.

$S(B, A) = S_3(B,A) = 3$ c.u. and so on.

Recording the results into the Ledger (Table 2) will result in a form of a Ledger for the total liabilities.

Table 2 – the Ledger for the players A,B, C (September)

№	Liabilities		Total, in c.u.
	To receive	To pay	
1	A	B	19
2	A	C	7
3	B	A	3
4	B	C	8
5	C	A	7
6	C	B	6
Total			50

Such Ledger refers to the group of systematic accounting registers as it allows to systematize the data obtained during the period.

Since the transaction log can include any number of the entries, as they can be repeated continuously, here the number of entries in the Ledger is limited to the number of possible pairs of players. Thus, this number is equal to six: AB, AC, BA, BC, CA, CB.

The Ledger’s data are enough to solve the task for the players A, B, and C. The closing statements can be represented in two ways:

1. By means of the gross settlement method which is broadly used for calculations between bank clients. In our example, one should fulfill the following operation for the calculations between A and B: A receives 19 c.u. from B, B receives 3 c.u. from A. The sum of the funds needed for closing calculations between all players is equal to the total of the Ledger, that is 50 c.u.

2. By means of netting or clearing debts. Here it is necessary to calculate the difference (balance) according to the next formula: $\Delta S (X,Y) = S (X,Y) - S (Y,X)$, where $X,Y:= A, B, C$.

$$\text{In our example: } \Delta S (A,B) = S (A,B) - S (B,A) = 19 - 3 = +16 \text{ c.u.},$$

$$\Delta S (A, B) = S (A,B) - S (B,A) = 19 - 3 = +16 \text{ c.u.} > 0,$$

$$\Delta S (B, A) = S (B,A) - S (A, B) = 3 - 19 = -16 \text{ c.u.} < 0$$

The symbol «+» means «To receive», the symbol « - », correspondingly, means «To pay».

Such calculations can be systematized by re-writing the data of the Ledger in the form of a **chess balance** which will be further called a matrix of debit turnovers (**MDT**). In such case it looks as a matrix of liabilities to receive funds and can be represented as follows:

	To receive	To pay			Total to receive
		A	B	C	
MDT =	A	0	19	7	26
	B	3	0	8	11
	C	7	6	0	13
	Total to pay	10	25	15	50

The transposed matrix will result with a matrix of liabilities to pay, which is a matrix of credit turnovers (**MCT**).

	To receive	To pay			Total to receive
		A	B	C	
MCT =	A	0	3	7	10
	B	19	0	6	25
	C	7	8	0	15
	Total to pay	26	11	13	50

Further we subtract a matrix of liabilities to pay (**MCT**) from a matrix of liabilities to receive (**MDT**) and, as a result, we obtain a matrix of Trial Balance (**MTB**):

$$\mathbf{MDT} - \mathbf{MCT} = \mathbf{MTB} \quad (1)$$

The difference obtained as a result of this subtraction is the value of a balance matrix? As follows:

$$\mathbf{MTB} =$$

To receive	To pay			Total to receive
	A	B	C	
A	0	+16	0	+16
B	-16	0	+2	-14
C	0	-2	0	-2
Total to pay	-16	+14	+2	0

Generally the basic accounting equation in a matrix form is as follows:

$$\mathbf{MTB}_0 + \mathbf{MDT} - \mathbf{MCT} = \mathbf{MTB}_1 \quad (2)$$

Here \mathbf{MTB}_0 – is a matrix of balances for the beginning of the period, \mathbf{MDT} – is a matrix of debit turnovers, $\mathbf{MCT} = \mathbf{MDT}'$ - is a matrix of credit turnovers for the same period which is transposed to \mathbf{MDT} , \mathbf{MTB}_1 is a matrix of trial balance for the end of the period.

If we suppose that the players of calculations did not have any liabilities at the beginning of the period, the matrix of income balances will be equal to zero, that will have zeros in all the positions. Then the general accounting equation using the data of our example will look as follows:

$$\underbrace{\begin{pmatrix} \textit{to} & \textit{to} & & & \\ \textit{receive} & \textit{pay} & & & \\ & A & B & C & \Sigma \\ A & 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 0 \\ \Sigma & 0 & 0 & 0 & 0 \end{pmatrix}}_{\mathbf{MTB}_0} + \underbrace{\begin{pmatrix} \textit{to} & \textit{to} & & & \\ \textit{receive} & \textit{pay} & & & \\ & A & B & C & \Sigma \\ A & 0 & 19 & 7 & 26 \\ B & 3 & 0 & 8 & 11 \\ C & 7 & 6 & 0 & 13 \\ \Sigma & 10 & 25 & 15 & 50 \end{pmatrix}}_{\mathbf{MDT}} - \underbrace{\begin{pmatrix} \textit{to} & \textit{to} & & & \\ \textit{receive} & \textit{pay} & & & \\ & A & B & C & \Sigma \\ A & 0 & 3 & 7 & 10 \\ B & 19 & 0 & 6 & 25 \\ C & 7 & 8 & 0 & 15 \\ \Sigma & 26 & 11 & 13 & 50 \end{pmatrix}}_{\mathbf{MCT}} = \underbrace{\begin{pmatrix} \textit{to} & \textit{to} & & & \\ \textit{receive} & \textit{pay} & & & \\ & A & B & C & \Sigma \\ A & 0 & +16 & 0 & +16 \\ B & -16 & 0 & +2 & -14 \\ C & 0 & -2 & 0 & -2 \\ \Sigma & -16 & +14 & +2 & 50 \end{pmatrix}}_{\mathbf{MTB}_1}$$

Conclusion

In such a way, the equations presented above are the equivalents of the Trial balance obtained by using T-accounts. This means that it is possible to use alternative approach which differs from T-account and vice versa.

