

# Multi-Objective Optimization for Metal Mine Production Technical

## Indicators with NSGA-II and ANN Algorithms

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**Abstract:** The selection of the best mine production technical indicators is crucial to increasing a mine's economic benefit and saving resources for sustainability. Therefore, this research proposes a 'multi-objective optimization model' based on a 'fast and elitist Non-dominated Sorting Genetic Algorithm' (NSGA-II) and 'Artificial Neural Networks' (ANN) for the optimization of production technical indicators in the entire geology, mining and beneficiation metal mine production processes. The multi-objective optimization model has decision variables including 'cut-off grade,' 'industrial grade' and 'loss rate,' with objectives being 'economic benefit (profit)' and 'resource benefit (metal volume).' First, the relationship between the technical indicators of mine production is studied. The REG model, MATLAB's own ksdensity function and the BP neural network are used to calculate the ore weight, the probability density of grade distribution, the dilution rate, the concentration ratio and the concentrate grade, and to further calculate geological reserves, profit and metal volume. Then, the NSGA-II is applied to maximize profit and metal volume simultaneously. Finally, the model is applied to the Huogeqi copper mine. The optimization result is a set of multiple optimal solutions called Pareto optimal solutions. Compared with the plan data, the profit and metal volume of partial optimization results increased by 2.89% and 2.64% simultaneously. These Pareto optimal solutions can help decision makers in bettering the actual process of metal mine production.

**Keywords:** Multi-objective optimization; metal mine; production technical indicators; NSGA-II; artificial neural networks

### 1. Introduction

Mine production technical indicators refer to those that can be artificially adjusted to control the production of mines, thus affecting the operation efficiency of a mining company. Such indicators include cut-off grade, industrial grade, geological grade, dilution rate, loss rate, mining grade, beneficiation grade, concentration ratio, concentrate grade and concentrate volume. These production technical indicators are closely linked and mutually restrained to form a system. A change of one indicator will cause changes to others. With market economy changes and the progress of production technology, it is necessary to adjust and optimize these indicators in time to obtain the best operation results, which is an inevitable link in the production of mine enterprises. The continuous mining of mineral resources increases the scarcity of resources, so the operating efficiency of mines should consider their economic and resource benefits. Therefore, in the whole geology, mining and beneficiation metal mine production processes, multi-objective optimization of production technical indicators has great practical significance.

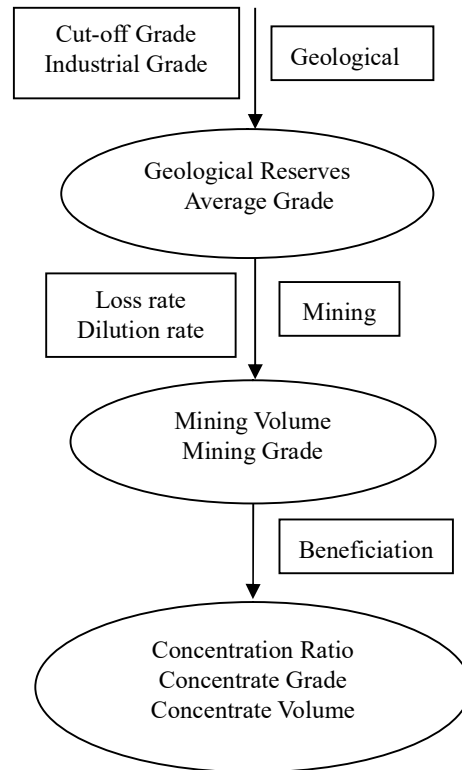
In recent years, researchers have studied the optimization of mine production technology indicators in three major aspects. The first is single-objective optimization of mine production technology indicators, without considering multiple objectives. Azimi et al. [1] established a nonlinear optimization model with objectives in which the maximum net present value and the cut-off grade is

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the decision variable. The model is optimized by an augmented Lagrangian genetic algorithm. He et al. [2] set up an optimization model in which resource utilization rate and concentrate volume are taken as the constraints, and the maximum net present value is taken as the objective. The PSO-ANN method is used to solve the model. The calculation results show that this method can effectively increase the resource utilization rate and concentrate volume and greatly increase the net present value of mine enterprises. Ahmadi et al. [3] optimized the cut-off grade of open pit mines by using genetic algorithms based on the capacity constraints of the mine smelter. The results obtained by the genetic algorithm are compared with the Lane model, which shows that the genetic algorithm can get results faster and more accurately. The second aspect is multi-objective optimization for mine production technical indicators of the entire mine production process using the traditional method. The traditional method converts multiple goals into single goals through weighted summation, but the optimization results largely depend on subjective weights [4]. Li et al. [5] and Liu et al. [6] proposed a multi-objective optimization model for metal mine production technical indicators on the basis of geology, mining and beneficiation for the entire production process and adopted the fuzzy comprehensive evaluation method to solve the model. The third aspect is the multi-objective optimization of production technologies using evolutionary algorithms, and this considers beneficiation process but not geology and mining process. Yu et al. [4] presented a nonlinear multi-objective programming model for a mineral processing production planning (MPPP) for optimizing five production indices, including concentrate grade, metal recovery, concentrate volume, concentration ratio and production cost. The model is solved using gradient-based NSGA-II and gradient-based SPEA2, respectively. Wang et al. [7] proposed a multi-objective optimization model based on data driven to optimize the concentrate grade and concentrate volume of complex mineral process and used the Reference Vector Guided Evolutionary Algorithm-Gaussian process (RVEA-GP) to solve the model. Although progress has been made in the optimization of above-mentioned mine production technical indicators and some results have been achieved in practice, few researchers have developed multi-objective optimization evolutionary algorithms for mine production technical indicators in the entire geology, mining and beneficiation metal mine production processes. Fig. 1 shows the geology – mining – beneficiation metal mine production processes.

