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THEORY OF $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-MAPS

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ABSTRACT. Several specific types of generalized maps of a generalized topological space have been defined and investigated for various purposes from time to time in the literature of topological spaces. Our recent research in the field of a new class of generalized maps of a generalized topological space is reported herein as a starting point for more generalized classes.

KEY WORDS AND PHRASES. *Generalized topological space, generalized sets, generalized maps, generalized continuous maps, generalized irresolute maps, generalized homeomorphism maps*

1. INTRODUCTION

The concepts[1] of $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map $\pi : \mathfrak{T}_{\Omega} \rightarrow \mathfrak{T}_{\Sigma}$ [1, 2], $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\Omega} \rightarrow \mathfrak{T}_{\Sigma}$ [9, 18], $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map $\pi : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ [3], and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ [21], called, respectively, ordinary and generalized maps (briefly, $\mathfrak{T}$-*map* and $\mathfrak{g}$-$\mathfrak{T}$-*map*, respectively) between $\mathcal{T}$-spaces $\mathfrak{T}_{\Omega}$ and $\mathfrak{T}_{\Sigma}$, and ordinary and generalized maps (briefly, $\mathfrak{T}_{\mathfrak{g}}$-*map* and $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-*map*, respectively) between $\mathcal{T}_{\mathfrak{g}}$-spaces $\mathfrak{T}_{\mathfrak{g},\Omega}$ and $\mathfrak{T}_{\mathfrak{g},\Sigma}$ are all fundamental concepts that have been introduced and investigated by several mathematicians [12, 16, 17, 20, 21, 23, 29, 31, 33].

Other concepts called $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-continuous and $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-irresolute maps and $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-homeomorphism (briefly, $\mathfrak{T}$-*continuous* and $\mathfrak{T}$-*irresolute maps*, and $\mathfrak{T}$-*homeomorphism*, respectively) [6, 24, 32], $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-continuous and $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-irresolute maps and $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-homeomorphism (briefly, $\mathfrak{g}$-$\mathfrak{T}$-*continuous* and $\mathfrak{g}$-$\mathfrak{T}$-*irresolute maps*, and $\mathfrak{g}$-$\mathfrak{T}$-*homeomorphism*, respectively) [6, 13, 14, 26], $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous and $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute maps, $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphism (briefly, $\mathfrak{T}_{\mathfrak{g}}$-*continuous*, $\mathfrak{T}_{\mathfrak{g}}$-*irresolute maps*, and $\mathfrak{T}_{\mathfrak{g}}$-*homeomorphism*, respectively) [14, 28] and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute maps, $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphism (briefly, $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-*continuous* and $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-*irresolute maps*, and $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-*homeomorphism*, respectively) [11, 22, 34] are all derived concepts based on the properties of $\mathfrak{T}$-map, $\mathfrak{g}$-$\mathfrak{T}$-map, $\mathfrak{T}_{\mathfrak{g}}$-map, and $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-map. Having received extensive studies, all these ordinary and generalized mappings are at this date well-known important notions in ordinary and generalized topologies and their applications.

In this paper, we will show how further contributions can be added to the field in a unified way.

[1]Notes to the reader: The structures $\mathfrak{T}_{\Omega} = (\Omega, \mathcal{T}_{\Omega})$ and $\mathfrak{T}_{\Sigma} = (\Sigma, \mathcal{T}_{\Sigma})$ are called ordinary topological spaces (briefly, $\mathcal{T}$-spaces), and the structures $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathcal{T}_{\mathfrak{g},\Sigma})$ are called generalized topological spaces (briefly, $\mathcal{T}_{\mathfrak{g}}$-spaces). The maps π, $\pi_{\mathfrak{g}} : \mathfrak{T}_{\Omega} \rightarrow \mathfrak{T}_{\Sigma}$ and π, $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$, respectively, stand for ordinary and generalized maps between $\mathcal{T}$-spaces and $\mathcal{T}_{\mathfrak{g}}$-spaces; the notations $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map (briefly, $\mathfrak{T}$-map), $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map (briefly, $\mathfrak{g}$-$\mathfrak{T}$-map), $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map (briefly, $\mathfrak{T}_{\mathfrak{g}}$-map), and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map (briefly, $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-map) emphasize their characters.

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2. Theory

2.1. PRELIMINARIES. Our discussion starts by recalling some basic definitions and notations of most essential concepts presented in the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-sets in a $\mathcal{T}_{\mathfrak{g}}$-space.

The set $\mathfrak{U}$ stands for the universe of discourse, fixed within the framework of the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps and containing as elements all sets (Λ-sets: $\Lambda \in \{\Omega, \Sigma, \Upsilon\}$; $\mathcal{T}_{\Lambda}$, $\mathfrak{g}$-$\mathcal{T}_{\Lambda}$, $\mathfrak{T}_{\Lambda}$, $\mathfrak{g}$-$\mathfrak{T}_{\Lambda}$-sets; $\mathcal{T}_{\mathfrak{g},\Lambda}$, $\mathfrak{g}$-$\mathcal{T}_{\mathfrak{g},\Lambda}$, $\mathfrak{T}_{\mathfrak{g},\Lambda}$, $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-sets) considered in this theory, and $I_n^0 \stackrel{\text{def}}{=} \{\nu \in \mathbb{N}^0 : \ \nu \leq n\}$; index sets I_{∞}^0, I_n^*, I_{∞}^* are defined similarly. Let $\Lambda \in \{\Omega, \Sigma, \Upsilon\} \subset \mathfrak{U}$ be a given set and let $\mathcal{P}(\Lambda) \stackrel{\text{def}}{=} \{\mathcal{O}_{\mathfrak{g},\nu} \subseteq \Lambda : \ \nu \in I_{\infty}^*\}$ be the family of all subsets $\mathcal{O}_{\mathfrak{g},1}$, $\mathcal{O}_{\mathfrak{g},2}$, $\ldots$, of Λ. Then every one-valued map of the type $\mathcal{T}_{\mathfrak{g},\Lambda} : \mathcal{P}(\Lambda) \to \mathcal{P}(\Lambda)$ satisfying $\mathcal{T}_{\mathfrak{g},\Lambda}(\emptyset) = \emptyset$, $\mathcal{T}_{\mathfrak{g},\Lambda}(\mathcal{O}_{\mathfrak{g}}) \subseteq \mathcal{O}_{\mathfrak{g}}$, and $\mathcal{T}_{\mathfrak{g},\Lambda}\big(\bigcup_{\nu \in I_{\infty}^*} \mathcal{O}_{\mathfrak{g},\nu}\big) = \bigcup_{\nu \in I_{\infty}^*} \mathcal{T}_{\mathfrak{g},\Lambda}(\mathcal{O}_{\mathfrak{g},\nu})$ is called a $\mathfrak{g}$-topology on Λ, and the structure $\mathfrak{T}_{\mathfrak{g},\Lambda} \stackrel{\text{def}}{=} (\Lambda, \mathcal{T}_{\mathfrak{g},\Lambda})$ is called a $\mathcal{T}_{\mathfrak{g},\Lambda}$-space, on which no separation axioms are assumed unless otherwise mentioned [8, 7, 27]. The operator $\mathrm{cl}_{\mathfrak{g},\Lambda} : \mathcal{P}(\Lambda) \to \mathcal{P}(\Lambda)$ carrying each $\mathfrak{T}_{\mathfrak{g},\Lambda}$-set $\mathcal{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g},\Lambda}$ into its closure $\mathrm{cl}_{\mathfrak{g},\Lambda}(\mathcal{S}_{\mathfrak{g}}) = \mathfrak{T}_{\mathfrak{g},\Lambda} - \mathrm{int}_{\mathfrak{g},\Lambda}(\mathfrak{T}_{\mathfrak{g},\Lambda} \setminus \mathcal{S}_{\mathfrak{g}}) \subset \mathfrak{T}_{\mathfrak{g},\Lambda}$ is called a $\mathfrak{g}$-closure operator and the operator $\mathrm{int}_{\mathfrak{g},\Lambda} : \mathcal{P}(\Lambda) \to \mathcal{P}(\Lambda)$ carrying each $\mathfrak{T}_{\mathfrak{g},\Lambda}$-set $\mathcal{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g},\Lambda}$ into its interior $\mathrm{int}_{\mathfrak{g},\Lambda}(\mathcal{S}_{\mathfrak{g}}) = \mathfrak{T}_{\mathfrak{g},\Lambda} - \mathrm{cl}_{\mathfrak{g},\Lambda}(\mathfrak{T}_{\mathfrak{g},\Lambda} \setminus \mathcal{S}_{\mathfrak{g}}) \subset \mathfrak{T}_{\mathfrak{g},\Lambda}$ is called a $\mathfrak{g}$-interior operator; for clarity, we will use $\mathrm{cl}_{\mathfrak{g}}(\cdot)$, $\mathrm{int}_{\mathfrak{g}}(\cdot)$, respectively, instead of $\mathrm{cl}_{\mathfrak{g},\Lambda}(\cdot)$, $\mathrm{int}_{\mathfrak{g},\Lambda}(\cdot)$.

Let $\mathfrak{T}_{\mathfrak{g},\Lambda}$ be a $\mathcal{T}_{\mathfrak{g},\Lambda}$-space, let $\complement_{\Lambda} : \mathcal{P}(\Lambda) \to \mathcal{P}(\Lambda)$ denotes the absolute complement with respect to the underlying set $\Lambda \subset \mathfrak{U}$, and let $\mathcal{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g},\Lambda}$ be any $\mathfrak{T}_{\mathfrak{g},\Lambda}$-set. The classes

$$\begin{aligned} \mathcal{T}_{\mathfrak{g},\Lambda} &\stackrel{\text{def}}{=} \{\mathcal{O}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g},\Lambda} : \ \mathcal{O}_{\mathfrak{g}} \in \mathcal{T}_{\mathfrak{g},\Lambda}\}, \\ \neg\mathcal{T}_{\mathfrak{g},\Lambda} &\stackrel{\text{def}}{=} \{\mathcal{K}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g},\Lambda} : \ \complement_{\Lambda}(\mathcal{K}_{\mathfrak{g}}) \in \mathcal{T}_{\mathfrak{g},\Lambda}\}, \end{aligned} \tag{2.1}$$

respectively, denote the classes of all $\mathcal{T}_{\mathfrak{g},\Lambda}$-open and $\mathcal{T}_{\mathfrak{g},\Lambda}$-closed sets relative to the $\mathfrak{g}$-topology $\mathcal{T}_{\mathfrak{g},\Lambda}$, and the classes

$$\begin{aligned} \mathrm{C}^{\mathrm{sub}}_{\mathcal{T}_{\mathfrak{g},\Lambda}}[\mathcal{S}_{\mathfrak{g}}] &\stackrel{\text{def}}{=} \{\mathcal{O}_{\mathfrak{g}} \in \mathcal{T}_{\mathfrak{g},\Lambda} : \ \mathcal{O}_{\mathfrak{g}} \subseteq \mathcal{S}_{\mathfrak{g}}\}, \\ \mathrm{C}^{\mathrm{sup}}_{\neg\mathcal{T}_{\mathfrak{g},\Lambda}}[\mathcal{S}_{\mathfrak{g}}] &\stackrel{\text{def}}{=} \{\mathcal{K}_{\mathfrak{g}} \in \neg\mathcal{T}_{\mathfrak{g},\Lambda} : \ \mathcal{K}_{\mathfrak{g}} \supseteq \mathcal{S}_{\mathfrak{g}}\}, \end{aligned} \tag{2.2}$$

respectively, denote the classes of $\mathcal{T}_{\mathfrak{g},\Lambda}$-open subsets and $\mathcal{T}_{\mathfrak{g},\Lambda}$-closed supersets (complements of the $\mathcal{T}_{\mathfrak{g},\Lambda}$-open subsets) of the $\mathfrak{T}_{\mathfrak{g},\Lambda}$-set $\mathcal{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g},\Lambda}$ relative to the $\mathfrak{g}$-topology $\mathcal{T}_{\mathfrak{g},\Lambda}$. To this end, the $\mathfrak{g}$-closure and the $\mathfrak{g}$-interior of a $\mathfrak{T}_{\mathfrak{g}}$-set $\mathcal{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ in a $\mathcal{T}_{\mathfrak{g},\Lambda}$-space [3] define themselves as

$$\mathrm{int}_{\mathfrak{g},\Lambda}(\mathcal{S}_{\mathfrak{g}}) \stackrel{\text{def}}{=} \bigcup_{\mathcal{O}_{\mathfrak{g}} \in \mathrm{C}^{\mathrm{sub}}_{\mathcal{T}_{\mathfrak{g},\Lambda}}[\mathcal{S}_{\mathfrak{g}}]} \mathcal{O}_{\mathfrak{g}}, \quad \mathrm{cl}_{\mathfrak{g},\Lambda}(\mathcal{S}_{\mathfrak{g}}) \stackrel{\text{def}}{=} \bigcap_{\mathcal{K}_{\mathfrak{g}} \in \mathrm{C}^{\mathrm{sup}}_{\neg\mathcal{T}_{\mathfrak{g},\Lambda}}[\mathcal{S}_{\mathfrak{g}}]} \mathcal{K}_{\mathfrak{g}}. \tag{2.3}$$

Throughout this work, by $\mathrm{cl}_{\mathfrak{g}} \circ \mathrm{int}_{\mathfrak{g}}(\cdot)$, $\mathrm{int}_{\mathfrak{g}} \circ \mathrm{cl}_{\mathfrak{g}}(\cdot)$, and $\mathrm{cl}_{\mathfrak{g}} \circ \mathrm{int}_{\mathfrak{g}} \circ \mathrm{cl}_{\mathfrak{g}}(\cdot)$, respectively, are meant $\mathrm{cl}_{\mathfrak{g}}(\mathrm{int}_{\mathfrak{g}}(\cdot))$, $\mathrm{int}_{\mathfrak{g}}(\mathrm{cl}_{\mathfrak{g}}(\cdot))$, and $\mathrm{cl}_{\mathfrak{g}}(\mathrm{int}_{\mathfrak{g}}(\mathrm{cl}_{\mathfrak{g}}(\cdot)))$; other composition operators are defined similarly. Also, the backslash $\mathfrak{T}_{\mathfrak{g}} \setminus \mathcal{S}_{\mathfrak{g}}$ refers to the set-theoretic difference $\mathfrak{T}_{\mathfrak{g}} - \mathcal{S}_{\mathfrak{g}}$. The mapping $\mathrm{op}_{\mathfrak{g}} : \mathcal{P}(\Lambda) \to \mathcal{P}(\Lambda)$ is called a $\mathfrak{g}$-operation on $\mathcal{P}(\Lambda)$ if the following statements hold:

$$\begin{aligned} &\forall \mathcal{S}_{\mathfrak{g}} \in \mathcal{P}(\Lambda) \setminus \{\emptyset\}, \ \exists (\mathcal{O}_{\mathfrak{g}}, \mathcal{K}_{\mathfrak{g}}) \in \mathcal{T}_{\mathfrak{g},\Lambda} \setminus \{\emptyset\} \times \neg\mathcal{T}_{\mathfrak{g},\Lambda} \setminus \{\emptyset\} : \\ &\big(\mathrm{op}_{\mathfrak{g}}(\emptyset) = \emptyset\big) \vee \big(\neg\,\mathrm{op}_{\mathfrak{g}}(\emptyset) = \emptyset\big), \ \big(\mathcal{S}_{\mathfrak{g}} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g}})\big) \vee \big(\mathcal{S}_{\mathfrak{g}} \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g}})\big), \end{aligned} \tag{2.4}$$

where $\neg\,\mathrm{op}_{\mathfrak{g}}:\mathcal{P}(\Lambda)\to\mathcal{P}(\Lambda)$ is called the "complementary $\mathfrak{g}$-operation" on $\mathcal{P}(\Lambda)$ and, for all $\mathfrak{T}_{\mathfrak{g}}$-sets $\mathcal{S}_{\mathfrak{g}}$, $\mathcal{S}_{\mathfrak{g},\nu}$, $\mathcal{S}_{\mathfrak{g},\mu}\in\mathcal{P}(\Lambda)\setminus\{\emptyset\}$, the following axioms are satisfied:

