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Article

Adaptive Trajectory Tracking Control of a Quadrotor Based on Iterative Learning Algorithm

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Abstract: This paper presents a new adaptive and optimal algorithm for the trajectory tracking control of a quadrotor using iterative learning algorithm (ILA) and enumerative learning algorithm. Ordinarily the ILA, as an adaptive method, can perform well with PID control to improve the controller's performance for a nonlinear system. Quadrotors are considered as non-linear and unstable systems which the use of an adaptive and optimal controller can increase its stability and decrease error level. In this method, a PID controller is proposed for the outer and inner control loops of a quadrotor and the ILA is used to adapt PID control gains. Subsequently, an enumerative learning algorithm is used to optimize the learning rates of the ILA. For this purpose, at first, the dynamic model of the quadrotor is acquired. After that, the structure of the inner and outer control loops is defined. In the end, the simulation results for the trajectory tracking control of a quadrotor are demonstrated. Through simulation, it is concluded that as time increases, the performance of the suggested control method in trajectory tracking control becomes better and better and error signals convergence to zero.

Keywords: Quadrotor; trajectory tracking control; PID control; iterative learning algorithm.

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1. Introduction

The quadrotor is the four-propeller type of unmanned aerial vehicles and has a wide range of utilities in civilian and military applications. Basically, the quadrotor differs from the common helicopter, which uses rotors with dynamically varying pitch blades. Quadrotor's smaller blades are a useful feature since they contain less kinetic energy thus reducing their ability to cause damage. Owing to the simple and low-cost build and high maneuverability, quadrotors are the most common and popular flying robots in the world. The quadrotor has four inputs and six coupled outputs. Moreover, it is considered as an under-actuated and highly nonlinear system. So, design and implement a control system for quadrotor had been a challenge in recent years.

PID control algorithm is a common method in control and robotics. It is simple to design in simulation and practice. There are several combinations of PID control algorithm with other methods which can improve the performance of the controller. Therefore, a cascade control based on PID controller and a generalized predictive controller was designed to regulate the position of quadrotor [1]. A parameterized transfer function was obtained for a quadrotor by the linear estimation and identification, with aerodynamic concepts to design controllers in an analytical way. Also, a PID controller was used by the root locus analysis and Ziegler-Nichols tuning rules based on the developed model [2]. A quadrotor model was created using a PD controller for the roll, pitch, and yaw control meanwhile a PID was necessary for the altitude control in order to remove the steady state error [3]. A recursive algorithm for a PID controller was designed to simplify the calculation stages concerning the tracking errors [4]. A PID controller with an extended Kalman filter was developed to filter out the sensors and system noises was studied [5]. A fuzzy control logic was developed to adjust classic PD and PID control for quadrotor stability control [6,7]. A bilinear PD controller was compared with a regular PD controller [8].

To deal with uncertainties in control of quadrotor, a linear quadratic regulator controller was applied for better stabilization and improving the flight quality of quadrotor under noisy sensor measurements [9]. A sliding mode controller was obtained based on the back-stepping method for stabilization of quadrotor. Also, a nonlinear observer was used in order to estimate unmeasured states and external additive disturbances [10, 11]. A model reference adaptive and fixed gain linear quadratic regulator controllers were implemented for a quadrotor with parametric uncertainties. Simulation and experiment results proved that the combination of these two controllers results in enhanced tracking performance and robustness to parametric uncertainties [12]. An intelligent control algorithm based on particle swarm optimization was used to increase the performance of controllers in the presence of disturbances like wind effect [13,14].

Another issue in quadrotor research field is to design an algorithm for the trajectory tracking control. Accordingly, a combination of the integral back-stepping control with the sliding mode control was studied to use for stabilization of a quadrotor attitude and to accomplish the task of trajectory tracking control [15]. A feedback linearization and an adaptive sliding mode controller were designed for an autonomous quadrotor [16]. A combination of integral back-stepping control with an adaptive terminal sliding mode was designed for the attitude control and an adaptive robust PID controller for the position control of quadrotor [17]. The effectiveness of a PID controller and a back-stepping controller for trajectory tracking control of quadrotor was evaluated by simulation results [18]. A new adaptive controller was designed for an autonomous quadrotor using nonlinear dynamic inversion and neuro-adaptive methods [19].

