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CoRisk: Measuring Contagion Risk with Correlation Network Models

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Abstract: We propose a novel credit risk measurement model for Corporate Default Swap spreads, that combines vector autoregressive regression with correlation networks. We focus on the sovereign CDS spreads of a collection of countries, that can be regarded as idiosyncratic measures of credit risk. We model them by means of a vector autoregressive regression model, composed by a time dependent country specific component, and by a contemporaneous component that describes contagion effects among countries. To disentangle the two components, we employ correlation networks, derived from the correlation matrix between the reduced form residuals. The proposed model is applied to ten countries that are representative of the recent financial crisis: top borrowing/lending countries, and peripheral European countries. The empirical findings show that the proposed model is a good predictor of CDS spreads movements, and that the contemporaneous component decreases prediction errors with respect to a simpler autoregressive model. From an applied viewpoint, core countries appear to import risk, as contagion increases their CDS spread, whereas peripheral countries appear as exporters of risk. Greece is an unfortunate exception, as its spreads seem to increase for both idiosyncratic factors and contagion effects.

Keywords: corporate default swap spreads, correlation networks, vector autoregressive regression

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1. Introduction

The global financial crisis and, more recently, the European sovereign crisis, have led to an increasing research literature on systemic risk, with different definitions and measurement models.

According to the ECB (2009): "Systemic risk is the risk of experiencing a strong systemic event, which adversely affects a number of systemically important intermediaries or markets". This definition introduces two key elements for the study of systemic risk: first, a trigger (extreme) event, transmitted to the system as a whole, and not only to individual institutions; second, financial instability, which is spread recursively through contagion from each financial institution to another.

While systemic risk definitions share this broad view, systemic risk measurement models are still quite divergent.

A first diversity concerns the economic environment as the context in which systemic risk arises and propagates: many models concentrate on the financial sector; others on the sovereign sector, with the two strongly linked to each other.

A second distinction derives from the use of a cross-sectional rather than a time-dynamic perspective: while the former mostly concentrates on contagion between the institutions operating

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in the market, the latter focuses on the generating cause-and-effect relationships over time. While contagion models identify transmission channels, thus describing how a crisis spreads through the whole system, time-dependent models associate a specific risk measure to individual institutions, with the aim of predicting what will happen to them in the nearby future, in an early-warning perspective.

A third distinction originates in the identification of the risk sources, thus setting endogenous against exogenous causes, as well as idiosyncratic against systematic shocks.

In this work we will develop a general measurement model that can be applied regardless of the underlying classification.

First, we will consider Credit Default Swap spreads, as a measure of credit risk of the underlying institution. CDS spreads can refer to any institution, whether a sovereign country, a financial institution or a non financial corporate and, therefore, there is no need to adopt different measurement models for each considered sector.

Second, we will model CDS spreads by means of a class of Vector Auto Regressive models that can account for both temporal and cross-sectional dependency. Doing so, we will be able to describe the contagion transmission mechanisms but also to construct early-warning predictive measures for single institutions. We will thus derive a contagion risk measure, called $CoRisk$, able to describe how many extra basis points should be paid to insure one monetary unit of a credit when contagion effects deriving from other countries are taken into account. Contagion that can be negative, thus causing the spread to increase (typical behaviour of "risk importer" countries), but also positive, determining the spread to decrease (typical behaviour of "risk exporter" countries).

Third, the proposed VAR model will be able to disentangle contagion components of risk from idiosyncratic ones, regardless of the nature of the original shock, whether endogeneous or exogeneous, individual or systematic.

To exemplify the methodology, the proposed model will be applied to the sovereign CDS spreads of ten countries. Five of them represent core countries: the United States, the United Kingdom, Japan, Germany and France. The other five are the European countries that most suffered from the recent sovereign crisis: Italy, Spain, Portugal, Ireland and Greece. First, by looking at correlation networks our results indicate that contagion induces a "clustering effect": core and peripheral countries further diverge through, and because of, contagion propagation, creating a sort of diabolic loop extremely difficult to be reversed. Second, by looking at the two components of risk our empirical findings show that: (a) the contagion effect, measured by $CoRisk$, is strongest for Euro area countries, and especially for the largest ones (Germany, France, Italy, Spain); (b) peripheral countries mostly behave as exporters, rather than importers of system risk: as a consequence, core economies are mostly affected by contagion risk, while peripheral countries strongly suffer high idiosyncratic default probabilities with the notable exception of Greece (strongly affected by both); (c) the model shows a good predictive performance, significantly improved with respect to the model that includes only the autoregressive, idiosyncratic component. Finally, our proposed methodology seems quite effective, since the inclusion of the contagion component in the model decreases the predictive errors in almost all cases (across countries and time). From an interpretational viewpoint, when the $CoRisk$ is considered as a contagion centrality measure, it has a clear advantage over classic measures, being expressed in basis points rather than general real numbers.

The paper is structured as follows. Section 2 contains the literature background necessary for the development of our research methodology, described in Section 3; Section 4 contains the main empirical findings from the application of the models, and Section 5 concludes with some final remarks.

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2. Background

The main contribution of this paper is a contagion model that improves credit risk prediction with the introduction of a systemic risk measurement.

From a chronological viewpoint, the first systemic risk measures have been proposed for the financial sector, in particular by Adrian and Brunnermeier (2011), Acharya et al. (2010), Acharya et al. (2012) and Brownlees and Engle (2012). On the basis of market share prices, these models consider systemic risk as endogenously determined and calculate the quantiles of the estimated loss probability distribution of a financial institution, conditional on an extreme event in the financial market (or viceversa).

The above approach is useful to establish policy thresholds aimed, in particular, at identifying the most systemic financial institutions, or the most systemic countries. However, it is a bivariate approach, which allows to calculate the risk of an institution conditional on a reference market but, on the other hand, it does not address the issue of how risks are transmitted between different institutions, in a multivariate framework.

A different stream of research considers systemic risks as exogenous, to be explained by causal factors. This is the approach proposed, in particular, by Ang and Longstaff (2012), Betz et al. (2014), Duprey et al. (2015), Schwaab et al. (2015), and Ramsay and Sarlin (2015), who explain whether the default probability of a bank, of a country, or of a company, depends on a set of exogenous risk sources, thus combining idiosyncratic and systematic factors.

While powerful from an early warning perspective, causal models, similarly to bivariate ones, concentrate on single institutions rather than on the economic system as a whole and, therefore, may underestimate systemic sources of risk arising from contagion effects within the system.

Trying to address the multivariate nature of systemic risk, researchers have recently proposed correlation network models, able to combine the rich structure of financial networks (see, e.g. Lorenz et al., 2009; Battiston et al., 2012) with a more parsimonious approach that can estimate contagion effects from the dependence structure among market prices. The first contributions in this framework are Billio et al. (2012) and Diebold and Yilmaz (2014), who derive contagion measures based on Granger-causality tests and variance decompositions. More recently, Ahelegbey et al. (2015) and Giudici and Spelta (2016) have extended this methodology introducing stochastic correlation networks. Another relevant reference consists in Das (2015), who derives a risk decomposition into individual and network (contagion) contributions.

While bivariate and causal models explain whether the risk of an institution is affected by a market crisis event or by a set of exogenous risk factors, correlation network models explain whether the same risk depends on endogeneous contagion effects, in a cross-sectional perspective. However, since they are built on cross-sectional data, they can not be used as predictive models or, more precisely, as early warning monitors.

Our aim is to improve correlation network models, allowing them to be predictive, and not only descriptive. To achieve this aim, we introduce partial correlation stochastic networks similar to those in Giudici and Spelta (2016) into the context of correlated VAR models, similar to those in Ahelegbey et al. (2015). Doing so, we merge the advantages of bivariate models (endogeneity), causal models (predictive capability) and correlation networks (identification of contagion channels).

Specifically, we extend (a) the approach of Ahelegby et al. (2015), by adding undirected graphical Gaussian models based on partial correlations into their graphical vector autoregressive; (b) the approach of Giudici and Spelta (2016), by enriching their undirected graphical Gaussian models with an autoregressive component derived through a VAR model; (c) the approach of Das (2015), implementing it with a probabilistic decomposition of risks into individual and contagion components.

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A further contribution of this paper consists in the extension of the application domain of correlation networks.

The available literature focuses on measuring market risk, related to the probability of a monetary loss due to adverse market conditions and, in this context, the dependence structure among market prices indicates how the market risk of a portfolio is affected by contagion. We instead focus on credit risk, related to the probability of a monetary loss due to the insolvency of the borrower.

In order to apply correlation networks within the credit risk context, a preliminary question is how to measure credit risk itself. Popescu and Turcu (2014), for example, use bond interest rates as a measure that reflects the credit risk of a country. However, we believe that interest rates are unlikely to express sensitivity to credit risk in the current scenario of interest rates at the zero lower bound. This is why we focus on Credit Default Swap spreads and build a correlation network model based on the dependence structure among CDS spreads to measure the effects of contagion on credit risk measurements.

