A review on the possible existence of strong elementary charge and its nuclear scale applications

U. V. S. Seshavatharam¹ & S. Lakshminarayana²

¹Honorary Faculty, I-SERVE, Survey no-42, Hitech city, Hyderabad-84, Telangana, India.
²Department of Nuclear Physics, Andhra University, Visakhapatnam-03, AP, India
Emails: seshavatharam.uvs@gmail.com and sln@auvsp.edu.in

Abstract: We review the basics of nuclear binding energy scheme assumed to be associated with the existence of a new strong elementary charge associated with square root of reciprocal of the strong coupling constant.

Keywords: strong coupling constant, strong elementary charge, nuclear binding energy.

1. Introduction

As strong interaction [1] is mostly hidden at low energy scales in the form of ‘residual nuclear force’ and Liquid drop model and Fermi gas model [2-5] are failing in understanding nuclear binding energy with ‘strong coupling constant’, in our earlier published paper [6] and recent submitted papers [7,8] we suggested that, by considering ‘square root’ of reciprocal of the strong coupling constant

$$\alpha_s \approx 0.1186$$

as an index of strength of nuclear elementary charge, nuclear binding energy and nuclear stability can be understood. Our model [6-11] seems to be simple and realistic compared to the new integrated model [12,13]. In this paper we review sections 6 and 7 with much better semi empirical relations.

2. About the semi empirical mass formula

Let $A$ be the total number of nucleons, $Z$ the number of protons and $N$ the number of neutrons. According to the semi-empirical mass formula [2,3,4], nuclear binding energy:

$$B \approx a_v A - a_s A^{2/3} - a_c \left( \frac{Z}{A^{1/3}} \right)^2 - a_p \left( \frac{A - 2Z}{A} \right) \pm \frac{a_p}{\sqrt{A}} \quad (1)$$

Here $a_v$ = volume energy coefficient, $a_s$ is the surface energy coefficient, $a_c$ is the coulomb energy coefficient, $a_p$ is the asymmetry energy coefficient and $a_p$ is the pairing energy coefficient. By maximizing $B(A,Z)$ with respect to $Z$,

$$Z \approx \frac{A}{2 + \left( \frac{a_c}{2a_s} \right) A^{2/3}} \quad \text{and} \quad A - 2Z = \frac{0.4A^2}{A + 200} \quad (2)$$

Maximizing $B(A)/A$ with respect to $A$ gives the nucleus which is most strongly bound or most stable.

3. New concepts and semi empirical relations of nuclear binding energy and stability

We would like to suggest that,

1) There exists a strong nuclear charge,

$$e_s \approx \frac{e}{\sqrt{\alpha_s}} \approx 4.652 \times 10^{-19} \text{ C}$$

2) Proton magnetic moment [1] can be addressed with

$$\mu_p \approx \frac{e_h}{2m_p} \approx 1.467 \times 10^{-26} \text{ J.T}^{-1}$$

3) Neutron magnetic moment [1] can be addressed with

$$\mu_n \approx (e_s - e) \frac{h}{2m_n} \approx 9.602 \times 10^{-27} \text{ J.T}^{-1}.$$ 

4) Characteristic nuclear radius can be expressed as,
\[ R_s \approx \left( \frac{1}{\sqrt{\alpha_s}} \right) \left( \frac{h}{m_c} + \frac{h}{m_p} \right) \approx \left( \frac{e}{e} \right) \left( \frac{2h}{m_c} \right) \approx 1.22 \text{ fm.} \]

where \( m_p \approx m_s \).

5) Nuclear beta stability line \([4]\) can be addressed with a relation of the form, \( A_s \approx 2Z + s(2Z)^2 \)

where \( s \approx \left( \frac{e_s}{m_p} + \frac{e_s}{m_e} \right) \approx 0.00158143 \).

6) Nuclear binding energy can be understood with a single energy coefficient of magnitude

\[ \frac{e_s e}{8\pi \hbar^2 (\hbar/m_c)} \approx 10.0 \text{ MeV}. \]

7) In deuteron, there exists no strong interaction in between neutron and proton.