In this paper, an adaptive and optimal control algorithm is designed for the trajectory tracking control of a quadrotor. Typically, the ILA is considered as an adaptive algorithm through which the performance of a control system becomes better and better as time increases. By this mean, a PID controller adapted by the P-type ILA is used for attitude control and a PID controller adapted by the PID-type ILA is used for altitude and position control. Therefore, the layout of this paper is as follows. In Section 2, the complete six-degree of freedom dynamic model of the quadrotor is discussed. The structures of the inner and outer control loops, including the ILA and PID control, are described in Section 3. Section 4 illustrates the performance of the baseline control method in trajectory tracking control through simulation. Finally, in Section 5 this document is terminated with the conclusion and the inspiration that can be derived from this work.

2. Quadrotor Modeling

The complete 6-DOF dynamic model of the quadrotor is presented in this section. For this mean, the system of coordinates to use must be defined first. There are two reference frames: an earth frame indicated by the “*e*” index and a body-fixed frame indicated by the “*b*” index. The earth frame is located at the operator’s position and the body fixed frame origin is coinciding with the center of the gravity of the quadrotor. Figure 1 shows a conceptual scheme of the quadrotor, reference frames, and Euler angles.

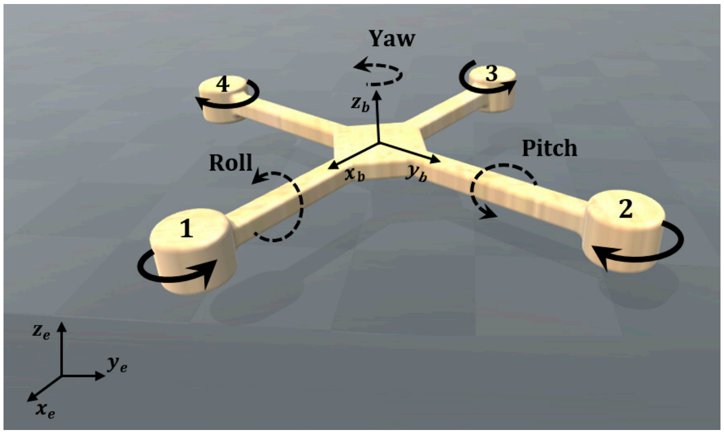


Figure 1. Quadrotor scheme

The quadrotor generally uses two pairs of rotors and to cancel the net moment along the z-axis, one pair (rotors 1 and 3) must rotate clockwise and one pair (rotors 2 and 4) counterclockwise. By changing the rotational speed of each rotor, the quadrotor supplants from a point to another through space. To make the pitch angle (θ), one can augment the speed of rotor 3 and reduce the speed of rotor 1. This action causes the quadrotor to move along the x-axis. Likewise, to make the roll angle (φ), one can augment the speed of rotor 2 and reduce the speed of rotor 4. This action also causes the quadrotor to move along the y-axis. Finally, to increase the yaw angle (ψ), one can augment the speed of rotors 1 and 3 and reduce the speed of rotors 2 and 4.

To drive dynamic equations of the quadrotor with respect to the earth frame, a rotation and a translational matrix should be used. The rotation matrix given by (1) is used to state linear velocities with respect to the earth frame. Also, the reverse translational matrix given by (2) is used to convert angular velocities with respect to the body-fixed frame to angular velocities with respect to the earth frame [20, 21].

$$R = \begin{bmatrix} \cos\theta\cos\psi & \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi \\ \cos\theta\sin\psi & \cos\varphi\cos\psi + \sin\varphi\sin\theta\sin\psi & \sin\theta\cos\varphi\sin\psi - \sin\varphi\cos\psi \\ -\sin\theta & \sin\varphi\cos\theta & \cos\varphi\cos\theta \end{bmatrix}, \tag{1}$$

$$T = \begin{bmatrix} 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi/\cos\theta & \cos\varphi/\cos\theta \end{bmatrix}, \tag{2}$$

Equation (1,2) shows the Newton-Euler equations which are used to drive the nonlinear dynamic model of the quadrotor.