If we assume that the market correctly prices the overall credit risk of an institution by means of the risk premia implied by the CDS spread, we can try to divide such risk premia into the idiosyncratic component, due to the individual characteristics of the borrower, from the systemic component, due to contagion effects. Doing so, the importance of each institution will depend not only on its position in the network, as in most financial network models, but also on its specific credit risk and on the credit risk of its neighbours.

We remark that Credit Default Swap spreads are available for several institutions, including sovereign countries, financial intermediaries and non financial corporates. Without loss of generality, this paper will focus on sovereign countries, with the aim of capturing the impact of contagion risk on their credit risk and, through that, on the capital flows towards and from each country, whose variations may severely impact financial stability. We thus contribute also to the stream of literature that employs financial networks in the modelling of international capital flows (see e.g. Minoiu and Reyes, 2013; Giudici and Spelta, 2016). While previous studies analyse the volumes of international claims between countries to infer how the distress in a specific region may affect the credit risk of the others, we examine CDS spreads to measure the same effect but in a direct (through prices), rather than indirect (through quantities) way.

We finally remark that a multivariate approach related to ours has been recently suggested by Mezei and Sarlin (2015) and Betz et al. (2016): the former define an aggregation operator in order to jointly estimate the importance of each single institution as well as contagion effects deriving from links between neighbours; the latter develop a tail risk analysis of networks in order to build a robust set of regressors for defining systemic contributions. We improve both approaches by deriving measures of contagion through partial correlations between the residuals from VAR processes. In such a way we can allow for non-linear effects and disentangle the idiosyncratic from the systemic component of each considered institution. In addition, our contagion measure ($CoRisk$) is allowed to be both positive or negative, meaning that the resulting default probability of each economic sector or country can be increased or decreased according to the sign of partial correlations. From an economic viewpoint, when a country is negatively related to troubled countries, its final default probability decreases because it is perceived as a flight-to-quality haven, meaning that it is positively affected by contagion effects. On the contrary, when countries are positively connected to troubled economies their default probability increases because they suffer negative contagion. Such a distinction between positive and negative contagion, to our knowledge, only appears in Grinis (2015).

3. Proposal

Let y_t^i be the Credit Default Swap (hereafter CDS) spread of a country $i\,(i = 1, \ldots, I)$, at time $t\,(t = 1, \ldots, T)$. We assume that y_t^i is a function of: (a) an autoregressive component, that expresses the dependency on the past CDS spread values of the same country; (b) a cross-sectional component, that expresses the contemporaneous dependency on the spreads of the other countries; (c) a stochastic residual. Formally, for each country i and time t we assume that the following holds:

$$y_t^i = \sum_{p=1}^{p_0} \alpha_p^i y_{t-p}^i + \sum_{j \neq i} \beta^{ij} y_t^j + \epsilon_t^i, \tag{3.1}$$

where p is a time lag (with $p_0 < t$), α_p^i and β^{ij} are unknown coefficients to be estimated from data, and ϵ_t^i are standard Gaussian residuals, which are independent across time and countries.

Equation (3.1) models the CDS spread dynamics as a structural VAR, in which the sovereign risk of each country depends on its past values through the idiosyncratic autoregressive component $\sum_{p=1}^{p_0} \alpha_p^i y_{t-p}^i$, and on the contemporary values of the other countries through the systemic component $\sum_{j \neq i} \beta^{ij} y_t^j$, that we name "Contemporary Risk" (*CoRisk* for brevity).

The previous model can be expressed in a more compact matrix form, as follows:

$$Y_t = \sum_{p=1}^{p_0} A_p Y_{t-p} + B_0 Y_t + E_t, \tag{3.2}$$

where Y_t is the I-dimensional vector containing the CDS spreads of all countries at time t, Y_{t-p} is the same vector, lagged at time $t-p$, A_p is a $p \times I$ matrix that contains the autoregressive coefficients, B_0 is a $I \times I$ symmetric matrix containing the contemporaneous coefficients and with null diagonal elements, and, finally, E_t is a vector of standard Gaussian residuals independent across time.

For estimation purposes, the model in (3.2) can be transformed in a reduced form, thus becoming:

$$Y_t = \Gamma_1 Y_{t-1} + \ldots + \Gamma_p Y_{t-p} + U_t, \tag{3.3}$$

with

$$\begin{cases} \Gamma_1 = (\mathbb{I} - B_0)^{-1} A_1, \\ \ldots \\ \Gamma_p = (\mathbb{I} - B_0)^{-1} A_p, \\ U_t = (\mathbb{I} - B_0)^{-1} E_t. \end{cases} \tag{3.4}$$

The previous formulation allows the estimation of the vectors of modified autoregressive coefficients $\Gamma_1, \ldots, \Gamma_p$, using time series data on CDS spreads contained in the vectors' ensemble $\{Y_1, \ldots, Y_t, \ldots, Y_T\}$.

However, our aim does not consist in estimating Γ_p, but in separately estimating its components $\{A_1, \ldots, A_p\}$ and B_0, thereby disentangling the autoregressive part from the contemporaneous one. Once B_0 is obtained, $\{A_1, \ldots, A_p\}$ can be derived from (3.4).

In order to estimate B_0, note that $(\mathbb{I} - B_0) U_t = E_t$, so that $U_t = B_0 U_t + E_t$. This implies that, for each country i,

$$U_t^i = \sum_{j \neq i} \beta^{ij} U_t^j + \epsilon_t^i, \tag{3.5}$$

meaning that the off-diagonal elements of B_0 can be obtained regressing each modified residual, derived from the application of (3.3), on those of the other countries.

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Note that the regression model in (3.5) is based on the transformation derived in equation (3.4), which makes the modified residuals correlated. The direction of such correlation is, however, unknown. In the application of (3.5) it is therefore not clear which country spread residual assumes the form of a response variable, and which one of an explanatory regressor.

A simplistic solution to this problem would be to estimate all possible regressions, i.e. to regress each CDS spread on all the others: but this procedure would be, besides illogic, computationally inefficient. To solve this issue we propose to approximate each pair of regression coefficients β^{ij} and β^{ji}, which represent the two opposite causality directions, with one correlation coefficient: this will be undirected but univocally determined by them, and it will turn out to be the partial correlation coefficient.

Formally, let $\Sigma = \mathrm{Corr}(U)$ be the correlation matrix between the modified residuals, and let Σ^{-1} be its inverse, with elements σ^{ij}. The partial correlation coefficient $\rho_{ij|S}$ between the residuals U^i and U^j, conditional on the remaining residuals ($U^s, s = 1, \ldots, S$, where $S = I \setminus \{i, j\}$), can be obtained as:

$$\rho_{ij|S} = \frac{-\sigma^{ij}}{\sqrt{\sigma^{ii}\sigma^{jj}}}. \tag{3.6}$$

It can then be shown that:

$$|\rho_{ij|S}| = \sqrt{\beta^{ij} \cdot \beta^{ji}}, \tag{3.7}$$

which means that the absolute value of the partial correlation coefficient between U^i and U^j, given all the other residuals, can be obtained as the geometric average between the coefficients β^{ij} and β^{ji} defined by equation (3.5), by setting, respectively, i rather than j as response variables. Equation (3.7) justifies the replacement of β^{ij} and β^{ji} with their corresponding partial correlation coefficient $\rho_{ij|S}$.

From an economic viewpoint, the partial correlation coefficient expresses how the CDS spread of a country i is affected by the contemporaneous spreads of the other countries $j \neq i$. The worse the countries to which i is more connected, the worse the default probability of i itself.

Indeed, the default probability of a country j at time t can be either greater or lower than its default probability at time $t-1$, depending on the sign of the partial correlation coefficient with the other countries. If $\rho_{ij|S} > 0$, the default probability of country i increases after the inclusion of contagion from j (positive contagion). Conversely, when $\rho_{ij|S} < 0$, the default probability of country i decreases after the inclusion of contagion from j (negative contagion).

A further advantage deriving from the employment of partial correlations lies in the possibility of employing correlation network models (see e.g. Giudici and Spelta, 2016), based on the conditional independence relationships described by partial correlations. Formally, let us assume that the vectors U_t are independently distributed according to a multivariate normal distribution $\mathcal{N}_I(0, \Sigma)$, where Σ represents the correlation matrix (that we assume to be non-singular). A correlation network model can be represented by an undirected graph G such that $G = (V, E)$, with a set of nodes $V = \{1, ..., I\}$, and an edge set $E = V \times V$ that describes the connections between the nodes. G can be represented by a binary adjacency matrix E with elements e_{ij}, each of them providing the information of whether a pair of vertices in G is (symmetrically) linked between each other ($e_{ij} = 1$) or not ($e_{ij} = 0$). If the nodes V of G are put in correspondence with the random variables $U_1, ..., U_I$, the edge set E induces conditional independences on U via the so-called Markov properties (see e.g. Lauritzen, 1996).

Let Σ^{-1} be the inverse of Σ, whose elements can be indicated as $\{\sigma^{ij}\}$. Whittaker (1990) proved that the following equivalence holds:

$$\rho_{ij|S} = 0 \Longleftrightarrow U_i \perp U_j | U_{V \setminus \{i,j\}} \Longleftrightarrow e_{ij} = 0$$

where the symbol $\perp$ indicates conditional independence.

