On the multiplicative degree-based topological indices of silicon-carbon $Si_2C_3\text{-}I[p,q]$ and $Si_2C_3\text{-}II[p,q]$

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Abstract: The application of graph theory in chemical and molecular structure research far exceeds people's expectations, and it has recently grown exponentially. In the molecular graph, atoms are represented by vertices and bonded by edges. Closed forms of multiplicative degree-based topological indices which are numerical parameters of the structure and determine physico-chemical properties of the concerned molecular compound. In this article, we compute and analyze many multiplicative degree-based topological indices of silicon-carbon $Si_2C_3\text{-}I[p,q]$ and $Si_2C_3\text{-}II[p,q]$.

Keywords: molecular graph, degree-based index, silicon-carbon.

Introduction

Mathematical chemistry provides tools such as polynomials and functions to capture information hidden in the symmetry of molecular graphs and thus predict properties of compounds without using quantum mechanics. A topological index isa numerical parameter of a graph and depicts its topology. It describes the structure of molecules numerically and are used in the development of qualitative structure activity relationships (QSARs). Most commonly known invariants of such kinds are degree-based topological indices. These are actually the numerical values that correlate the structure with various physical properties, chemical reactivity and biological activities [1-5]. It is an established fact that many properties such as heat of formation, boiling point, strain energy, rigidity and fracture toughness of a molecule are strongly connected to its graphical structure.

Hosoya polynomial, (Wiener polynomial), [6] plays a pivotal role in distance-based topological indices. A long list of distance-based indices can be easily evaluated from

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Hosoya polynomial. A similar breakthrough was obtained recently by Klavzar *et. al.* [7], in the context of degree-based indices. Authors in [7] introduced M-polynomial in, 2015, to play a role, parallel to Hosoya polynomial to determine closed form of many degree-based topological indices [8-12]. The real power of M-polynomial is its comprehensive nature containing healthy information about degree-based graph invariants. These invariants are calculated on the basis of symmetries present in the 2d-molecular lattices and collectively determine some properties of the material under observation.

Figure 1 and Figure 2 show two-dimensional molecular diagrams of silicon carbide $Si_2C_3\text{-}I$. In order to describe its molecular graph, we have set this way: we define p as the number of connected units in a row (chain), and q as the number of connected rows, the number of p cells per connection. In Figures 3 and 4, we demonstrate how cells are connected in one row (chain) and how one row is connected to another row. We will use $Si_2C_3\text{-}I[p,q]$ to represent this molecular graph.

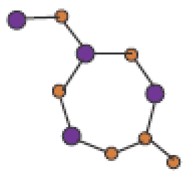

Figure 1. Unit Cell of $Si_2C_3\text{-}I[p,q]$

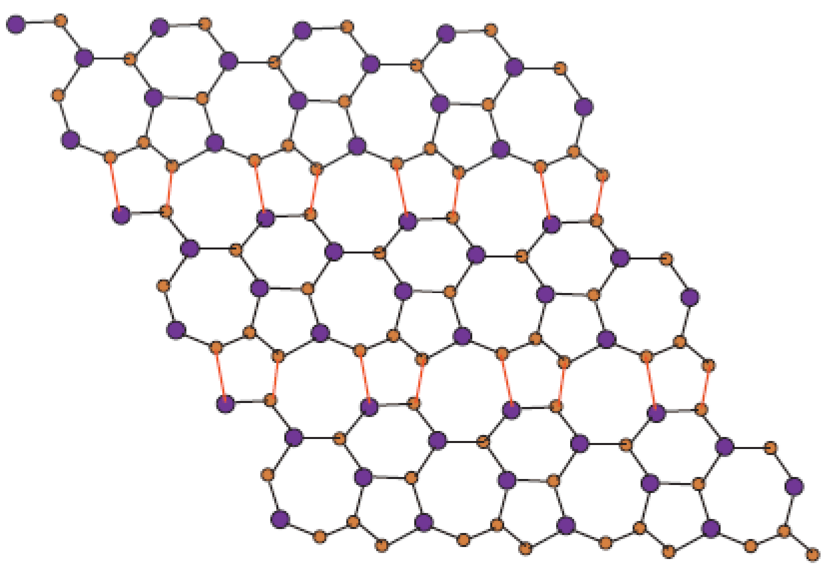

Figure 2. Sheet of $Si_2C_3\text{-}I[p,q]$ for p=4 and q=3

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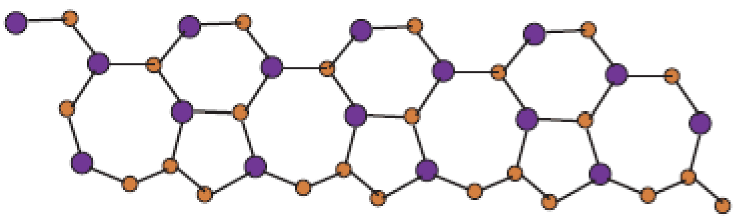

Figure 3. Sheet of $Si_2C_3\text{-}I[p,q]$ for p=4 and q=1

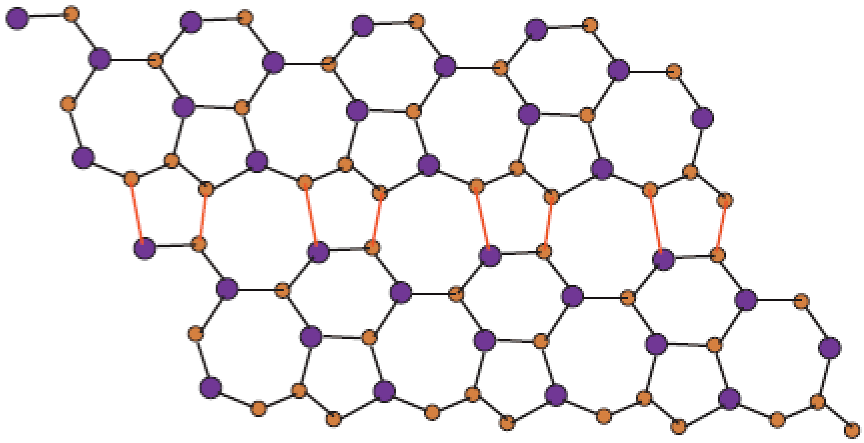

Figure 4. Sheet of $Si_2C_3\text{-}I[p,q]$ for p=4 and q=2

In figures 1-4 Carbon atom C are shown as brown and Silicon atom Si are shown as blue. Figure 5-8 shows a two-dimensional molecular map of silicon carbide $Si_2C_3\text{-}II$. In order to describe its molecular graph, we set this way [13]: we define p as the number of unit cells connected in a row (chain), and with q, we represent the number of connected rows, the number of rows per connection It is p units. In Figures 7 and 8, we demonstrate how cells are connected in a row (chain) and how a row is connected to another row. We will use $Si_2C_3\text{-}II[p,q]$ to represent this molecular graph.

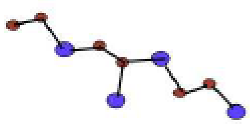

Figure 5. One Unit of $Si_2C_3\text{-}II[p,q]$

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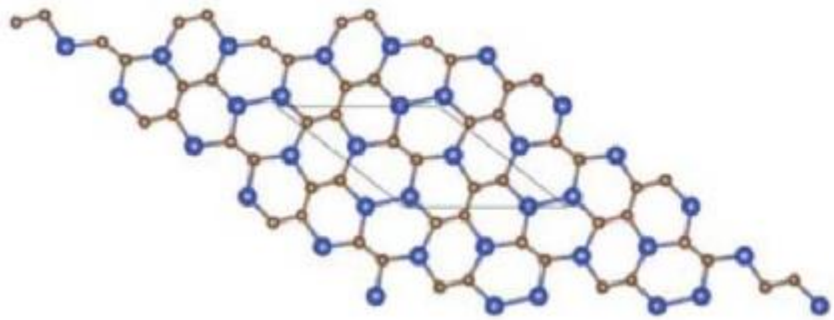

Figure 6. Sheet of $Si_2C_3\text{-}II[p,q]$ for p=3 and q=3

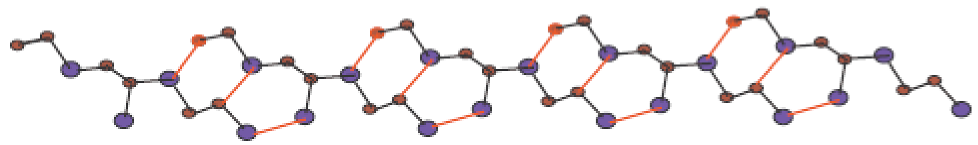

Figure 7. Sheet of $Si_2C_3\text{-}II[p,q]$ for p=5 and q=1

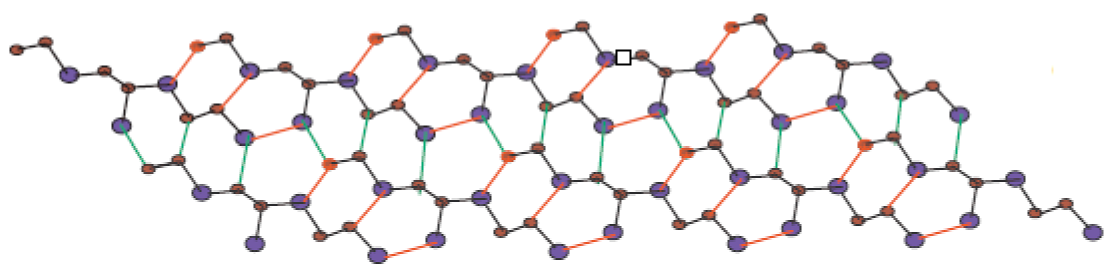

Figure 8. Sheet of $Si_2C_3\text{-}II[p,q]$ for p=5 and q=2

In this paper we closed form of many multiplicative degree based topological indices of silicon-carbon $Si_2C_3\text{-}I[p,q]$ and $Si_2C_3\text{-}II[p,q]$. For the application of these indices we refer [14-16] and the references therein.

