

Article

Quantum Nonlocality From Higher-Derivative Gravity

Gregory S. Duane^{1,2}

¹Geophysics Institute
University of Bergen
Postboks 7803
5020 Bergen, Norway

²Dept. of Atmospheric and Oceanic Sciences
UCB 311
University of Colorado
Boulder, CO 80309-0311

* gregory.duane@colorado.edu, 720-987-3760

Abstract: A classical origin for the Bohmian quantum potential, as that potential term arises in the quantum mechanical treatment of black holes and Einstein-Rosen (ER) bridges, can be based on 4th-order extensions of Einstein's equations. In Bohm's ontological interpretation, black hole radiation, and the analogous tunneling process of quantum transmission through an ER bridge, are classically allowed if the dynamics are modified to include such a quantum potential. The 4th-order extension of general relativity required to generate the quantum potential is given by adding quadratic curvature terms with coefficients that maintain a fixed ratio, as their magnitudes approach zero. Quantum transmission through the classically non-traversable bridge is replaced by classical transmission through a traversable wormhole. If entangled particles are connected by a Planck-width ER bridge, as conjectured by Maldacena and Susskind, then the classical wormhole transmission effect gives the ontological nonlocal connection between the particles posited in Bohm's interpretation of their entanglement. It is hypothesized that higher-derivative extensions of classical gravity can account for the nonlocal part of the quantum potential generally.

Keywords: 4th order gravity, quantum potential, ER=EPR, wormholes

1. Motivation and Introduction

Nonlocality is a prominent feature of quantum mechanics both in the standard, probabilistic Copenhagen interpretation and in the deterministic re-interpretation by Bohm [1]. In the former case, the spin of each member of an EPR pair, along any measurement axis, is determined at the time of measurement of either member's spin along that axis, despite any space-like separation, however large, between the measurement apparatus and the remote spin. Bell's Theorem [2] precludes any understanding of this result in terms of spin values that exist at the time of creation of the pair, independently of observation. In Bohm's interpretation, the effect of measurement on the remote spin is taken to be a real physical effect, mediated by a quantum potential.

With either interpretation, one is left to ask if any sort of generalization of the classical, and relativistic, notion of locality can be articulated that does not depend on the full apparatus of standard quantum theory. If observations trigger the collapse of a globally defined wave packet, as in the Copenhagen interpretation, can one be more specific about the aspect of the observational process that is responsible for the collapse? If one adopts Bohm's quantum potential interpretation, can such a

29 non-local potential be linked to interactions that are familiar in classical physics? One must account
30 for the impossibility of non-local transmission of information either through the alleged effect of
31 observation or through the quantum potential.

32 A picture of quantum nonlocality emerges from the Maldacena-Susskind “ER=EPR” conjecture
33 that was originally put forward to resolve the black-hole information loss paradox [3]. We do not
34 review the original motivation for ER=EPR, which involved entangled macroscopic black holes, but
35 simply note that what emerged was a suggestion that any pair of entangled particles is connected by
36 an Einstein-Rosen (ER) bridge of Planck-scale width. Non-locality, in the proposed view, is mediated
37 by ER bridges, which intimately link space-like separated regions. Indeed, the members of an EPR pair
38 have zero separation, as computed along a path through the bridge and thus are, in effect, the same
39 particle. An Einstein-Rosen bridge is an unfolding of a black hole in a multiply-connected space time
40 topology, so information that enters the bridge is not transmitted for the same reason that it cannot
41 escape a black hole. That would explain why the non-local connections cannot be used for signaling.

42 It is not clear how Einstein-Rosen bridges, while offering a more specific and ontological picture,
43 could lead to the kind of nonlocality that would be induced by observations Copenhagen-style. But
44 it is possible to compare the physical effects of such bridges with those of the quantum potential in
45 the Bohm interpretation. Effects like those of the quantum potential would require that the bridges
46 be traversable, in order for the measurement outcome for one member of an EPR pair to feel the
47 orientation of the measurement apparatus for its partner, as required by Bell’s Theorem. Classical
48 ER bridges are not traversable. However we note that just as quantum black holes emit radiation,
49 quantum ER bridges are crudely traversable. That is, particles can tunnel out of the black hole or
50 through the bridge. There are severe restrictions on the kind of information that can be re-emitted in
51 one case, or transmitted, in the other, but neither effect requires essentially new physics that is beyond
52 general relativity or quantum mechanics.