4. Beta stability line with respect to strong coupling constant

If \( \alpha_s \approx 0.1186 \), for \( Z > 8 \), close to the line of beta stability,

\[ A_s \approx \left[ Z + \left( \frac{e_s}{e} \right)^{6/5} \right] \approx \left( Z + 2.904 \right)^{1/2} \]

(3)

See table 1, column-2.

For \( Z > 16 \), close to the line of beta stability,

\[ (A_s - 2Z)^2 \approx (Z \beta + 1)^2 - 4 \]

where \( \beta = \left( \frac{3}{5} \right) \alpha_s \approx 0.07116 \).

(4)

See table 1, column-3.

5. Beta stability line with respect to nucleon mass difference

With reference to nucleon and electron rest masses \([1]\), we noticed that,

\[ \exp \left( \frac{(m_n - m_p)c^2}{m_e c^2} \right) \approx 12.5659102 \approx 4\pi \]

(5)

where, \( m_s c^2 \approx 939.575413 \text{ MeV} \),
\( m_p c^2 \approx 938.272081 \text{ MeV} \); \( m_e c^2 \approx 0.51099895 \text{ MeV} \)

Based on this observation, beta stability line can be understood with the following empirical relations.

Let, \( k \approx (1/4\pi)^2 \approx 0.006333 \)

(6)

\begin{align*}
A_s & \approx 2Z + (Z/4\pi)^2 \approx 2Z + kZ^2 \\
N_s & \approx Z + (Z/4\pi)^2 \approx Z + kZ^2 \\
\left( Z/\sqrt{A_s - 2Z} \right) & \approx 4\pi
\end{align*}

(7)

See table 1, column-4. Based on these relations,

\[ \frac{A_s - 2Z}{A_s} \approx k^2 A_s N_s \sqrt{Z} \]

A) \[ \frac{A_s - 2Z}{A_s} \approx k^2 A_s N_s \sqrt{Z} \]
B) \[ \frac{A_s^{1/2} N_s^{1/2} Z^{1/2}}{\sqrt{A_s - 2Z}} \approx \frac{1}{k} \approx 4\pi \]

(8)

<table>
<thead>
<tr>
<th>Proton number ( Z )</th>
<th>Stable mass number ( A_s )</th>
<th>Relation (3)</th>
<th>Relation (4)</th>
<th>Relation (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>30</td>
<td>28</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>36</td>
<td>35</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>43</td>
<td>42</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>50</td>
<td>49</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>57</td>
<td>56</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>64</td>
<td>63</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>71</td>
<td>71</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>86</td>
<td>86</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>94</td>
<td>93</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>101</td>
<td>101</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>109</td>
<td>109</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>117</td>
<td>117</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>125</td>
<td>125</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>133</td>
<td>133</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>141</td>
<td>141</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>150</td>
<td>149</td>
<td>148</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>158</td>
<td>158</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>166</td>
<td>166</td>
<td>165</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>175</td>
<td>175</td>
<td>174</td>
<td></td>
</tr>
<tr>
<td>74</td>
<td>183</td>
<td>183</td>
<td>183</td>
<td></td>
</tr>
<tr>
<td>77</td>
<td>192</td>
<td>192</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>201</td>
<td>201</td>
<td>201</td>
<td></td>
</tr>
</tbody>
</table>
6. Semi empirical relation for nuclear binding energy

Based on the new integrated model proposed by N. Ghahramany et al [12,13],

\[
B(Z,N) = \left[ A - \left( \frac{N^2 - Z^2}{3Z} + \delta(N - Z) + 3 \right) \right] \frac{m_e c^2}{\gamma}
\]

(9)

where, \( \gamma = \) Adjusting coefficient \( \approx \) (90 to 100).

if \( N \neq Z \), \( \delta(N - Z) = 0 \) and if \( N = Z \), \( \delta(N - Z) = 1 \).