Definitions and Literature Review

A molecular graph is a simple graph in chemical graph theory, in which atoms are represented by vertices and chemical bonds are represented by edges. A graph is connected if there is a connection between any pair of vertices. A network is a connected graph which has no multiple edge and loop. The number of vertices which are connected to a fixed v vertex is called the degree of v and is denoted by d_v. The distance between two vertices is the length of shortest path between them. The concept of valence in chemistry and concept of degree is somewhat close. For details on bases of

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graph theory, we refer to the book [17]. Quantitative structure-activity and Structure-property relationships predict the properties and biological activities of unstudied material. In these studies, topological indices and some Physico-chemical properties are used to predict bioactivity of the chemical compounds [18-21].

Throughout this paper, G denotes connected graph, V and E denote the vertex set and the edge set and d_v denotes the degree of a vertex. A topological index of the graph of a chemical compound is a number, which can be used to characterize the underlined chemical compound and help to predict its physiochemical properties. Weiner laid the foundation of Topological index in 1947. He was approximated the boiling point of alkanes and introduced the Weiner index [22]. Till now more than 140 topological indices are defined but no single index is enough to determine all physico-chemical properties, but, these topological indices together can do this to some extent. Later, in 1975, Milan Randić introduced Randić index, [23]. In 1998, Bollobas and Erdos [24] and Amic *et al*. [25] proposed the generalized Randić index and has been studied by both chemists and mathematicians [26]. The Randić index is one of the most popular and most studied and applied topological index. Many reviews, papers and books [27-32] are written on this simple graph invariant. Some indices related to Wiener's work are the first and second multiplicative Zagreb indices [33], respectively

$$II_1(G) = \prod_{u\in V(G)} (d_u)^2$$

$$II_2(G) = \prod_{uv\in E(G)} d_u \cdot d_v$$

and the Narumi-Katayama index [34]

$$NK(G) = \prod_{u\in V(G)} d_u$$

Like the Wiener index, these types of indices are the focus of considerable research in computational chemistry [35--38]. For example, in 2011 I. Gutman [35] characterized the multiplicative Zagreb indices for trees and determined the unique trees that obtained maximum and minimum values for M1(G) and M2(G), respectively. S. Wang and the last author [38] then extended Gutman's result to the following index for k-trees,

$$W_1^s(G) = \prod_{u\in V(G)} (d_u)^s .$$

Notice that s = 1, 2 is the Narumi-Katayama and Zagreb index, respectively. Based on the successful consideration of multiplicative Zagreb indices, M. Eliasi et al [39] continued to define a new multiplicative version of the first Zagreb index as

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$$II_1^*(G)=\prod_{uv\in E(G)}(d_u+d_v).$$

Furthering the concept of indexing with the edge set, the first author introduced the first and second hyper-Zagreb indices of a graph [40]. They are defined as

$$HII_1(G)=\prod_{uv\in E(G)}(d_u+d_v)^2$$

$$HII_2(G)=\prod_{uv\in E(G)}(d_u\cdot d_v)^2$$

In [41] Kulli et al. defined the first and second generalized Zagreb indices

$$MZ_1^a(G)=\prod_{uv\in E(G)}(d_u+d_v)^a$$

$$MZ_2^a(G)=\prod_{uv\in E(G)}(d_u\cdot d_v)^a$$

Multiplicative sum connectivity and multiplicative product connectivity indices [42] are define as:

$$SCII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_u+d_v}}$$

$$PCII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_u\cdot d_v}}$$

Multiplicative atomic bond connectivity index and multiplicative Geometric arithmetic index are defined as

$$ABCII(G)=\prod_{uv\in E(G)}\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}$$

$$GAII(G)=\prod_{uv\in E(G)}\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}$$

$$GA^aII(G)=\prod_{uv\in E(G)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^a$$

Computational Results

In this section we give our main results.

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Theorem 1: Let $Si_2C_3\text{-}I[p,q]$ be the Silicon Carbide. Then

1. $II_2\left(Si_2C_3\text{-}I\right)=(2)^{4(2p+3q-2)}\times(3)^{6(5pq+2p-3q+1)}$.
2. $II_1^*\left(Si_2C_3\text{-}I\right)=(2)^{15pq-7p-9q+9}\times(3)^{15pq-9p-13q+8}\times(5)^{6p+8q-9}$.
3. $HII_1\left(Si_2C_3\text{-}I\right)=(3)^{30pq-18p-26q+16}\times(2)^{30pq-14p-18q+18}\times(5)^{12p+16q-18}$.
4. $HII_2\left(Si_2C_3\text{-}I\right)=(2)^{8(2p+3q-2)}\times(3)^{12(5pq-2p-3q+6)}$.
5. $MZ_1^a\left(Si_2C_3\text{-}I\right)=(2)^{\alpha(15pq-7p-9q+9)}\times(3)^{\alpha(15pq-9p-13q+8)}\times(5)^{\alpha(6p+8q-9)}$.
6. $MZ_2^a\left(Si_2C_3\text{-}I\right)=(2)^{4\alpha(2p+3q-2)}\times(3)^{6\alpha(5pq+2p-3q+1)}$.
7. $XII\left(Si_2C_3\text{-}I\right)=\left(\frac{1}{2}\right)^{1+p+2q}\times\left(\frac{1}{\sqrt{3}}\right)\times\left(\frac{1}{\sqrt{5}}\right)^{6p+8q-9}\times\left(\frac{1}{\sqrt{6}}\right)^{15pq-9p-13q+7}$.
8. $\chi II\left(Si_2C_3\text{-}I\right)=\left(\frac{1}{2}\right)^{2(2p+3q-2)}\times\left(\frac{1}{3}\right)^{3(5pq-2p-3q+1)}$.
9. $ABCII\left(Si_2C_3\text{-}I\right)=\left[\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]^{16p+23q-15pq-16}\times\left[\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]^{15pq-9p-13q+8}$.
10. $GAII\left(Si_2C_3\text{-}I\right)=(2)^{9p+12q-13}\times(3)^{3p+4q-5}\times(5)^{9-6p-8q}$.
11. $G^{\alpha}AII(Si_2C_3\text{-}I)=(2)^{\alpha(9p+12q-13)}\times(3)^{\alpha(3p+4q-5)}\times(5)^{\alpha(9-6p-8q)}$.

Proof: Let G be the graph of $Si_2C_3\text{-}I[p,q]$ where we define p as the number of connected unit cells in a row (chain) and by q we represents the number of connected rows each with p number of cells. From the graph of $Si_2C_3\text{-}I[p,q]$ (figure 1-4) we can see that the total number of vertices are $10pq$ and total number of edges are $15pq-2p-3q$.

The edge set of $Si_2C_3\text{-}I[p,q]$ with $p,q\geq 1$ has following five partitions,

$$E_{\{1,2\}}=\left\{e=uv\in E\left(Si_2C_3\text{-}I[p,q]\right)|d_u=1,d_v=2\right\},$$
$$E_{\{1,3\}}=\left\{e=uv\in E\left(Si_2C_3\text{-}I[p,q]\right)|d_u=1,d_v=3\right\},$$
$$E_{\{2,2\}}=\left\{e=uv\in E\left(Si_2C_3\text{-}I[p,q]\right)|d_u=2,d_v=2\right\},$$
$$E_{\{2,3\}}=\left\{e=uv\in E\left(Si_2C_3\text{-}I[p,q]\right)|d_u=2,d_v=3\right\}.$$

And

$$E_{\{3,3\}}=\left\{e=uv\in E\left(Si_2C_3\text{-}I[p,q]\right)|d_u=3,d_v=3\right\}.$$

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Now

$$\left|E_{\{1,2\}}\right| = 1,$$
$$\left|E_{\{1,3\}}\right| = 1,$$
$$\left|E_{\{2,2\}}\right| = p + 2q,$$
$$\left|E_{\{2,3\}}\right| = 6p - 1 + 8(q-1),$$