$$F_{net} = \frac{d}{dt}[mV]_b + \Omega \times [mV]_b, \tag{3}$$

$$M_{net} = \frac{d}{dt}[I\Omega]_b + \Omega \times [I\Omega]_b, \tag{4}$$

The quadrotor is assumed to be symmetric with respect to the x and y-axes. It means that the center of gravity is located at the center of the quadrotor. Also, angles variations are small and the motor inertia is negligible rather than the quadrotor inertia. By considering these assumptions, the 6-DOF model of the quadrotor with respect to the body-fixed frame can be expressed as equations (1-6) [22, 23]:

$$\ddot{X} = \frac{1}{m}\, U_1\, (\cos\varphi\, \sin\theta\, \cos\psi + \sin\varphi\, \sin\psi), \tag{5}$$

$$\ddot{Y} = \frac{1}{m}\, U_1\, (\sin\theta\, \cos\varphi\, \sin\psi - \sin\varphi\, \cos\psi), \tag{6}$$

$$\ddot{Z} = \frac{1}{m}\, U_1\, (\cos\varphi\, \cos\theta) - g, \tag{7}$$

$$\ddot{\varphi} = \frac{1}{I_{xx}} U_2 - (\frac{I_{zz} - I_{yy}}{I_{xx}})\dot{\theta}\dot{\psi}, \tag{8}$$

$$\ddot{\theta} = \frac{1}{I_{yy}} U_3 - \left(\frac{I_{xx} - I_{zz}}{I_{xx}}\right)\dot{\varphi}\dot{\psi}, \tag{9}$$

$$\ddot{\psi} = \frac{1}{I_{zz}} U_4 - \left(\frac{I_{yy} - I_{xx}}{I_{zz}}\right)\dot{\varphi}\dot{\theta}, \tag{10}$$

where m and I stand for the quadrotor's mass and inertia matrix respectively. Also, U_i's are manipulated variables and are related to the thrust force and axial torques. Basically quadrotor motion is achieved by changing the combination and varying the speed of the rotors described as (5) [24]:

$$U_1 = k_T(w_1^2 + w_2^2 + w_3^2 + w_4^2), \tag{11}$$

$$U_2 = k_T l(w_3^2 - w_1^2), \tag{12}$$

$$U_3 = k_T l(w_2^2 - w_4^2), \tag{13}$$

$$U_4 = k_D(w_1^2 - w_2^2 + w_3^2 - w_4^2), \tag{14}$$

where "l", "k_T", "k_D", and "w" stand for the quadrotor's radius, thrust coefficient, drag factor, and propeller's speed respectively.

3. Control Scheme

The quadrotor has four inputs and six coupled outputs and is considered as an under-actuated and highly nonlinear system. So a classical PID controller cannot handle it's nonlinear inherent as it is expected. One solution is to use an adaptive algorithm that can adapt PID control in different modes. Fuzzy control, neural network control, linear quadratic regulator (LQR), and genetic algorithm are some algorithms that can improve PID control's performance. Equation (7) is a general form of the control law for a classical PID controller:

$$U_i = K_P . e + \frac{1}{T_I}\int e \, dt + T_D \frac{de}{dt}, \tag{15}$$

where "K_P" is the proportional gain, "T_I" is the reset time, and "T_D" is the rate time.

ILA or iterative learning control (ILC) is an adaptive control method through which the performance of a control system becomes better and better as time increases and can improve tracking performance of a nonlinear system. This algorithm can perform well with the various type of controllers. Equation (8) shows a general form of the control update law describing a PID-type ILA.

$$G_{k+1}^i = G_k^i + \alpha^i . e_k^i + \beta^i \int e_k^i \, dt + \gamma^i \frac{de_k^i}{dt}, \tag{16}$$

where "α", "β" and "γ" are the learning rates. For a P-type ILA; gains β and γ are equal to zero. To adapt a PID controller; gains K_P, T_I , and T_D are set as "G" in equation (8) each in a separate equation.

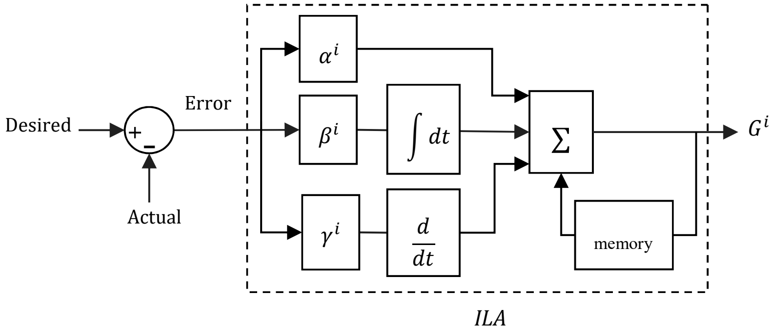


Figure 2. Quadrotor and reference frames

To tune the learning rates of ILA, an optimal method is used named as the enumerative learning algorithm. In this method, a normalized boundary with specific steps selects for each parameter. Then the algorithm run until the minimum total error is acquired and the values in that step will choose as the optimal values.