However, the difference of the approaches lays in the manner of the solution of the case. Besides the T-accounts methods is used primarily for teaching purposes, since a variety of accounting forms are used in practice.

Still the matrix model of accounting technology can be considered as an example of solving the case of mutual calculations.

On this basis we have developed and introduced in 2000 a principally new course that is constantly being revised and developed with the participation of the other course staff.

References

1. Sangster, A. and G. Scataglinibelghitar, (2010). Luca Pacioli: The Father of Accounting Education. *Accounting Education*, 2010(19), issue 4, 423-438
2. Leech, A., (1986). The Theory and Development of a Matrix-Based Accounting System, *Accounting and Business Research*, 16:64, 327-341, DOI: 10.1080/00014788.1986.9729333
3. Stoner, G., Vysotskaya, A. (2012). Introductory Accounting, with Matrices, at the Southern Federal University, Russia. *Issues in Accounting Education*, November 2012, Vol. 27, No. 4, pp. 1019-1044.
4. De Morgan, A. (1846). *Elements of Arithmetic*, 5th edition, Annex "The basic principle of accounting", London: Taylor and Walton.
5. Mepham, M.J., (Autumn 1988) "Matrix-Based Accounting - A Comment" *Accounting and Business Research*, Vol. 18, No. 72, pp. 375-378
6. Kolvakh O. (1996) *Computer Accounting for all*. - Rostov n/D: Publishing house "Feniks", P.416. (Кольвах О.И. Компьютерная бухгалтерия для всех.»—Ростов-н/Д: Изд-во «Феникс»,1996.—416с)

7. Kolvakh, O. (1999) “Situational-matrix accounting: models and conceptual solvings” – Rostov-on-Don: Pub. SKNC VSH, -243p. (Кольвах О.И. «Ситуационно–матричная бухгалтерия: модели и концептуальные решения.»–Ростов-н/Д: Изд–во СКНЦ ВШ,1999.–243с).
8. Kolvakh, O.I. (2011, April). Matrix Models as the Metamodel of Accounting. Paper presented at the European Accounting Association (EAA) annual Conference, Rome, Italy.
9. Vysotskaya, A. (2011). Taxation and accounting by matrix simulating. The success of modern sciences, №6., pp.84-88. (Высотская А. Б. Налогообложение и бухгалтерский учет в разрезе матричного моделирования. Успехи современного естествознания, №6. pp.84-88).
10. Vysotskaya, A. and V. Aleshin. (2013, May). Situationally matrix modelling in tax planning for SMEs. Evidence from Russia. Paper presented at the EAA Annual Congress, Paris, France.
11. Vysotskaya, A. (2014). Financial Planning by Means of Situational Matrix Modelling in the Economic Globalization Process. Economics and Management: Problems and Solutions, №3 (27), pp.173-178. (Высотская А.Б. Финансовое прогнозирование средствами ситуационно-матричного моделирования в условиях экономической глобализации.// Экономика и управление: проблемы, решения. 2014. № 3 (27). С. 173-178).
12. Sbitneva, S. (2013) Some aspects of constant accounting for analysis and prognozing of business-processes. The success of modern sciences, № 12; pp. 104-108. (Сбитнева С.А. Некоторые аспекты константной бухгалтерии для

- анализа и прогнозирования бизнес-процессов // Успехи современного естествознания. – 2013. – № 12. – С. 104-108).
13. Vysotskaya, A., Kolvakh O., and G. Stoner, (2016). "Mutual Calculations in Creating Accounting Models: a Demonstration of the Power of Matrix Mathematics in Accounting Education," *Accounting Education*, 2016, Vol. 25, Issue 4, pp. 396-413
 14. Smirnova, I.A., Sokolov, J.V. and Emmanuel, C.R. (1995) Accounting education in Russia today, *European Accounting Review*, 4- 4, pp. 833 – 846
 15. ICAEW (2010). *Business models in accounting: the theory of the firm and financial reporting*.
 16. Churchill N. (1964), *Linear Algebra and Cost Allocation: Some Examples // The accounting Review*.- October. – pp. 894-903
 17. Demski J., FitzGerald S. (2008), *Quantum Information and Accounting Information: Their Salient Features and Applications // Journal of Accounting and Public Policy*. – pp. 435-464.
 18. Didier Leclère et Jean-Guy Degos. (1990), *Methodes matricielles de gestion comptable approfondie*, EYROLLES Management, Paris. – 192 p.
 19. Didier Leclère et Jean-Guy Degos. (1995), *Comptabilité matricielle // Bernard co l a ssf. encyclopedie DE comptabilite, controle DE gestion et audit*. – p. 383.
 20. Gardner M. Jones. (1965), *Linear Algebra for the Neophyte // The Accounting Review*, Vol. 40, No. 3 (Jul., 1965), pp. 636-640

21. Sorter G.H. (1969), An «Events» Approach to Basic Accounting Theory // The accounting Review. - January,- pp.12-19
22. Schroderheim G. (1964), Using mathematical probability to estimate the allowance for doubtful accounts // The accounting Review. - July,- pp.679-684
23. Williams T. (1964), Matrix theory and cost allocation// The Accounting Review.- October. – pp. 671-678.
24. Mattessich R. (1994), The Number concept in Business and Concern Economics // Leonardo Fibonacci. Il tempo, le opera, l'eredità scientifica. – Pisa: Pacini editore,. – pp. 109 – 137.
25. Mattessich R., Galassi G. (2000), History of the Spreadsheet: from Matrix Accounting to Budget Simulation and Computerization //Accounting and History A Selection of paper presented at the World Congress of Accounting Historians. – Vadrid – Spain, 19-21 July.