And

$$\left|E_{\{3,3\}}\right| = 15pq - 9p - 13q + 7.$$

1. $$\begin{aligned}
II_2\left(Si_2C_{3^-}I\right) &= \prod_{uv\in E\left(Si_2C_{3^-}I\right)} d_u \cdot d_v \\
&= \prod_{u\in E_1\left(Si_2C_{3^-}I\right)} \left(d_u \cdot d_v\right) \times \prod_{u\in E_2\left(Si_2C_{3^-}I\right)} \left(d_u \cdot d_v\right) \times \prod_{u\in E_3\left(Si_2C_{3^-}I\right)} \left(d_u \cdot d_v\right) \\
&\times \prod_{u\in E_4\left(Si_2C_{3^-}I\right)} \left(d_u \cdot d_v\right) \times \prod_{u\in E_5\left(Si_2C_{3^-}I\right)} \left(d_u \cdot d_v\right) \\
&= \left(d_u \cdot d_v\right)^{\left|E_1\left(Si_2C_{3^-}I\right)\right|} \times \left(d_u \cdot d_v\right)^{\left|E_2\left(Si_2C_{3^-}I\right)\right|} \times \left(d_u \cdot d_v\right)^{\left|E_3\left(Si_2C_{3^-}I\right)\right|} \\
&\times \left(d_u \cdot d_v\right)^{\left|E_4\left(Si_2C_{3^-}I\right)\right|} \times \left(d_u \cdot d_v\right)^{\left|E_5\left(Si_2C_{3^-}I\right)\right|} \\
&= (1\times 2)\times(1\times 3)\times(2\times 2)^{p+2q} \times (2\times 3)^{6p+8q-9} \times (3\times 3)^{15pq-9p-13q+7} \\
&= (2)\times(3)\times(2)^{2p+4q} \times (2)^{6p+8q-9} \times (3)^{6p+8q-9} \times (3)^{30pq-18p-26q+14} \\
&= (2)^{1+2p+6p+4q+8q-9} \times (3)^{1+6p+8q-9+30pq-18p-26q+14} \\
&= (2)^{8p+12q-8} \times (3)^{30pq-12p-18q+6} \\
&= (2)^{4(2p+3q-2)} \times (3)^{6(5pq+2p-3q+1)}.
\end{aligned}$$

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$$2.\ II_1^*\left(Si_2C_{3-}I\right)=\prod_{uv\in E\left(Si_2C_{3-}I\right)}\left(d_u+d_v\right)$$

$$=\left(d_u+d_v\right)^{\left|E_1\left(Si_2C_{3-}I\right)\right|}\times\left(d_u+d_v\right)^{\left|E_2\left(Si_2C_{3-}I\right)\right|}\times\left(d_u+d_v\right)^{\left|E_3\left(Si_2C_{3-}I\right)\right|}$$

$$\times\left(d_u+d_v\right)^{\left|E_4\left(Si_2C_{3-}I\right)\right|}\times\left(d_u+d_v\right)^{\left|E_5\left(Si_2C_{3-}I\right)\right|}$$

$$=\left(1+2\right)\times\left(1+3\right)\times\left(2+2\right)^{p+2q}\times\left(2+3\right)^{6p+8q-9}\times\left(3+3\right)^{15pq-9p-13q+7}$$

$$=\left(3\right)\times\left(4\right)\times\left(2\right)^{2p+4q}\times\left(5\right)^{6p+8q-9}\times\left(6\right)^{15pq-9p-13q+7}$$

$$=\left(2\right)^{15pq-7p-9q+9}\times\left(3\right)^{15pq-9p-13q+8}\times\left(5\right)^{6p+8q-9}$$

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$$3.\ HII_1\left(Si_2C_{3^-}I\right)=\prod_{uv\in E\left(Si_2C_{3^-}I\right)}\left(d_u+d_v\right)^2$$

$$=\prod_{u\in E_1\left(Si_2C_{3^-}I\right)}\left(d_u+d_v\right)^2\times\prod_{u\in E_2\left(Si_2C_{3^-}I\right)}\left(d_u+d_v\right)^2\times\prod_{u\in E_3\left(Si_2C_{3^-}I\right)}\left(d_u+d_v\right)^2$$

$$\times\prod_{u\in E_4\left(Si_2C_{3^-}I\right)}\left(d_u+d_v\right)^2\times\prod_{u\in E_5\left(Si_2C_{3^-}I\right)}\left(d_u+d_v\right)^2$$

$$=\left[\left(d_u+d_v\right)^2\right]^{\left|E_1\left(Si_2C_{3^-}I\right)\right|}\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_2\left(Si_2C_{3^-}I\right)\right|}\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_3\left(Si_2C_{3^-}I\right)\right|}$$

$$\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_4\left(Si_2C_{3^-}I\right)\right|}\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_4\left(Si_2C_{3^-}I\right)\right|}$$

$$=\left[(1+2)^2\right]\times\left[(1+3)^2\right]\times\left[(2+2)^2\right]^{p+2q}\times\left[(2+3)^2\right]^{6p+8q-9}\times\left[(3+3)^2\right]^{15pq-9p-13q+7}$$

$$=(3)^2\times(2)^4\times(2)^{4p+8q}\times(5)^{12p+16q-18}\times(6)^{30pq-18p-26q+14}$$

$$=(3)^{2+30pq-18p-26q+14}\times(2)^{4+4p+8q+30pq-18p-26q+14}\times(5)^{12p+16q-18}$$

$$=(3)^{30pq-18p-26q+16}\times(2)^{30pq-14p-18q+18}\times(5)^{12p+16q-18}.$$

$$4.\ HII_2\left(Si_2C_{3^-}I\right)=\prod_{uv\in E\left(Si_2C_{3^-}I\right)}\left(d_u\cdot d_v\right)^2$$

$$=\prod_{u\in E_1\left(Si_2C_{3^-}I\right)}\left(d_u\cdot d_v\right)^2\times\prod_{u\in E_2\left(Si_2C_{3^-}I\right)}\left(d_u\cdot d_v\right)^2\times\prod_{u\in E_3\left(Si_2C_{3^-}I\right)}\left(d_u\cdot d_v\right)^2$$

$$\times\prod_{u\in E_4\left(Si_2C_{3^-}I\right)}\left(d_u\cdot d_v\right)^2\times\prod_{u\in E_5\left(Si_2C_{3^-}I\right)}\left(d_u\cdot d_v\right)^2$$

$$=\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_1\left(Si_2C_{3^-}I\right)\right|}\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_2\left(Si_2C_{3^-}I\right)\right|}\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_3\left(Si_2C_{3^-}I\right)\right|}$$

$$\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_4\left(Si_2C_{3^-}I\right)\right|}\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_5\left(Si_2C_{3^-}I\right)\right|}$$

$$=\left[(1\times2)^2\right]\times\left[(1\times3)^2\right]\times\left[(2\times2)^2\right]^{p+2q}\times\left[(2\times3)^2\right]^{6p+8q-9}\times\left[(3\times3)^2\right]^{15pq-9p-13q+7}$$

$$=(2)^2\times(3)^2\times(2)^{4p+8q}\times(6)^{12p+16q-18}\times(3)^{60pq-36p-52q+28}$$

$$=(2)^{2+4p+8q+12p+16q-18}\times(3)^{2+12p+16q-18+60pq-36p-52q+28}$$

$$=(2)^{16p+24q-16}\times(3)^{60pq-24p-36q+12}$$

$$=(2)^{8(2p+3q-2)}\times(3)^{12(5pq-2p-3q+6)}.$$

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$$5.\ II_1^*\left(Si_2C_3\text{-}I\right)=\prod_{uv\in E\left(Si_2C_3\text{-}I\right)}\left(d_u+d_v\right)^{\alpha}$$

$$=\prod_{u\in E_1\left(Si_2C_3\text{-}I\right)}\left(d_u+d_v\right)^{\alpha}\times\prod_{u\in E_2\left(Si_2C_3\text{-}I\right)}\left(d_u+d_v\right)^{\alpha}\times\prod_{u\in E_3\left(Si_2C_3\text{-}I\right)}\left(d_u+d_v\right)^{\alpha}$$

$$\times\prod_{u\in E_4\left(Si_2C_3\text{-}I\right)}\left(d_u+d_v\right)\times\prod_{u\in E_5\left(Si_2C_3\text{-}I\right)}\left(d_u+d_v\right)^{\alpha}$$

$$=\left(d_u+d_v\right)^{\alpha\left|E_1\left(Si_2C_3\text{-}I\right)\right|}\times\left(d_u+d_v\right)^{\alpha\left|E_2\left(Si_2C_3\text{-}I\right)\right|}\times\left(d_u+d_v\right)^{\alpha\left|E_3\left(Si_2C_3\text{-}I\right)\right|}$$

$$\times\left(d_u+d_v\right)^{\alpha\left|E_4\left(Si_2C_3\text{-}I\right)\right|}\times\left(d_u+d_v\right)^{\alpha\left|E_5\left(Si_2C_3\text{-}I\right)\right|}$$

$$=\left(1+2\right)^{\alpha}\times\left(1+3\right)^{\alpha}\times\left(2+2\right)^{\alpha(p+2q)}\times\left(2+3\right)^{\alpha(6p+8q-9)}\times\left(3+3\right)^{\alpha(15pq-9p-13q+7)}$$