In recent years, researchers have developed a variety of multi-objective evolutionary algorithms (MOEAs), such as the fast and elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) [8], multi-objective particle swarm optimization (MOPSO) [9] and multi-objective differential evolution (MODE) [10]. The common goal of these methods is finding the optimal distribution of the Pareto optimal frontier. Among these methods, the NSGA-II algorithm has the advantages of good robustness, high computational efficiency and diversity and has been widely used to solve multi-objective optimization problems [11-13]. Artificial neural networks are considered to be effective tools for engineering modeling, mainly because of their ability to maximize the approximation of complex problems [14-16]. Some researchers combined ANN with NSGA-II to solve the multi-objective optimization problem of complex process and achieved good results [17-19]. Therefore, this paper uses ANN combined with NSGA-II to optimize the metal mine production technical indicators.

The purpose of this study is to establish a multi-objective optimization model with economic and resource benefits as the objectives in the whole geology, mining and beneficiation metal mine production processes. The ANN and NSGA-II are combined to optimize the metal mine production technical indicators.



**Fig. 1** The geological – mining – beneficiation production processes of a metal mine

## 2. Multi-objective optimization model for metal mine production technical indicators

### 2.1. Mathematical model of metal mine production technical indicators process

#### 2.1.1. Geological Reserves Model

In the dual-grade system, when the cut-off grade and industrial grade combination is different, there is a need for re-delineating the ore body and estimating the new grade of geological reserves and average grade. Due to the cut-off grade and industrial grade combination, a great deal of engineering is required to estimate the solid and block model with the help of 3DMine or Geovia Surpac. To estimate the geological reserves and the average grade, the mathematical statistics modeling process is relatively simple. The specific method is as follows.

(1) Calculation of geological reserves:

$$Q = Q(p_1, p_2) = Q_0 \times \frac{\int_{p_1}^{p_2} \varphi(x) \cdot D(x) \cdot f(x) dx + \int_{p_2}^{\infty} D(x) \cdot f(x) dx}{\int_{p_a}^{p_b} \varphi(x) \cdot D(x) \cdot f(x) dx + \int_{p_b}^{\infty} D(x) \cdot f(x) dx} \quad (1)$$

where  $p_a$  is the original cut-off grade;  $p_b$  is the original industrial grade;  $Q_0$  is the original geological reserve;  $p_1$  is the new cut-off grade;  $p_2$  is the new industrial grade;  $\varphi(x)$  is the probability function of the non-economic reserve sample entering the economic reserve sample between the cut-off and industrial grade;  $D(x)$  is the function with the sample grade ( $x$ ) as an independent variable and ore body weight as a dependent variable;  $f(x)$  is the probability function

on the sample grade distribution;  $Q$  is the geological reserve value when the cut-off grade is  $p_1$  and the industrial grade is  $p_2$ ; and  $Q(p_1, p_2)$  is the corresponding function of  $p_1$  and  $p_2$  as independent variables and  $Q$  as the dependent variable.

$$\varphi(x) = \left(\frac{x - p_1}{p_2 - p_1}\right)^m \quad (p_1 \leq x \leq p_2) \quad (2)$$

where  $m$  is constant, and the value depends on the geological conditions of the mine.

(2) Calculation of average grade

$$p_3 = P(p_1, p_2) = \frac{\int_{p_1}^{p_2} x \cdot \varphi(x) \cdot D(x) \cdot f(x) dx + \int_{p_2}^{\infty} x \cdot D(x) \cdot f(x) dx}{\int_{p_1}^{p_2} \varphi(x) \cdot D(x) \cdot f(x) dx + \int_{p_2}^{\infty} D(x) \cdot f(x) dx} \quad (3)$$

where  $p_3$  is the geological average grade value when the cut-off grade is  $p_1$  and the industrial grade is  $p_2$ ;  $P(p_1, p_2)$  is the corresponding function of  $p_1$  and  $p_2$  as independent variables and  $p_3$  as a dependent variable.

### 2.1.2. Mining model

The mining model mainly includes the relationship model on the loss rate – dilution rate, and the calculation model of mining grade and mining volume. In general, the loss rate and dilution rate of ore mining have a certain correlation; the greater the dilution rate, the smaller the loss rate. Thus, a linear or non-linear regression function of loss rate and depletion rate is established:

$$c_2 = f_1(c_1) \quad (4)$$

where  $c_1$  is the loss rate;  $c_2$  is the depletion rate.

Calculation of mining volume:

$$Q_1 = Q \cdot \frac{1 - c_1}{1 - c_2} \quad (5)$$

where  $Q_1$  is the mining volume.

Calculation of mining grade:

$$p_4 = p_3 \cdot (1 - c_2) \quad (6)$$

where  $p_4$  is the mining grade.