- Ax. I. $(\mathcal{S}_{\mathfrak{g}}\subseteq\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g}}))\vee(\mathcal{S}_{\mathfrak{g}}\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g}}))$,
- Ax. II. $(\mathrm{op}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}})\subseteq\mathrm{op}_{\mathfrak{g}}\circ\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g}}))\vee(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}})\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\circ\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g}}))$,
- Ax. III. $(\mathcal{S}_{\mathfrak{g},\nu}\subseteq\mathcal{S}_{\mathfrak{g},\mu}\ \rightarrow\ \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\nu})\subseteq\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\mu}))\vee(\mathcal{S}_{\mathfrak{g},\mu}\subseteq\mathcal{S}_{\mathfrak{g},\nu}\ \leftarrow\ \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\mu})\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\nu}))$,
- Ax. IV. $(\mathrm{op}_{\mathfrak{g}}(\bigcup_{\sigma=\nu,\mu}\mathcal{S}_{\mathfrak{g},\sigma})\subseteq\bigcup_{\sigma=\nu,\mu}\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma}))\vee(\neg\,\mathrm{op}_{\mathfrak{g}}(\bigcup_{\sigma=\nu,\mu}\mathcal{S}_{\mathfrak{g},\sigma})\supseteq\bigcup_{\sigma=\nu,\mu}\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma}))$,

for some $\mathcal{T}_{\mathfrak{g},\Lambda}$-open sets $\mathcal{O}_{\mathfrak{g}}$, $\mathcal{O}_{\mathfrak{g},\nu}$, $\mathcal{O}_{\mathfrak{g},\mu}\in\mathcal{T}_{\mathfrak{g},\Lambda}\setminus\{\emptyset\}$ and $\mathcal{T}_{\mathfrak{g},\Lambda}$-closed sets $\mathcal{K}_{\mathfrak{g}}$, $\mathcal{K}_{\mathfrak{g},\nu}$, $\mathcal{K}_{\mathfrak{g},\mu}\in\neg\mathcal{T}_{\mathfrak{g},\Lambda}$ [4, 19]. The class $\mathcal{L}_{\mathfrak{g}}[\Omega]=\mathcal{L}_{\mathfrak{g}}^{\omega}[\Lambda]\times\mathcal{L}_{\mathfrak{g}}^{\kappa}[\Omega]$, where

$$\mathcal{L}_{\mathfrak{g}}[\Lambda]\stackrel{\text{def}}{=}\left\{\mathbf{op}_{\mathfrak{g},\nu\mu}(\cdot)=(\mathrm{op}_{\mathfrak{g},\nu}(\cdot),\neg\,\mathrm{op}_{\mathfrak{g},\mu}(\cdot)):\ (\nu,\mu)\in I_3^0\times I_3^0\right\}\tag{2.5}$$

in the $\mathcal{T}_{\mathfrak{g},\Lambda}$-space $\mathfrak{T}_{\mathfrak{g},\Lambda}$, stands for the class of all possible $\mathfrak{g}$-operators and their complementary $\mathfrak{g}$-operators in the $\mathcal{T}_{\mathfrak{g},\Lambda}$-space $\mathfrak{T}_{\mathfrak{g},\Lambda}$. Its elements are defined as:

$$\begin{aligned}\mathrm{op}_{\mathfrak{g}}(\cdot)&\in\mathcal{L}_{\mathfrak{g}}^{\omega}[\Lambda]\stackrel{\text{def}}{=}\{\mathrm{op}_{\mathfrak{g},0}(\cdot),\ \mathrm{op}_{\mathfrak{g},1}(\cdot),\ \mathrm{op}_{\mathfrak{g},2}(\cdot),\ \mathrm{op}_{\mathfrak{g},3}(\cdot)\}\\&=\{\mathrm{int}_{\mathfrak{g}}(\cdot),\ \mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}(\cdot),\ \mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}(\cdot),\ \mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}(\cdot)\};\\\neg\,\mathrm{op}_{\mathfrak{g}}(\cdot)&\in\mathcal{L}_{\mathfrak{g}}^{\kappa}[\Lambda]\stackrel{\text{def}}{=}\{\neg\,\mathrm{op}_{\mathfrak{g},0}(\cdot),\ \neg\,\mathrm{op}_{\mathfrak{g},1}(\cdot),\ \neg\,\mathrm{op}_{\mathfrak{g},2}(\cdot),\ \neg\,\mathrm{op}_{\mathfrak{g},3}(\cdot)\}\\&=\{\mathrm{cl}_{\mathfrak{g}}(\cdot),\ \mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}(\cdot),\ \mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}(\cdot),\ \mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}(\cdot)\}.\end{aligned}\tag{2.6}$$

A $\mathfrak{T}_{\mathfrak{g},\Lambda}$-set $\mathcal{S}_{\mathfrak{g},\Lambda}\subset\mathfrak{T}_{\mathfrak{g}}$ in a $\mathcal{T}_{\mathfrak{g},\Lambda}$-space is called a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-set if and only if there exist a pair $(\mathcal{O}_{\mathfrak{g}},\mathcal{K}_{\mathfrak{g}})\in\mathcal{T}_{\mathfrak{g},\Lambda}\times\neg\mathcal{T}_{\mathfrak{g},\Lambda}$ of $\mathcal{T}_{\mathfrak{g},\Lambda}$-open and $\mathcal{T}_{\mathfrak{g},\Lambda}$-closed sets, and a $\mathfrak{g}$-operator $\mathbf{op}_{\mathfrak{g}}(\cdot)\in\mathcal{L}_{\mathfrak{g}}[\Lambda]$ such that the following statement holds:

$$(\exists\xi)\left[(\xi\in\mathcal{S}_{\mathfrak{g}})\wedge((\mathcal{S}_{\mathfrak{g}}\subseteq\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g}}))\vee(\mathcal{S}_{\mathfrak{g}}\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g}})))\right].\tag{2.7}$$

The $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-set $\mathcal{S}_{\mathfrak{g}}\subset\mathfrak{T}_{\mathfrak{g},\Lambda}$ is said to be of category ν if and only if it belongs to the following class of $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-sets:

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-S}[\mathfrak{T}_{\mathfrak{g},\Lambda}]\stackrel{\text{def}}{=}\{\mathcal{S}_{\mathfrak{g}}\subset\mathfrak{T}_{\mathfrak{g},\Lambda}:\ (\exists\mathcal{O}_{\mathfrak{g}},\mathcal{K}_{\mathfrak{g}},\mathbf{op}_{\mathfrak{g},\nu}(\cdot))\\\left[(\mathcal{S}_{\mathfrak{g}}\subseteq\mathrm{op}_{\mathfrak{g},\nu}(\mathcal{O}_{\mathfrak{g}}))\vee(\mathcal{S}_{\mathfrak{g}}\supseteq\neg\,\mathrm{op}_{\mathfrak{g},\nu}(\mathcal{K}_{\mathfrak{g}}))\right]\}.\end{aligned}\tag{2.8}$$

It is called a $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-open set if it satisfies the first property in $\mathfrak{g}$-ν-S$[\mathfrak{T}_{\mathfrak{g},\Lambda}]$ and a $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-closed set if it satisfies the second property in $\mathfrak{g}$-ν-S$[\mathfrak{T}_{\mathfrak{g},\Lambda}]$. The classes of $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-open and $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-closed sets, respectively, are defined by

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-O}[\mathfrak{T}_{\mathfrak{g},\Lambda}]&\stackrel{\text{def}}{=}\{\mathcal{S}_{\mathfrak{g}}\subset\mathfrak{T}_{\mathfrak{g},\Lambda}:\ (\exists\mathcal{O}_{\mathfrak{g}},\mathbf{op}_{\mathfrak{g},\nu}(\cdot))\left[\mathcal{S}_{\mathfrak{g}}\subseteq\mathrm{op}_{\mathfrak{g},\nu}(\mathcal{O}_{\mathfrak{g}})\right]\},\\\mathfrak{g}\text{-}\nu\text{-K}[\mathfrak{T}_{\mathfrak{g},\Lambda}]&\stackrel{\text{def}}{=}\{\mathcal{S}_{\mathfrak{g}}\subset\mathfrak{T}_{\mathfrak{g},\Lambda}:\ (\exists\mathcal{K}_{\mathfrak{g}},\mathbf{op}_{\mathfrak{g},\nu}(\cdot))\left[\mathcal{S}_{\mathfrak{g}}\supseteq\neg\,\mathrm{op}_{\mathfrak{g},\nu}(\mathcal{K}_{\mathfrak{g}})\right]\}.\end{aligned}\tag{2.9}$$

From these classes, the following relation holds:

$$\begin{aligned}\mathfrak{g}\text{-S}[\mathfrak{T}_{\mathfrak{g},\Lambda}]&=\bigcup_{\nu\in I_3^0}\mathfrak{g}\text{-}\nu\text{-S}[\mathfrak{T}_{\mathfrak{g},\Lambda}]\\&=\bigcup_{\nu\in I_3^0}(\mathfrak{g}\text{-}\nu\text{-O}[\mathfrak{T}_{\mathfrak{g}}]\cup\mathfrak{g}\text{-}\nu\text{-K}[\mathfrak{T}_{\mathfrak{g}}])\\&=(\bigcup_{\nu\in I_3^0}\mathfrak{g}\text{-}\nu\text{-O}[\mathfrak{T}_{\mathfrak{g},\Lambda}])\cup(\bigcup_{\nu\in I_3^0}\mathfrak{g}\text{-}\nu\text{-K}[\mathfrak{T}_{\mathfrak{g},\Lambda}])\\&=\mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g},\Lambda}]\cup\mathfrak{g}\text{-K}[\mathfrak{T}_{\mathfrak{g},\Lambda}].\end{aligned}\tag{2.10}$$

By omitting the subscript $\mathfrak{g}$ in almost all symbols of the above definitions, we obtain very similar definitions but in a $\mathcal{T}_{\Lambda}$-space.

A $\mathfrak{T}_{\Lambda}$-set $\mathcal{S} \subset \mathfrak{T}_{\Lambda}$ in a $\mathcal{T}_{\Lambda}$-space is called a $\mathfrak{g}$-$\mathfrak{T}_{\Lambda}$-set if and only if there exists a pair $(\mathcal{O}, \mathcal{K}) \in \mathcal{T}_{\Lambda} \times \neg \mathcal{T}_{\Lambda}$ of $\mathcal{T}_{\Lambda}$-open and $\mathcal{T}_{\Lambda}$-closed sets, and an operator $\mathbf{op}(\cdot) \in \mathcal{L}[\Lambda]$ such that the following statement holds:

$$(\exists \xi) \left[(\xi \in \mathcal{S}) \wedge \left((\mathcal{S} \subseteq \mathrm{op}(\mathcal{O})) \vee (\mathcal{S} \supseteq \neg\, \mathrm{op}(\mathcal{K})) \right) \right]. \tag{2.11}$$

The $\mathfrak{g}$-$\mathfrak{T}_{\Lambda}$-set $\mathcal{S} \subset \mathfrak{T}_{\Lambda}$ is said to be of category ν if and only if it belongs to the following class of $\mathfrak{g}$-ν-$\mathcal{T}_{\Lambda}$-sets:

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-}\mathrm{S}\big[\mathfrak{T}_{\Lambda}\big] \overset{\mathrm{def}}{=} \big\{ \mathcal{S} \subset \mathfrak{T}_{\Lambda} :\ (\exists \mathcal{O}, \mathcal{K}, \mathbf{op}_{\nu}(\cdot)) \\ \big[(\mathcal{S} \subseteq \mathrm{op}_{\nu}(\mathcal{O})) \vee (\mathcal{S} \supseteq \neg\, \mathrm{op}_{\nu}(\mathcal{K})) \big] \big\}. \end{aligned} \tag{2.12}$$

It is called a $\mathfrak{g}$-ν-$\mathfrak{T}_{\Lambda}$-open set if it satisfies the first property in $\mathfrak{g}$-ν-$\mathrm{S}\big[\mathfrak{T}_{\Lambda}\big]$ and a $\mathfrak{g}$-ν-$\mathfrak{T}_{\Lambda}$-closed set if it satisfies the second property in $\mathfrak{g}$-ν-$\mathrm{S}\big[\mathfrak{T}_{\Lambda}\big]$. The classes of $\mathfrak{g}$-ν-$\mathfrak{T}_{\Lambda}$-open and $\mathfrak{g}$-ν-$\mathfrak{T}_{\Lambda}$-closed sets, respectively, are defined by

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}\big[\mathfrak{T}_{\Lambda}\big] &\overset{\mathrm{def}}{=} \big\{ \mathcal{S} \subset \mathfrak{T}_{\Lambda} :\ (\exists \mathcal{O}, \mathbf{op}_{\nu}(\cdot)) \big[\mathcal{S} \subseteq \mathrm{op}_{\nu}(\mathcal{O}) \big] \big\}, \\ \mathfrak{g}\text{-}\nu\text{-}\mathrm{K}\big[\mathfrak{T}_{\Lambda}\big] &\overset{\mathrm{def}}{=} \big\{ \mathcal{S} \subset \mathfrak{T}_{\Lambda} :\ (\exists \mathcal{K}, \mathbf{op}_{\nu}(\cdot)) \big[\mathcal{S} \supseteq \neg\, \mathrm{op}_{\nu}(\mathcal{K}) \big] \big\}. \end{aligned} \tag{2.13}$$

As in the previous definitions, from these classes, the following relation holds:

$$\begin{aligned} \mathfrak{g}\text{-}\mathrm{S}\big[\mathfrak{T}_{\Lambda}\big] &= \textstyle\bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{S}\big[\mathfrak{T}_{\Lambda}\big] \\ &= \textstyle\bigcup_{\nu \in I_3^0} \big(\mathfrak{g}\text{-}\nu\text{-}\mathrm{O}\big[\mathfrak{T}_{\Lambda}\big] \cup \mathfrak{g}\text{-}\nu\text{-}\mathrm{K}\big[\mathfrak{T}_{\Lambda}\big] \big) \\ &= \big(\textstyle\bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}\big[\mathfrak{T}_{\Lambda}\big] \big) \cup \big(\textstyle\bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{K}\big[\mathfrak{T}_{\Lambda}\big] \big) \\ &= \mathfrak{g}\text{-}\mathrm{O}\big[\mathfrak{T}_{\Lambda}\big] \cup \mathfrak{g}\text{-}\mathrm{K}\big[\mathfrak{T}_{\Lambda}\big]. \end{aligned} \tag{2.14}$$

The classes $\mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Lambda}]$ and $\mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Lambda}]$ denote the families of $\mathfrak{T}_{\mathfrak{g},\Lambda}$-open and $\mathfrak{T}_{\mathfrak{g},\Lambda}$-closed sets, respectively, in $\mathfrak{T}_{\mathfrak{g},\Lambda}$, with $\mathrm{S}[\mathfrak{T}_{\mathfrak{g},\Lambda}] = \mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Lambda}] \cup \mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Lambda}]$; the classes $\mathrm{O}[\mathfrak{T}_{\Lambda}]$ and $\mathrm{K}[\mathfrak{T}_{\Lambda}]$ denote the families of $\mathfrak{T}$-open and $\mathfrak{T}_{\Lambda}$-closed sets, respectively, in $\mathfrak{T}_{\Lambda}$, with $\mathrm{S}[\mathfrak{T}_{\Lambda}] = \mathrm{O}[\mathfrak{T}_{\Lambda}] \cup \mathrm{K}[\mathfrak{T}_{\Lambda}]$. (Whenever we feel that the subscript $\Lambda \in \{\Omega, \Sigma, \Upsilon\}$ is understood from the context, it will be omitted for clarity.) We are now in a position to present a carefully chosen set of terms used in the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps between $\mathcal{T}_{\mathfrak{g}}$-spaces.

A $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map and a $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map, respectively, are mappings in the usual sense between $\mathcal{T}$-spaces and $\mathcal{T}_{\mathfrak{g}}$-spaces.