$$=\left(3\right)^{\alpha}\times\left(4\right)^{\alpha}\times\left(2\right)^{\alpha(2p+4q)}\times\left(5\right)^{\alpha(6p+8q-9)}\times\left(6\right)^{\alpha(15pq-9p-13q+7)}$$

$$=\left(2\right)^{\alpha(15pq-7p-9q+9)}\times\left(3\right)^{\alpha(15pq-9p-13q+8)}\times\left(5\right)^{\alpha(6p+8q-9)}.$$

$$6.\ II_2\left(Si_2C_3\text{-}I\right)=\prod_{uv\in E\left(Si_2C_3\text{-}I\right)}\left(d_u\cdot d_v\right)^{\alpha}$$

$$=\prod_{u\in E_1\left(Si_2C_3\text{-}I\right)}\left(d_u\cdot d_v\right)^{\alpha}\times\prod_{u\in E_2\left(Si_2C_3\text{-}I\right)}\left(d_u\cdot d_v\right)^{\alpha}\times\prod_{u\in E_3\left(Si_2C_3\text{-}I\right)}\left(d_u\cdot d_v\right)^{\alpha}$$

$$\times\prod_{u\in E_4\left(Si_2C_3\text{-}I\right)}\left(d_u\cdot d_v\right)^{\alpha}\times\prod_{u\in E_5\left(Si_2C_3\text{-}I\right)}\left(d_u\cdot d_v\right)^{\alpha}$$

$$=\left(d_u\cdot d_v\right)^{\alpha\left|E_1\left(Si_2C_3\text{-}I\right)\right|}\times\left(d_u\cdot d_v\right)^{\alpha\left|E_2\left(Si_2C_3\text{-}I\right)\right|}\times\left(d_u\cdot d_v\right)^{\alpha\left|E_3\left(Si_2C_3\text{-}I\right)\right|}$$

$$\times\left(d_u\cdot d_v\right)^{\alpha\left|E_4\left(Si_2C_3\text{-}I\right)\right|}\times\left(d_u\cdot d_v\right)^{\alpha\left|E_5\left(Si_2C_3\text{-}I\right)\right|}$$

$$=\left(1\times2\right)^{\alpha}\times\left(1\times3\right)^{\alpha}\times\left(2\times2\right)^{\alpha(p+2q)}\times\left(2\times3\right)^{\alpha(6p+8q-9)}\times\left(3\times3\right)^{\alpha(15pq-9p-13q+7)}$$

$$=\left(2\right)^{\alpha}\times\left(3\right)^{\alpha}\times\left(2\right)^{\alpha(2p+4q)}\times\left(2\right)^{\alpha(6p+8q-9)}\times\left(3\right)^{\alpha(6p+8q-9)}\times\left(3\right)^{\alpha(30pq-18p-26q+14)}$$

$$=\left(2\right)^{\alpha(1+2p+6p+4q+8q-9)}\times\left(3\right)^{\alpha(1+6p+8q-9+30pq-18p-26q+14)}$$

$$=\left(2\right)^{\alpha(8p+12q-8)}\times\left(3\right)^{\alpha(30pq-12p-18q+6)}$$

$$=\left(2\right)^{4\alpha(2p+3q-2)}\times\left(3\right)^{6\alpha(5pq+2p-3q+1)}.$$

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$$7.\ XII\left(Si_2C_{3^-}I\right)=\prod_{uv\in E\left(Si_2C_{3^-}I\right)}\frac{1}{\sqrt{d_u+d_v}}$$

$$=\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_1\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_2\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_3\left(Si_2C_{3^-}I\right)\right|}$$

$$\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_4\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_5\left(Si_2C_{3^-}I\right)\right|}$$

$$=\left(\frac{1}{\sqrt{1+2}}\right)\times\left(\frac{1}{\sqrt{1+3}}\right)\times\left(\frac{1}{\sqrt{2+2}}\right)^{p+2q}\times\left(\frac{1}{\sqrt{2+3}}\right)^{6p+8q-9}\times\left(\frac{1}{\sqrt{3+3}}\right)^{15pq-9p-13q+7}$$

$$=\left(\frac{1}{\sqrt{3}}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{p+2q}\times\left(\frac{1}{\sqrt{5}}\right)^{6p+8q-9}\times\left(\frac{1}{\sqrt{6}}\right)^{15pq-9p-13q+7}$$

$$=\left(\frac{1}{2}\right)^{1+p+2q}\times\left(\frac{1}{\sqrt{3}}\right)\times\left(\frac{1}{\sqrt{5}}\right)^{6p+8q-9}\times\left(\frac{1}{\sqrt{6}}\right)^{15pq-9p-13q+7}.$$

$$8.\ \chi II\left(Si_2C_{3^-}I\right)=\prod_{uv\in E\left(Si_2C_{3^-}I\right)}\frac{1}{\sqrt{d_u\cdot d_v}}$$

$$=\left(\frac{1}{\sqrt{d_u\cdot d_v}}\right)^{\left|E_1\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{1}{\sqrt{d_u\cdot d_v}}\right)^{\left|E_2\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{1}{\sqrt{d_u\cdot d_v}}\right)^{\left|E_3\left(Si_2C_{3^-}I\right)\right|}$$

$$\times\left(\frac{1}{\sqrt{d_u\cdot d_v}}\right)^{\left|E_4\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{1}{\sqrt{d_u\cdot d_v}}\right)^{\left|E_5\left(Si_2C_{3^-}I\right)\right|}$$

$$=\left(\frac{1}{\sqrt{1\cdot 2}}\right)\times\left(\frac{1}{\sqrt{1\cdot 3}}\right)\times\left(\frac{1}{\sqrt{2\cdot 2}}\right)^{p+2q}\times\left(\frac{1}{\sqrt{2\cdot 3}}\right)^{6p+8q-9}\times\left(\frac{1}{\sqrt{3\cdot 3}}\right)^{15pq-9p-13q+7}$$

$$=\left(\frac{1}{\sqrt{2}}\right)\times\left(\frac{1}{\sqrt{3}}\right)\times\left(\frac{1}{2}\right)^{p+2q}\times\left(\frac{1}{\sqrt{6}}\right)^{6p+8q-9}\times\left(\frac{1}{3}\right)^{15pq-9p-13q+7}$$

$$=\left(\frac{1}{2}\right)^{\frac{1}{2}+p+2q}\times\left(\frac{1}{3}\right)^{\frac{1}{2}+15pq-9p-13q+7}\times\left(\frac{1}{2}\right)^{3p+4q-\frac{9}{2}}\times\left(\frac{1}{3}\right)^{3p+4q-\frac{9}{2}}$$

$$=\left(\frac{1}{2}\right)^{4p+6q-4}\times\left(\frac{1}{3}\right)^{15pq-6p-9q+3}$$

$$=\left(\frac{1}{2}\right)^{2(2p+3q-2)}\times\left(\frac{1}{3}\right)^{3(5pq-2p-3q+1)}.$$

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9. $$ABCII\left(Si_2C_{3^-}I\right)=\prod_{uv\in E\left(Si_2C_{3^-}I\right)}\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}$$

$$=\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_1\left(Si_2C_{3^-}I\right)\right|}\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_2\left(Si_2C_{3^-}I\right)\right|}\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_3\left(Si_2C_{3^-}I\right)\right|}$$

$$\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_4\left(Si_2C_{3^-}I\right)\right|}\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_5\left(Si_2C_{3^-}I\right)\right|}$$

$$=\left(\sqrt{\frac{1+2-2}{1\cdot 2}}\right)\times\left(\sqrt{\frac{1+3-2}{1\cdot 3}}\right)\times\left(\sqrt{\frac{2+2-2}{2\cdot 2}}\right)^{p+2q}\times\left(\sqrt{\frac{2+3-2}{2\cdot 3}}\right)^{6p+8q-9}\times\left(\sqrt{\frac{3+3-2}{2\cdot 3}}\right)^{15pq-9p-13q+7}$$

$$=\left(\sqrt{\frac{1}{2}}\right)\times\left(\sqrt{\frac{2}{3}}\right)\times\left(\sqrt{\frac{1}{2}}\right)^{p+2q}\times\left(\sqrt{\frac{1}{2}}\right)^{6p+8q-9}\times\left(\sqrt{\frac{2}{3}}\right)^{15pq-9p-13q+7}$$

$$=\left[\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]^{1+p+2q+6p+8q-9-1-15pq+9p+13q-7}\times\left[\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]^{1+15pq-9p-13q+7}$$

$$=\left[\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]^{16p+23q-15pq-16}\times\left[\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]^{15pq-9p-13q+8}.$$