The previous equivalence implies that, if the partial correlation between two measures is equal to zero, the corresponding CDS spread residuals are conditionally independent and, therefore, the corresponding countries do not (directly) impact each other. Thus, a correlation network model among all countries can be estimated on the basis of the partial correlation coefficients between the modified residuals, which can be calculated by inverting the correlation matrix among the modified residuals.

From a statistical viewpoint, it is possible to test the null hypotheses that a partial correlation coefficient is equal to zero by means of a pairwise $t - test$, as described in Whittaker (1990) or in Giudici (2003). As a consequence, a correlation network model can be built by placing a link between two countries if and only if the corresponding partial correlation coefficient is significantly different from zero.

4. Empirical Findings

4.1. Data

What developed in the last Section can be applied to any set of institutions, as long as they have CDS priced by the market. Here we focus the application of the proposed methodology to sovereign institutions, in the period that precedes and follows the global financial crisis.

We focus on 10 countries: five of them correspond to the world's largest borrowers/lenders: the United States (US), the United Kingdom (UK), Germany (DE), France (FR) and Japan (JP). They can be considered "core" countries, with a relatively low probability of default. On the other hand, we consider the five "peripheral" European countries that have been more impacted by the recent sovereign crisis: Greece (GR), Ireland (IR), Italy (IT), Portugal (PT) and Spain (SP).

For each country, we collected data on their daily CDS spreads for the period that goes from July 1st, 2006, to December 31st, 2016. A summary statistics for such CDS spreads is reported in Table 1.

[Table 1 about here]

From Table 1 note the difference between peripheral countries, whose average CDS spreads range from the 133.18 of Italy to the 3708.36 basis points of Greece, and core countries, whose average CDS spreads range from the 20.30 basis points of Germany to the 51.68 basis points of Japan. Indeed, it is well known that Greece presents the most critical situation, with the highest CDS spread values throughout the considered period. Portugal has a similar dynamics, although with a smaller magnitude. Ireland initially behaves similarly to Portugal but then experiences a strong recovery that brings its CDS spread values in line with those of core countries. Italy and Spain show rather similar values, on average and through time, with CDS spreads that are lower then the former ones but considerably higher than those of core countries. All core countries present relatively low CDS spread values; among them, Japan, the United Kingdom and France exhibit the highest values, while the United States and Germany seem to be the safest economies.

4.2. Correlation networks

We now present the results from the application of the proposed model. Figure 1 presents the partial correlation network between the considered countries, estimated using the entire time series. In the graph, the larger a node, the larger its idiosyncratic default probability. Concerning edges between nodes, green lines stand for positive partial correlations, while red lines indicate negative partial correlations; the ticker the line, the stronger the connection. Absent connections correspond to conditional independences.

[Figure 1 about here]

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Figure 1 clearly shows that the countries included in our sample are highly interconnected, with most partial correlations having a positive sign. The most central countries appear to be France and Italy: this is not a surprising result, and it is mainly due to the relevance of the European sovereign crisis within the considered time period.

For a more precise understanding of the results shown in Figure 1, Table 2 presents the partial correlation values with the corresponding significance $t-tests$.

[Table 2 about here]

Table 2 indicates that the CDS spreads of some pairs of countries are indeed conditionally independent (their link is missing in Figure 1). From an economic viewpoint, this means that they do not directly have an impact on each other. Such conditional independence affects the pairs (Greece, United States), (Greece, Japan), (Spain, United Kingdom), (Germany, United States). Table 2 also reports countries with a negative direct correlation, for example: Germany with Spain and Portugal; Greece with France and Ireland; Ireland with Italy and Japan; the United States with France and Portugal. All these pairs of countries negatively impact each other, consistently with a "flight to quality" effect. Last, some countries appear to be strongly positively related, thus indicating a direct reciprocal contagion. This is the case of some pairs of core countries, such as (France, Germany), (Germany, UK), (UK, USA); but it also involves peripheral countries, like (Greece, Spain), (Ireland, Spain), (Spain, Portugal), (Italy, Spain), (Italy, Portugal).

The latter two results indicate that contagion induces a "clustering effect": core and peripheral countries further diverge through, and because of, contagion propagation, creating a sort of diabolic loop extremely difficult to be reversed. Core countries, with low CDS spreads, are unaffected by contagion, as they depend positively on each other and negatively with the peripherals. On the other hand, peripheral countries, with high CDS spreads, are impacted by contagion from other peripheral countries, and this effect is not mitigated by core countries as correlations with them are mostly negative.

We remark that, to our knowledge, ours is one of the first papers that allow for negative contagion. Negative contagion can be explained thinking at capital flows: when a country i is facing a crisis period, investors tend to shift their portfolio towards "safer" places in order to reduce risk, and such places are the countries negatively related to i which, therefore, show an improvement in their survival probability. This mechanism justifies the difference between *positive* and *negative* risk propagation.

For robustness purposes, we also remark that we have calculated the partial correlation network separately for three time periods, more precisely: 1/7/2006 - 31/12/2009 (financial crisis), 1/1/2010 - 31/12/2012 (sovereign crisis) and 1/1/2013 - 31/12/2016 (post-crisis). Figure 2 shows the corresponding results.

[Figure 2 about here]

Figure 2 emphasises that the risk transmission mechanism has changed over the years: in the pre-crisis period the overall number of significant partial correlations is quite high; during the financial crisis it decreases; during the sovereign crisis it further decreases and the "clustering effect" that separates core and peripheral economies in two quite distinct subgroups emerges even stronger. Last, in the post-crisis period the partial correlation pattern returns to the pre-crisis situation, however with a persisting clustering effect identifiable not only by positive within group correlations, but also by negative ones across the two groups. This overall mechanism finds the exception of Ireland, which seems to "migrate" from the periphery to the core.

4.3. VAR model estimation

As described in Section 3, by means of partial correlations we can derive the B_0 matrix and, then, the autoregressive parameters $A_1, \ldots, A_p$. We are thus able to estimate the time-dependent CDS spreads of each country i, disentangling the autoregressive idiosyncratic component from the contemporaneous *CoRisk* according to equation (3.2).

Figure 3 represents, for each of the five peripheral countries (namely Portugal, Ireland, Italy, Greece and Spain), the time evolution of the observed CDS spreads and of their estimated autoregressive and contemporaneous (*CoRisk*) components. The same graphs referred to core countries (France, Germany, Japan, the United Kingdom and the United States) are reported in Figure 4.

[Figure 3 about here]

[Figure 4 about here]

From Figure 3 note that the autoregressive component prevails in peripheral countries, with a limited *CoRisk* effect in Ireland, Portugal and Spain. Figure 4, instead, shows that although the autoregressive component prevails, there is a substantial contagion propagation affecting core countries: first and foremost France and Germany, the most concerned by the European sovereign crisis. The Unites States and the UK have been somewhat impacted during the global financial crisis.

From an economic viewpoint, Figures 3 and 4 can be jointly interpreted together with the network structure presented in Figure 1: considering that peripheral economies are characterised by the highest idiosyncratic risk component, we can overall state that they are mainly "exporters" rather than "importers" of credit risk; on the contrary, core countries appear to be potential "importers" of credit risk.

4.4. CoRisk as a new centrality measure

In the financial network literature a strong importance is assumed by the concept of "centrality measure" (of each node in the network, corresponding to single countries in our context). In qualitative terms, one node can be considered as "central" in a network if it is "highly interconnected" with the others, where the meaning of "highly interconnected" typically depends on the definition of "centrality" we assume: a centrality measure thus aims at mapping such "centrality" feature be means of real numbers. As an example, the eigenvector centrality (see e.g. Furfine, 2003; Billio et al., 2012) considers as "more central" those nodes which are connected to other "highly central" nodes: it thus assigns a normalised score to each node which takes the form of a real number $\in [0, 1]$. This centrality measure can be easily calculated, for each country, from the correlation matrix between CDS spreads (see e.g. Giudici and Spelta, 2016).

Another measure of centrality is called degree of centrality, and it can also be easily obtained from the edge set E by considering the number of significant links each country has with its neighbours. Note that the *CoRisk* component of equation (3.2) gives, for each sovereign, a weighted (or normalised) degree of centrality, in which each binary link is replaced by the corresponding partial correlation coefficient. The resulting measure is intuitively expressed in basis points rather than in absolute numbers $\in \mathbb{R}$.

Table 3 reports the comparison between the eigenvector centrality measure, calculated on the partial correlation matrix, and our *CoRisk*. It also reports the autoregressive component of equation (3.2).

[Table 3 about here]

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Table 3 reveals that the most central countries in terms of eigenvector centrality are France, Germany, Italy and Spain: exactly the main players in the European sovereign crisis, either as importers (France, Germany) or exporters (Italy, Spain) of risk. Smaller countries such as Portugal and Ireland follow. The United Kingdom and, more evidently, the United States and Japan, are less central in explaining the variations of CDS spreads in the considered period, consistently with the observed economic facts. Last, Greece appears quite isolated, being its crisis mainly due to idiosyncratic causes.