Definition 2.1 ($(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$, $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-Maps). Let $\mathfrak{T}_{\Omega} = (\Omega, \mathcal{T}_{\Omega})$ and $\mathfrak{T}_{\Sigma} = (\Sigma, \mathcal{T}_{\Sigma})$ be $\mathcal{T}$-spaces and, let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces. Then, a map:

- I. $\pi : \mathfrak{T}_{\Omega} \to \mathfrak{T}_{\Sigma}$ is called a $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map from $\mathfrak{T}_{\Omega}$ into $\mathfrak{T}_{\Sigma}$.
- II. $\pi : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is called a $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map from $\mathfrak{T}_{\mathfrak{g},\Omega}$ into $\mathfrak{T}_{\mathfrak{g},\Sigma}$.

A $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map is a generalization of a $(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map and, hence, is a distinguished mapping between $\mathcal{T}$-spaces which does not exhibit mapping properties in the usual sense but does exhibit mapping properties in the generalized sense.

Definition 2.2 ($\mathfrak{g}$-ν-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-Map). Let $\mathfrak{T}_{\Omega} = (\Omega, \mathcal{T}_{\Omega})$ and $\mathfrak{T}_{\Sigma} = (\Sigma, \mathcal{T}_{\Sigma})$ be $\mathcal{T}$-spaces, and let $\mathbf{op}(\cdot) \in \mathcal{L}[\Sigma]$. Then, a map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\Omega} \to \mathfrak{T}_{\Sigma}$ is called a $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map if and only if, for every pair $(\mathcal{O}_{\omega}, \mathcal{K}_{\omega}) \in \mathcal{T}_{\Omega} \times \neg \mathcal{T}_{\Omega}$ of $\mathcal{T}_{\Omega}$-open and $\mathcal{T}_{\Omega}$-closed sets in $\mathfrak{T}_\Omega$ there corresponds a pair $(\mathcal{O}_\sigma, \mathcal{K}_\sigma) \in \mathcal{T}_\Sigma \times \neg\mathcal{T}_\Sigma$ of $\mathcal{T}_\Sigma$-open and $\mathcal{T}_\Sigma$-closed sets in $\mathfrak{T}_\Sigma$ such that the following statement holds:

$$\left[\pi_{\mathfrak{g}}(\mathcal{O}_\omega) \subseteq \mathrm{op}(\mathcal{O}_\sigma)\right] \vee \left[\pi_{\mathfrak{g}}(\mathcal{K}_\omega) \supseteq \neg\,\mathrm{op}(\mathcal{K}_\sigma)\right]. \tag{2.15}$$

A $\mathfrak{g}$- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-map is said to be of category ν if and only if it belongs to the following class of $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-maps:

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-M}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right] \stackrel{\text{def}}{=} \big\{\pi_{\mathfrak{g}}:\ &(\forall \mathcal{O}_\omega, \mathcal{K}_\omega)\left(\exists \mathcal{O}_\sigma, \mathcal{K}_\sigma, \mathbf{op}_\nu(\cdot)\right) \\ &\left[\left(\pi_{\mathfrak{g}}(\mathcal{O}_\omega) \subseteq \mathrm{op}_\nu(\mathcal{O}_\sigma)\right) \vee \left(\pi_{\mathfrak{g}}(\mathcal{K}_\omega) \supseteq \neg\,\mathrm{op}_\nu(\mathcal{K}_\sigma)\right)\right]\big\}. \end{aligned} \tag{2.16}$$

It is called a $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-open map if it satisfies the first property in $\mathfrak{g}$-ν-M $[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma]$ and a $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-closed map if it satisfies the second property in $\mathfrak{g}$-ν-M $[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma]$. The classes of $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-open and $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-closed maps, respectively, are defined by

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right] &\stackrel{\text{def}}{=} \left\{\pi_{\mathfrak{g}} : (\forall \mathcal{O}_\omega)\left(\exists \mathcal{O}_\sigma, \mathbf{op}_\nu(\cdot)\right)\left[\pi_{\mathfrak{g}}(\mathcal{O}_\omega) \subseteq \mathrm{op}_\nu(\mathcal{O}_\sigma)\right]\right\}, \\ \mathfrak{g}\text{-}\nu\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right] &\stackrel{\text{def}}{=} \left\{\pi_{\mathfrak{g}} : (\forall \mathcal{K}_\omega)\left(\exists \mathcal{K}_\sigma, \mathbf{op}_\nu(\cdot)\right)\left[\pi_{\mathfrak{g}}(\mathcal{K}_\omega) \supseteq \mathrm{op}_\nu(\mathcal{K}_\sigma)\right]\right\}. \end{aligned} \tag{2.17}$$

From the class $\mathfrak{g}$-ν-M $[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma]$, consisting of the classes $\mathfrak{g}$-ν-$\mathrm{M_O}$ $[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma]$ and $\mathfrak{g}$-ν-$\mathrm{M_K}$ $[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma]$, respectively, of $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-open and $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-closed maps, where $\nu \in I_3^0$, there results in the following definition.

Definition 2.3. Let $\mathfrak{T}_\Omega = (\Omega, \mathcal{T}_\Omega)$ and $\mathfrak{T}_\Sigma = (\Sigma, \mathcal{T}_\Sigma)$ be $\mathcal{T}$-spaces. If, for each $\nu \in I_3^0$, $\mathfrak{g}$-ν-$\mathrm{M_O}$ $[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma]$ and $\mathfrak{g}$-ν-$\mathrm{M_K}$ $[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma]$, respectively, denote the classes of $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-open and $\mathfrak{g}$-ν- $(\mathfrak{T}_\Omega, \mathfrak{T}_\Sigma)$-closed maps, then

$$\begin{aligned} \mathfrak{g}\text{-M}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right] &= \textstyle\bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-M}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right] \\ &= \textstyle\bigcup_{\nu \in I_3^0} \left(\mathfrak{g}\text{-}\nu\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right] \cup \mathfrak{g}\text{-}\nu\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right]\right) \\ &= \left(\textstyle\bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right]\right) \cup \left(\textstyle\bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right]\right) \\ &= \mathfrak{g}\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right] \cup \mathfrak{g}\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_\Omega; \mathfrak{T}_\Sigma\right]. \end{aligned} \tag{2.18}$$

As above, the $\mathfrak{g}$- $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map is a generalization of the $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map and, thus, is a distinguished mapping between $\mathcal{T}_{\mathfrak{g}}$-spaces which does not exhibit mapping properties in the usual sense but does exhibit mapping properties in the generalized sense.

Definition 2.4 ($\mathfrak{g}$-ν- $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-Map). Let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces, and let $\mathbf{op}_{\mathfrak{g}}(\cdot) \in \mathcal{L}_{\mathfrak{g}}[\Sigma]$. Then, a map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is called a $\mathfrak{g}$- $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map if and only if, for every pair $(\mathcal{O}_{\mathfrak{g},\omega}, \mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ of $\mathcal{T}_{\mathfrak{g},\Omega}$-open and $\mathcal{T}_{\mathfrak{g},\Omega}$-closed sets in $\mathfrak{T}_{\mathfrak{g},\Omega}$ there corresponds a pair $(\mathcal{O}_{\mathfrak{g},\sigma}, \mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ of $\mathcal{T}_{\mathfrak{g},\Sigma}$-open and $\mathcal{T}_\Sigma$-closed sets in $\mathfrak{T}_{\mathfrak{g},\Sigma}$ such that the following statement holds:

$$\left[\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right] \vee \left[\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\right]. \tag{2.19}$$

A $\mathfrak{g}$- $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map is said to be of category ν if and only if it belongs to the following class of $\mathfrak{g}$-ν- $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-maps:

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}\right] \stackrel{\text{def}}{=} \big\{\pi_{\mathfrak{g}}:\ &(\forall \mathcal{O}_{\mathfrak{g},\omega}, \mathcal{K}_{\mathfrak{g},\omega})\left(\exists \mathcal{O}_{\mathfrak{g},\sigma}, \mathcal{K}_{\mathfrak{g},\sigma}, \mathbf{op}_{\mathfrak{g},\nu}(\cdot)\right) \\ &\left[\left(\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g},\nu}(\mathcal{O}_{\mathfrak{g},\sigma})\right) \vee \left(\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}(\mathcal{K}_{\mathfrak{g},\sigma})\right)\right]\big\}. \end{aligned} \tag{2.20}$$

It is called a $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-open map if it satisfies the first property in the class $\mathfrak{g}$-ν-$\mathrm{M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ and a $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-closed map if it satisfies the second property in $\mathfrak{g}$-ν-$\mathrm{M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$. The classes of $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-open maps and $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-closed maps, respectively, are defined by

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{O}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] &\stackrel{\text{def}}{=} \{\pi_{\mathfrak{g}} : (\forall\mathcal{O}_{\mathfrak{g},\omega})(\exists\mathcal{O}_{\mathfrak{g},\sigma},\mathbf{op}_{\mathfrak{g},\nu}(\cdot)) \\ &\qquad [\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g},\nu}(\mathcal{O}_{\mathfrak{g},\sigma})]\}, \\ \mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{K}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] &\stackrel{\text{def}}{=} \{\pi_{\mathfrak{g}} : (\forall\mathcal{K}_{\omega})(\exists\mathcal{K}_{\mathfrak{g},\sigma},\mathbf{op}_{\mathfrak{g},\nu}(\cdot)) \\ &\qquad [\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \mathrm{op}_{\mathfrak{g},\nu}(\mathcal{K}_{\mathfrak{g},\sigma})]\}. \end{aligned} \tag{2.21}$$

From the class $\mathfrak{g}$-ν-$\mathrm{M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, consisting of the classes $\mathfrak{g}$-ν-$\mathrm{M}_{\mathrm{O}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ and $\mathfrak{g}$-ν-$\mathrm{M}_{\mathrm{K}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ of $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-open and $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-closed maps, where $\nu \in I_3^0$, respectively, there results in the following definition.

Definition 2.5. Let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega,\mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma,\mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces. If, for each $\nu \in I_3^0$, $\mathfrak{g}$-ν-$\mathrm{M}_{\mathrm{O}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ and $\mathfrak{g}$-ν-$\mathrm{M}_{\mathrm{K}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, respectively, denote the classes of $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-open and $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-closed maps, then

$$\begin{aligned}\mathfrak{g}\text{-}\mathrm{M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] &= \textstyle\bigcup_{\nu\in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \\ &= \textstyle\bigcup_{\nu\in I_3^0}(\mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{O}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \cup \mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{K}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]) \\ &= (\textstyle\bigcup_{\nu\in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{O}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]) \cup (\textstyle\bigcup_{\nu\in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{K}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]) \\ &= \mathfrak{g}\text{-}\mathrm{M}_{\mathrm{O}}[\mathfrak{T}_{\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \cup \mathfrak{g}\text{-}\mathrm{M}_{\mathrm{K}}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]. \end{aligned} \tag{2.22}$$

Definition 2.6 ($\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-Continuous). Let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega,\mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma,\mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces, and let $\mathbf{op}_{\mathfrak{g}}(\cdot) \in \mathcal{L}_{\mathfrak{g}}[\Omega]$. Then, a map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is said to be $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous if and only if, for every pair $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ of $\mathcal{T}_{\mathfrak{g},\Sigma}$-open and $\mathcal{T}_{\Sigma}$-closed sets in $\mathfrak{T}_{\mathfrak{g},\Sigma}$ there corresponds a pair $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ of $\mathcal{T}_{\mathfrak{g},\Omega}$-open and $\mathcal{T}_{\mathfrak{g},\Omega}$-closed sets in $\mathfrak{T}_{\mathfrak{g},\Omega}$ such that the following statement holds:

$$[\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})] \vee [\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})]. \tag{2.23}$$

A $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous map is said to be of category ν if and only if it belongs to the following class of $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous maps:

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] &\stackrel{\text{def}}{=} \{\pi_{\mathfrak{g}} : (\forall\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma})(\exists\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega},\mathbf{op}_{\mathfrak{g},\nu}(\cdot)) \\ &[(\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g},\nu}(\mathcal{O}_{\mathfrak{g},\omega})) \vee (\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}(\mathcal{K}_{\mathfrak{g},\omega}))]\}. \end{aligned} \tag{2.24}$$

Definition 2.7. Let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega,\mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma,\mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces. If, for each $\nu \in I_3^0$, $\mathfrak{g}$-ν-$\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ denotes the class of $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous maps, then

$$\mathfrak{g}\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] = \textstyle\bigcup_{\nu\in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]. \tag{2.25}$$

Definition 2.8 ($\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-Irresolute). Let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega,\mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma,\mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces, and let $\mathbf{op}_{\mathfrak{g}}(\cdot) \in \mathcal{L}_{\mathfrak{g}}[\Omega]$. Then, a map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is said to be $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute if and only if, for every pair $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ of $\mathcal{T}_{\mathfrak{g},\Sigma}$-open and $\mathcal{T}_{\Sigma}$-closed sets in $\mathfrak{T}_{\mathfrak{g},\Sigma}$ there corresponds a pair

$(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega})\in\mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$ of $\mathcal{T}_{\mathfrak{g},\Omega}$-open and $\mathcal{T}_{\mathfrak{g},\Omega}$-closed sets in $\mathfrak{T}_{\mathfrak{g},\Omega}$ such that the following statement holds:

$$\left[\pi_{\mathfrak{g}}^{-1}\left(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right]\vee\left[\pi_{\mathfrak{g}}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right]. \tag{2.26}$$

A $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute map is said to be of category ν if and only if it belongs to the following class of $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute maps:

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-}\mathrm{I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]&\stackrel{\text{def}}{=}\big\{\pi_{\mathfrak{g}}:\ \left(\forall\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}\right)\left(\exists\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega},\mathbf{op}_{\mathfrak{g},\nu}\left(\cdot\right)\right)\\&\left[\left(\pi_{\mathfrak{g}}^{-1}\left(\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\subseteq\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right)\vee\left(\pi_{\mathfrak{g}}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\supseteq\right.\right.\\&\left.\left.\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right)\right]\big\}.\end{aligned} \tag{2.27}$$

DEFINITION 2.9. Let $\mathfrak{T}_{\mathfrak{g},\Omega}=(\Omega,\mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma}=(\Sigma,\mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces. If, for each $\nu\in I_3^0$, $\mathfrak{g}$-ν-$\mathrm{I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ denotes the class of $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute maps, then

$$\mathfrak{g}\text{-}\mathrm{I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]=\bigcup\nolimits_{\nu\in I_3^0}\mathfrak{g}\text{-}\nu\text{-}\mathrm{I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]. \tag{2.28}$$

DEFINITION 2.10. Let $\mathfrak{T}_{\mathfrak{g},\Omega}=(\Omega,\mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma}=(\Sigma,\mathcal{T}_{\mathfrak{g},\Sigma})$ be $\mathcal{T}_{\mathfrak{g}}$-spaces and, let $\mathfrak{T}_{\Omega}=(\Omega,\mathcal{T}_{\Omega})$ and $\mathfrak{T}_{\Sigma}=(\Sigma,\mathcal{T}_{\Sigma})$ be $\mathcal{T}$-spaces.

- I. The classes $\mathrm{M_O}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$ and $\mathrm{M_K}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$ denote the families of $\mathfrak{T}$-open and $\mathfrak{T}$-closed maps, respectively, from $\mathfrak{T}_{\Omega}$ into $\mathfrak{T}_{\Sigma}$, with $\mathrm{M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]=\mathrm{M_O}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]\cup\mathrm{M_K}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$.
- II. The classes $\mathrm{M_O}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\mathrm{M_K}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ denote the families of $\mathfrak{T}_{\mathfrak{g}}$-open and $\mathfrak{T}_{\mathfrak{g}}$-closed maps, respectively, from $\mathfrak{T}_{\mathfrak{g},\Omega}$ into $\mathfrak{T}_{\mathfrak{g},\Sigma}$, with $\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]=\mathrm{M_O}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\cup\mathrm{M_K}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$.

The following sections present the main results of the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps.