53 Further, if the Bohm interpretation can be applied to field theory and to gravity in particular, then
54 tunneling through quantum ER bridges should itself be explicable in terms of a quantum potential
55 added to classical general relativity. Thus an extension of general relativity could account for quantum
56 entanglement. Here, we assume that general relativity describes phenomena down to well below the
57 Planck scale L_p . In bootstrap fashion, the quantum behavior of black holes and ER bridges would
58 provide an explanation for quantum nonlocality, at least as it appears strikingly in EPR correlations.
59 On the occasion of the Bohm Centennial, it is suggested that an ontological re-interpretation of the
60 quantum ER bridge behavior would then also explain nonlocal quantum entanglement generally. But
61 what kind of geometrodynamics would give rise to the quantum potential required in the specific case
62 of an ER bridge? The purpose of this short paper is to examine an extension of Einstein’s equations
63 that will account for the extra term and will thus allow us to interpret the quantum ER transmission
64 effect classically.

65 The proposed ontological account of quantum nonlocality tends to resolve the contradiction
66 between quantum mechanics and general relativity in a way that gives primacy to the latter. It can
67 be compared to Penrose’s suggestion that the collapse of the wavefunction is a gravitational effect
68 [4] in this regard. After reviewing both the quantum potential interpretation of black hole radiation
69 and classical higher derivative gravity in the next section, we compare the two in the following
70 section. We derive a result that only infinitesimal higher-derivative corrections are required (so that
71 the standard second-order theory is a singular limit). Implications for a re-interpretation of quantum
72 theory generally are discussed in the final section.

73 2. Background

74 2.1. Quantum potential interpretation of black hole radiation

If black hole radiation, like other quantum processes, can be described by an objective
deterministic theory, it should be possible to augment Einstein’s classical equations with terms that

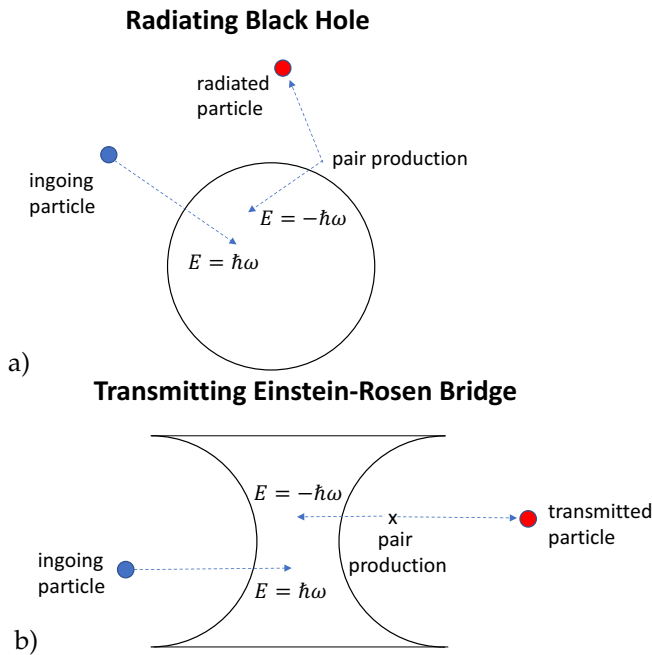


Figure 1. Steady states of a) a radiating black hole, and b) a transmitting Einstein-Rosen bridge, result from pair production just outside the horizon and ongoing classical ingestion.

give such radiation, which can indeed be characterized as a form of tunneling [5]. A quantum potential for black hole radiation was in fact derived by deBarros *et al.* [6,7] in work that has not been widely cited. Recall that for a simple quantum mechanical system with a wave-function Ψ written in polar form as $\Psi = \mathcal{R} \exp(iS)$, the quantum potential is $Q = -(\nabla^2 \mathcal{R})/2m\mathcal{R}$, meaning that the Schrodinger equation for Ψ implies that particle motion is governed by the Hamilton-Jacobi equation with an extra potential term:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{1}{2m} \frac{\nabla^2 \mathcal{R}}{\mathcal{R}} = 0 \quad (1)$$