### 2.1.3. Beneficiation model

In this paper, there is no pre-beneficiation in the mine production system, so the beneficiation grade and the beneficiation volume are equal to the mining grade and mining volume:

$$p_5 = p_4 \quad (7)$$

$$Q_2 = Q_1 \quad (8)$$

where  $p_5$  is the beneficiation grade;  $Q_2$  is the beneficiation volume.

Among the influencing factors of beneficiation indicators, the mathematical model of the beneficiation grade, concentration ratio and concentrate grade are analyzed emphatically, and concentrate volume are calculated.

Beneficiation grade and concentration ratio than the general reverse relationship establish the linear or non-linear regression function on the beneficiation grade and concentration ratio:

$$c_3 = f_2(p_5) \quad (9)$$

where  $c_3$  is the concentration ratio.

The relationship between concentrate grade and beneficiation grade and concentration ratio is complex and obviously nonlinear. It is difficult to establish multiple linear or nonlinear regression functions to reflect the relationship between them. Therefore, artificial neural networks are used to establish the functional relationship between them:

$$p_6 = f_3(p_5, c_3) \quad (10)$$

where  $p_6$  is the concentrate grade.

Calculation of concentrate volume:

$$Q_3 = Q_2 / c_3 \quad (11)$$

where  $Q_3$  is the concentrate volume.

## 2.2. Decision variables, constraints and objective functions

### 2.2.1. Decision variables and constraints

#### (1) Decision variables

Based on the metal mine production process, select the corresponding decision variables by analyzing the entire production process. Select the cut-off grade ( $p_1$ ), the industrial grade ( $p_2$ ) and the loss rate ( $c_1$ ) as the decision variables of this study.

#### (2) Constraints

Values of the decision variables must be within a given range, for the cut-off grade of equation (12), the industrial grade equation (13) and the loss rate equation (14).

$$p_{1\min} \leq p_1 \leq p_{1\max} \quad (12)$$

$$p_{2\min} \leq p_2 \leq p_{2\max} \quad (13)$$

$$c_{1\min} \leq c_1 \leq c_{1\max} \quad (14)$$

The cut-off grade has an equation relationship with the industrial grade as equation (15); the copper concentrate grade is above its market minimum grade standard as equation (16).

$$p_1 \leq p_2 \quad (15)$$

$$p_{6\min} \leq p_6 \quad (16)$$

### 2.2.2. The objective functions

#### (1) The resource benefit objective function

Metal ore is a non-renewable resource; when mining, its resource benefit should be considered. The metal volume can measure the resource benefit of mine production and reflect the mine's utilization of resources. The larger metal volume are, the better the corresponding solution.

The objective function of metal volume is expressed by equation (17):

$$\max MQ = Q_3 \times p_6 \quad (17)$$

where  $MQ$  is the metal volume; other symbols are consistent with the previous equations.

#### (2) The economic benefit objective function

The economic benefit is to maximize profit.

The objective function of profit is represented by equation (18):

$$\max \theta = Q_3 \cdot f_4(p_6) - (h_1 \cdot Q_1 + h_2 \cdot Q_2) \quad (18)$$

where  $f_4(p_6)$  is the function of  $p_6$  as an independent variable and  $q$  as a dependent variable;

$q$  is the concentrate transaction price;  $h_1$  is the unit mining cost;  $h_2$  is the unit beneficiation cost.

## 3. Multi-objective optimization for metal mining production based on NSGA-II and ANN algorithm

### 3.1. Methods description

#### 3.1.1. Multi-objective optimization

In recent years, multi-objective optimization has been successfully applied in many fields such as management [20], chemistry [21], biology [22], machine [23] and civil engineering [24]. In general, a constrained multi-objective optimization problem can be mathematically formulated as follows [25]:

$$\begin{cases} \text{minimize} & F(x) = \{f_1(x), f_2(x), \dots, f_n\} \\ \text{subject to} & g_i(x) \leq 0, \quad i = 1, 2, \dots, m \\ & x_j^l \leq x_j \leq x_j^u \quad j = 1, 2, \dots, k \end{cases} \quad (19)$$

where  $x$  is the set of decision variables;  $x_j^l$  and  $x_j^u$  are the minimum and maximum values of each

decision variables  $x_j$ ;  $k$  is the number of decision variables;  $F(x)$  and  $g_i(x)$  are the set of

objective and constraint functions;  $n \geq 2$ ;  $m$  are the number of objective and constraint functions.

Compared with single-objective optimization, the complexity of multi-objective optimization is greatly increased, and it can optimize multiple objectives at the same time. These objectives are often irreversible or conflicting, and a goal for their improvement may lead to a reduction in the performance

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of another target. The essential difference with the single-objective optimization problem is that the solution of the multi-objective optimization problem is not unique; it is a set of Pareto optimal solutions. The academic community generally believes that Vilfredo Pareto proposed a multi-objective solution. A Pareto solution cannot be improved in any objective without damaging at least one other objective. The purpose of multi-objective optimization is to find the Pareto solutions. The set of those Pareto solutions is called the Pareto Front.