10. $$GAII\left(Si_2C_{3^-}I\right)=\prod_{uv\in E\left(Si_2C_{3^-}I\right)}\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}$$

$$=\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_1\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_2\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_3\left(Si_2C_{3^-}I\right)\right|}$$

$$\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_4\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_5\left(Si_2C_{3^-}I\right)\right|}$$

$$=\left(\frac{2\sqrt{1\cdot 2}}{1+2}\right)\times\left(\frac{2\sqrt{1\cdot 3}}{1+3}\right)\times\left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)^{p+2q}\times\left(\frac{2\sqrt{2\cdot 3}}{2+3}\right)^{6p+8q-9}\times\left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{15pq-9p-13q+7}$$

$$=\left(\frac{2\sqrt{2}}{3}\right)\times\left(\frac{\sqrt{3}}{2}\right)\times\left(\frac{2\sqrt{4}}{4}\right)^{p+2q}\times\left(\frac{2\sqrt{6}}{5}\right)^{6p+8q-9}\times\left(\frac{2\sqrt{9}}{6}\right)^{15pq-9p-13q+7}$$

$$=\left(\frac{2^{\frac{3}{2}}}{3}\right)\times\left(\frac{\sqrt{3}}{2}\right)\times(1)\times\left(\frac{2\sqrt{6}}{5}\right)^{6p+8q-9}\times(1)$$

$$=(2)^{\frac{3}{2}-1+6p+8q-9+3p+4q-\frac{9}{2}}\times(3)^{\frac{1}{2}-1+3p+4q-\frac{9}{2}}\times(5)^{9-6p-8q}$$

$$=(2)^{9p+12q-13}\times(3)^{3p+4q-5}\times(5)^{9-6p-8q}.$$

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11. $G^{\alpha}AII\left(Si_2C_{3^-}I\right)=\prod_{uv\in E\left(Si_2C_{3^-}I\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}$

$$=\prod_{uv\in E_1\left(Si_2C_{3^-}I\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}\times\prod_{uv\in E_2\left(Si_2C_{3^-}I\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}\times\prod_{uv\in E_3\left(Si_2C_{3^-}I\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}$$

$$\times\prod_{uv\in E_4\left(Si_2C_{3^-}I\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}\times\prod_{uv\in E_5\left(Si_2C_{3^-}I\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}$$

$$=\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_1\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_2\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_3\left(Si_2C_{3^-}I\right)\right|}$$

$$\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_4\left(Si_2C_{3^-}I\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_5\left(Si_2C_{3^-}I\right)\right|}$$

$$=\left(\frac{2\sqrt{1\cdot 2}}{1+2}\right)^{\alpha}\times\left(\frac{2\sqrt{1\cdot 3}}{1+3}\right)^{\alpha}\times\left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)^{\alpha(p+2q)}\times\left(\frac{2\sqrt{2\cdot 3}}{2+3}\right)^{\alpha(6p+8q-9)}\times\left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{\alpha(15pq-9p-13q+7)}$$

$$=\left(\frac{2\sqrt{2}}{3}\right)^{\alpha}\times\left(\frac{\sqrt{3}}{2}\right)^{\alpha}\times\left(\frac{2\sqrt{4}}{4}\right)^{\alpha(p+2q)}\times\left(\frac{2\sqrt{6}}{5}\right)^{\alpha(6p+8q-9)}\times\left(\frac{2\sqrt{9}}{6}\right)^{\alpha(15pq-9p-13q+7)}$$

$$=\left(\frac{2^{\frac{3}{2}}}{3}\right)^{\alpha}\times\left(\frac{\sqrt{3}}{2}\right)^{\alpha}\times(1)\times\left(\frac{2\sqrt{6}}{5}\right)^{\alpha(6p+8q-9)}\times(1)$$

$$=(2)^{\alpha(\frac{3}{2}-1+6p+8q-9+3p+4q-\frac{9}{2})}\times(3)^{\alpha(\frac{1}{2}-1+3p+4q-\frac{9}{2})}\times(5)^{\alpha(9-6p-8q)}$$

$$=(2)^{\alpha(9p+12q-13)}\times(3)^{\alpha(3p+4q-5)}\times(5)^{\alpha(9-6p-8q)}.$$

Theorem 2: Let $Si_2C_{3^-}II[p,q]$ be the Silicon Carbide. Then

1. $II_2\left(Si_2C_{3^-}\mathrm{II}\right)=(2)^{12(p+q-1)}\times(3)^{3(10pq+6p-6q+3)}$.
2. $II_1^*\left(Si_2C_{3^-}\mathrm{II}\right)=(2)^{15pq-9p-9q+13}\times(3)^{15pq-13p-13q+13}\times(5)^{8p+8q-14}$.
3. $HII_1\left(Si_2C_{3^-}\mathrm{II}\right)=(2)^{30pq-18p-18q+26}\times(3)^{30pq-26p-26q+26}\times(5)^{16p+16q-28}$.
4. $HII_2\left(Si_2C_{3^-}\mathrm{II}\right)=(2)^{24(p+q-1)}\times(3)^{6(10pq-6p-6q+3)}$.
5. $MZ_1^a\left(Si_2C_{3^-}\mathrm{II}\right)=(2)^{\alpha(15pq-9p-9q+13)}\times(3)^{\alpha(15pq-13p-13q+13)}\times(5)^{\alpha(8p+8q-14)}$.
6. $MZ_2^a\left(Si_2C_{3^-}\mathrm{II}\right)=(2)^{12\alpha(p+q-1)}\times(3)^{3\alpha(10pq+6p-6q+3)}$.

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7. $XII\left(Si_2C_{3^-}\text{II}\right)=\left(\frac{1}{2}\right)^{1+2p+2q}\times\left(\frac{1}{3}\right)\times\left(\frac{1}{5}\right)^{4p+4q-7}\times\left(\frac{1}{\sqrt{6}}\right)^{15pq-13p-13q+11}.$

8. $\chi II\left(Si_2C_{3^-}\text{II}\right)=\left(\frac{1}{2}\right)^{6(p+q-1)}\times\left(\frac{1}{3}\right)^{3(5pq-3p-3q+\frac{1}{2})}.$

9. $ABCII\left(Si_2C_{3^-}\text{II}\right)=\left[\left(\frac{1}{2}\right)\right]^{\frac{23}{2}p+\frac{23}{2}q-\frac{15}{2}pq-12}\times\left[\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]^{15pq-13p-13q+11}.$

10. $GAII\left(Si_2C_{3^-}\text{II}\right)=(2)^{12p+12q-19}\times(3)^{4p+4q-\frac{17}{2}}\times(5)^{14-8p-8q}.$

11. $\text{G}^{\alpha}\text{AII}(Si_2C_{3^-}\text{II})=(2)^{\alpha(12p+12q-19)}\times(3)^{\alpha(4p+4q-\frac{17}{2})}\times(5)^{\alpha(14-8p-8q)}.$

Proof: Let G be the graph of $Si_2C_{3^-}II[p,q]$ where we define p as the number of connected unit cells in a row (chain) and by q we represents the number of connected rows each with p number of cells. From the graph of $Si_2C_{3^-}II[p,q]$ (figure 5-8) we can see that the total number of vertices are $10pq$ and total number of edges are $15pq-3p-3q$.

The edge set of $Si_2C_{3^-}II[p,q]$ with $p,q\geq 1$ has following five partitions,

$$E_{\{1,2\}}=\left\{e=uv\in E\left(Si_2C_{3^-}II[p,q]\right)|d_u=1,d_v=2\right\},$$
$$E_{\{1,3\}}=\left\{e=uv\in E\left(Si_2C_{3^-}II[p,q]\right)|d_u=1,d_v=3\right\},$$
$$E_{\{2,2\}}=\left\{e=uv\in E\left(Si_2C_{3^-}II[p,q]\right)|d_u=2,d_v=2\right\},$$
$$E_{\{2,3\}}=\left\{e=uv\in E\left(Si_2C_{3^-}II[p,q]\right)|d_u=2,d_v=3\right\}.$$

And

$$E_{\{3,3\}}=\left\{e=uv\in E\left(Si_2C_{3^-}II[p,q]\right)|d_u=3,d_v=3\right\}.$$

Now

$$\left|E_{\{1,2\}}\right|=2,$$
$$\left|E_{\{1,3\}}\right|=1,$$
$$\left|E_{\{2,2\}}\right|=2p+2q,$$
$$\left|E_{\{2,3\}}\right|=8p+8q-14,$$