If it is true that the *CoRisk* measure in Table 3 can be interpreted as a graphical centrality, it also indicates how much "net" contagion each country receives from/exports to the others, as measured by the increase/decrease of the CDS spread due to the contemporary component. In this sense, the *CoRisk* centrality measure indicates that Greece is the country mostly impacted by contagion, followed by Ireland, Germany and France: they are all affected by positive changes, indicating they are "importers" of risk (in line with the previous Section).

From an economic viewpoint, these results mean that Greece is negatively affected by the difficulties of the countries to which it is most connected (see Table 2), possibly because of a funding risk. France and Germany, on the other hand, import risk from peripheral countries through their credit risk. Ireland, probably for its recent economic recovery, is an importer of risk similarly to core countries. On the other hand, Italy, Portugal and Spain have a negative sign, thus behaving as countries that are "exporters" of contagion. This effect is stronger for Spain, whereas the other two peripheral countries appear to act as intermediaries since they import risk from other peripherals (including Greece) and export it to core countries. Last, the United States, Japan and the United Kingdom appear less central, in line with the results obtained through the eigenvector centrality measure and with the network results showing they are relatively less linked to European peripheral sovereigns.

We remark that, summing together the *CoRisk* and the autoregressive component, we obtain an "average" estimate of CDS spreads that can be compared to the observed spreads reported in Table 1: we can thus derive an overall measure of fit of the model. It can be shown that the overall mean square error is equal to 274 b.p. against the 398 b.p. obtained with the model that contains only the autoregressive component: moreover, the almost all the difference between the two errors is related to the contagion contributions of Greece, Portugal, Spain and Italy.

4.5. Predictive performance

We now assess the predictive performance of our proposed model, for two reasons: (a) understanding if the structural VAR proposed in Section 3 is able to well predict CDS spreads, and (b) assessing the improvement of the proposed model due to the contagion component with respect to a simple autoregressive formulation, from a predictive-performance viewpoint. More precisely, we aim at estimating the month in which the European sovereign crisis has peaked: August 2011. We build our model using one year of daily data up to July 31st, 2011. With the obtained model, we predict the CDS spreads for the whole month ahead, and compare the predictions with the observed spreads. We also compare our model with a simpler one, based only on the autoregressive component without the contemporaneous *CoRisk* effect.

Figure 5 and 6 represent, respectively for the five peripheral (Greece, Ireland, Italy, Portugal and Spain) and core (France, Germany, Japan, the United Kingdom and the United State) countries, a comparison between the observed and the predicted CDS spreads, using both the proposed model and the simpler model containing only the autoregressive component.

[Figure 5 about here]

[Figure 6 about here]

Both Figures 5 and 6 reveal that the full model is always more conservative than the simpler autoregressive model: while the former always slightly overestimated CDS spreads, the latter underestimates. Such effect is slightly smaller for core countries with respect to peripheral ones, mainly because of their different magnitude scale of overall risk.

In order to compare the two models and understand which one better performs in a predictive sense, we need a summary metric. Table 4 contains, for each country as well as for the whole sample, the mean square errors referred to August 2011.

[Table 4 about here]

Consistently with the previous Figures, Table 4 shows that the full model over-performs the simple autoregressive model, as it has lower mean square errors for most countries, as well as a smaller overall error.

For robustness purposes, we now predict a "normal" period such as August 2012, that followed the ECB President Draghi's "whatever it takes" famous speech. Also in this case we build the model using one year of daily data up to July 31st, 2012. With the obtained model, we predict the CDS spreads for August 2012 and we compare the predictions with the observed spreads by using our model and the simple autoregressive one.

Figure 7 and 8 represent, respectively for the five peripheral (Greece, Ireland, Italy, Portugal and Spain) and core (France, Germany, Japan, the United Kingdom and the United State) countries, a comparison between the observed and the predicted CDS spreads, using both the proposed model and the simpler model containing only the autoregressive component.

[Figure 7 about here]

[Figure 8 about here]

From both Figures 7 and 8 we first that the situation is opposite with respect to the previous case, with all risk components decreasing rather than increasing. However, the predictive performances of the models are similar as before, with the full model always being more "conservative" than the simpler autoregressive one. Table 5 summarises the mean square errors referred to August 2012, separately for.each country as well as for the whole sample.

[Table 5 about here]

By comparing Tables 5 and 4, it is clear that the predictive errors for August 2012 are on overage higher than the errors obtained in the same month of the previous year: this is not surprising, since all countries (especially Greece) suffer from the sovereign crisis affecting Europe in 2012 and thus experience a higher idiosyncratic risk. However, the full model again outperforms the simpler model in all countries, with the only exception of Italy.

We finally remark that the predictive errors obtained in this section are lower than those commented when discussing centrality measures: this can be explained recalling that in this final robustness check, models' estimates and partial correlation matrices have been obtained by using only the previous year's daily data instead of the whole time-sample. We are aware this is a simplistic assumption that makes predictions more static and, therefore, less performant, but it does not affect the comparison between estimates related to different years (time-dimension), countries (cross-sectional dimension) or models (forecasting).

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5. Conclusions

In this work we have proposed a new systemic risk measurement model, based on structural VAR processes for CDS spreads. Our main methodological contribution consists in the introduction of partial correlations and correlation networks into VAR models, so to disentangle the autoregressive component (interpreted as idiosyncratic risk) and the contemporaneous part (interpreted as contemporaneous contagion effect), that we have named *CoRisk*. In the context of CDS spreads, our proposed *CoRisk* measure describes how many extra basis points should be paid to insure one monetary unit of a credit, when contagion is taken into account. Such contagion effect, moreover, can be both positive or negative: in the former case it determines the spread to increase, typical behaviour of "risk-importer" countries; in the latter situation it causes the spread to decrease, typical behaviour of "risk-exporter" countries.

By means of partial correlation networks, our results indicate that contagion induces a "clustering effect": core and peripheral countries further diverge through, and because of, contagion propagation, creating a sort of diabolic loop extremely difficult to be reversed. Core countries, with low CDS spreads, are unaffected by contagion, as they depend positively on each other and negatively with the peripherals. On the other hand, peripheral countries, with high CDS spreads, are impacted by contagion from other peripheral countries, and this effect is not mitigated by core countries as correlations with them are mostly negative.

By looking at the predictive performance, our proposed methodology seems quite effective, since the inclusion of the contagion component in the model decreases the predictive errors in almost all cases. From an interpretational viewpoint, when the *CoRisk* is considered as a contagion centrality measure, it has a clear advantage over classic measures, being expressed in basis points rather than as an absolute real number.

From an applied viewpoint, our empirical findings show that (a) the contagion effect, measured by *CoRisk*, is strongest for Euro area countries, and especially for the largest ones (Germany, France, Italy, Spain); (b) peripheral countries mostly behave as exporters, rather than importers of system risk: as a consequence, core economies are mostly affected by contagion risk, while peripheral countries strongly suffer high idiosyncratic default probabilities with the notable exception of Greece (strongly affected by both); (c) the model shows a good predictive performance, significantly improved with respect to the model that includes only the autoregressive, idiosyncratic component.

6. Acknowledgements

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Tables

Table 1: summary statistics for the CDS spreads. Means, standard deviations, minimum and maximum values are all expressed in Basis Points.

Country	Mean	St. Dev.	Min	Max
France	40.17	34.12	1.47	198.68
Germany	20.30	17.48	1.29	91.37
Greece	3708.36	7534.43	4.71	23571.95
Ireland	179.22	218.78	1.66	1193.98
Italy	133.18	105.94	5.29	501.52
Japan	51.68	32.99	2.45	160.43
Portugal	276.86	308.39	3.86	1554.02
Spain	124.26	105.85	2.34	504.15
United Kingdom	41.45	31.32	1.19	164.63
United States	26.96	17.36	1.07	100.25

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Table 2: estimated partial correlations between contemporaneous country spread effects, and the corresponding significance $t - test$ values (in parentheses). Bold values indicate not significant values at the 1% level