3.1. Inner Control Loop

Altitude and attitude control of the quadrotor is usually named as the inner control loop. For this part, a PID controller adapted by a PID-type ILA is employed for altitude control while a PID controller adapted by a P-type ILA is used for attitude control of the quadrotor. The overall block diagram of the proposed control algorithm for the inner control loop box is shown in Fig. 3.

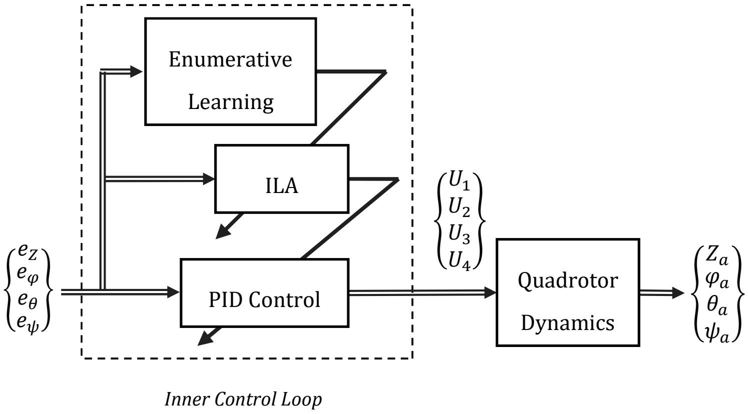


Figure 3. Inner control loop

3.1. Outer Control Loop

Position control of the quadrotor is usually named as the outer control loop. For this part, a PID controller adapted by a PID-type ILA is employed for position control. The overall block diagram of the proposed control algorithm for the outer control loop box is shown in Fig. 4.

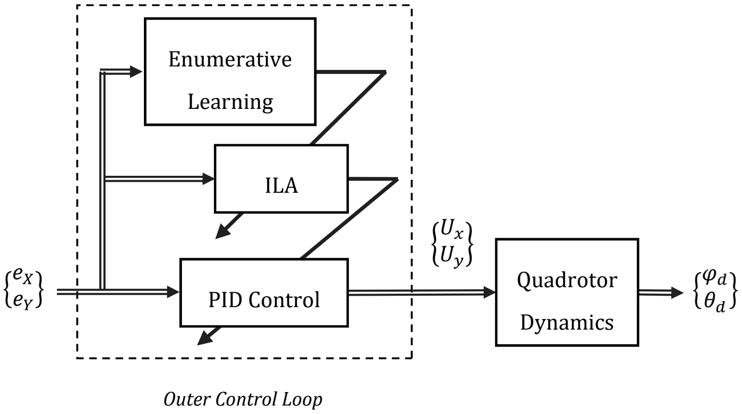


Figure 4. Outer control loop

where the relation between virtual forces "U_x" and "U_y" by desired roll and pitch angles can be expressed as (8) [25].