We noticed that,

\[
\frac{m_e c^2}{\gamma} \approx \frac{m_e c^2}{(90 \text{ to } 100)} \approx \left( \frac{e e}{8 \pi \varepsilon_0 (\hbar / m_c)} \right) \approx 10.0 \text{ MeV}
\]

(10)

And with reference to relation (7), it is also possible to show that, for \( Z \approx (40 \text{ to } 83) \), close to the beta stability line [7],

\[
\left( \frac{N^2 - Z^2}{Z} \right) \approx kA Z
\]

(11)

Based on the above relations and proposed concepts, and with reference to the first four terms of the semi empirical mass formula, close to the beta stability line [8], if \( \alpha \approx 0.1186 \) and \( R_c \approx 1.22 \text{fm} \), semi empirically, we developed the following relations.

a) Starting from \( Z=3 \), close to the beta stability line,

\[
(B) = \left( (Z-1) \left( \frac{e e}{4 \pi \varepsilon_0 (\hbar / m_c)} \right) \right) \mp 10.0 \text{ MeV}
\]

\[
\approx \left( (Z-1) \times 20.0 \text{ MeV} \right) \mp 10.0 \text{ MeV}
\]

(12)

For example, binding energy of Oxygen (O) close to its stable atomic nuclides can be estimated to be \( [(8 - 1) \times 20] \mp 10.0 \text{ MeV} \approx (130 \text{ to } 150) \text{ MeV} \).

Binding energy of Iron (Fe) close to its stable atomic nuclides can be estimated to be \( [(26 - 1) \times 20] \mp 10.0 \text{ MeV} \approx (490 \text{ to } 510) \text{ MeV} \).

Binding energy of Tin (Sn) close to its stable atomic nuclides can be estimated to be \( [(50 - 1) \times 20] \mp 10.0 \text{ MeV} \approx (970 \text{ to } 990) \text{ MeV} \).

Binding energy of Lead (Pb) close to its stable atomic nuclides can be estimated to be \( [(82 - 1) \times 20] \mp 10.0 \text{ MeV} \approx (1610 \text{ to } 1630) \text{ MeV} \).

b) For \( A \approx (4 \text{ to } 80) \), close to the beta stability line.

\[
(B) = \left( A - \sqrt{A} \left( \frac{e e}{8 \pi \varepsilon_0 (\hbar / m_c)} \right) \right) \mp 10.0 \text{ MeV}
\]

\[
\approx \left( A - \sqrt{A} \times 10.0 \text{ MeV} \right)
\]

(13)

Binding energy of Helium-4 can be approximately estimated to be \( (4 - \sqrt{4}) \times 10.0 \text{ MeV} \approx 20.0 \text{ MeV} \).

Actual binding energy is 28.296 MeV.

Binding energy of Carbon-12 can be approximately estimated to be \( (12 - \sqrt{12}) \times 10.0 \text{ MeV} \approx 95.5 \text{ MeV} \).

Actual binding energy is 92.162 MeV.

Binding energy of Scandium-45 can be approximately estimated to be \( (45 - \sqrt{45}) \times 10.0 \text{ MeV} \approx 382.92 \text{ MeV} \). Actual binding energy is 387.848 MeV.

<table>
<thead>
<tr>
<th>83</th>
<th>209</th>
<th>210</th>
<th>210</th>
</tr>
</thead>
<tbody>
<tr>
<td>86</td>
<td>218</td>
<td>219</td>
<td>219</td>
</tr>
<tr>
<td>89</td>
<td>227</td>
<td>228</td>
<td>228</td>
</tr>
<tr>
<td>92</td>
<td>236</td>
<td>237</td>
<td>238</td>
</tr>
<tr>
<td>95</td>
<td>245</td>
<td>246</td>
<td>247</td>
</tr>
<tr>
<td>98</td>
<td>254</td>
<td>256</td>
<td>257</td>
</tr>
<tr>
<td>101</td>
<td>263</td>
<td>265</td>
<td>267</td>
</tr>
<tr>
<td>107</td>
<td>281</td>
<td>284</td>
<td>287</td>
</tr>
<tr>
<td>110</td>
<td>291</td>
<td>294</td>
<td>297</td>
</tr>
<tr>
<td>113</td>
<td>300</td>
<td>304</td>
<td>307</td>
</tr>
<tr>
<td>116</td>
<td>309</td>
<td>314</td>
<td>317</td>
</tr>
</tbody>
</table>
Binding energy of Manganese-55 approximately can be estimated to be,
\[
(55 - \sqrt{55}) 	imes 10.0 \text{ MeV} \approx 475.84 \text{ MeV}.
\]
Actual binding energy is 482.072 MeV.