Country	France	Germany	Greece	Ireland	Italy	Japan	Portugal	Spain	UK	USA
France	1.00	0.67	-0.34	0.05	0.35	0.21	0.17	0.21	-0.22	- 0.19
		(47.90)	(-19.48)	(2.86)	(19.80)	(11.57)	(9.34)	(11.59)	(-12.00)	(-10.36)
Germany	0.67	1.00	0.15	0.27	-0.05	-0.05	-0.24	-0.27	0.61	- 0.04
	(47.95)		(8.26)	(14.80)	(-2.69)	(-2.91)	(-13.32)	(-15.13)	(41.04)	(**2.21**)
Greece	-0.34	0.15	1.00	-0.24	0.10	-0.01	0.14	0.50	-0.10	0.01
	(-19.48)	(8.26)		(-13.35)	(5.71)	(**-0.91**)	(7.75)	(30.73)	(-5.56)	(**0.84**)
Ireland	0.05	0.27	-0.24	1.00	-0.61	-0.24	0.73	0.54	-0.10	0.28
	(2.86)	(14.80)	(-13.80)		(-40.99)	(-13.33)	(55.88)	(33.71)	(-5.36)	(15.24)
Italy	0.35	-0.05	0.10	-0.61	1.00	-0.18	0.55	0.55	0.12	0.15
	(19.80)	(-2.69)	(5.71)	(-40.99)		(-9.58)	(34.81)	(34.66)	(6.57)	(8.17)
Japan	0.21	-0.05	-0.01	-0.24	-0.18	1.00	0.36	0.21	0.13	0.28 (15.31)
	(11.57)	(-2.91)	(**-0.91**)	(-13.33)	(-9.58)		(20.23)	(11.33)	(7.06)	(15.31)
Portugal	0.17	-0.24	0.14	0.73	0.55	0.36	1.00	-0.33	-0.07	-0.21 (-11.52)
	(9.34)	(-13.32)	(7.75)	(55.88)	(34.81)	(20.23)		(-18.59)	(-3.90)	(-11.52)
Spain	0.21	-0.27	0.50	0.54	0.55	0.21	-0.33	1.00	0.02	0.06
	(11.59)	(-15.13)	(30.73)	(33.71)	(34.66)	(11.33)	(-18.59)		(**1.12**)	(3.43)
UK	-0.22	0.61	-0.10	-0.10	0.12	0.13	-0.07	0.02	1.00	0.52
	(-12.00)	(41.04)	(-5.56)	(-5.36)	(6.57)	(7.06)	(-3.90)	(**1.12**)		(32.06)
USA	-0.19	0.04	0.01	0.28	0.15	0.28	-0.21	0.06	0.52	1.00
	(-10.36)	(**2.21**)	(**0.84**)	(15.24)	(8.17)	(15.31)	(-11.52)	(3.43)	(32.06)	

Table 3: comparison between the two components of risk obtained with our proposed structural VAR model (*CoRisk* and autoregressive parts) and the eigenvector centrality measures.

Country	Corisk	Autoreg	Eigenvector Centrality
France	18.49	23.43	0.98
Germany	17.43	12.12	0.82
Greece	400.80	2454.28	0.16
Ireland	58.77	118.89	0.60
Italy	-5.98	86.46	0.84
Japan	-2.62	33.44	0.51
Portugal	-9.14	183.05	0.66
Spain	-45.28	81.70	0.81
United Kingdom	0.23	27.59	0.63
United States	3.94	17.59	0.53

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Table 4: comparison between the root mean square errors (RMSE) obtained with our full structural VAR model (RMSE full) and with a model composed by the solely autoregressive component (RMSE only autoregressive).

Country	RMSE only autoregressive	RMSE full
France	3.86	3.42
Germany	0.36	0.29
Greece	5.65	4.33
Ireland	2.86	2.82
Italy	1.07	0.42
Japan	0.22	0.23
Portugal	1.63	2.38
Spain	0.13	0.16
United Kingdom	0.36	0.25
United States	0.07	0.08

Table 5: comparison between the root mean square errors (RMSE) obtained with our full structural VAR model (RMSE full) and with a model composed by the solely autoregressive component (RMSE only autoregressive).

Country	RMSE only autoregressive	RMSE full
France	0.83	0.57
Germany	0.25	0.18
Greece	104.31	71.04
Ireland	0.82	0.18
Italy	0.56	1.32
Japan	0.55	0.53
Portugal	1.43	1.19
Spain	2.15	1.89
United Kingdom	0.08	0.04
United States	0.21	0.19

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Figures

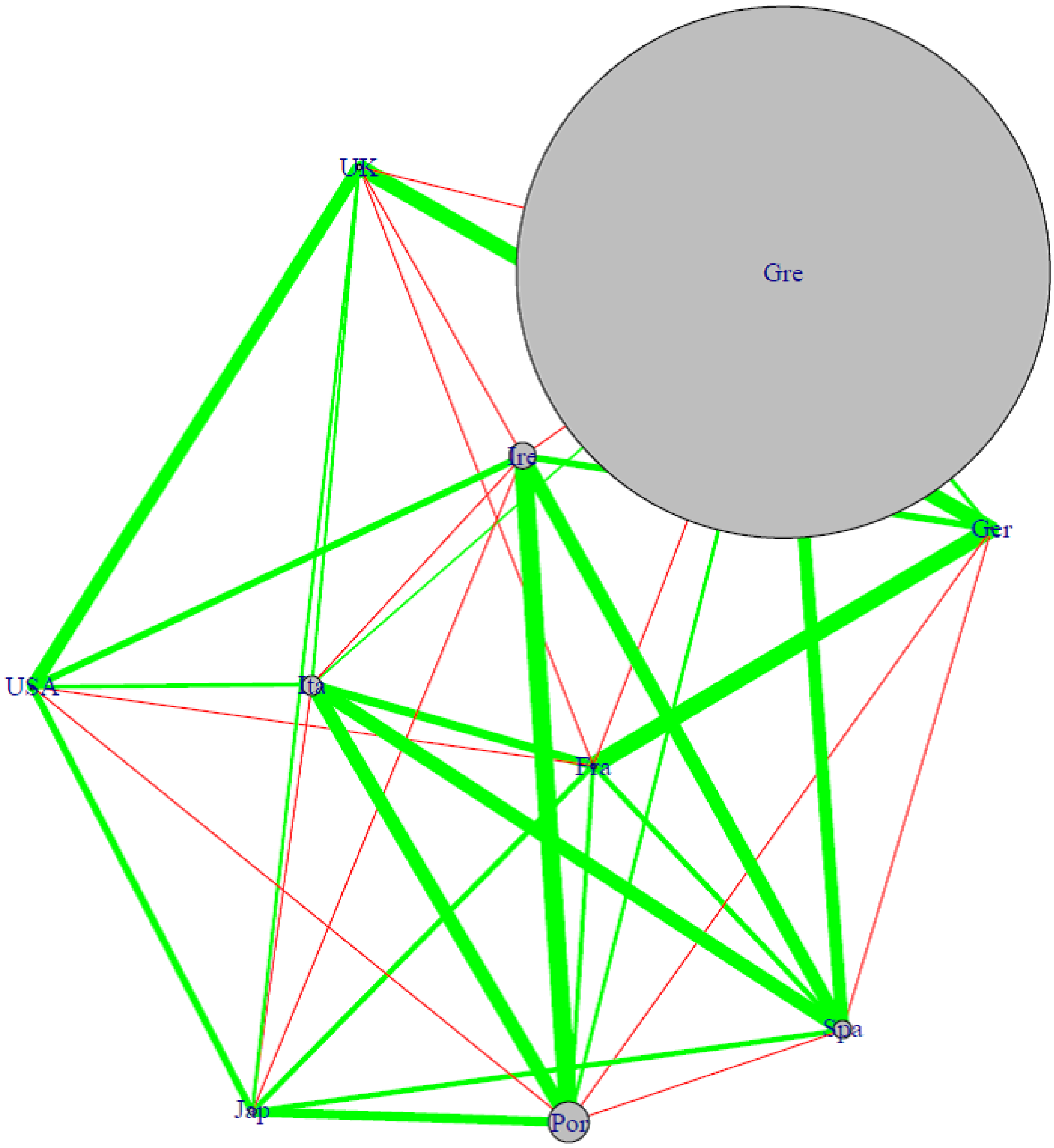

Figure 1: partial correlation network referred to 10 countries: five core economies (France, Germany, Japan, the United Kingdom and the United State) and five peripheral ones (Greece, Ireland, Italy, Portugal and Spain). Green lines stand for positive partial correlations, red lines for negative correlations; the ticker the line, the stronger the connection; the larger a node, the higher the corresponding idiosyncratic probability of default.