$$U_x = \cos\varphi \sin\theta \cos\psi + \sin\varphi \sin\psi, \tag{17}$$

$$U_y = \sin\theta \cos\varphi \sin\psi - \sin\varphi \cos\psi, \tag{18}$$

4. Discussion Simulation Results

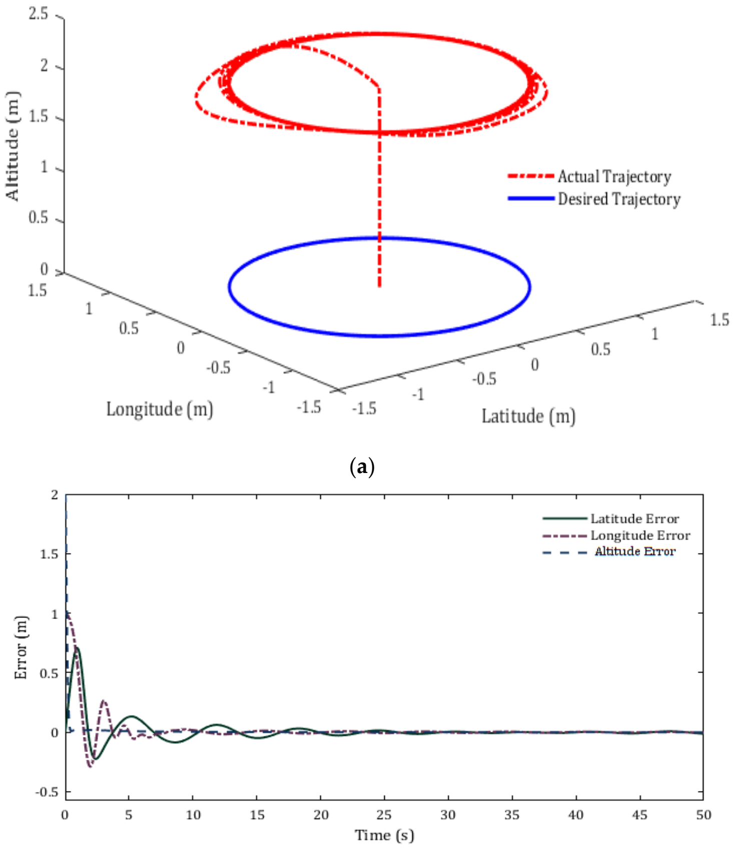
In this section, the performance of the proposed control method in trajectory tracking control and achieving minimum error is illustrated using Simulink MATLAB. The desired trajectories are selected as a circular path, a butterfly path, and a spiral path. Also for the simulation purpose, the imaginary quadrotor parameters are chosen as Table I [20].

TABLE I

SYSTEM PARAMETERS FOR SIMULATION PURPOSES

Parameter	Description	Value	Units
$\boldsymbol{m}$	Quadrotor mass	0.14	kg
$\boldsymbol{l}$	Quadrotor arm	0.17	m
$\boldsymbol{I_{xx}}$	Inertia moment with respect to the x-axis	0.002	$\mathrm{kg.m^2}$
$\boldsymbol{I_{yy}}$	Inertia moment with respect to the y-axis	0.002	$\mathrm{kg.m^2}$
$\boldsymbol{I_{zz}}$	Inertia moment with respect to the z-axis	0.004	$\mathrm{kg.m^2}$
$\boldsymbol{g}$	Gravity acceleration	9.8	$\mathrm{m.s^{-2}}$

The simulation results for tracking a circular path and its longitude and latitude error signals are shown in Fig. 5. It can be seen that as time increases, the proposed controller is able to move the quadrotor to the desired path and minimize the error signals.

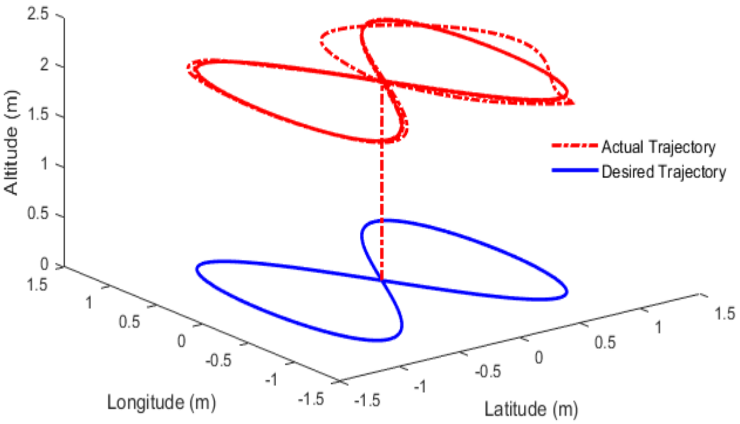


(**a**)

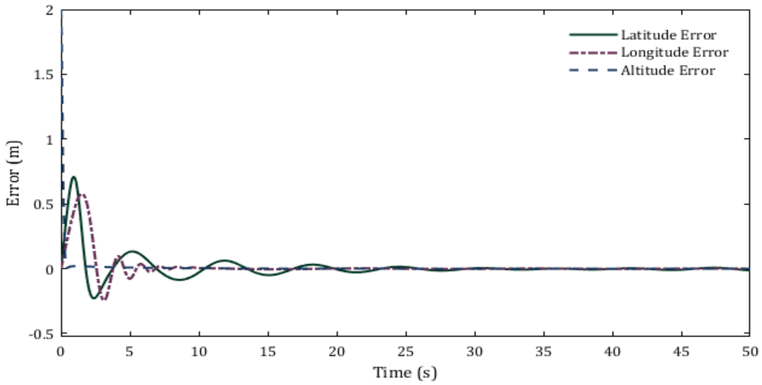
(**b**)

Figure 5. (**a**) Trajectory tracking control for a circular path; (**b**) Error Signals for tracking a circular path.