2.2. MAIN RESULTS. The purpose of the following lines is to explore properties and characterizations of $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma})$-maps $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ belonging to the class $\mathfrak{g}$-$\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$.

THEOREM 2.11. *If $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ is a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-open or a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-closed map, then*

$$\pi_{\mathfrak{g}}\in\mathfrak{g}\text{-}\mathrm{M_O}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\cup\mathfrak{g}\text{-}\mathrm{M_K}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]. \tag{2.29}$$

PROOF. Let $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ be a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map. Then, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega})\in\mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$ there exists $(\mathcal{O}_{\mathfrak{g},},\mathcal{K}_{\mathfrak{g},\sigma})\in\mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that

$$\left[\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathcal{O}_{\mathfrak{g},\sigma}\right]\vee\left[\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\mathcal{K}_{\mathfrak{g},\sigma}\right].$$

But, $\mathcal{O}_{\mathfrak{g},\sigma}\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)$ and $\mathcal{K}_{\mathfrak{g},\sigma}\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)$. Consequently,

$$\left[\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right]\vee\left[\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right].$$

Hence, $\pi_{\mathfrak{g}}\in\mathfrak{g}\text{-}\mathrm{M_O}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\cup\mathfrak{g}\text{-}\mathrm{M_K}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Q.E.D.

The converse of THM. 2.11 is clearly false, because the statement "$\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ is a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map and $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ is not a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map" is untrue. The following theorem states that, the image of a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-set in a $\mathcal{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g},\Omega}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-set in a $\mathcal{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g},\Sigma}$ if and only if the map $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ is a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map.

THEOREM 2.12. *A necessary and sufficient condition for* $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ *to be a* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$*-map is that, for every* $(\mathcal{O}_{\mathfrak{g},\omega}, \mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$,

$$\begin{aligned}&\left[\pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}))\right] \vee \left[\pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\big) \supseteq \right.\\ &\left. \neg\,\mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}))\right].\end{aligned} \tag{2.30}$$

PROOF. *Necessity.* Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$. Then for $(\mathcal{O}_{\mathfrak{g},\omega}, \mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ there corresponds $(\mathcal{O}_{\mathfrak{g},}, \mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that

$$\left[\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right] \vee \left[\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\right].$$

Because $\left[\mathcal{O}_{\mathfrak{g},\omega} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\mathcal{K}_{\mathfrak{g},\omega} \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right]$, it consequently follows that,

$$\begin{aligned}&\left[\pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\big) \subseteq \mathrm{op}_{\mathfrak{g}} \circ \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right] \vee \left[\pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\big)\right.\\ &\left.\supseteq \neg\,\mathrm{op}_{\mathfrak{g}} \circ \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\right].\end{aligned}$$

But, since

$$\mathrm{op}_{\mathfrak{g}} \circ \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})), \quad \neg\,\mathrm{op}_{\mathfrak{g}} \circ \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})),$$

the proof at once follows.

Sufficiency. For every $(\mathcal{O}_{\mathfrak{g},\omega}, \mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$, let

$$\begin{aligned}&\left[\pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}))\right] \vee \left[\pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\big) \supseteq \right.\\ &\left. \neg\,\mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}))\right].\end{aligned}$$

Then,

$$\begin{aligned}&\left[\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right]\\ &\vee\left[\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right],\end{aligned}$$

because, $\mathcal{O}_{\mathfrak{g},\omega} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})$, $\mathcal{K}_{\mathfrak{g},\omega} \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})$, $\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})$, and $\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})$. Therefore,

$$\left[\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right] \vee \left[\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right].$$

Thus, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$, which completes the proof. Q.E.D.

THEOREM 2.13. *If* $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$ *and* $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Sigma}; \mathfrak{T}_{\mathfrak{g},\Upsilon}]$*, then* $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Upsilon}]$.

PROOF. Let $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$ and $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Sigma}; \mathfrak{T}_{\mathfrak{g},\Upsilon}]$. Then, for every $(\mathcal{O}_{\mathfrak{g},\omega}, \mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ there exists $(\mathcal{O}_{\mathfrak{g},\sigma}, \mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ and, for every $(\mathcal{O}_{\mathfrak{g},\sigma}, \mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ there exists $(\mathcal{O}_{\mathfrak{g},\upsilon}, \mathcal{K}_{\mathfrak{g},\upsilon}) \in \mathcal{T}_{\mathfrak{g},\Upsilon} \times \neg\mathcal{T}_{\mathfrak{g},\Upsilon}$ such that

$$\begin{aligned}&\left[\pi_{\mathfrak{g},\alpha}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right] \vee \left[\pi_{\mathfrak{g},\alpha}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\right],\\ &\left[\pi_{\mathfrak{g},\beta}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\upsilon})\right] \vee \left[\pi_{\mathfrak{g},\beta}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\upsilon})\right].\end{aligned}$$

From the first line, aided with the second, the logical statement preceding $\vee$ becomes

$$\begin{aligned}\pi_{\mathfrak{g},\alpha}(\mathcal{O}_{\mathfrak{g},\omega}) &\subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\\ \pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\mathcal{O}_{\mathfrak{g},\omega}) &\subseteq \pi_{\mathfrak{g},\beta}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g},\beta}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}\big(\mathcal{O}_{\mathfrak{g},\upsilon}\big),\end{aligned}$$

and, that following $\vee$ becomes

$$\begin{aligned}\pi_{\mathfrak{g},\alpha}(\mathcal{K}_{\mathfrak{g},\omega}) &\supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\\ \pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\mathcal{K}_{\mathfrak{g},\omega}) &\supseteq \pi_{\mathfrak{g},\beta}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g},\beta}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\big(\mathcal{K}_{\mathfrak{g},\upsilon}\big).\end{aligned}$$

Thus, $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$, which proves the theorem. Q.E.D.

THEOREM 2.14. *Let $\pi_{\mathfrak{g},\alpha}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ be a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map and let $\pi_{\mathfrak{g},\beta}:\mathfrak{T}_{\mathfrak{g},\Sigma}\rightarrow\mathfrak{T}_{\mathfrak{g},\Upsilon}$ be a $(\mathfrak{T}_{\mathfrak{g},\Sigma},\mathfrak{T}_{\mathfrak{g},\Upsilon})$-map. Then:*

- I. $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ *implies* $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$.
- II. $\pi_{\mathfrak{g},\beta}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$ *implies* $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$.

PROOF. I. Let $\pi_{\mathfrak{g},\alpha}$ be a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map and $\pi_{\mathfrak{g},\beta}$ a $(\mathfrak{T}_{\mathfrak{g},\Sigma},\mathfrak{T}_{\mathfrak{g},\Upsilon})$-map. Then, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega})\in\mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$, there exists $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma})\in\mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$ and, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma})\in\mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$, there exists $(\mathcal{O}_{\mathfrak{g},\upsilon},\mathcal{K}_{\mathfrak{g},\upsilon})\in\mathcal{T}_{\mathfrak{g},\Upsilon}\times\neg\mathcal{T}_{\mathfrak{g},\Upsilon}$ such that

$$\left[\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathcal{O}_{\mathfrak{g},\sigma}\right]\vee\left[\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\mathcal{K}_{\mathfrak{g},\sigma}\right],$$
$$\left[\pi_{\mathfrak{g},\beta}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\subseteq\mathcal{O}_{\mathfrak{g},\upsilon}\right]\vee\left[\pi_{\mathfrak{g},\beta}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\supseteq\mathcal{K}_{\mathfrak{g},\upsilon}\right].$$

The logical statements expressing the relations $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$ are, respectively,

$$\left[\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\big(\mathcal{O}_{\mathfrak{g},\sigma}\big)\right]\quad\vee\quad\left[\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\big(\mathcal{K}_{\mathfrak{g},\sigma}\big)\right],$$
$$\left[\pi_{\mathfrak{g},\beta}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\big(\mathcal{O}_{\mathfrak{g},\upsilon}\big)\right]\quad\vee\quad\left[\pi_{\mathfrak{g},\beta}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\big(\mathcal{K}_{\mathfrak{g},\upsilon}\big)\right].$$

Therefore, if only the relation $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ holds, then

$$\begin{aligned}
&\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\pi_{\mathfrak{g},\beta}\big(\mathrm{op}_{\mathfrak{g}}\big(\mathcal{O}_{\mathfrak{g},\sigma}\big)\big)\right]\vee\Big[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\\
&\qquad\qquad\supseteq\pi_{\mathfrak{g},\beta}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\big(\mathcal{K}_{\mathfrak{g},\sigma}\big)\big)\Big]\\
\Rightarrow\quad&\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g},\beta}\big(\mathcal{O}_{\mathfrak{g},\sigma}\big)\big)\right]\vee\Big[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\\
&\qquad\qquad\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g},\beta}\big(\mathcal{K}_{\mathfrak{g},\sigma}\big)\big)\Big]\\
\Rightarrow\quad&\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\big(\mathcal{O}_{\mathfrak{g},\upsilon}\big)\big)\right]\vee\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\big(\mathcal{K}_{\mathfrak{g},\upsilon}\big)\right],
\end{aligned}$$

and, hence, $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$.

II. If only the relation $\pi_{\mathfrak{g},\beta}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$ holds, then

$$\begin{aligned}
&\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\pi_{\mathfrak{g},\beta}\big(\mathcal{O}_{\mathfrak{g},\sigma}\big)\right]\vee\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\pi_{\mathfrak{g},\beta}\big(\mathcal{K}_{\mathfrak{g},\sigma}\big)\right]\\
\Rightarrow\quad&\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\big(\mathcal{O}_{\mathfrak{g},\upsilon}\big)\big)\right]\vee\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\big(\mathcal{K}_{\mathfrak{g},\upsilon}\big)\right],
\end{aligned}$$

and, hence, $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$. Q.E.D.

PROPOSITION 2.15. *Let $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$, satisfying*

$$\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right]\vee\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right],$$
$$\left[\pi_{\mathfrak{g},\alpha}\circ\pi_{\mathfrak{g},\beta}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right]\vee\left[\pi_{\mathfrak{g},\alpha}\circ\pi_{\mathfrak{g},\beta}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right],\tag{2.31}$$

respectively. Then, there exist inverse maps $\pi_{\mathfrak{g},\alpha}^{-1}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta}^{-1}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$ such that $\pi_{\mathfrak{g},\beta}=\pi_{\mathfrak{g},\alpha}^{-1}$ and $\pi_{\mathfrak{g},\alpha}=\pi_{\mathfrak{g},\beta}^{-1}$.

PROOF. It is clear that, $\mathcal{O}_{\mathfrak{g},\mu}\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\mu}\right)$ or $\mathcal{K}_{\mathfrak{g},\mu}\supseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\mu}\right)$ for every $\mu\in\{\omega,\sigma\}$. But, $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ or $\pi_{\mathfrak{g},\beta}\in\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$ satisfy

$$\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right]\vee\left[\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right],$$
$$\left[\pi_{\mathfrak{g},\alpha}\circ\pi_{\mathfrak{g},\beta}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right]\vee\left[\pi_{\mathfrak{g},\alpha}\circ\pi_{\mathfrak{g},\beta}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right].$$

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Hence, there exist $\pi_{\mathfrak{g},\alpha}^{-1} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta}^{-1} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$ such that $\pi_{\mathfrak{g},\beta} = \pi_{\mathfrak{g},\alpha}^{-1}$ and $\pi_{\mathfrak{g},\alpha} = \pi_{\mathfrak{g},\beta}^{-1}$. This proves the proposition. Q.E.D.

THEOREM 2.16. *Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Given any $\mathfrak{T}_{\mathfrak{g},\Sigma}$-set $\mathcal{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ and any pair $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ of $\mathcal{T}_{\mathfrak{g},\Omega}$-open and $\mathcal{T}_{\mathfrak{g},\Omega}$-closed sets satisfying*

$$\left[\pi_{\mathfrak{g}}^{-1}(\mathcal{S}_{\mathfrak{g}}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\pi_{\mathfrak{g}}^{-1}(\mathcal{S}_{\mathfrak{g}}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right], \tag{2.32}$$

then:

- I. $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ *implies the existence of a $\mathcal{T}_{\mathfrak{g},\Sigma}$-open set $\mathcal{O}_{\mathfrak{g},\sigma} \supseteq \mathcal{S}_{\mathfrak{g}}$ such that $\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})$.*
- II. $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ *implies the existence of a $\mathcal{T}_{\mathfrak{g},\Sigma}$-closed set $\mathcal{K}_{\mathfrak{g},\sigma} \supseteq \mathcal{S}_{\mathfrak{g}}$ such that $\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})$.*

PROOF. I. Let $\mathcal{O}_{\mathfrak{g},\sigma} = \Sigma - \pi_{\mathfrak{g}}\big(\Omega - \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\big)$. Then, since $\pi_{\mathfrak{g}}^{-1}(\mathcal{S}_{\mathfrak{g}}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})$ and $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, there exists a $\mathcal{T}_{\mathfrak{g},\Sigma}$-open set $\mathcal{O}_{\mathfrak{g},\sigma} \in \mathcal{T}_{\mathfrak{g},\Sigma}$ such that $\mathcal{O}_{\mathfrak{g},\sigma} \supseteq \mathcal{S}_{\mathfrak{g}}$. But, since

$$\begin{aligned}\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) &= \Omega - \pi_{\mathfrak{g}}^{-1} \circ \pi_{\mathfrak{g}}\big(\Omega - \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\big)\\ &\subseteq \Omega - \big(\Omega - \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\big) = \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}),\end{aligned}$$

the proof of I. follows.

II. Let $\mathcal{K}_{\mathfrak{g},\sigma} = \Sigma - \pi_{\mathfrak{g}}\big(\Omega - \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\big)$. Then, because $\pi_{\mathfrak{g}}^{-1}(\mathcal{S}_{\mathfrak{g}}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})$ and $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, there exists a $\mathcal{T}_{\mathfrak{g},\Sigma}$-closed $\mathcal{K}_{\mathfrak{g},\sigma} \in \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that $\mathcal{K}_{\mathfrak{g},\sigma} \supseteq \mathcal{S}_{\mathfrak{g}}$. But, since

$$\begin{aligned}\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) &= \Omega - \pi_{\mathfrak{g}}^{-1} \circ \pi_{\mathfrak{g}}\big(\Omega - \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\big)\\ &\supseteq \Omega - \big(\Omega - \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\big) = \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}),\end{aligned}$$

the proof of II. follows. Q.E.D.

We next investigate further properties and give characterizations of those elements which belong to the class $\mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$.

THEOREM 2.17. *If $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous map, then $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$.*

PROOF. If $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous map, then, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$, there exists $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that

$$\left[\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathcal{O}_{\mathfrak{g},\omega}\right] \vee \left[\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \mathcal{K}_{\mathfrak{g},\omega}\right].$$

But, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$,

$$\left[\mathcal{O}_{\mathfrak{g},\omega} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\mathcal{K}_{\mathfrak{g},\omega} \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right],$$

and, consequently,

$$\left[\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right].$$

Hence, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Q.E.D.

THEOREM 2.18. *If $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map satisfying, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$,*

$$\left[\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma})\big)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma})\big)\right],$$

then $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$.

PROOF. For every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$, it is evident that

$$\begin{aligned} & \left[\mathcal{O}_{\mathfrak{g},\sigma} \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\mathcal{K}_{\mathfrak{g},\sigma} \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right] \\ \Rightarrow\quad & \left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \subseteq \pi_{\mathfrak{g}}^{-1}\left(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \supseteq \pi_{\mathfrak{g}}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right] \\ \Rightarrow\quad & \left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right]. \end{aligned}$$

Hence, there exists $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that $\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \subseteq \mathcal{O}_{\mathfrak{g},\omega}$ and $\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \supseteq \mathcal{K}_{\mathfrak{g},\omega}$. Consequently, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, which completes the proof. Q.E.D.