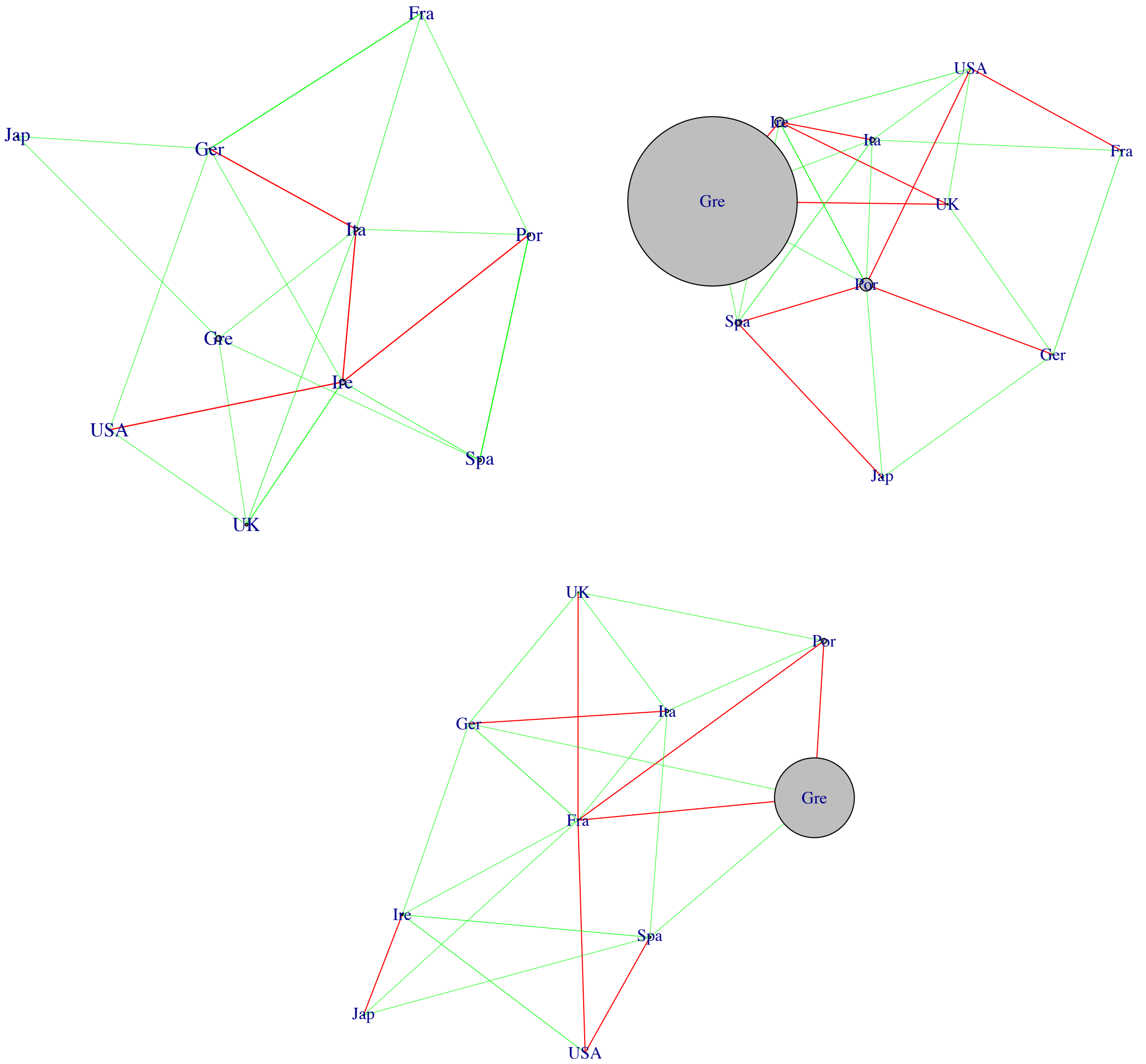

Figure 2: partial correlation network referred to the 10 considered countries in three different time periods (financial-crisis, sovereign-crisis, post-crisis). Green lines stand for positive partial correlations, red lines for negative correlations; the ticker the line, the stronger the connection; the larger a node, the higher the corresponding idiosyncratic probability of default.

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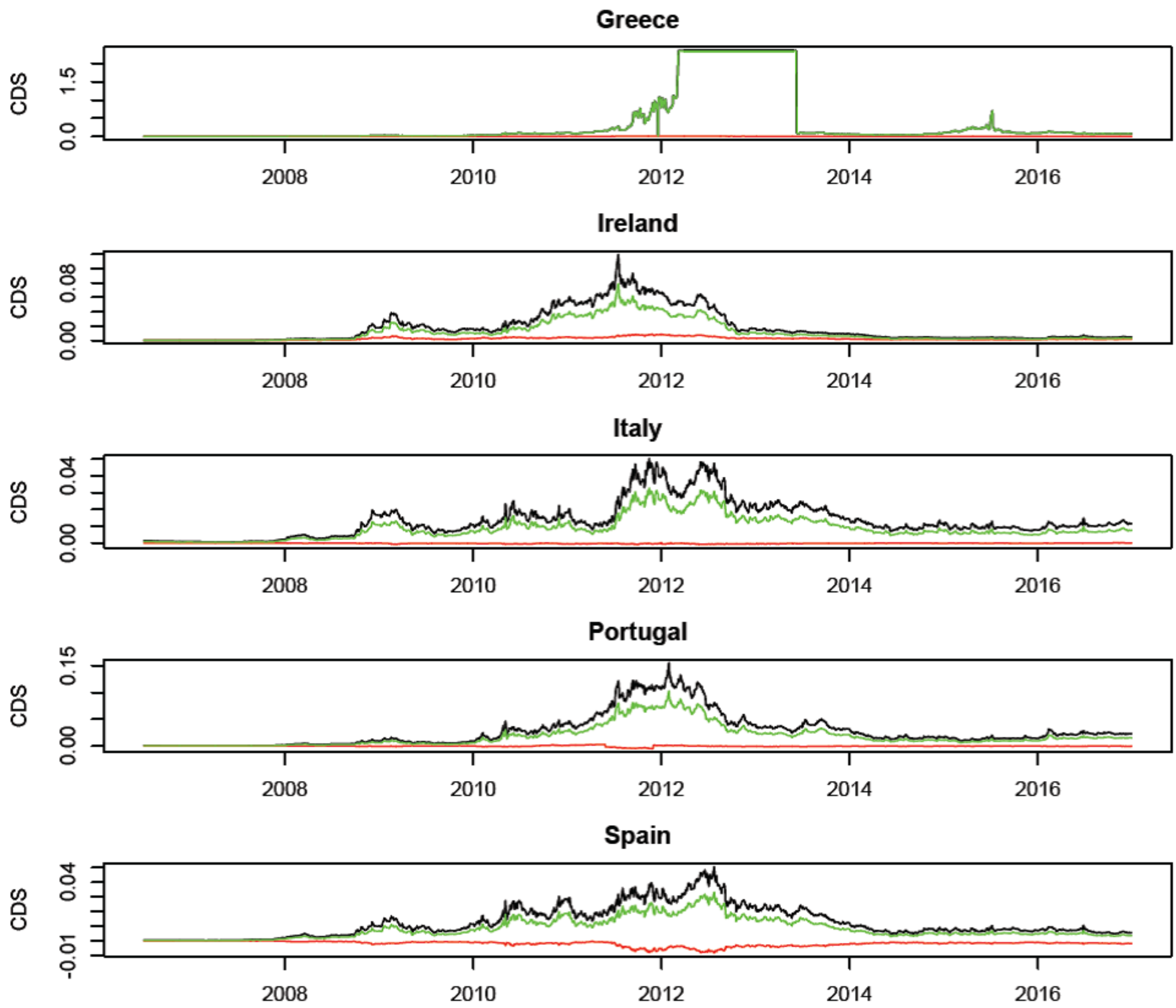
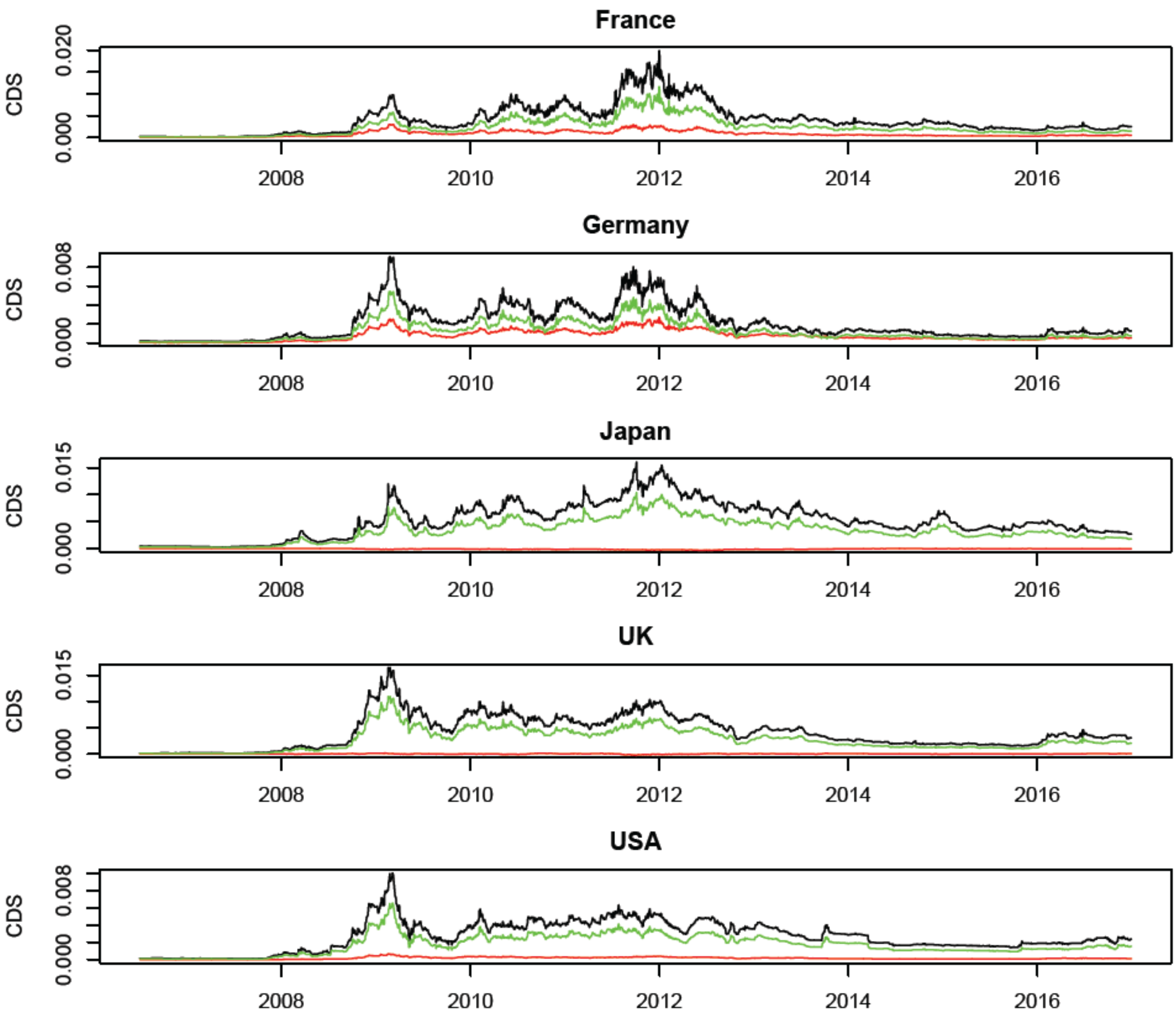