And

$$\left|E_{\{3,3\}}\right|=15pq-13p-13q+11.$$

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$$1.\ II_2\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)} d_u\cdot d_v$$

$$=\prod_{u\in E_1\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)\times\prod_{u\in E_2\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)\times\prod_{u\in E_3\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)$$

$$\times\prod_{u\in E_4\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)\times\prod_{u\in E_5\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)$$

$$=\left(d_u\cdot d_v\right)^{\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u\cdot d_v\right)^{\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u\cdot d_v\right)^{\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$

$$\times\left(d_u\cdot d_v\right)^{\left|E_4\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u\cdot d_v\right)^{\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$

$$=\left(1\times2\right)^2\times\left(1\times3\right)\times\left(2\times2\right)^{2p+2q}\times\left(2\times3\right)^{8p+8q-14}\times\left(3\times3\right)^{15pq-13p-13q+11}$$

$$=\left(2\right)^2\times\left(3\right)\times\left(2\right)^{4p+4q}\times\left(2\right)^{8p+8q-14}\times\left(3\right)^{8p+8q-14}\times\left(3\right)^{30pq-26p-26q+22}$$

$$=\left(2\right)^{2+4p+8p+4q+8q-14}\times\left(3\right)^{1+8p+8q-14+30pq-26p-26q+22}$$

$$=\left(2\right)^{12p+12q-12}\times\left(3\right)^{30pq-18p-18q+9}$$

$$=\left(2\right)^{12(p+q-1)}\times\left(3\right)^{3(10pq+6p-6q+3)}.$$

$$2.\ II_1^*\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)$$

$$=\prod_{u\in E_1\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)\times\prod_{u\in E_2\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)\times\prod_{u\in E_3\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)$$

$$\times\prod_{u\in E_4\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)\times\prod_{u\in E_5\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)$$

$$=\left(d_u+d_v\right)^{\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u+d_v\right)^{\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u+d_v\right)^{\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$

$$\times\left(d_u+d_v\right)^{\left|E_4\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u+d_v\right)^{\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$

$$=\left(1+2\right)^2\times\left(1+3\right)\times\left(2+2\right)^{2p+2q}\times\left(2+3\right)^{8p+8q-14}\times\left(3+3\right)^{15pq-13p-13q+11}$$

$$=\left(3\right)^2\times\left(4\right)\times\left(2\right)^{4p+4q}\times\left(5\right)^{8p+8q-14}\times\left(6\right)^{15pq-13p-13q+11}$$

$$=\left(2\right)^{2+4p+4q+15pq-13p-13q+11}\times\left(3\right)^{2+15pq-13p-13q+11}\times\left(5\right)^{8p+8q-14}$$

$$=\left(2\right)^{15pq-9p-9q+13}\times\left(3\right)^{15pq-13p-13q+13}\times\left(5\right)^{8p+8q-14}.$$

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3. $$HII_1\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^2$$

$$=\prod_{u\in E_1\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^2\times\prod_{u\in E_2\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^2\times\prod_{u\in E_3\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^2$$

$$\times\prod_{u\in E_4\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^2\times\prod_{u\in E_5\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^2$$

$$=\left[\left(d_u+d_v\right)^2\right]^{\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$

$$\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_4\left(Si_2C_{3^-}II\right)\right|}\times\left[\left(d_u+d_v\right)^2\right]^{\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$

$$=\left[\left(1+2\right)^2\right]^2\times\left[\left(1+3\right)^2\right]\times\left[\left(2+2\right)^2\right]^{2p+2q}\times\left[\left(2+3\right)^2\right]^{8p+8q-14}\times\left[\left(3+3\right)^2\right]^{15pq-13p-13q+11}$$

$$=(3)^4\times(2)^4\times(2)^{8p+8q}\times(5)^{16p+16q-28}\times(6)^{30pq-26p-26q+22}$$

$$=(3)^{4+30pq-26p-26q+22}\times(2)^{4+8p+8q+30pq-26p-26q+22}\times(5)^{16p+16q-28}$$

$$=(2)^{30pq-18p-18q+26}\times(3)^{30pq-26p-26q+26}\times(5)^{16p+16q-28}.$$

4. $$HII_2\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^2$$

$$=\prod_{u\in E_1\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^2\times\prod_{u\in E_2\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^2\times\prod_{u\in E_3\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^2$$

$$\times\prod_{u\in E_4\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^2\times\prod_{u\in E_5\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^2$$

$$=\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$

$$\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_4\left(Si_2C_{3^-}II\right)\right|}\times\left[\left(d_u\cdot d_v\right)^2\right]^{\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$

$$=\left[\left(1\times2\right)^2\right]^2\times\left[\left(1\times3\right)^2\right]\times\left[\left(2\times2\right)^2\right]^{2p+2q}\times\left[\left(2\times3\right)^2\right]^{8p+8q-14}\times\left[\left(3\times3\right)^2\right]^{15pq-13p-13q+11}$$

$$=(2)^4\times(3)^2\times(2)^{8p+8q}\times(6)^{16p+16q-28}\times(3)^{60pq-52p-52q+44}$$

$$=(2)^{4+8p+8q+16p+16q-28}\times(3)^{2+16p+16q-28+60pq-52p-52q+44}$$

$$=(2)^{24p+24q-24}\times(3)^{60pq-36p-36q+18}$$

$$=(2)^{24(p+q-1)}\times(3)^{6(10pq-6p-6q+3)}.$$

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5. $$II_1^*\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^{\alpha}$$
$$=\prod_{u\in E_1\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^{\alpha}\times\prod_{u\in E_2\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^{\alpha}\times\prod_{u\in E_3\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^{\alpha}$$
$$\times\prod_{u\in E_4\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)\times\prod_{u\in E_5\left(Si_2C_{3^-}II\right)}\left(d_u+d_v\right)^{\alpha}$$
$$=\left(d_u+d_v\right)^{\alpha\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u+d_v\right)^{\alpha\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u+d_v\right)^{\alpha\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$
$$\times\left(d_u+d_v\right)^{\alpha\left|E_4\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u+d_v\right)^{\alpha\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$
$$=\left(1+2\right)^{2\alpha}\times\left(1+3\right)^{\alpha}\times\left(2+2\right)^{\alpha(2p+2q)}\times\left(2+3\right)^{\alpha(8p+8q-14)}\times\left(3+3\right)^{\alpha(15pq-13p-13q+11)}$$
$$=\left(3\right)^{2\alpha}\times\left(4\right)^{\alpha}\times\left(2\right)^{\alpha(4p+4q)}\times\left(5\right)^{\alpha(8p+8q-14)}\times\left(6\right)^{\alpha(15pq-13p-13q+11)}$$
$$=\left(2\right)^{\alpha(15pq-9p-9q+13)}\times\left(3\right)^{\alpha(15pq-13p-13q+13)}\times\left(5\right)^{\alpha(8p+8q-14)}.$$

6. $$II_2\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^{\alpha}$$
$$=\prod_{u\in E_1\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^{\alpha}\times\prod_{u\in E_2\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^{\alpha}\times\prod_{u\in E_3\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^{\alpha}$$
$$\times\prod_{u\in E_4\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^{\alpha}\times\prod_{u\in E_5\left(Si_2C_{3^-}II\right)}\left(d_u\cdot d_v\right)^{\alpha}$$
$$=\left(d_u\cdot d_v\right)^{\alpha\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u\cdot d_v\right)^{\alpha\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u\cdot d_v\right)^{\alpha\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$
$$\times\left(d_u\cdot d_v\right)^{\alpha\left|E_4\left(Si_2C_{3^-}II\right)\right|}\times\left(d_u\cdot d_v\right)^{\alpha\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$
$$=\left(1\times2\right)^{2\alpha}\times\left(1\times3\right)^{\alpha}\times\left(2\times2\right)^{\alpha(2p+2q)}\times\left(2\times3\right)^{\alpha(8p+8q-14)}\times\left(3\times3\right)^{\alpha(15pq-13p-13q+11)}$$
$$=\left(2\right)^{2\alpha}\times\left(3\right)^{\alpha}\times\left(2\right)^{\alpha(4p+4q)}\times\left(2\right)^{\alpha(8p+8q-14)}\times\left(3\right)^{\alpha(8p+8q-14)}\times\left(3\right)^{\alpha(30pq-26p-26q+22)}$$
$$=\left(2\right)^{\alpha(2+4p+8p+4q+8q-14)}\times\left(3\right)^{\alpha(1+8p+8q-14+30pq-26p-26q+22)}$$
$$=\left(2\right)^{\alpha(12p+12q-12)}\times\left(3\right)^{\alpha(30pq-18p-18q+9)}$$
$$=\left(2\right)^{12\alpha(p+q-1)}\times\left(3\right)^{3\alpha(10pq+6p-6q+3)}.$$

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$$7.\ XII\left(Si_2C_{3-}II\right)=\prod_{uv\in E\left(Si_2C_{3-}II\right)}\frac{1}{\sqrt{d_u+d_v}}$$

$$=\prod_{u\in E_1\left(Si_2C_{3-}II\right)}\frac{1}{\sqrt{d_u+d_v}}\times\prod_{u\in E_2\left(Si_2C_{3-}II\right)}\frac{1}{\sqrt{d_u+d_v}}\times\prod_{u\in E_3\left(Si_2C_{3-}II\right)}\frac{1}{\sqrt{d_u+d_v}}$$

$$\times\prod_{u\in E_4\left(Si_2C_{3-}II\right)}\frac{1}{\sqrt{d_u+d_v}}\times\prod_{u\in E_5\left(Si_2C_{3-}II\right)}\frac{1}{\sqrt{d_u+d_v}}$$

$$=\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_1\left(Si_2C_{3-}II\right)\right|}\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_2\left(Si_2C_{3-}II\right)\right|}\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_3\left(Si_2C_{3-}II\right)\right|}$$

$$\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_4\left(Si_2C_{3-}II\right)\right|}\times\left(\frac{1}{\sqrt{d_u+d_v}}\right)^{\left|E_5\left(Si_2C_{3-}II\right)\right|}$$