### 3.1.2. Artificial neural networks

Artificial neural networks require the use of physical devices or computers to simulate the biological neural network structure and are widely used in control, classification, nonlinear prediction, optimization and many other areas. In 1943, McCulloch et al. [26] created the mathematical model description of neurons for the first time and offered groundbreaking work for neural network technology. In 1949, Hebb et al. [27] proposed the Hebb learning rule, which was the earliest learning rule of artificial neural networks. In 1958, Rosenblatt [28] proposed a perceptron network model, from which the first complete artificial neural network model was born. In 1974, Werbos [29] proposed the back propagation (BP) algorithm, which is currently the most influential artificial neural network learning algorithm. In 1982, Hopfield [30,31] proposed the Hopfield neural network model, an interconnected neural network, and the introduction of energy function, making the stability of the network a criterion. After many scientists' efforts and research, the study of artificial neural networks has experienced breakthroughs and important achievements in military, communications, industrial and other fields of application where it is highly effective. We apply the BP neural network to the metal mine production process to calculate the concentrate grade in this study.

### 3.1.3. NSGA-II

In 1989, Goldberg [32] proposed a method to compute the fitness based on the Pareto optimal theory, using a non-inferior solution and selection operator to optimize the population in the Pareto optimal direction. Based on Goldberg's idea, many multi-objective optimization algorithms were proposed, including the Niche Pareto Genetic Algorithm (NPGA) [33], Non-dominated Sorting Genetic Algorithm (NSGA) [34] and Strength Pareto Evolutionary Algorithm (SPEA) [35]. These algorithms can obtain uniformly distributed non-inferior optimal solutions when solving multi-objective optimization, but sharing parameters and high computational complexity. Based on NSGA, Deb [8] proposed NSGA-II with an elite strategy, which reduces the computational complexity and avoids the setting of shared parameters. Generally, the NSGA-II algorithm consists of the following basic steps:

Step 1: Initialize the parameters of NSGA-II and randomly generate the parent population  $P_0$  with the population size of  $N$ .

Step 2: Through the basic operation of traditional genetic algorithms, such as mutation and crossover, produce the next generation of offspring population  $Q_k$  with a population size of  $N$ . The two populations are mixed together to form a population  $R_k$  with a population size of  $2N$ .

Step 3: Sort the new population  $R_k$  based on non-domination criteria.

Step 4: Calculate the crowding distance value for all individuals with different order values. Choose the better individual as the new parent population  $P_{k+1}$  using order values and crowding

distance value.

Step 5: Determine whether the algorithm satisfies the termination condition and if so, output the result; otherwise, return to Step 2.

In recent years, noteworthy research has been done for process optimization by the NSGA-II algorithm. Among those studies are multi-objective process optimizations of oil and gas production [36], multi-objective process optimizations of friction stir welding [37], multi-objective process optimizations of electrical discharge machining [38] and multi-objective process optimization of Laser-magnetic [39] applications. These prior studies have proven the reliability and effectiveness of NSGA-II for solving multi-objective process optimization problems.

### 3.2. The ANN-NSGA-II algorithm for metal mine production technical indicator optimization

#### 3.2.1. Formulations of the multi-objective optimization for metal mine production technical indicators

The objectives of this study are to maximize the  $MQ$  and  $\theta$  value simultaneously. These objective functions are conflicting. To convert the objective functions for minimization, they are suitably modified. The objective functions are as given follows:

$$\text{objective 1} = -MQ \quad (20)$$

$$\text{objective 2} = -\theta \quad (21)$$

The multi-objective optimization for metal mine production can be formulated as follows:

$$\left\{ \begin{array}{l} \text{minimize} \quad \{-MQ(p_1, p_2, c_1)\} \\ \text{subject to} \quad p_{1\min} \leq p_1 \leq p_{1\max} \\ \quad \quad \quad p_{2\min} \leq p_2 \leq p_{2\max} \\ \quad \quad \quad c_{1\min} \leq p_2 \leq c_{1\max} \\ \quad \quad \quad p_1 \leq p_2 \\ \quad \quad \quad p_{6\min} \leq p_6 \end{array} \right. \quad (22)$$

#### 3.2.2. The ANN-NSGA-II algorithm

This paper combines ANN with the NSGA-II algorithm to form an ANN-NSGA-II algorithm, which is used to optimize metal mine production technical indicators. The flowchart of the ANN-NSGA-II algorithm for the optimization of metal mine production technical indicators is shown in Fig. 2.