Figure 3: for each considered peripheral country, the graphs report the observed CDS spreads (black line), the estimated autoregressive CDS spread components (green line) and the estimated contemporary CDS spread component (*CoRisk*, red line).



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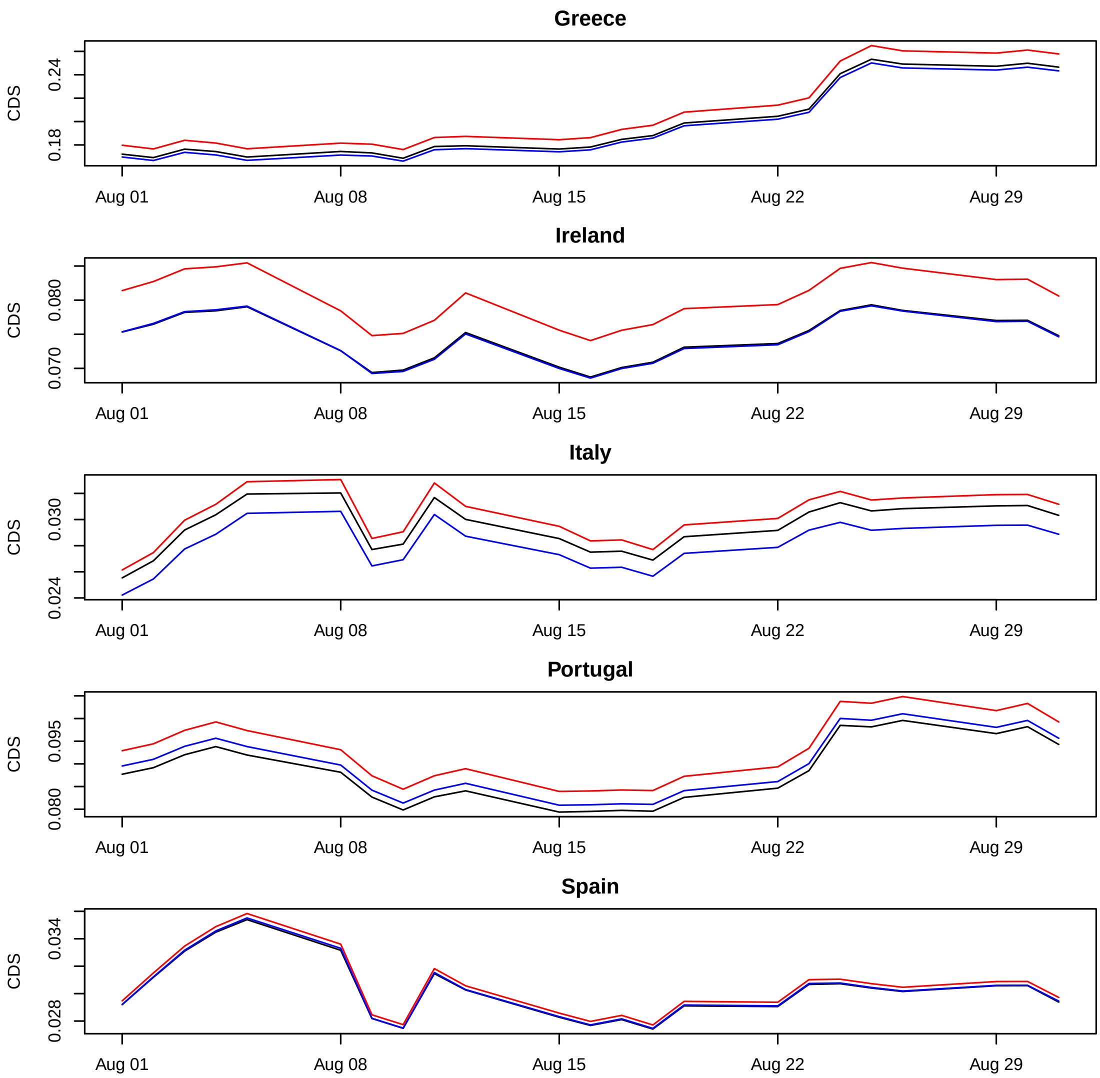

Figure 5: for each considered peripheral country, the graphs report the observed CDS spreads at time t (black line), the CDS spreads predicted at time $t-1$ using the proposed model (red line) and the CDS spreads predicted at time $t-1$ using only the autoregressive component (blue line).

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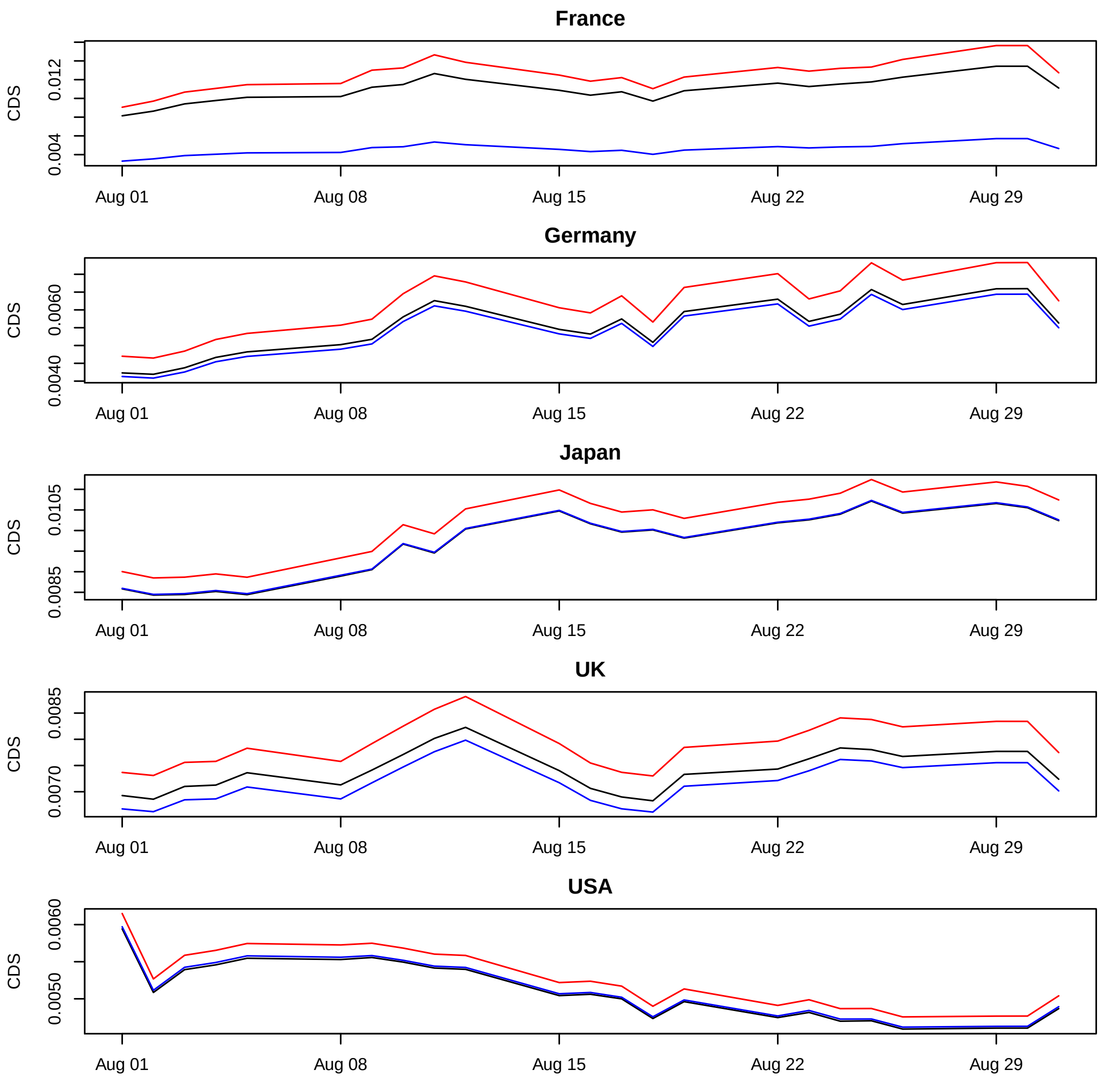

Figure 6: for each considered core country, the graphs report the observed CDS spreads at time t (black line), the CDS spreads predicted at time $t-1$ using the proposed model (red line) and the CDS spreads predicted at time $t-1$ using only the autoregressive component (blue line).