$$=\left(\frac{1}{\sqrt{1+2}}\right)^{2}\times\left(\frac{1}{\sqrt{1+3}}\right)\times\left(\frac{1}{\sqrt{2+2}}\right)^{2p+2q}\times\left(\frac{1}{\sqrt{2+3}}\right)^{8p+8q-14}\times\left(\frac{1}{\sqrt{3+3}}\right)^{15pq-13p-13q+11}$$

$$=\left(\frac{1}{\sqrt{3}}\right)^{2}\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{2p+2q}\times\left(\frac{1}{\sqrt{5}}\right)^{8p+8q-14}\times\left(\frac{1}{\sqrt{6}}\right)^{15pq-13p-13q+11}$$

$$=\left(\frac{1}{2}\right)^{1+2p+2q}\times\left(\frac{1}{3}\right)\times\left(\frac{1}{5}\right)^{4p+4q-7}\times\left(\frac{1}{\sqrt{6}}\right)^{15pq-13p-13q+11}.$$

$$8.\ \chi II\left(Si_2C_{3-}II\right)=\prod_{uv\in E\left(Si_2C_{3-}II\right)}\frac{1}{\sqrt{d_u\cdot d_v}}$$

$$=\left(\frac{1}{\sqrt{1\cdot 2}}\right)^{2}\times\left(\frac{1}{\sqrt{1\cdot 3}}\right)\times\left(\frac{1}{\sqrt{2\cdot 2}}\right)^{2p+2q}\times\left(\frac{1}{\sqrt{2\cdot 3}}\right)^{8p+8q-14}\times\left(\frac{1}{\sqrt{3\cdot 3}}\right)^{15pq-13p-13q+11}$$

$$=\left(\frac{1}{2}\right)\times\left(\frac{1}{\sqrt{3}}\right)\times\left(\frac{1}{2}\right)^{2p+2q}\times\left(\frac{1}{\sqrt{6}}\right)^{8p+8q-14}\times\left(\frac{1}{3}\right)^{15pq-13p-13q+11}$$

$$=\left(\frac{1}{2}\right)^{1+2p+2q}\times\left(\frac{1}{3}\right)^{\frac{1}{2}+15pq-13p-13q+11}\times\left(\frac{1}{2}\right)^{4p+4q-7}\times\left(\frac{1}{3}\right)^{4p+4q-7}$$

$$=\left(\frac{1}{2}\right)^{6p+6q-6}\times\left(\frac{1}{3}\right)^{15pq-9p-9q+\frac{9}{2}}$$

$$=\left(\frac{1}{2}\right)^{6(p+q-1)}\times\left(\frac{1}{3}\right)^{3(5pq-3p-3q+\frac{1}{2})}.$$

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9. $$ABCII\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)}\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}$$

$$=\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$

$$\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_4\left(Si_2C_{3^-}I\right)I\right|}\times\left(\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}\right)^{\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$

$$=\left(\sqrt{\frac{1+2-2}{1\cdot 2}}\right)\times\left(\sqrt{\frac{1+3-2}{1\cdot 3}}\right)\times\left(\sqrt{\frac{2+2-2}{2\cdot 2}}\right)^{p+2q}\times\left(\sqrt{\frac{2+3-2}{2\cdot 3}}\right)^{6p+8q-9}\times\left(\sqrt{\frac{3+3-2}{2\cdot 3}}\right)^{15pq-9p-13q+7}$$

$$=\left(\sqrt{\frac{1}{2}}\right)^{2}\times\left(\sqrt{\frac{2}{3}}\right)\times\left(\sqrt{\frac{1}{2}}\right)^{2p+2q}\times\left(\sqrt{\frac{1}{2}}\right)^{8p+8q-14}\times\left(\sqrt{\frac{2}{3}}\right)^{15pq-13p-13q+11}$$

$$=\left[\left(\frac{1}{2}\right)\right]^{1+p+q+4p+4q-7-\frac{1}{2}-\frac{15}{2}pq+\frac{13}{2}p+\frac{13}{2}q-\frac{11}{2}}\times\left[\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]^{1+15pq-13p-13q+11}$$

$$=\left[\left(\frac{1}{2}\right)\right]^{\frac{23}{2}p+\frac{23}{2}q-\frac{15}{2}pq-12}\times\left[\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]^{15pq-13p-13q+11}.$$

10. $$GAII\left(Si_2C_{3^-}II\right)=\prod_{uv\in E\left(Si_2C_{3^-}II\right)}\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}$$

$$=\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_1\left(Si_2C_{3^-}II\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_2\left(Si_2C_{3^-}II\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_3\left(Si_2C_{3^-}II\right)\right|}$$

$$\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_4\left(Si_2C_{3^-}II\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\left|E_5\left(Si_2C_{3^-}II\right)\right|}$$

$$=\left(\frac{2\sqrt{1\cdot 2}}{1+2}\right)^{2}\times\left(\frac{2\sqrt{1\cdot 3}}{1+3}\right)\times\left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)^{2p+2q}\times\left(\frac{2\sqrt{2\cdot 3}}{2+3}\right)^{8p+8q-14}\times\left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{15pq-13p-13q+11}$$

$$=\left(\frac{2\sqrt{2}}{3}\right)^{2}\times\left(\frac{\sqrt{3}}{2}\right)\times\left(\frac{2\sqrt{4}}{4}\right)^{2p+2q}\times\left(\frac{2\sqrt{6}}{5}\right)^{8p+8q-14}\times\left(\frac{2\sqrt{9}}{6}\right)^{15pq-13p-13q+11}$$

$$=\left(\frac{2^3}{3^2}\right)\times\left(\frac{\sqrt{3}}{2}\right)\times(1)\times\left(\frac{2\sqrt{6}}{5}\right)^{8p+8q-14}\times(1)$$

$$=(2)^{3-1+8p+8q-14+4p+4q-7}\times(3)^{\frac{1}{2}-2+4p+4q-7}\times(5)^{14-8p-8q}$$

$$=(2)^{12p+12q-19}\times(3)^{4p+4q-\frac{17}{2}}\times(5)^{14-8p-8q}.$$

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11. $$G^{\alpha}AII\left(Si_2C_{3-}II\right)=\prod_{uv\in E\left(Si_2C_{3-}II\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}$$

$$=\prod_{uv\in E_1\left(Si_2C_{3-}II\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}\times\prod_{uv\in E_2\left(Si_2C_{3-}II\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}\times\prod_{uv\in E_3\left(Si_2C_{3-}II\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}$$

$$\times\prod_{uv\in E_4\left(Si_2C_{3-}II\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}\times\prod_{uv\in E_5\left(Si_2C_{3-}II\right)}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}$$

$$=\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_1\left(Si_2C_{3-}II\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_2\left(Si_2C_{3-}II\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_3\left(Si_2C_{3-}II\right)\right|}$$

$$\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_4\left(Si_2C_{3-}II\right)\right|}\times\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha\left|E_5\left(Si_2C_{3-}II\right)\right|}$$

$$=\left(\frac{2\sqrt{1\cdot 2}}{1+2}\right)^{2\alpha}\times\left(\frac{2\sqrt{1\cdot 3}}{1+3}\right)^{\alpha}\times\left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)^{\alpha(2p+2q)}\times\left(\frac{2\sqrt{2\cdot 3}}{2+3}\right)^{\alpha(8p+8q-14)}\times\left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{\alpha(15pq-13p-13q+11)}$$

$$=\left(\frac{2\sqrt{2}}{3}\right)^{\alpha}\times\left(\frac{\sqrt{3}}{2}\right)^{\alpha}\times\left(\frac{2\sqrt{4}}{4}\right)^{\alpha(2p+2q)}\times\left(\frac{2\sqrt{6}}{5}\right)^{\alpha(8p+8q-14)}\times\left(\frac{2\sqrt{9}}{6}\right)^{\alpha(15pq-13p-13q+11)}$$

$$=\left(\frac{2^3}{3^2}\right)^{\alpha}\times\left(\frac{\sqrt{3}}{2}\right)^{\alpha}\times(1)\times\left(\frac{2\sqrt{6}}{5}\right)^{\alpha(6p+8q-9)}\times(1)$$

$$=(2)^{\alpha(3-1+8p+8q-14+4p+4q-7)}\times(3)^{\alpha(\frac{1}{2}-2+4p+4q-7)}\times(5)^{\alpha(14-8p-8q)}$$

$$=(2)^{\alpha(12p+12q-19)}\times(3)^{\alpha(4p+4q-\frac{17}{2})}\times(5)^{\alpha(14-8p-8q)}.$$

Conclusion

In the present article, we computed closed form of multiplicative degree-based topological indices for silicon-carbon $Si_2C_{3}\text{-}I[p,q]$. Topological indices thus calculated for these silicon-carbon can help us to understand the physical features, chemical reactivity, and biological activities. In this point of view, a topological index can be regarded as a score function which maps each molecular structure to a real number and is used as descriptors of the molecule under testing. These results can also play a vital part in the determination of the significance of silicon-carbon in electronics and industry.