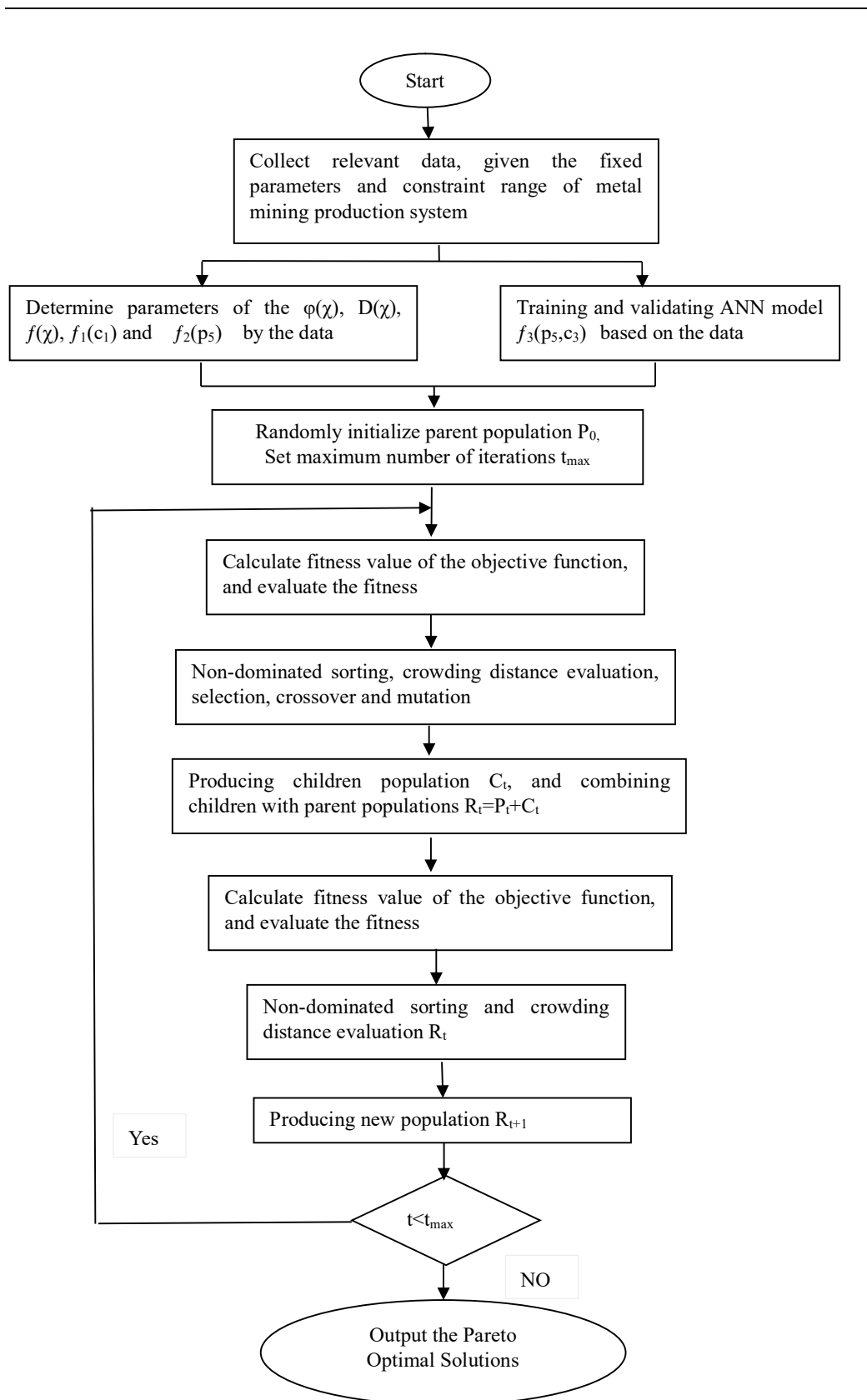


Fig. 2 The ANN-NSGA-II algorithm for optimization metal mine production technical indicator  
4. Example analysis

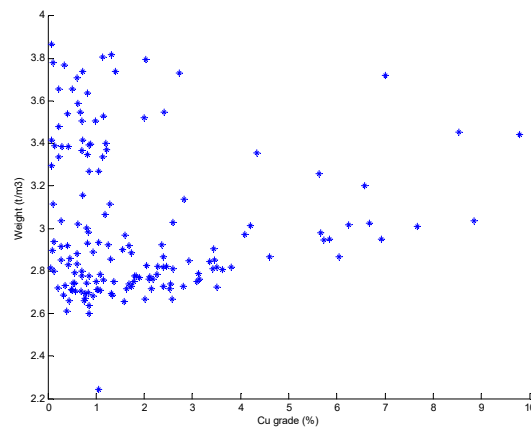
The Huogeqi copper mine is a geology, mining and beneficiation system of mines, which is a well-known large-scale enterprise in China. At present, the following problems exist in the production of mines. First, current production indicators are determined according to the mining and beneficiation processes of the late last century. In recent years, mining and beneficiation technologies and process have improved, and it is necessary to conduct research now. Second, in order to achieve the sustainable development of mines, the resource benefit should be considered in the production process. The current production technical indicators have not considered the resource benefit. Therefore, it is necessary for the Huogeqi copper mine to carry out the multi-objective optimization of production technical indicators. In the next five years, the Huogeqi copper mine will be mined between 600 and 900 meters underground for ore; the objective of this article is to carry out the multi-objective optimization for it.

#### 4.1. Computation models

##### 4.1.1. Ore weight computation model

Based on the data of 156 sets of copper ore weight and grade collected from the Huogeqi stage of -600 ~ -900 m, the scatter plot of weight and grade is drawn as shown in Fig. 3. It can be clearly seen from Fig. 3 that the distribution of scatter points is relatively scattered with no obvious distribution. The data of weight and grade were statistically analyzed, and the Spearman correlation coefficient and significance level between the data were -0.0532 and 0.5085, respectively. As the significant level of 0.5085 is greater than 0.05, it did not pass the test of significance, indicating that there is no correlation between copper ore weight and grade, so that for the function of weight for the average ore weight, the mathematical expression is as follows:

$$D(x) = 2.76 \quad (23)$$

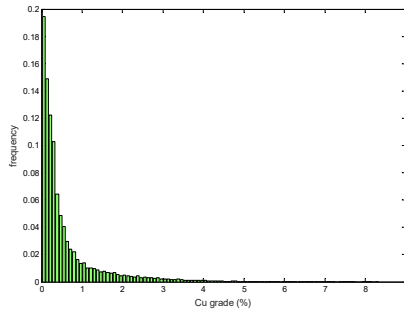


**Fig. 3** Scatter plot of copper ore weight and grade

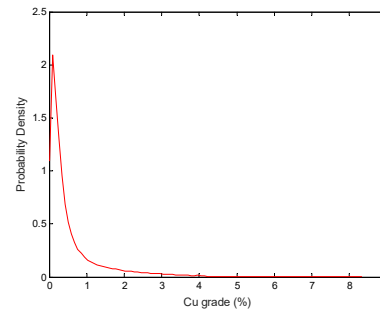
##### 4.1.2. Probability density of grade distribution computation model