The simulation results for tracking a butterfly path and its longitude and latitude error signals are shown in Fig. 7. Just like the circular path, it can be seen that the proposed controller is able to move the quadrotor to the desired path and minimize the error signals as time increases.



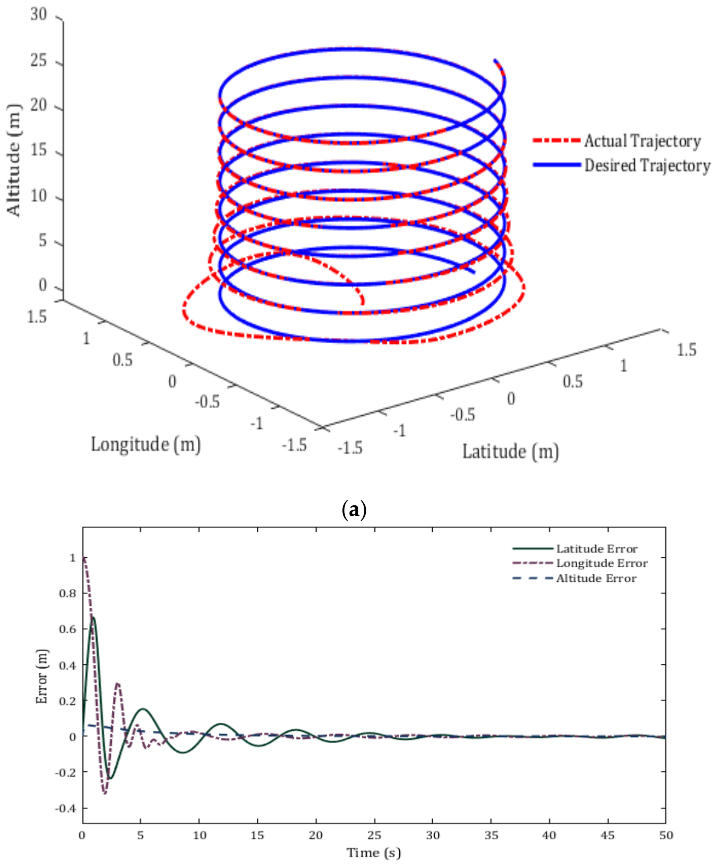
(**a**)



(**b**)

Figure 6. (**a**) Trajectory tracking control for a butterfly path; (**b**) Error signals for tracking a butterfly path.

The simulation results for tracking a spiral path and its longitude, latitude, and altitude error signals are shown in Fig. 9. Usually, for tracking a trajectory, roll and pitch angles change and this change affects the total thrust which may cause instability. However, it can be seen that the proposed control structure is able to preserve the desired altitude for the quadrotor, move it to the desired path, and minimize the error signals.



(**a**)

(**b**)

Figure 7. (**a**) Trajectory tracking control for a spiral path; (**b**) Error signals for tracking a spiral path.

5. Conclusions

In this paper, an optimal ILA was presented to adapt PID control using the enumerative learning algorithm for the trajectory tracking control of a quadrotor. In this method, a PID controller adapted by the PID-type ILA was proposed for altitude and position control and a PID controller adapted by the P-type ILA was proposed for attitude control of a quadrotor. Also, the enumerative learning algorithm was used to choose the optimal values of the learning rates of the ILA. For this purpose, at first, the dynamic model of the quadrotor was acquired based on the Newton-Euler equations. After that, the adaptive and optimal control structure of the inner and outer control loops was defined. In the end, the simulation results for the trajectory tracking control of a quadrotor were demonstrated. Through simulation, it is concluded that as time increases, the performance of the suggested control method in trajectory tracking control becomes better and better and error signals convergence to zero.

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