Binding energy of Kripton-80 approximately can be estimated to be,
\[
(80 - \sqrt{80}) 	imes 10.0 \text{ MeV} \approx 710.56 \text{ MeV}.
\]
Actual binding energy is 695.434 MeV.

c) For \((Z \geq 4)\), close to the beta stability line,

\[
B_{(Z,A)} \approx \left\{ A - \left[ \left( \frac{kAZ}{2.531} + 2.531 \right) + 1 \right] \right\} \cdot 10.06 \text{ MeV}
\]
where \(\left( \frac{m_e - m_p}{m_c^2} \right) \approx \ln(4\pi) \approx 2.531\) and
\[
\left( \frac{kAZ}{2.531} + 2.531 \right) = "\text{New term" needs explanation} \]

We are working on understanding the physical significance of \(\left( \frac{kAZ}{2.531} + 2.531 \right)\). It needs further study at basic level.

See the following figure-1. Green curve represents the binding energy per nucleon estimated with the first four terms of SEMF relation (1) and (7). Dashed red curve represents the binding energy per nucleon estimated with relations (7) and (14).

Figure 1: Comparison of estimated and SEMF binding energy per nucleon

See table-2 for the isotopic binding energy of Z=50

<table>
<thead>
<tr>
<th>Mass number</th>
<th>Estimated binding energy (MeV) Relation(14)</th>
<th>Actual binding energy (MeV)</th>
<th>Error (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>950.2</td>
<td>953.532</td>
<td>3.3</td>
</tr>
<tr>
<td>114</td>
<td>967.8</td>
<td>971.574</td>
<td>3.7</td>
</tr>
<tr>
<td>115</td>
<td>976.6</td>
<td>979.121</td>
<td>2.5</td>
</tr>
<tr>
<td>116</td>
<td>985.4</td>
<td>988.684</td>
<td>3.2</td>
</tr>
<tr>
<td>117</td>
<td>994.2</td>
<td>995.627</td>
<td>1.4</td>
</tr>
<tr>
<td>118</td>
<td>1003.0</td>
<td>1004.955</td>
<td>1.9</td>
</tr>
<tr>
<td>119</td>
<td>1011.8</td>
<td>1011.438</td>
<td>-0.4</td>
</tr>
<tr>
<td>120</td>
<td>1020.6</td>
<td>1020.546</td>
<td>-0.1</td>
</tr>
<tr>
<td>122</td>
<td>1038.2</td>
<td>1035.53</td>
<td>-2.7</td>
</tr>
<tr>
<td>124</td>
<td>1055.9</td>
<td>1049.963</td>
<td>-5.9</td>
</tr>
</tbody>
</table>

7. To understand the binding energies of Deuteron, Triton and \(^4\)He.

If it is assumed that there exists no strong interaction in between neutron and proton, above relation (13) can be expressed as follows.

\[
(B) \approx \left\{ \left( A - \sqrt{A} \right) \left( \frac{e^2}{8\pi\hbar c} \right) \right\}
\]

Based on this relation (15), Deuteron \((^2\text{H})\) binding energy can be estimated to be 2.02 MeV and actual binding energy is 2.225 MeV.

From relation (15), Triton \((^3\text{H})\) binding energy can be estimated to be 4.37 MeV. From relation (13), Triton \((^3\text{H})\) binding energy can be estimated to be 12.68 MeV. Actual binding energy (8.482 MeV) seems to be close to the average of (4.37 and 12.68) MeV = 8.525 MeV. Clearly speaking, binding energy of \((^3\text{H})\) seems to follow electromagnetic interaction as well as strong interaction and needs further study.
In the similar way, \((^{3}\text{He})\) binding energy can be understood in terms of the combined effect of electromagnetic and strong interactions.

8. Conclusion

Nowadays, estimating and understanding nuclear binding energy with ‘strong interaction’ seems to attract many nuclear physicists. In this context, by considering the proposed semi empirical relations, existence of the ‘strong elementary charge’ can be confirmed. With further research, a realistic nuclear model pertaining to strong interaction can be developed.

Acknowledgements

Author Seshavatharam is indebted to his brother B.Vamsi Krishna for his valuable support.

References