The data of the grades and sample lengths of -600 ~ -900 m obtained from the Huogeqi copper mine are provided by the mine's geological department, with a total of 48332 sets of data. The frequency histogram of the sample grade is shown in Fig. 4. MATLAB's own ksdensity function was used to calculate the sample size of the probability density function, and the probability density function curve is shown in Fig. 5. Since the probability density function sought by this method is an implicit function, there is no specific mathematical expression. To verify the fitting accuracy of the probability density function, the maximum points of Fig. 4 and Fig. 5 are moved to the same point for

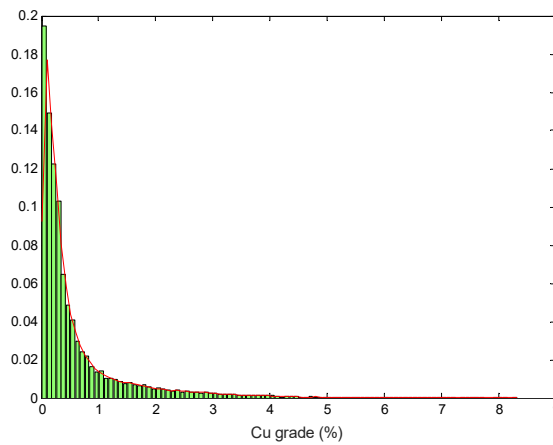
combination, as shown in Fig. 6, which shows that the probability density function fitting effect is good.



**Fig. 4** Copper grade frequency distribution histogram



**Fig. 5** Copper grade probability density

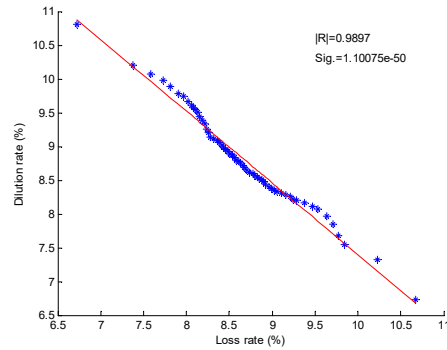


**Fig. 6** Copper grade frequency distribution histogram and probability density

#### 4.1.3. Dilution Rate computation model

The monthly data on loss and dilution rates from January 2012 to December 2016 provided by the Huogeqi copper mine are plotted on a scatter plot of loss and dilution rates, as shown in Fig. 7. Fig. 7 shows that the depletion rate is linearly distributed with the loss rate. The linear correlation coefficient is -0.9897 and the significance level  $1.0075e-50$  between them by calculation. As the significance level  $1.0075e-50$  is far less than 0.05, the significance test shows that the dilution rate has a strong linear relationship with the loss rate. The mathematical expression for the depletion rate calculation model is as follows:

$$c_2 = f_1(c_1) = -1.0631 \times c_1 + 18.0268 \quad (24)$$

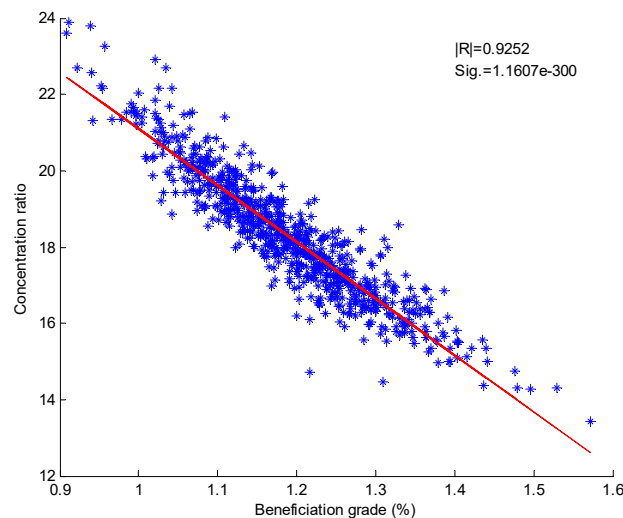


**Fig. 7** Linear fit of dilution rate and loss rate

#### 4.1.4. Concentration ratio computation model

The daily data of 711 groups' beneficiation production from January 2015 to December 2016 was provided by the Huogeqi copper mine, and the scatter plot of concentration ratio and beneficiation grade is drawn as shown in Fig. 8. Fig. 8 shows a clear linear distribution between the concentration ratio and the beneficiation grade. The linear correlation coefficient is -0.9252 and the significance level is 1.1607e-300 between them. As the significance level of 1.1607e-300 is far less than 0.05, the significance test shows that the concentration ratio has a strong linear relationship with the beneficiation grade. The mathematical expression for the calculation model of concentration ratio is as follows:

$$c_3 = f_2(p_5) = -1482.7903 \times p_5 + 35.9238 \quad (25)$$



**Fig. 8** Linear fit of concentration ratio and beneficiation grade

#### 4.1.5. Concentrate grade computation model

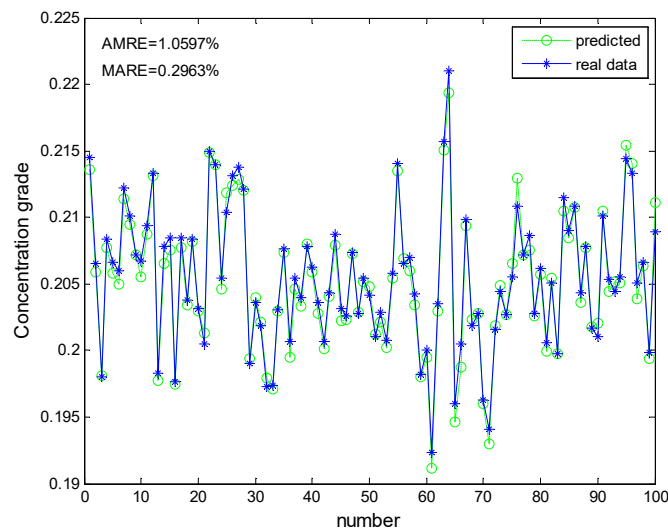
We built a back-propagation neural network using the concentration ratio and beneficiation grade as inputs and the concentration grade as the output. According to 711 groups' daily data of beneficiation production, we used data from the 1st to the 611th day as training samples and data from the 612th to the 711th day as test samples.

We used the newff() function to build the back-propagation neural network, which contains 2

input nodes, 1 hidden layer, and 1 output node. The 'tansig' and 'purelin' functions are selected as the transfer functions of the hidden layer and the output layer, respectively; the 'traingdm' is selected as the learning algorithm, the precision is selected as 0.0000001 and the maximum number of iterations selected is as 2500. To choose the best hidden nodes, two statistical parameters called the Mean Absolute Relative Error (MARE) and the Absolute Maximum Relative Error (AMRE) are used. The referenced statistical parameters have been calculated for concentration grade with different hidden nodes and presented in Tab. 1. From comparing MARE and AMRE, the hidden node 3 is superior to others, and the hidden node is chosen to be 3. The modeling accuracy of the back-propagation neural network models in predicting the concentration grade has been demonstrated in Fig. 9. As the figure shows, the ANN models predict the concentration grade with high degree of accuracy.

**Tab. 1** The BP network results comparison with different nodes

Hidden nodes	Concentration grade			
	Train MARE (%)	Test MARE (%)	Train AMRE (%)	Test AMRE (%)
1	0.8417	0.7491	7.3575	4.7698
2	0.3057	0.2979	1.4701	1.0916
3	0.3049	0.2963	1.4543	1.0597
4	0.3124	0.3019	1.6102	1.0677
5	0.3215	0.3025	1.8151	1.4596



**Fig. 9** Concentration grade predicted by ANN

#### 4.1.6. Copper concentrate transaction price calculation model

The market transaction prices of Chinese copper concentrates are mainly based on 1 # copper. The transaction price of different concentrate grades are adjusted on this basis. The compensation price and pricing coefficient corresponding to the grade of copper concentrate obtained from the Huogeqi mine are shown in Tab. 2. The mathematical calculation of the transaction price is as follows:

$$q = f_4(p_6) = q_1 \times p_6 \times \lambda + q_2 \quad (26)$$

where  $q_1$  is the Shanghai Stock Exchange # 1 copper settlement price;  $\lambda$  is the pricing

coefficient and  $q_2$  is the compensation price.

**Tab.2** Different copper grade corresponds to compensation price and pricing coefficient

Grade (%)	compensation price ( $\$ \cdot t^{-1}$ )	pricing coefficient
$\geq 23$	47.4	0.86
22.00~22.99	31.6	0.85
21.00~21.99	15.8	0.84
20.00~20.99	0	0.83
19.00~19.99	-15.8	0.81
18.00~18.99	-31.6	0.795
17.00~17.99	-47.4	0.78
16.00~16.99	-63.2	0.77

#### 4.2. Results and discussion

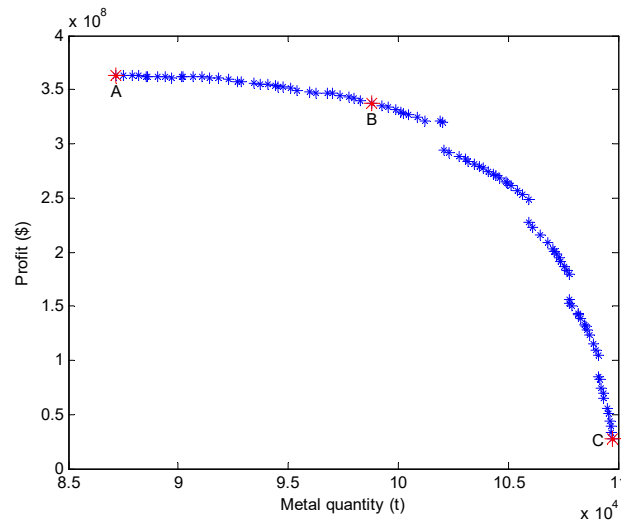
We use the proposed the ANN-NSGA-II model to optimize production technical indicators of the Huogeqi copper mine over the next five years. According to the production requirements of the Huogeqi copper mines, the industrial grade ranges from 0.1% to 0.9%, the boundary grade ranges from 0.1% to 0.9% and the dilution rate ranges from 6% to 12%. The parameters for the proposed model of the Huogeqi copper mine and NSGA-II model are shown in Tab. 3.