DEFINITION 2.19 ($(\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma})$-Bijective Map). A $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ is said to be bijective if and only if it belongs the following class:

$$\mathfrak{g}\text{-B}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] \stackrel{\mathrm{def}}{=} \left\{\pi_{\mathfrak{g}}:\ (\forall\zeta \in \mathfrak{T}_{\mathfrak{g},\Sigma})\,(\exists!\xi \in \mathfrak{T}_{\mathfrak{g},\Omega})\left[\pi_{\mathfrak{g}}(\xi)=\zeta\right]\right\}. \tag{2.33}$$

THEOREM 2.20. *If* $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-B}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, *then*

$$\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] \ \Leftrightarrow\ \pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]. \tag{2.34}$$

PROOF. *Necessity.* Let $\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$. Then

$$\left[\left(\pi_{\mathfrak{g}}^{-1}\right)^{-1}\left(\mathcal{O}_{\mathfrak{g},\omega}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\left(\pi_{\mathfrak{g}}^{-1}\right)^{-1}\left(\mathcal{K}_{\mathfrak{g},\omega}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right].$$

But $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-B}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ implies $\left(\pi_{\mathfrak{g}}^{-1}\right)^{-1}\left(\mathcal{S}_{\mathfrak{g}}\right) = \pi_{\mathfrak{g}}\left(\mathcal{S}_{\mathfrak{g}}\right)$ for every $\mathcal{S}_{\mathfrak{g}} \in \mathcal{T}_{\mathfrak{g},\Omega} \cup \neg\mathcal{T}_{\mathfrak{g},\Omega}$. Consequently,

$$\left[\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right].$$

Hence, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$.

Sufficiency. Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Then,

$$\left[\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right].$$

But $\pi_{\mathfrak{g}}\left(\mathcal{S}_{\mathfrak{g}}\right) = \left(\pi_{\mathfrak{g}}^{-1}\right)^{-1}\left(\mathcal{S}_{\mathfrak{g}}\right)$ for every $\mathcal{S}_{\mathfrak{g}} \in \mathcal{T}_{\mathfrak{g},\Omega}\cup\neg\mathcal{T}_{\mathfrak{g},\Omega}$, since $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-B}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Consequently,

$$\left[\left(\pi_{\mathfrak{g}}^{-1}\right)^{-1}\left(\mathcal{O}_{\mathfrak{g},\omega}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\left(\pi_{\mathfrak{g}}^{-1}\right)^{-1}\left(\mathcal{K}_{\mathfrak{g},\omega}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right].$$

Thus, $\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$. Q.E.D.

THEOREM 2.21. *If* $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ *and* $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$, *then* $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$.

PROOF. Let $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$. Then $\pi_{\mathfrak{g},\alpha}^{-1} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$ and $\pi_{\mathfrak{g},\beta}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Upsilon};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, implying

$$\begin{aligned} & \left[\pi_{\mathfrak{g},\alpha}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \quad\vee\quad \left[\pi_{\mathfrak{g},\alpha}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right], \\ & \left[\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \quad\vee\quad \left[\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right], \end{aligned}$$

respectively. Combining both logical statements, there follows that

$$\begin{aligned} & \left[\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right] \\ \Rightarrow \quad & \left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right) \subseteq \pi_{\mathfrak{g},\alpha}^{-1}\left(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right)\right. \\ & \qquad \left.\supseteq \pi_{\mathfrak{g},\alpha}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right] \\ \Rightarrow \quad & \left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g},\alpha}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right)\right. \\ & \qquad \left.\supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g},\alpha}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right] \\ \Rightarrow \quad & \left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \vee \left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right]. \end{aligned}$$

Since $\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{S}_{\mathfrak{g}}\right) = \left(\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{S}_{\mathfrak{g}}\right)$ for every $\mathcal{S}_{\mathfrak{g}} \in \mathcal{T}_{\mathfrak{g},\Upsilon} \cup \neg\mathcal{T}_{\mathfrak{g},\Upsilon}$, there follows that $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$, which was to be proved. Q.E.D.

Theorem 2.22. *Let* $\pi_{\mathfrak{g},\alpha} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ *be a* $(\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma})$*-map and, let the collection* $\left\{\left\langle\mathcal{O}_{\mathfrak{g},\alpha}\right\rangle_{\alpha\in I_n^*} : \Omega \subseteq \mathrm{op}_{\mathfrak{g}}\left(\bigcup_{\alpha\in I_n^*}\mathcal{O}_{\mathfrak{g},\alpha}\right)\right\}$ *and the collection* $\left\{\left\langle\mathcal{K}_{\mathfrak{g},\alpha}\right\rangle_{\alpha\in I_n^*} : \Omega \subseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\bigcup_{\alpha\in I_n^*}\mathcal{K}_{\mathfrak{g},\alpha}\right)\right\}$*, respectively, be* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-open and* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-closed coverings of* Ω*, where* $\left\langle\mathcal{O}_{\mathfrak{g},\alpha}\right\rangle_{\alpha\in I_n^*}$ *and* $\left\langle\mathcal{K}_{\mathfrak{g},\alpha}\right\rangle_{\alpha\in I_n^*}$*, respectively, denote sequences of* $\mathcal{T}_{\mathfrak{g}}$*-open sets and* $\mathcal{T}_{\mathfrak{g}}$*-closed sets. If, for every* $\alpha \in I_n^*$*,* $\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$*, where* $\iota_{\mathfrak{g},\alpha} : \mathcal{O}_{\mathfrak{g},\alpha} \hookrightarrow \mathfrak{T}_{\mathfrak{g},\Omega}$ *or* $\iota_{\mathfrak{g},\alpha} : \mathcal{O}_{\mathfrak{g},\alpha} \hookrightarrow \mathfrak{T}_{\mathfrak{g},\Omega}$*, then* $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$.

Proof. For every $\alpha \in I_n^*$, let $\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Then, for every pair $\left(\mathcal{O}_{\mathfrak{g},\sigma(\alpha)},\mathcal{K}_{\mathfrak{g},\sigma(\alpha)}\right) \in \mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$, there exists $\left(\mathcal{O}_{\mathfrak{g},\omega(\alpha)},\mathcal{K}_{\mathfrak{g},\omega(\alpha)}\right) \in \mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that

$$\begin{aligned} & \left[\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma(\alpha)}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega(\alpha)}\right)\right] \vee \left[\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma(\alpha)}\right)\right. \\ & \qquad \left.\supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega(\alpha)}\right)\right] \\ \Rightarrow \quad & \left[\bigcup_{\alpha\in I_n^*}\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma(\alpha)}\right) \subseteq \bigcup_{\alpha\in I_n^*}\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega(\alpha)}\right)\right] \\ & \vee\left[\bigcup_{\alpha\in I_n^*}\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma(\alpha)}\right) \supseteq \bigcup_{\alpha\in I_n^*}\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega(\alpha)}\right)\right] \\ \Rightarrow \quad & \left[\bigcup_{\alpha\in I_n^*}\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma(\alpha)}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\bigcup_{\alpha\in I_n^*}\mathcal{O}_{\mathfrak{g},\omega(\alpha)}\right)\right] \\ & \vee\left[\bigcup_{\alpha\in I_n^*}\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma(\alpha)}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\bigcup_{\alpha\in I_n^*}\mathcal{K}_{\mathfrak{g},\omega(\alpha)}\right)\right]. \end{aligned}$$

Since the following relations hold

$$\begin{aligned} \pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma(\alpha)}\right) &= \bigcup_{\alpha\in I_n^*}\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma(\alpha)}\right), \\ \pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma(\alpha)}\right) &= \bigcup_{\alpha\in I_n^*}\left(\pi_{\mathfrak{g}}\circ\iota_{\mathfrak{g},\alpha}\right)^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma(\alpha)}\right), \end{aligned}$$

the proof of the theorem follows. Q.E.D.

Henceforth, we investigate some properties and give some characterizations of $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute maps.

Theorem 2.23. *A* $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-map* $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ *is a* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-irresolute map if and only if, for every* $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$*,*

$$\begin{aligned} & \left[\pi_{\mathfrak{g}}^{-1}\left(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right) \supseteq\right. \\ & \qquad \left.\neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right]. \end{aligned} \tag{2.35}$$

PROOF. *Necessity.* Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Then, there exists $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$,

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right].$$

But since $\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)$ and $\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)$, it follows that

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \Big[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\Big].$$

Sufficiency. Let $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ be a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map satisfying, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$,

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \Big[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\Big].$$

But, $\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)$ and $\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)$. Therefore, there exists $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that $\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)$ and $\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)$. Consequently,

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right].$$

Thus, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, which completes the proof. Q.E.D.

THEOREM 2.24. *A* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-map* $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ *is a* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-irresolute map if and only if, for every* $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$,

$$\left[\pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\big) \supseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right)\right] \vee \Big[\pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\big) \subseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right)\Big]. \tag{2.36}$$

PROOF. *Necessity.* Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Then, there exists $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$,

$$\begin{aligned} &\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right] \\ \Rightarrow\quad &\left[\pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\big) \supseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\big) \subseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right]. \end{aligned}$$

But since $\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \supseteq \pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\big)$ and $\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \subseteq \pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\big)$, it follows that

$$\left[\pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\big) \supseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right)\right] \vee \Big[\pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\big) \subseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right)\Big].$$

Sufficiency. Let $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ be a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-map satisfying, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$,

$$\left[\pi_{\mathfrak{g}}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\big) \supseteq \mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right)\right] \vee \Big[\pi_{\mathfrak{g}}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\big) \subseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right)\Big].$$

Then,

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \vee \Big[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\Big].$$

But, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ equivalently implies the existence of $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$, $\mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})$ or $\neg\,\mathrm{op}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})$. Consequently,

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right].$$

Thus, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, which completes the proof. Q.E.D.

THEOREM 2.25. *Let* $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\to\mathfrak{T}_{\mathfrak{g},\Sigma}$ *be a* $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-map. Then,*

$$\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \cap \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \;\Rightarrow\; \pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]. \tag{2.37}$$

PROOF. Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \cap \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$. Then, there exists a pair $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$,

$$\left[\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right] \vee \left[\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\right],$$

and there exists $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$,

$$\left[\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right].$$

From the first statement, there follows that

$$\left[\mathcal{O}_{\mathfrak{g},\omega} \subseteq \pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\big)\right] \vee \left[\mathcal{K}_{\mathfrak{g},\omega} \supseteq \pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\big)\right].$$

But,

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma})\big)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma})\big)\right];$$

and, from the second statement, there follows that

$$\left[\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\neg\,\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}).\right.$$

From these last two logical statements, it consequently follows that

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right],$$

and, hence, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, which completes the proof. Q.E.D.

THEOREM 2.26. *If* $\mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ *and* $\mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, *respectively, denote the classes of* $\mathfrak{g}$*-* $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-continuous and* $\mathfrak{g}$*-* $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-irresolute maps, then*

$$\mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \supseteq \mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]. \tag{2.38}$$

PROOF. Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$. Then, there exists $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$,

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right].$$

But, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ is equivalent to

$$\left[\pi_{\mathfrak{g}}^{-1}\big(\mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma})\big)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma})\big)\right],$$

and, $\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma})\big) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})$ and $\neg\,\mathrm{op}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma})\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})$. Consequently, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ implies

$$\left[\pi_{\mathfrak{g}}^{-1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega})\right] \vee \left[\pi_{\mathfrak{g}}^{-1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega})\right],$$

and, hence, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, which completes the proof. Q.E.D.

THEOREM 2.27. *If* $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ *and* $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$*, then* $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$.

PROOF. Let $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$. Then, for every $(\mathcal{O}_{\mathfrak{g},\upsilon},\mathcal{K}_{\mathfrak{g},\upsilon}) \in \mathcal{T}_{\mathfrak{g},\Upsilon} \times \neg\mathcal{T}_{\mathfrak{g},\Upsilon}$ there exists $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ and for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ there exists $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ such that

$$\left[\pi_{\mathfrak{g},\beta}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \quad\vee\quad \left[\pi_{\mathfrak{g},\beta}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right],$$

$$\left[\pi_{\mathfrak{g},\alpha}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \quad\vee\quad \left[\pi_{\mathfrak{g},\alpha}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right],$$

respectively. Consequently,

$$\begin{aligned}
&\left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right)\big) \subseteq \pi_{\mathfrak{g},\alpha}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\big)\right] \\
&\qquad \vee\left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right)\big) \supseteq \pi_{\mathfrak{g},\alpha}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\big)\right] \\
\Rightarrow\quad &\left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\big(\mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\upsilon}\right)\big) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \\
&\qquad \vee\left[\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1}\big(\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\upsilon}\right)\big) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right].
\end{aligned}$$

But $\pi_{\mathfrak{g},\alpha}^{-1}\circ\pi_{\mathfrak{g},\beta}^{-1} = \left(\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\right)^{-1}$. Hence, $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$. Q.E.D.

We generalize the notion of $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphism in a natural way and then investigate some properties and give some characterizations of such generalization on this basis.

DEFINITION 2.28. Two $\mathcal{T}_{\mathfrak{g}}$-spaces $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega,\mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma,\mathcal{T}_{\mathfrak{g},\Sigma})$ are called "$\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphic," written $\mathfrak{T}_{\mathfrak{g},\Omega} \cong \mathfrak{T}_{\mathfrak{g},\Sigma}$, if and only if

$$\big(\exists\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-B}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\big)\left[\big(\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\big) \wedge \big(\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]\big)\right].$$

(2.39)

The map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ is called a "$\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphism," written $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \cong \mathfrak{T}_{\mathfrak{g},\Sigma}$, and belongs to the following class:

$$\mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] \stackrel{\text{def}}{=} \left\{\pi_{\mathfrak{g}} :\ \pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \cong \mathfrak{T}_{\mathfrak{g},\Sigma}\right\}. \tag{2.40}$$

THEOREM 2.29. *If* $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ *is a* $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-homeomorphism, then it is a* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-homeomorphism:* $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$

PROOF. Let $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ be a $(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphism. Then, for every $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ there exists $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ and for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$ there exists $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that

$$\left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \subseteq \mathcal{O}_{\mathfrak{g},\omega}\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \supseteq \mathcal{K}_{\mathfrak{g},\omega}\right],$$

$$\left[\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right) \subseteq \mathcal{O}_{\mathfrak{g},\sigma}\right] \vee \left[\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right) \supseteq \mathcal{K}_{\mathfrak{g},\sigma}\right],$$

respectively. But, $\left[\mathcal{O}_{\mathfrak{g},\nu} \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\nu}\right)\right] \vee \left[\mathcal{K}_{\mathfrak{g},\nu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\nu}\right)\right]$ for every $\nu \in \{\omega,\sigma\}$. Consequently,

$$\left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right],$$

$$\left[\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right] \vee \left[\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right) \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right].$$

Therefore, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$; thus, it follows that $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Q.E.D.

THEOREM 2.30. *If* $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ *and* $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$, *then* $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$.

PROOF. Let $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$. Then, there exists exactly one $\xi \in \mathfrak{T}_{\mathfrak{g},\Omega}$ such that, for all $\zeta \in \mathfrak{T}_{\mathfrak{g},\Sigma}$, $\pi_{\mathfrak{g},\alpha}(\xi) = \zeta$ and, there exists exactly one $\zeta \in \mathfrak{T}_{\mathfrak{g},\Sigma}$ such that, for all $\eta \in \mathfrak{T}_{\mathfrak{g},\Upsilon}$, $\pi_{\mathfrak{g},\beta}(\zeta) = \eta$. Therefore there exists exactly one $\xi \in \mathfrak{T}_{\mathfrak{g},\Omega}$ such that, for all $\eta \in \mathfrak{T}_{\mathfrak{g},\Upsilon}$, $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\xi) = \eta$; hence, $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-B}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$. On the one hand, $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$ implies $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$ and, on the other hand, $\pi_{\mathfrak{g},\alpha}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$ and $\pi_{\mathfrak{g},\beta}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Upsilon};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ implies $\pi_{\mathfrak{g},\alpha}^{-1} \circ \pi_{\mathfrak{g},\beta}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Upsilon};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$. But, $\pi_{\mathfrak{g},\alpha}^{-1} \circ \pi_{\mathfrak{g},\beta}^{-1} = \left(\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}\right)^{-1}$. Hence, $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$, which proves the theorem. Q.E.D.