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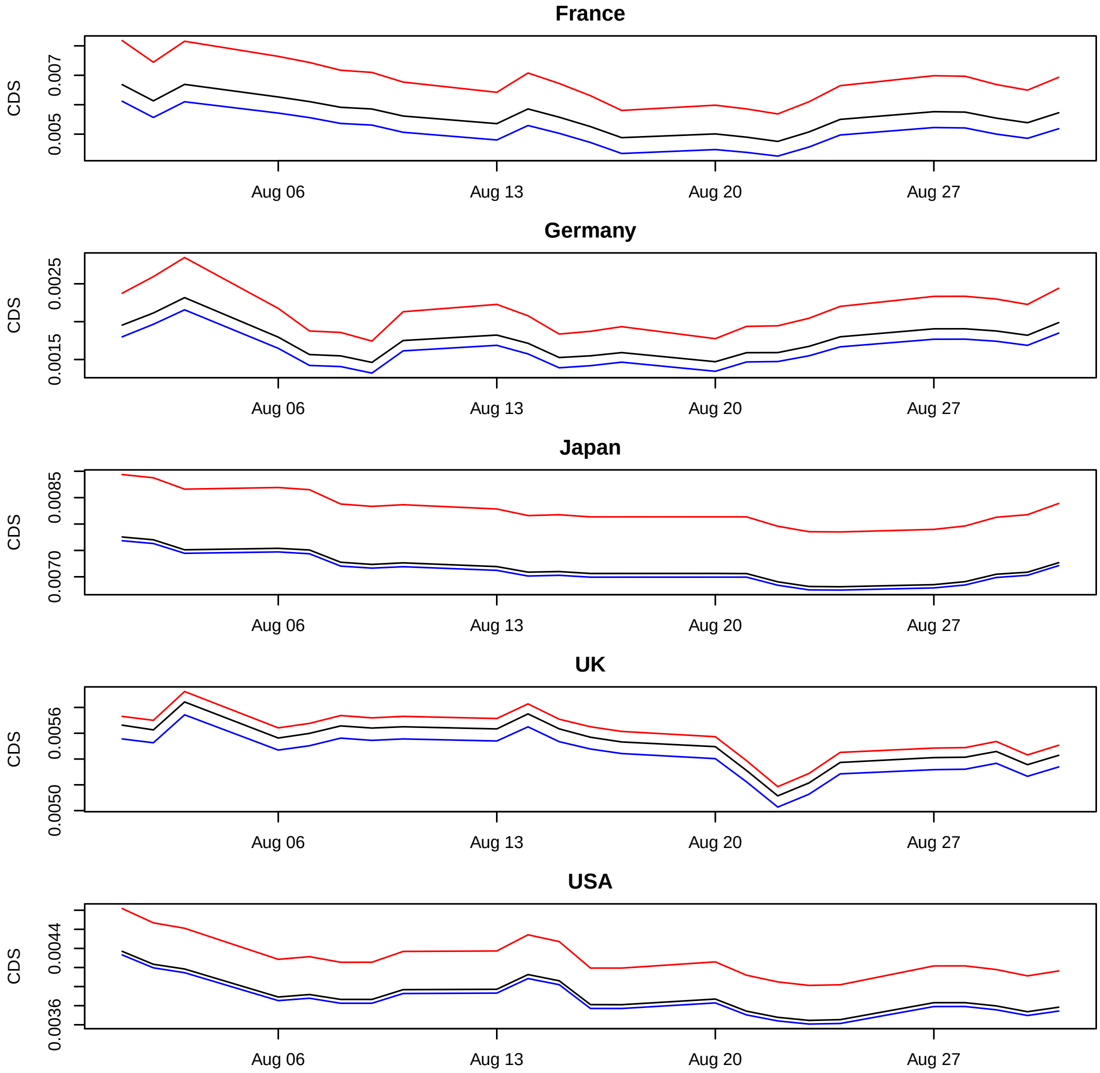

Figure 7: for each considered peripheral country, the graphs report the observed CDS spreads at time t (black line), the CDS spreads predicted at time $t-1$ using the proposed model (red line) and the CDS spreads predicted at time $t-1$ using only the autoregressive component (blue line).

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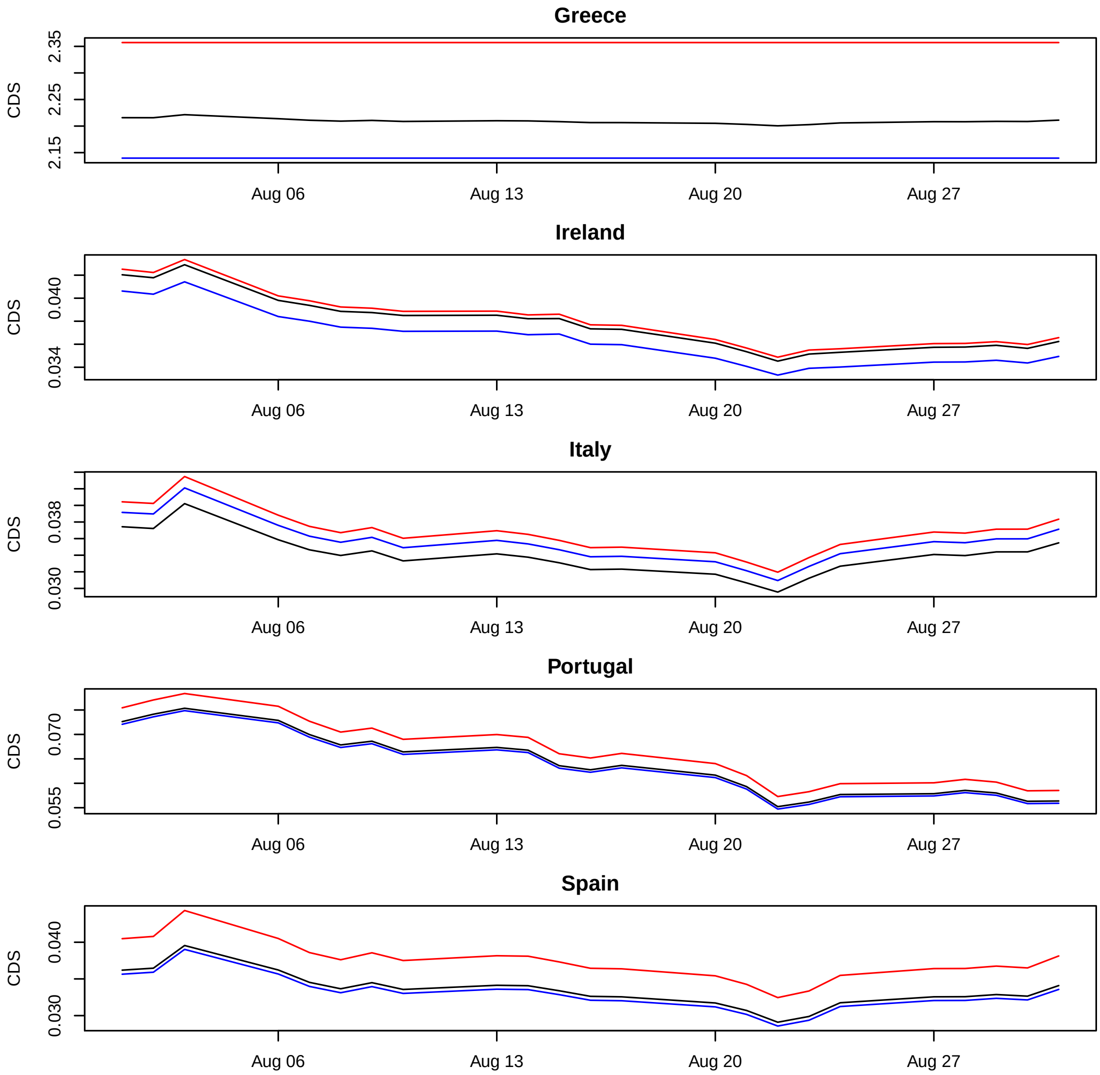

Figure 8: for each considered core country, the graphs report the observed CDS spreads at time t (black line), the CDS spreads predicted at time $t-1$ using the proposed model (red line) and the CDS spreads predicted at time $t-1$ using only the autoregressive component (blue line).