**Tab. 3** Parameters of Huogeqi copper mine and NSGA-II model

Parameter	Notation	Value
Huogeqi copper mine		
The original grade of cut-off (%)	$p_a$	0.30
The original grade industrial (%)	$p_b$	0.50
The original geological reserve(t)	$Q_0$	9000000
Constant	$m$	0.66
Unit mining cost ( $\$/t$ )	$h_1$	15.8
Unit beneficiation cost( $\$/t$ )	$h_2$	18.96
The settlement price of # 1 copper( $\$/t$ )	$q_1$	8088.02
Lower bound of cut-off grade (%)	$p_{1\min}$	0.10
Upper bound of cut-off grade (%)	$p_{1\max}$	0.90
Lower bound of industrial grade (%)	$p_{2\min}$	0.10
Upper bound of industrial grade (%)	$p_{2\max}$	0.90
Lower bound of loss rate (%)	$c_{1\min}$	6

Upper bound of loss rate (%)	$C_{1\min}$	12
Lower bound of concentrate grade (%)	$p_{6\min}$	16
<b>NSGA-II</b>		
Number of decision variables	n	3
Number of objective functions	k	2
Population size	NP	100
Maximum number of iterations	$t_{\max}$	150
Crossover index (SBX)	$\eta_c$	20
Mutation index (polynomial mutation)	$\eta_w$	100
Crossover probabilities	CP	0.5
Mutation probabilities	MP	1/3

The results of the best profit and metal volume for all points evaluated after 150 generations are shown in Fig. 10. The Pareto-optimal curve is clearly visible in Fig. 10. The Pareto optimal results clearly reveal the conflict between two objectives, the profit and the metal volume. Any change in profit will lead to a decline in metal volume and vice versa. No solution is better than the others; neither solution is acceptable. This shows that multi-objective optimization techniques are needed in the optimization of metal mine production. As shown in Fig. 10, there is a maximum profit at point A, and the metal volume is the smallest at this point. On the other hand, maximum metal volume at point C, with the least profit at this point. Point A is the best value for a single objective function for profit, and point C is the optimal value for a single objective function for the metal volume.



**Fig. 10** The Pareto-front for the Huogeqi copper mine production using NSGA-II

Tab. 4 shows the optimum values of two objectives for three typical points from A to C (Pareto-front) as well as the plan data from the Huogeqi copper mine production. The result of the optimization at point B is 2.89% more profit and 2.64% more metal volume than the plan data for the Huogeqi copper mine. Obviously, point B is superior in terms of profit and metal volume in the plan

data for the Huogeqi copper mine.

**Tab. 4** Three typical points from A to C in Pareto- optimal fronts and plan data

Parameters	A	B	C	Plan data
Profit(\$)	$3.6353 \times 10^8$	$3.3756 \times 10^8$	$2.7763 \times 10^7$	$3.2809 \times 10^8$
Metal quantity(t)	$8.7186 \times 10^4$	$9.8799 \times 10^4$	$1.0974 \times 10^4$	$9.6259 \times 10^4$
Cut-off grade (%)	0.594	0.365	0.120	0.3
Industrial grade (%)	0.628	0.410	0.130	0.5
Loss rate (%)	6.016	6.027	6.002	8

## 5. Conclusions

In this paper, by analyzing the characteristics of the metal mine production process, a multi-objective optimization model is built to maximize profit and metal volume. The hybrid algorithm of ANN-NSGA-II is used to solve the proposed optimization model. After modeling and optimizing the metal mine production process, the following conclusions are drawn.

First, aiming at the shortcomings of single-objective optimization or weighted multi-objective optimization of metal mine production technical indicators, a hybrid ANN-NSGA-II algorithm is proposed. The outer layer includes the NSGA-II algorithm, which directs the particle to the optimal solution. The inner layer includes the REG model, MATLAB's own ksdensity function and the BP neural network, which are used to calculate weight density, probability density of grade distribution, dilution rate, concentration ratio and concentrate grade, further calculation of geological reserves, profit and metal volume. The outer layer is searched globally, and the inner layer is locally fitted. The two layers jointly achieve the multi-objective optimization of metal mine production technical indicators.

Second, using the Huogeqi copper mine as an example, the proposed ANN-NSGA-II model is simulated, and the Pareto optimal solution set is obtained. The obtained Pareto solution sets have good distribution and convergence and meet the multi-objective optimization theory. The Pareto optimal solution set reflects the conflict of the objective solution set, indicating that the optimization of metal mine production technical indicators needs multi-objective optimization techniques.

Third, using the Huogeqi copper mine as an example, compared with the plan data, the profit and metal volume of partial optimization results increased by 2.89% and 2.64% simultaneously. The fitting results of artificial neural network established by this model are in agreement with the real data. The absolute maximum relative error and mean absolute relative error of the test are 1.0597% and 0.2963%, respectively. The hybrid algorithm that combines ANN with NSGA-II has proven to be effective and can help decision makers in bettering the actual process of metal mine production.

## Competing Interest

The authors declare that there are no competing interests regarding the publication of this paper.

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