THEOREM 2.31. *If* $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, *then, for every* $(\mathcal{O}_{\mathfrak{g},\sigma}, \mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma} \times \neg\mathcal{T}_{\mathfrak{g},\Sigma}$,

$$\begin{aligned}\left[\pi_{\mathfrak{g}}^{-1}\left(\operatorname{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right) = \operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\neg\operatorname{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right.\\ \left. = \neg\operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right].\end{aligned} \tag{2.41}$$

PROOF. Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Then, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-B}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and

$$\left(\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\right) \wedge \left(\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]\right).$$

Consequently,

$$\begin{aligned}\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] \cap \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] \quad &\Rightarrow \quad \pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\\ \pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right] \cap \mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right] \quad &\Rightarrow \quad \pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right].\end{aligned}$$

But, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$ are equivalent to

$$\begin{aligned}&\left[\pi_{\mathfrak{g}}^{-1}\left(\operatorname{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right) \subseteq \operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\neg\operatorname{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right) \supseteq\right.\\ &\qquad\left.\neg\operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right],\\ &\left[\pi_{\mathfrak{g}}^{-1}\left(\operatorname{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right) \supseteq \operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma}\right)\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\neg\operatorname{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right) \subseteq\right.\\ &\qquad\left.\neg\operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma}\right)\right)\right],\end{aligned}$$

respectively. Hence, equality holds. Q.E.D.

COROLLARY 2.32. *If* $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$, *then, for every* $(\mathcal{O}_{\mathfrak{g},\omega}, \mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega} \times \neg\mathcal{T}_{\mathfrak{g},\Omega}$,

$$\begin{aligned}\left[\pi_{\mathfrak{g}}\left(\operatorname{op}_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right) = \operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{O}_{\mathfrak{g},\omega}\right)\right)\right] \vee \left[\pi_{\mathfrak{g}}\left(\neg\operatorname{op}_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right)\right.\\ \left. = \neg\operatorname{op}_{\mathfrak{g}}\left(\pi_{\mathfrak{g}}\left(\mathcal{K}_{\mathfrak{g},\omega}\right)\right)\right].\end{aligned} \tag{2.42}$$

THEOREM 2.33. *A* $\mathfrak{g}\text{-}(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$*-homeomorphism is an equivalence relation between* $\mathcal{T}_{\mathfrak{g}}$*-spaces.*

PROOF. *Reflexivity.* The identity map $\operatorname{id}_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Omega}$ is a bicontinuous bijection. Therefore, it is a $\mathfrak{g}\text{-}(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphism $\operatorname{id}_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \cong \mathfrak{T}_{\mathfrak{g},\Omega}$ and, hence, $\operatorname{id}_{\mathfrak{g}}(\cdot) \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$.

Symmetry. Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Then, the map $\pi_{\mathfrak{g}}^{-1} : T_{\mathfrak{g},\Sigma} \to \mathfrak{T}_{\mathfrak{g},\Omega}$ is a $\mathfrak{g}\text{-}(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-homeomorphism $\pi_{\mathfrak{g}}^{-1} : \mathfrak{T}_{\mathfrak{g},\Sigma} \cong \mathfrak{T}_{\mathfrak{g},\Omega}$ and, thus, it follows that $\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}\right]$.

Transitivity. The proof follows from $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$ imply $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-Hom}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}\right]$. Q.E.D.

3. Discussion

3.1. Categorical Classifications. Having adopted a categorical approach in the classifications of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps between any two of such $\mathcal{T}_{\mathfrak{g}}$-spaces $\mathfrak{T}_{\mathfrak{g},\Omega}$, $\mathfrak{T}_{\mathfrak{g},\Sigma}$, and $\mathfrak{T}_{\mathfrak{g},\Upsilon}$, the twofold purposes of the following developments are to establish the various relationships between the classes of $(\mathfrak{T}_{\mathfrak{g},\Lambda},\mathfrak{T}_{\mathfrak{g},\Theta})$-maps and $\mathfrak{g}$-$(\mathfrak{T}_{\Lambda},\mathfrak{T}_{\Theta})$-maps, the classes of $(\mathfrak{T}_{\mathfrak{g},\Lambda},\mathfrak{T}_{\mathfrak{g},\Theta})$-continuous maps and $\mathfrak{g}$-$(\mathfrak{T}_{\Lambda},\mathfrak{T}_{\Theta})$-continuous maps, the classes of $(\mathfrak{T}_{\mathfrak{g},\Lambda},\mathfrak{T}_{\mathfrak{g},\Theta})$-irresolute maps and $\mathfrak{g}$-$(\mathfrak{T}_{\Lambda},\mathfrak{T}_{\Theta})$-irresolute maps, and the classes of $(\mathfrak{T}_{\mathfrak{g},\Lambda},\mathfrak{T}_{\mathfrak{g},\Theta})$-homeomorphism maps and $\mathfrak{g}$-$(\mathfrak{T}_{\Lambda},\mathfrak{T}_{\Theta})$-homeomorphism maps, where $\Lambda,\ \Theta \in \{\Omega,\Sigma,\Upsilon\}$, and to illustrate them through specific diagrams called, *map*, *categorical map*, *continuous map*, *irresolute map*, *homeomorphism map*, and *continuous-irresolute map diagrams.*

We have seen that, $\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] \subseteq \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. But,

$$\begin{aligned}\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] &= \mathrm{M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\cup\mathrm{M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\\ \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] &= \mathfrak{g}\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\cup\mathfrak{g}\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right].\end{aligned}$$

Consequently,

$$\begin{aligned}\mathrm{M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\ \mathrm{M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] &\subseteq \mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\\ \mathfrak{g}\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\ \mathfrak{g}\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] &\subseteq \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\end{aligned}$$

which, in turn, imply

$$\begin{aligned}\mathrm{M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\subseteq\mathfrak{g}\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] &\subseteq \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\\ \mathrm{M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\subseteq\mathfrak{g}\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] &\subseteq \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right],\end{aligned}$$

respectively. In Fig. 1, we present the relations between the class $\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] = \mathrm{M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\cup\mathrm{M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ of $(\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma})$-open and $(\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma})$-closed maps and, also, the class $\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] = \mathfrak{g}\text{-M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\cup\mathfrak{g}\text{-M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ of $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma})$-open and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma})$-closed maps. The diagram in Fig. 1 shall be termed a *map diagram.*

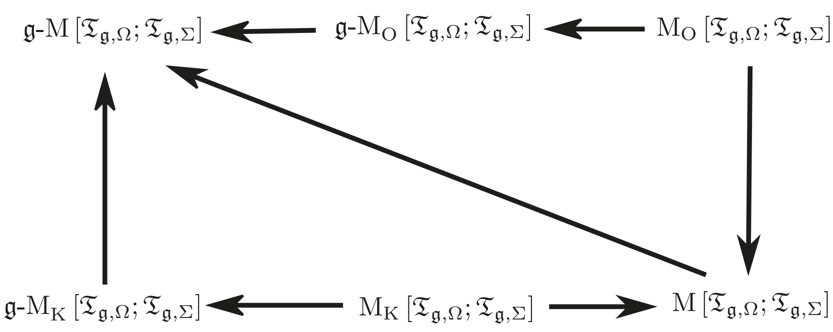

Figure 1. Relationships: Map diagram.

For every $\nu \in I_3^0$, it is plain that, $\mathfrak{g}\text{-}\nu\text{-M}\left[\mathfrak{T}\Omega;\mathfrak{T}_{\Sigma}\right] \subseteq \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$ and, also, $\mathfrak{g}\text{-}\nu\text{-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right] \subseteq \mathfrak{g}\text{-}\nu\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right] \subseteq \mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Further, $\mathfrak{g}\text{-2-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right] \subseteq \mathfrak{g}\text{-3-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$; likewise, $\mathfrak{g}\text{-0-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right] \subseteq \mathfrak{g}\text{-1-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right] \subseteq \mathfrak{g}\text{-3-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$ and

$\mathfrak{g}$-2-M $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \subseteq \mathfrak{g}$-3-M $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ and also, the relation $\mathfrak{g}$-0-M $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \subseteq \mathfrak{g}$-1-M $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] \subseteq \mathfrak{g}$-3-M $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ holds. In fact, for every $\mathfrak{T}_{\mathfrak{g}}$-set $\mathcal{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$, the following relations hold:

$$\begin{aligned}
\mathrm{int}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}) &\subseteq \mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}) \subseteq \mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}) \supseteq \mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}),\\
\mathrm{cl}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}) &\supseteq \mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}) \supseteq \mathrm{int}_{\mathfrak{g}}\circ\mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}) \subseteq \mathrm{cl}_{\mathfrak{g}}\circ\mathrm{int}_{\mathfrak{g}}(\mathcal{S}_{\mathfrak{g}}).
\end{aligned}$$

Consequently, for every $(\mathcal{O}_{\mathfrak{g},\omega},\mathcal{K}_{\mathfrak{g},\omega}) \in \mathcal{T}_{\mathfrak{g},\Omega}\times\neg\mathcal{T}_{\mathfrak{g},\Omega}$ there exists $(\mathcal{O}_{\mathfrak{g},\sigma},\mathcal{K}_{\mathfrak{g},\sigma}) \in \mathcal{T}_{\mathfrak{g},\Sigma}\times\neg\mathcal{T}_{\mathfrak{g},\Sigma}$ such that

$$\begin{aligned}
\pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}) &\subseteq \mathrm{op}_{\mathfrak{g},0}(\mathcal{O}_{\mathfrak{g},\sigma})\\
&\subseteq \mathrm{op}_{\mathfrak{g},1}(\mathcal{O}_{\mathfrak{g},\sigma}) \subseteq \mathrm{op}_{\mathfrak{g},3}(\mathcal{O}_{\mathfrak{g},\sigma}) \supseteq \mathrm{op}_{\mathfrak{g},2}(\mathcal{O}_{\mathfrak{g},\sigma}) \supseteq \pi_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\omega}),\\
\pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}) &\supseteq \neg\,\mathrm{op}_{\mathfrak{g},0}(\mathcal{K}_{\mathfrak{g},\sigma})\\
&\supseteq \neg\,\mathrm{op}_{\mathfrak{g},1}(\mathcal{K}_{\mathfrak{g},\sigma}) \supseteq \neg\,\mathrm{op}_{\mathfrak{g},3}(\mathcal{K}_{\mathfrak{g},\sigma}) \subseteq \neg\,\mathrm{op}_{\mathfrak{g},2}(\mathcal{K}_{\mathfrak{g},\sigma}) \subseteq \pi_{\mathfrak{g}}(\mathcal{K}_{\mathfrak{g},\omega}).
\end{aligned}$$

In FIG. 2, we present the relationships between the class $\mathfrak{g}$-M$[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}] = \bigcup_{\nu\in I_3^0}\mathfrak{g}$-$\nu$-M$[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ of $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-maps of categories 0, 1, 2 and 3, and the class $\mathfrak{g}$-M$[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}] = \bigcup_{\nu\in I_3^0}\mathfrak{g}$-$\nu$-M$[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}]$ of $\mathfrak{g}$-$(\mathfrak{T}_{\Omega},\mathfrak{T}_{\Sigma})$-maps of categories 0, 1, 2 and 3. These characteristics may be indicated, as in FIG. 2, by what we shall term a *categorical map diagram.*

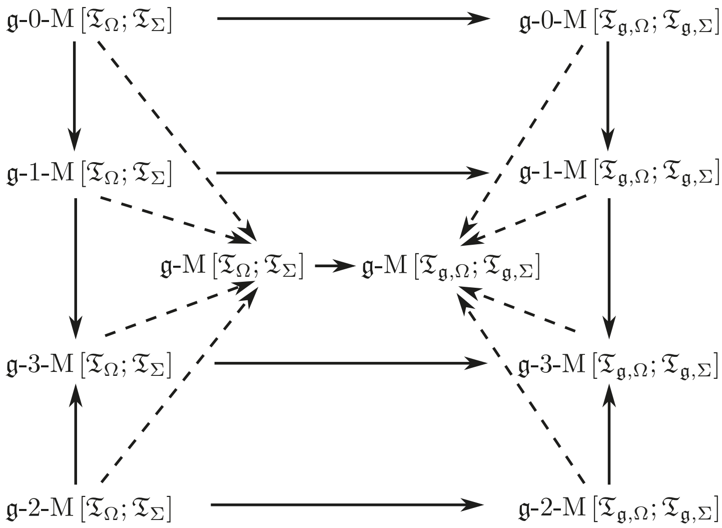

FIGURE 2. Relationships: Categorical Map diagram.

Now, suppose we are given $\pi_{\mathfrak{g},\alpha}:\mathfrak{T}_{\mathfrak{g},\Omega}\cong\mathfrak{T}_{\mathfrak{g},\Sigma}$ and $\pi_{\mathfrak{g},\alpha}:\mathfrak{T}_{\mathfrak{g},\Sigma}\cong\mathfrak{T}_{\mathfrak{g},\Upsilon}$. Then, by virtue of previous theorems, $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$. Also, $\pi_{\mathfrak{g},\alpha} \in$ Hom $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in$ Hom $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in$ Hom $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$ imply $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and the relation $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, respectively. These features may be indicated, as in FIG. 3, by what we shall term a *homeomorphism map diagram.*

Next, suppose we are given $\pi_{\mathfrak{g},\alpha}:\mathfrak{T}_{\mathfrak{g},\Omega}\to\mathfrak{T}_{\mathfrak{g},\Sigma}$ and $\pi_{\mathfrak{g},\alpha}:\mathfrak{T}_{\mathfrak{g},\Sigma}\to\mathfrak{T}_{\mathfrak{g},\Upsilon}$. If $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ and $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$ then, by virtue of previous theorems, $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$. Moreover, $\pi_{\mathfrak{g},\alpha} \in$ C $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in$ C $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$,

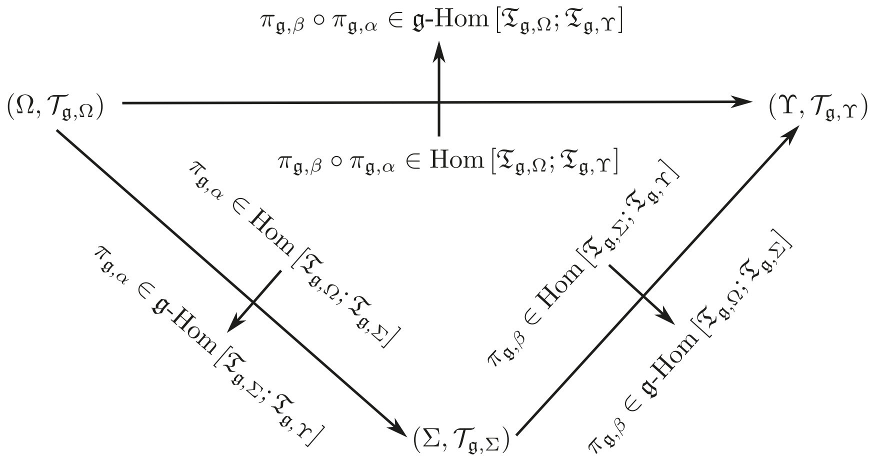

FIGURE 3. Relationships: Homeomorphism map diagram.

and $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in \mathrm{C}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, respectively, imply $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-}\mathrm{C}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta}\in\mathfrak{g}\text{-}\mathrm{C}\,[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-}\mathrm{C}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$. These features may be indicated, as in FIG. 4, by what we shall term a *continuous map diagram*.