For example Randic index is useful for determining physio-chemical properties of alkanes as noticed by chemist Melan Randic in 1975. He noticed the correlation between

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the Randic index R and several physico–chemical properties of alkanes like, enthalpies of formation, boiling points, chromatographic retention times, vapor pressure and surface areas. Following figure 9 is adapted from [21] relating to boiling point of some Alkanes and its correlation with Randic index.

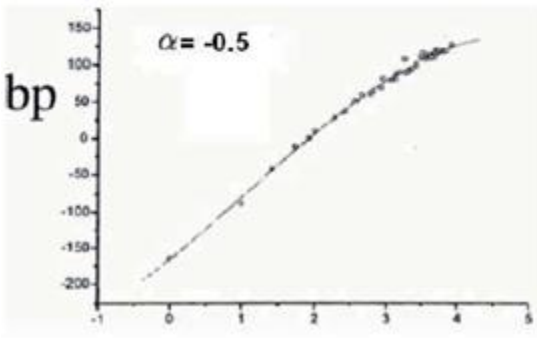

Figure 9 correlation of the Randic index with the boiling point of a selected set of Alkanes

Completing Interests

Authors do not have any competing interest.

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Data Availability

Not applicable

References

1. G. Rucker, C. Rucker, On topological indices, boiling points, and cycloalkanes. J. *Chem. Inf. Comput. Sci.*1999, 39, 788–802.
2. S. Klavžar, I. Gutman, A Comparison of the Schultz molecular topological index with the Wiener index. *J. Chem. Inf. Comput. Sci*. 1996, 36, 1001–1003.
3. F. M. Brückler, T. Došli´c, A. Graovac, I. Gutman, On a class of distance-based molecular structure descriptors. *Chem. Phys. Lett*. 2011, 503, 336–338.
4. H. Deng, J. Yang, F. Xia, F. A general modeling of some vertex-degree based topological indices in benzenoid systems and phenylenes. *Comp. Math. Appl.*2011, 61, 3017–3023.

Peer-reviewed version available at *Symmetry* **2018**, *10*, 320; doi:10.3390/sym10080320

5. H. Zhang, F. Zhang, The Clar covering polynomial of hexagonal systems. *Discret. Appl. Math*. 1996, 69, 147–167.
6. I. Gutman, Some properties of the Wiener polynomials. *Graph Theory Notes New York*1993, 125, 13–18.
7. E. Deutsch, S. Klavzar, M-Polynomial and degree-based topological indices. *Iran. J. Math. Chem*. 2015, 6, 93–102.
8. M. Munir, W. Nazeer,S. Rafique, S.M. Kang, M-polynomial and related topological indices of Nanostar dendrimers. *Symmetry*2016, 8, 97.
9. M. Munir, W. Nazeer,S. Rafique, A. R. Nizami, S.M. Kang, M-polynomial and degree-based topological indices of titania nanotubes. *Symmetry* 2016, 8, 117.
10. Kwun, Y. ,Munir, M. Nazeer, W; Rafique, S.; Kang, S. M. M-polynomial and degree-based topological indices of V-phenalinic nanotubes and nanotori, Scientific Reports 7, 8756, doi:10.1038/s41598-017-08309-y2017
11. M. Munir, W. Nazeer,S. Rafique, A. R. Nizami, S.M. Kang, Some Computational Aspects of Triangular Boron Nanotubes. *Symmetry* **2016**, *9*, 6; doi: 10.3390/sym9010006.
12. M. Munir, W. Nazeer, S. Rafique, S.M. Kang, M-Polynomial and Degree-Based Topological Indices of Polyhex Nanotubes. Symmetry 2016, 8, 149.
13. Imran, M., Siddiqui, M. K., Naeem, M., & Iqbal, M. A. (2018). On Topological Properties of Symmetric Chemical Structures. Symmetry, 10(5), 173.
14. Alaeiyan, M., Farahani, M. R., & Jamil, M. K. (2016). Computation of the fifth geometric-arithmetic index for polycyclic aromatic hydrocarbons pahk. Applied Mathematics and Nonlinear Sciences, 1(1), 283-290.
15. Jamil, M. K., Farahani, M. R., Imran, M., & Malik, M. A. (2016). Computing eccentric version of second zagreb index of polycyclic aromatic hydrocarbons pahkpahk. Applied Mathematics and Nonlinear Sciences, 1(1), 247-252.
16. Farahani, M. R., Jamil, M. K., & Imran, M. (2016). Vertex PIv topological index of titania carbon nanotubes TiO2 (m, n). Appl. Math. Nonl. Sc, 1(1), 175-182.
17. West D.B., Introduction to graph theory (Vol. 2). Upper Saddle River: Prentice hall, 2001.
18. Gao W., Wang Y., Wang W., Shi L., The first multiplication atom-bond connectivity index of molecular structures in drugs, Saudi Pharmaceutical Journal, 2017,25(4), 548-555.
19. Gao W., Wang Y., Basavanagoud B. and Jamil M.K., Characteristics studies of molecular structures in drugs, Saudi Pharmaceutical Journal, 2017, 25(4), pp.580-586.
20. Gao W., Wang W., Dimitrov D., Wang, Y., 2017. Nano Properties Analysis via Fourth Multiplicative ABC Indicator Calculating, Arabian Journal of Chemistry.
21. Deutsch E., Klavzar S., M-Polynomial and degree-based topological indices, Iranian Journal of Mathematical Chemistry, (2015), 6: 93–102

Peer-reviewed version available at Symmetry 2018, 10, 320; doi:10.3390/sym10080320

22. Wiener H., Structural determination of paraffin boiling points, Journal of the American Chemical Society, 1947, 69(1), pp.17-20.
23. Randic M., Characterization of molecular branching, Journal of the American Chemical Society, 1975, 97(23), pp.6609-6615.
24. Bollobás B. and Erdös P., Graphs of extremal weights, Ars Combinatoria, 1998, 50, 225-233.
25. Amić D., Bešlo D., Lučić, B., Nikolić S., Trinajstić, N., The vertex-connectivity index revisited, Journal of chemical information and computer sciences, 1998, 38(5), pp.819-822.
26. Hu Y., Li X., Shi Y., Xu T., Gutman I., On molecular graphs with smallest and greatest zeroth-order general Randic index, MATCH Commun. Math. Comput. Chem, 2005, 54(2), 425-434.
27. Li X., Gutman, I., Randić M., Mathematical aspects of Randić-type molecular structure descriptors. University, Faculty of Science, 2006,.
28. Randić M., On history of the Randić index and emerging hostility toward chemical graph theory, MATCH Commun. Math. Comput. Chem, 2008, 59, 5-124.
29. Randić M., The connectivity index 25 years after, Journal of Molecular Graphics and Modelling, 20(1), 2001, 19-35.
30. Gutman I., Furtula, B. eds., Recent results in the theory of Randić index. University, Faculty of Science.
31. Li X., Shi Y., A survey on the Randic index, MATCH Commun. Math. Comput. Chem, 2008, 59(1), pp.127-156.
32. Gutmann I. (Ed.). (2008). Recent Results in the Theory of Randić Index, Kragujevac University
33. Gutman I., Ruscić B., Trinajstić N., Wilcox Jr C.F., Graph theory and molecular orbitals. XII. Acyclic polyenes, The Journal of Chemical Physics, 1975, 62(9), pp.3399-3405.
34. Narumi H. and Katayama M., Simple topological index: A newly devised index characterizing the topological nature of structural isomers of saturated hydrocarbons, Memoirs of the Faculty of Engineering, Hokkaido University, 1984, 16(3), 209-214.
35. Gutman I., Multiplicative Zagreb indices of trees, Bull. Soc. Math. Banja Luka, 2011, 18, 17-23.
36. Todeschini R., Ballabio D. and Consonni V., Novel molecular descriptors based on functions of new vertex degrees, Mathematical Chemistry Monographs, 2010, 73-100.
37. Todeschini, R., Consonni V., New local vertex invariants and molecular descriptors based on functions of the vertex degrees, MATCH Commun. Math. Comput. Chem, 2010, 64(2), 359-372.

Peer-reviewed version available at Symmetry 2018, 10, 320; doi:10.3390/sym10080320

38. Wang S., Wei B., Multiplicative Zagreb indices of k-trees, Discrete Applied Mathematics, 2015, 180,168-175.
39. Eliasi, M., Iranmanesh, A., Gutman, I., Multiplicative versions of first Zagreb index, Match-Communications in Mathematical and Computer Chemistry, 2012, 68(1), 217.
40. Kulli, V.R., 2016. • MULTIPLICATIVE HYPER-ZAGREB INDICES AND COINDICES OF GRAPHS: COMPUTING THESE INDICES OF SOME NANOSTRUCTURES. International Research Journal of Pure Algebra ISSN: 2248-9037, 6(7).
41. Kulli V.R., Stone B., Wang S., Wei, B., Generalised multiplicative indices of polycyclic aromatic hydrocarbons and benzenoid systems, Zeitschrift für Naturforschung A, 2017, 72(6),573-576.
42. Kulli V.R., Multiplicative connectivity indices of TUC4C8 [m, n] and TUC4 [m, n] nanotubes, Journal of Computer and Mathematical Sciences, 2016, 7(11), 599-605.