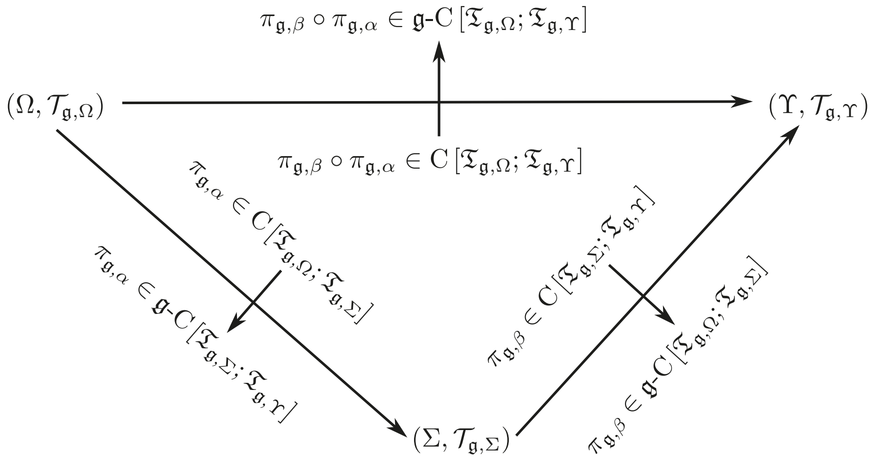

FIGURE 4. Relationships: Continuous map diagram.

Finally, suppose we are given $\pi_{\mathfrak{g},\alpha}:\mathfrak{T}_{\mathfrak{g},\Omega}\rightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ and $\pi_{\mathfrak{g},\alpha}:\mathfrak{T}_{\mathfrak{g},\Sigma}\rightarrow\mathfrak{T}_{\mathfrak{g},\Upsilon}$. If $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ and $\pi_{\mathfrak{g},\beta}\in\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$ then, by virtue of previous theorems, $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$. On the other hand, $\pi_{\mathfrak{g},\alpha}\in\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta}\in\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, respectively, imply $\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta}\in\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta}\circ\pi_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$. These features may be indicated, as in FIG. 5, by what we shall term a *irresolute map diagram*.

Let us end this discussion section with a concise summary of the principal implications of the findings regardless of categorical classifications. We have the relations $\mathfrak{g}\text{-}\mathrm{C}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]\supseteq\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ and $\mathfrak{g}\text{-}\mathrm{C}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\mathfrak{g}\text{-}\mathrm{I}\,[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]\subseteq$

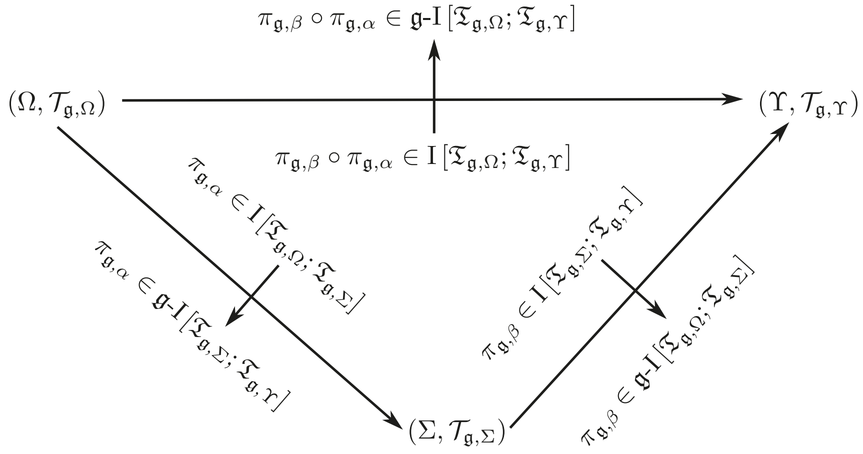

FIGURE 5. Relationships: Irresolute map diagram.

$\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$; the relation $\mathfrak{g}\text{-C}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]\supseteq\mathfrak{g}\text{-I}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$ and, also, $\mathfrak{g}\text{-C}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$, $\mathfrak{g}\text{-I}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]\subseteq\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$; $\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]\subseteq\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$. Consequently, it follows that $\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ is related with $\mathfrak{g}\text{-C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ and $\mathfrak{g}\text{-I}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$; $\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$ is related with $\mathfrak{g}\text{-C}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$ and $\mathfrak{g}\text{-I}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$; $\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]$ is related with $\mathfrak{g}\text{-M}\left[\mathfrak{T}_{\Omega};\mathfrak{T}_{\Sigma}\right]$. These relations may be indicated, as in FIG. 6, by what we shall term a *continuity-irresolute map diagram*.

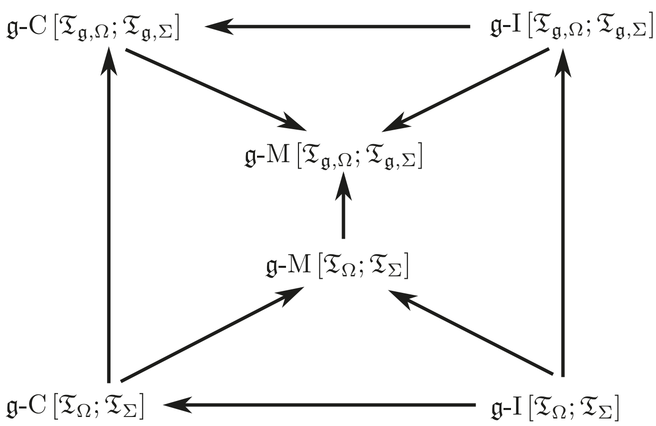

FIGURE 6. Relationships: Continuous-Irresolute map diagram.

As in the papers of [5, 10, 15, 29, 30], among others, the manner we have positioned the arrows is solely to stress that, in general, none of the implications in FIGS 1, 2 and 1 is reversible.

At this stage, a nice application is worth considering, and is presented in the following section.

3.2. A NICE APPLICATION. By focusing on important concepts from the viewpoint of the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps, we shall now present a nice application based upon five-point sets. Let $\Omega=\{\xi_{\nu}:\ \nu\in I_5^*\}$, $\Sigma=\{\zeta_{\nu}:\ \nu\in I_5^*\}$, and $\Upsilon=\{\eta_{\nu}:\ \nu\in I_5^*\}$

denote the underlying sets, and consider the $\mathcal{T}_{\mathfrak{g}}$-spaces $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathcal{T}_{\mathfrak{g},\Omega})$, $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathcal{T}_{\mathfrak{g},\Sigma})$, and $\mathfrak{T}_{\mathfrak{g},\Upsilon} = (\Upsilon, \mathcal{T}_{\mathfrak{g},\Upsilon})$, where

$$\begin{aligned} \mathcal{T}_{\mathfrak{g}}(\Omega) &= \{\emptyset, \{\xi_1\}, \{\xi_2,\xi_3\}, \{\xi_1,\xi_2,\xi_3\}\} \\ &= \{\mathcal{O}_{\mathfrak{g},\omega_1}, \mathcal{O}_{\mathfrak{g},\omega_2}, \mathcal{O}_{\mathfrak{g},\omega_3}, \mathcal{O}_{\mathfrak{g},\omega_4}\}, \\ \neg\mathcal{T}_{\mathfrak{g}}(\Omega) &= \{\Omega, \{\xi_2,\xi_3,\xi_4,\xi_5\}, \{\xi_1,\xi_4,\xi_5\}, \{\xi_4,\xi_5\}\} \\ &= \{\mathcal{K}_{\mathfrak{g},\omega_1}, \mathcal{K}_{\mathfrak{g},\omega_2}, \mathcal{K}_{\mathfrak{g},\omega_3}, \mathcal{K}_{\mathfrak{g},\omega_4}\}, \end{aligned} \tag{3.1}$$

$$\begin{aligned} \mathcal{T}_{\mathfrak{g}}(\Sigma) &= \{\emptyset, \{\zeta_2\}, \{\zeta_3,\zeta_4\}, \{\zeta_2,\zeta_3,\zeta_4\}\} \\ &= \{\mathcal{O}_{\mathfrak{g},\sigma_1}, \mathcal{O}_{\mathfrak{g},\sigma_2}, \mathcal{O}_{\mathfrak{g},\sigma_3}, \mathcal{O}_{\mathfrak{g},\sigma_4}\}, \\ \neg\mathcal{T}_{\mathfrak{g}}(\Sigma) &= \{\Sigma, \{\zeta_1,\zeta_3,\zeta_4,\zeta_5\}, \{\zeta_1,\zeta_2,\zeta_5\}, \{\zeta_1,\zeta_5\}\} \\ &= \{\mathcal{K}_{\mathfrak{g},\sigma_1}, \mathcal{K}_{\mathfrak{g},\sigma_2}, \mathcal{K}_{\mathfrak{g},\sigma_3}, \mathcal{K}_{\mathfrak{g},\sigma_4}\}, \end{aligned} \tag{3.2}$$

$$\begin{aligned} \mathcal{T}_{\mathfrak{g}}(\Upsilon) &= \{\emptyset, \{\eta_3\}, \{\eta_4,\eta_5\}, \{\eta_3,\eta_4,\eta_5\}\} \\ &= \{\mathcal{O}_{\mathfrak{g},\upsilon_1}, \mathcal{O}_{\mathfrak{g},\upsilon_2}, \mathcal{O}_{\mathfrak{g},\upsilon_3}, \mathcal{O}_{\mathfrak{g},\upsilon_4}\}, \\ \neg\mathcal{T}_{\mathfrak{g}}(\Upsilon) &= \{\Upsilon, \{\eta_1,\eta_2,\eta_4,\eta_5\}, \{\eta_1,\eta_2,\eta_3\}, \{\eta_1,\eta_2\}\} \\ &= \{\mathcal{K}_{\mathfrak{g},\upsilon_1}, \mathcal{K}_{\mathfrak{g},\upsilon_2}, \mathcal{K}_{\mathfrak{g},\upsilon_3}, \mathcal{K}_{\mathfrak{g},\upsilon_4}\}, \end{aligned} \tag{3.3}$$

respectively, stand for the classes of $\mathcal{T}_{\mathfrak{g}}$-open and $\mathcal{T}_{\mathfrak{g}}$-closed sets relative to the $\mathcal{T}_{\mathfrak{g}}$-spaces $\mathfrak{T}_{\mathfrak{g},\Omega}$, $\mathfrak{T}_{\mathfrak{g},\Sigma}$, and $\mathfrak{T}_{\mathfrak{g},\Upsilon}$. For any $\mathcal{T}_{\mathfrak{g}} \in \{\mathcal{T}_{\mathfrak{g},\Omega}, \mathcal{T}_{\mathfrak{g},\Sigma}, \mathcal{T}_{\mathfrak{g},\Upsilon}\}$, since conditions $\mathcal{T}_{\mathfrak{g}}(\emptyset) = \emptyset$, $\mathcal{T}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\nu}) \subseteq \mathcal{O}_{\mathfrak{g},\nu}$ for every $\nu \in I_4^*$, and $\mathcal{T}_{\mathfrak{g}}\big(\bigcup_{\nu\in I_4^*}\mathcal{O}_{\mathfrak{g},\nu}\big) = \bigcup_{\nu\in I_4^*}\mathcal{T}_{\mathfrak{g}}(\mathcal{O}_{\mathfrak{g},\nu})$ are satisfied, it is evident that, for every $\Lambda \in \{\Omega, \Sigma, \Upsilon\}$, the one-valued map $\mathcal{T}_{\mathfrak{g}} : \mathcal{P}(\Lambda) \to \mathcal{P}(\Lambda)$ is a $\mathfrak{g}$-topology. Furthermore, for any $\mathfrak{T} \in \{\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma}, \mathfrak{T}_{\Upsilon}\}$, it is easily checked that, $\mathcal{O}_{\mathfrak{g},\mu} \in \mathfrak{g}$-$\nu$-$\mathrm{O}[\mathfrak{T}]$ for every $(\nu,\mu) \in I_3^0 \times I_4^*$. Hence, the $\mathcal{T}_{\mathfrak{g}}$-open sets forming the $\mathfrak{g}$-topology $\mathcal{T}_{\mathfrak{g}} : \mathcal{P}(\Lambda) \to \mathcal{P}(\Lambda)$ of the $\mathcal{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Lambda, \mathcal{T}_{\mathfrak{g}})$ are $\mathfrak{g}$-$\mathfrak{T}$-open sets relative to the $\mathcal{T}$-space $\mathfrak{T} = (\Lambda, \mathcal{T})$, where $\Lambda \in \{\Omega, \Sigma, \Upsilon\}$, $\mathcal{T} \in \{\mathcal{T}_{\Omega}, \mathcal{T}_{\Sigma}, \mathcal{T}_{\Upsilon}\}$, and $\mathfrak{T} \in \{\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma}, \mathfrak{T}_{\Upsilon}\}$.

After calculations, the classes $\mathfrak{g}$-ν-$\mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Lambda}]$ and $\mathfrak{g}$-ν-$\mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Lambda}]$, respectively, of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-open and $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g},\Lambda}$-closed sets of categories $\nu \in \{0,2\}$, where $\Lambda \in \{\Omega, \Sigma, \Upsilon\}$, then take the following forms:

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Omega}] &= \mathcal{T}_{\mathfrak{g},\Omega} \cup \{\{\xi_2\}, \{\xi_3\}, \{\xi_1,\xi_2\}, \{\xi_1,\xi_3\}\}; \\ \mathfrak{g}\text{-}\nu\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Omega}] &= \neg\mathcal{T}_{\mathfrak{g},\Omega} \cup \{\{\xi_3,\xi_4,\xi_5\}, \{\xi_1,\xi_2,\xi_4,\xi_5\}, \\ &\qquad \{\xi_1,\xi_3,\xi_4,\xi_5\}, \{\xi_2,\xi_4,\xi_5\}\} \ \forall\nu \in \{0,2\}; \end{aligned} \tag{3.4}$$

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Sigma}] &= \mathcal{T}_{\mathfrak{g},\Sigma} \cup \{\{\zeta_3\}, \{\zeta_4\}, \{\zeta_2,\zeta_3\}, \{\zeta_2,\zeta_4\}\}; \\ \mathfrak{g}\text{-}\nu\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Sigma}] &= \neg\mathcal{T}_{\mathfrak{g},\Sigma} \cup \{\{\zeta_1,\zeta_4,\zeta_5\}, \{\zeta_1,\zeta_2,\zeta_3,\zeta_5\}, \\ &\qquad \{\zeta_1,\zeta_2,\zeta_4,\zeta_5\}, \{\zeta_1,\zeta_3,\zeta_5\}\} \ \forall\nu \in \{0,2\}; \end{aligned} \tag{3.5}$$

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Upsilon}] &= \mathcal{T}_{\mathfrak{g},\Upsilon} \cup \{\{\eta_4\}, \{\eta_5\}, \{\eta_3,\eta_4\}, \{\eta_3,\eta_5\}\}; \\ \mathfrak{g}\text{-}\nu\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Upsilon}] &= \neg\mathcal{T}_{\mathfrak{g},\Upsilon} \cup \{\{\eta_1,\eta_2,\eta_5\}, \{\eta_1,\eta_2,\eta_3,\eta_4\}, \\ &\qquad \{\eta_1,\eta_2,\eta_3,\eta_5\}, \{\eta_1,\eta_2,\eta_4\}\} \ \forall\nu \in \{0,2\}. \end{aligned} \tag{3.6}$$

On the other hand, those of categories $\nu \in \{1,3\}$ take the following forms:

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-O}\left[\mathfrak{T}_{\mathfrak{g},\Lambda}\right] &= \mathcal{T}_{\mathfrak{g},\Lambda} \cup \left\{\mathcal{O}_{\mathfrak{g}} : \mathcal{O}_{\mathfrak{g}} \in \mathcal{P}(\Lambda) \setminus \mathcal{T}_{\mathfrak{g},\Lambda}\right\}; \\ \mathfrak{g}\text{-}\nu\text{-K}\left[\mathfrak{T}_{\mathfrak{g},\Lambda}\right] &= \neg\mathcal{T}_{\mathfrak{g},\Lambda} \cup \left\{\mathcal{K}_{\mathfrak{g}} : \mathcal{K}_{\mathfrak{g}} \in \mathcal{P}(\Lambda) \setminus \neg\mathcal{T}_{\mathfrak{g},\Lambda}\right\} \quad \forall \nu \in \{1,3\}, \end{aligned} \tag{3.7}$$

where $\Lambda \in \{\Omega, \Sigma, \Upsilon\}$. We choose to consider the $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-map $\pi_{\mathfrak{g},\alpha} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ and the $\mathfrak{g}$-$(\mathfrak{T}_{\Sigma}, \mathfrak{T}_{\Upsilon})$-map $\pi_{\mathfrak{g},\beta} : \mathfrak{T}_{\mathfrak{g},\Sigma} \rightarrow \mathfrak{T}_{\mathfrak{g},\Upsilon}$ defined, respectively, by

$$\begin{aligned} \pi_{\mathfrak{g},\alpha}(\xi_1) &= \zeta_2,\ \pi_{\mathfrak{g},\alpha}(\xi_2) = \zeta_3,\ \pi_{\mathfrak{g},\alpha}(\xi_3) = \zeta_4,\ \pi_{\mathfrak{g},\alpha}(\xi_4) = \zeta_1,\ \pi_{\mathfrak{g},\alpha}(\xi_5) = \zeta_5; \\ \pi_{\mathfrak{g},\beta}(\zeta_1) &= \eta_1,\ \pi_{\mathfrak{g},\beta}(\zeta_2) = \eta_3,\ \pi_{\mathfrak{g},\beta}(\zeta_3) = \eta_4,\ \pi_{\mathfrak{g},\beta}(\zeta_4) = \eta_5,\ \pi_{\mathfrak{g},\beta}(\zeta_5) = \eta_2. \end{aligned}$$

Finally, we set the relations $\pi_{\mathfrak{g},\alpha}(\mathcal{O}_{\mathfrak{g},\omega_1}) = \mathcal{O}_{\mathfrak{g},\sigma_1}$ and $\pi_{\mathfrak{g},\beta}(\mathcal{O}_{\mathfrak{g},\sigma_1}) = \mathcal{O}_{\mathfrak{g},\upsilon_1}$ so that $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\mathcal{O}_{\mathfrak{g},\omega_1}) = \mathcal{O}_{\mathfrak{g},\upsilon_1}$. As for the composite $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Upsilon})$-map $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} : \mathfrak{T}_{\mathfrak{g},\Omega} \rightarrow \mathfrak{T}_{\mathfrak{g},\Upsilon}$, a simple calculation shows that

$$\begin{aligned} \pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\xi_1) &= \eta_3,\ \pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\xi_2) = \eta_4,\ \pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\xi_3) = \eta_5, \\ \pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\xi_4) &= \eta_1,\ \pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha}(\xi_5) = \eta_2. \end{aligned}$$

At this stage, we have all the basic ingredients to discuss any class of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps between any two of such $\mathcal{T}_{\mathfrak{g}}$-spaces $\mathfrak{T}_{\mathfrak{g},\Omega}$, $\mathfrak{T}_{\mathfrak{g},\Sigma}$, and $\mathfrak{T}_{\mathfrak{g},\Upsilon}$. We choose to discuss some elements of the classes $\mathfrak{g}$-M $[\mathfrak{T}_{\mathfrak{g},\Lambda}; \mathfrak{T}_{\mathfrak{g},\Theta}]$, $\mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Lambda}; \mathfrak{T}_{\mathfrak{g},\Theta}]$, $\mathfrak{g}$-I $[\mathfrak{T}_{\mathfrak{g},\Lambda}; \mathfrak{T}_{\mathfrak{g},\Theta}]$, and $\mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Lambda}; \mathfrak{T}_{\mathfrak{g},\Theta}]$ of $(\mathfrak{T}_{\mathfrak{g},\Lambda}, \mathfrak{T}_{\mathfrak{g},\Theta})$-maps, $(\mathfrak{T}_{\mathfrak{g},\Lambda}, \mathfrak{T}_{\mathfrak{g},\Theta})$-continuous, $(\mathfrak{T}_{\mathfrak{g},\Lambda}, \mathfrak{T}_{\mathfrak{g},\Theta})$-irresolute, and $(\mathfrak{T}_{\mathfrak{g},\Lambda}, \mathfrak{T}_{\mathfrak{g},\Theta})$-homeomorphism maps, respectively, where Λ, $\Theta \in \{\Omega, \Sigma, \Upsilon\}$. A first sequence of calculations shows that

$$\begin{aligned} \pi_{\mathfrak{g},\alpha}\left(\mathcal{O}_{\mathfrak{g},\omega_\mu}\right) &= \mathcal{O}_{\mathfrak{g},\sigma_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{O}_{\mathfrak{g},\sigma_\mu}\right), \\ \pi_{\mathfrak{g},\alpha}\left(\mathcal{K}_{\mathfrak{g},\omega_\mu}\right) &= \mathcal{K}_{\mathfrak{g},\sigma_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{K}_{\mathfrak{g},\sigma_\mu}\right) \quad \forall (\nu,\mu) \in I_3^0 \times I_4^*; \\ \pi_{\mathfrak{g},\beta}\left(\mathcal{O}_{\mathfrak{g},\sigma_\mu}\right) &= \mathcal{O}_{\mathfrak{g},\upsilon_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{O}_{\mathfrak{g},\upsilon_\mu}\right), \\ \pi_{\mathfrak{g},\beta}\left(\mathcal{K}_{\mathfrak{g},\sigma_\mu}\right) &= \mathcal{K}_{\mathfrak{g},\upsilon_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{K}_{\mathfrak{g},\upsilon_\mu}\right) \quad \forall (\nu,\mu) \in I_3^0 \times I_4^*. \end{aligned}$$

Hence, we conclude that, $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-M $[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-M $[\mathfrak{T}_{\mathfrak{g},\Sigma}; \mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-M $[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Upsilon}]$. On the other hand, a second sequence of calculations shows that

$$\begin{aligned} \pi_{\mathfrak{g},\alpha}^{-1}\left(\mathcal{O}_{\mathfrak{g},\sigma_\mu}\right) &= \mathcal{O}_{\mathfrak{g},\omega_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{O}_{\mathfrak{g},\omega_\mu}\right), \\ \pi_{\mathfrak{g},\alpha}^{-1}\left(\mathcal{K}_{\mathfrak{g},\sigma_\mu}\right) &= \mathcal{K}_{\mathfrak{g},\omega_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{K}_{\mathfrak{g},\omega_\mu}\right) \quad \forall (\nu,\mu) \in I_3^0 \times I_4^*; \\ \pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{O}_{\mathfrak{g},\upsilon_\mu}\right) &= \mathcal{O}_{\mathfrak{g},\sigma_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{O}_{\mathfrak{g},\sigma_\mu}\right), \\ \pi_{\mathfrak{g},\beta}^{-1}\left(\mathcal{K}_{\mathfrak{g},\upsilon_\mu}\right) &= \mathcal{K}_{\mathfrak{g},\sigma_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathcal{K}_{\mathfrak{g},\sigma_\mu}\right) \quad \forall (\nu,\mu) \in I_3^0 \times I_4^*. \end{aligned}$$

From the above expressions, it then follows that, $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Sigma}; \mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Upsilon}]$. A third sequence of calculations shows that

$$\begin{aligned}
\pi_{\mathfrak{g},\alpha}^{-1}\left(\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma_\mu}\right)\right) &= \mathscr{O}_{\mathfrak{g},\omega_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\omega_\mu}\right), \\
\pi_{\mathfrak{g},\alpha}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma_\mu}\right)\right) &= \mathscr{K}_{\mathfrak{g},\omega_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\omega_\mu}\right) \quad \forall\,(\nu,\mu) \in \{0,2\} \times I_4^*; \\
\pi_{\mathfrak{g},\alpha}^{-1}\left(\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma_{5-\mu}}\right)\right) &= \mathscr{K}_{\mathfrak{g},\omega_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\omega_\mu}\right), \\
\pi_{\mathfrak{g},\alpha}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma_{5-\mu}}\right)\right) &= \mathscr{O}_{\mathfrak{g},\omega_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\omega_\mu}\right) \quad \forall\,(\nu,\mu) \in \{1,3\} \times I_4^*; \\
\pi_{\mathfrak{g},\beta}^{-1}\left(\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\upsilon_\mu}\right)\right) &= \mathscr{O}_{\mathfrak{g},\sigma_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma_\mu}\right), \\
\pi_{\mathfrak{g},\beta}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\upsilon_\mu}\right)\right) &= \mathscr{K}_{\mathfrak{g},\sigma_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma_\mu}\right) \quad \forall\,(\nu,\mu) \in \{0,2\} \times I_4^*; \\
\pi_{\mathfrak{g},\beta}^{-1}\left(\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\upsilon_{5-\mu}}\right)\right) &= \mathscr{K}_{\mathfrak{g},\sigma_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma_\mu}\right), \\
\pi_{\mathfrak{g},\beta}^{-1}\left(\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\upsilon_{5-\mu}}\right)\right) &= \mathscr{O}_{\mathfrak{g},\sigma_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma_\mu}\right) \quad \forall\,(\nu,\mu) \in \{1,3\} \times I_4^*.
\end{aligned}$$

From the properties of $\mathfrak{g}$-ν-I $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\mathfrak{g}$-ν-I $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\mathfrak{g}$-ν-I $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, where $\nu \in I_3^0$, we have $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-$\nu$-I $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-$\nu$-I $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-$\nu$-I $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$ only for every $\nu \in \{0,2\}$; none of these membership relations holds for any $\nu \in \{1,3\}$, as is easily seen by inspection. On the other hand, by virtue of the definitions of the $\mathfrak{g}$-$(\mathfrak{T}_\Omega,\mathfrak{T}_\Sigma)$-map $\pi_{\mathfrak{g},\alpha} : \mathfrak{T}_{\mathfrak{g},\Omega} \to \mathfrak{T}_{\mathfrak{g},\Sigma}$ and the $\mathfrak{g}$-$(\mathfrak{T}_\Sigma,\mathfrak{T}_\Upsilon)$-map $\pi_{\mathfrak{g},\beta} : \mathfrak{T}_{\mathfrak{g},\Sigma} \to \mathfrak{T}_{\mathfrak{g},\Upsilon}$, it is clear that the membership relations $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-B $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-B $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-B $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$ hold.

Having discussed $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, to discuss the $\mathfrak{g}$-$(\mathfrak{T}_\Omega,\mathfrak{T}_\Sigma)$-homeomorphism map $\pi_{\mathfrak{g},\alpha} : \mathfrak{T}_{\mathfrak{g},\Omega} \cong \mathfrak{T}_{\mathfrak{g},\Sigma}$ and the $\mathfrak{g}$-$(\mathfrak{T}_\Sigma,\mathfrak{T}_\Upsilon)$-homeomorphism map $\pi_{\mathfrak{g},\beta} : \mathfrak{T}_{\mathfrak{g},\Sigma} \cong \mathfrak{T}_{\mathfrak{g},\Upsilon}$ we must first discuss the relations $\pi_{\mathfrak{g},\alpha}^{-1} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}]$ and $\pi_{\mathfrak{g},\beta}^{-1} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Upsilon};\mathfrak{T}_{\mathfrak{g},\Sigma}]$. A fourth sequence of calculations shows that

$$\begin{aligned}
\left(\pi_{\mathfrak{g},\alpha}^{-1}\right)^{-1}\left(\mathscr{O}_{\mathfrak{g},\omega_\mu}\right) &= \mathscr{O}_{\mathfrak{g},\sigma_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma_\mu}\right), \\
\left(\pi_{\mathfrak{g},\alpha}^{-1}\right)^{-1}\left(\mathscr{K}_{\mathfrak{g},\omega_\mu}\right) &= \mathscr{K}_{\mathfrak{g},\sigma_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma_\mu}\right) \quad \forall\,(\nu,\mu) \in I_3^0 \times I_4^*; \\
\left(\pi_{\mathfrak{g},\beta}^{-1}\right)^{-1}\left(\mathscr{O}_{\mathfrak{g},\sigma_\mu}\right) &= \mathscr{O}_{\mathfrak{g},\upsilon_\mu} \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\upsilon_\mu}\right), \\
\left(\pi_{\mathfrak{g},\beta}^{-1}\right)^{-1}\left(\mathscr{K}_{\mathfrak{g},\sigma_\mu}\right) &= \mathscr{K}_{\mathfrak{g},\upsilon_\mu} \supseteq \neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\upsilon_\mu}\right) \quad \forall\,(\nu,\mu) \in I_3^0 \times I_4^*.
\end{aligned}$$

From these, it clearly follows that the relations $\pi_{\mathfrak{g},\alpha}^{-1} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Omega}]$, $\pi_{\mathfrak{g},\beta}^{-1} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Upsilon};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, and $(\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha})^{-1} \in \mathfrak{g}$-C $[\mathfrak{T}_{\mathfrak{g},\Upsilon};\mathfrak{T}_{\mathfrak{g},\Omega}]$ hold. Hence, it follows that $\pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\pi_{\mathfrak{g},\beta} \in \mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Sigma};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$, and, also, $\pi_{\mathfrak{g},\beta} \circ \pi_{\mathfrak{g},\alpha} \in \mathfrak{g}$-Hom $[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Upsilon}]$.

The discussions carried out in the preceding sections can be easily verified from this nice application. The next section provides concluding remarks and future directions of the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-sets discussed in the preceding sections.

3.3. Concluding Remarks. In this chapter, we developed a new theory, called *Theory of* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-*Maps* that is founded upon the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-sets. In its own rights, the proposed theory has several advantages. The very first advantage is that the theory holds equally well when $(\Lambda, \mathcal{T}_{\mathfrak{g},\Lambda}) = (\Lambda, \mathcal{T}_\Lambda)$, where $\Lambda \in \{\Omega, \Sigma, \Upsilon\}$, and other features adapted on this ground, in which case it might be called *Theory of* $\mathfrak{g}$-$\mathfrak{T}$-*Maps*.

Hence, between any two such $\mathcal{T}_{\mathfrak{g}}$-spaces $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathcal{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathcal{T}_{\mathfrak{g},\Sigma})$ the theoretical framework categorises such pairs of concepts as $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-open and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-closed maps, $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-semi-open and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-semi-closed maps, $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-preopen, $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-preclosed maps, and, finally, $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-semi-preopen and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-semi-preclosed maps as $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps of categories 0, 1, 2, and 3, respectively, and theorises the concepts in a unified way; between any two such $\mathcal{T}$-spaces $\mathfrak{T}_{\Omega} = (\Omega, \mathcal{T}_{\Omega})$ and $\mathfrak{T}_{\Sigma} = (\Sigma, \mathcal{T}_{\Sigma})$ the theoretical framework categorises such pairs of concepts as $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-open and $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-closed maps, $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-semi-open and $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-semi-closed maps, $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-preopen, $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-preclosed maps, and $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-semi-preopen and $\mathfrak{g}$-$(\mathfrak{T}_{\Omega}, \mathfrak{T}_{\Sigma})$-semi-preclosed maps as $\mathfrak{g}$-$\mathfrak{T}$-maps of categories 0, 1, 2, and 3, respectively, and theorises the concepts in a unified way.

It is an interesting topic for future research to develop the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-maps of mixed categories. More precisely, for some pair $(\nu, \mu) \in I_3^0 \times I_3^0$ such that $\nu \neq \mu$, to develop the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open maps based on the elements of the class $\{\mathcal{O}_{\mathfrak{g}} = \mathcal{O}_{\mathfrak{g},\nu} \cup \mathcal{O}_{\mathfrak{g},\mu} : (\mathcal{O}_{\mathfrak{g},\nu}, \mathcal{O}_{\mathfrak{g},\mu}) \in \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-}\mu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]\}$ and the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-closed maps based on the elements of the class $\{\mathcal{K}_{\mathfrak{g}} = \mathcal{K}_{\mathfrak{g},\nu} \cup \mathcal{K}_{\mathfrak{g},\mu} : (\mathcal{K}_{\mathfrak{g},\nu}, \mathcal{K}_{\mathfrak{g},\mu}) \in \mathfrak{g}\text{-}\nu\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-}\mu\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g}}]\}$, as [25] developed the theory of weakly b-open functions. Such two theories are what we thought would certainly be worth considering, and the discussion of this paper ends here.

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