75 In the common example of tunneling through a potential barrier, we imagine that the particles each
 76 possess an energy $E < V$, where V is now the height of the barrier. There are no classically allowed
 77 trajectories through the barrier, but nevertheless, according to standard quantum mechanics, and as
 78 experimentally confirmed, particles that start on one side do appear on the opposite side. There are
 79 no definite trajectories in standard quantum mechanics (although the result could be obtained as a
 80 sum over alternative classically forbidden trajectories in the path integral formalism). But in Bohm's
 81 ontological interpretation, each particle follows a definite, classically allowed trajectory in a dynamical
 82 system governed by the potential $V + Q$. That is possible because $E > V + Q$ over the entirety of at
 83 least some paths through the barrier, as illustrated in [8].

In the current context, the quantum potential is gleaned from the Wheeler-Dewitt equation $\hat{H}\Psi = 0$, obtained by quantizing the hamiltonian constraint in the ADM formulation of general relativity, in which spacetime is foliated into a temporal sequence of spacelike hypersurfaces. For a general wave-functional Ψ of the spatial metric on a hypersurface and of the matter fields, this equation is:

$$[G_{ijkl} \frac{\delta}{\delta h_{ij}} \frac{\delta}{\delta h_{kl}} - \sqrt{h} ({}^3R(\mathbf{h}) + H_{\text{matter}})]\Psi(\mathbf{h}, \text{matter}) = 0 \quad (2)$$

84 where h is the determinant of the space metric h_{ij} , $G_{ijkl} \equiv \frac{1}{2}h^{-1/2}(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl})$ and ${}^{(3)}\mathcal{R}$ is
 85 the intrinsic curvature of the evolving space-like hypersurface. Inserting the polar decomposition
 86 $\Psi = \mathcal{R} \exp(i\mathcal{S})$ in (2), we get:

$$G_{ijkl} \frac{\delta \mathcal{S}}{\delta h_{ij}} \frac{\delta \mathcal{S}}{\delta h_{kl}} - \sqrt{h} {}^{(3)}\mathcal{R}(h) + \sqrt{h} Q + \text{matter terms} = 0 \quad (3)$$

87 for a quantum potential Q that is defined in terms of derivatives of \mathcal{R} both with respect to the metric
 88 and with respect to the matter fields. For $Q = 0$, Eq. (3) is the Hamilton-Jacobi equation for the usual
 89 action S in the ADM formalism. For $Q \neq 0$, the dynamics are modified, but all fields still follow
 90 well-defined trajectories, as for a single particle in the Bohmian interpretation.

Tomimatsu [9] solved the Wheeler-Dewitt equation (2) for a spherically symmetric case, using boundary conditions that describe a scalar field in an evaporating black-hole metric, limiting attention to the region near the apparent horizon. He found:

$$\Psi(r_o, \Phi) = C \exp\left[i\left(\frac{r_o}{4} + \frac{k^2}{2r_o}\right) - |k\Phi|\right] \quad (4)$$

giving the wave-functional $\Psi(r_o, \Phi)$ as a function of the instantaneous apparent black-hole radius r_o and the scalar field Φ at the apparent horizon, with k an eigenvalue to be determined. We take the wave functional to have the same form for a transmitting ER bridge, since only the topology is different from that of a black hole. Traversability can indeed be determined at the horizon. For the wave function (4), we have $\mathcal{R} = C \exp(-|k\Phi|)$, and so the quantum potential depends only on derivatives with respect to the scalar field and not with respect to the metric. As found by deBarros *et al.* [7], the term in the matter sector of the hamiltonian, $(1/2r_o^2)\hat{P}_\Phi^2 = -(1/2r_o^2)\delta^2/\delta\Phi^2$, contributes a quantum potential

$$Q = -\frac{k^2}{2r_o^2} \quad (5)$$

91 to the Hamilton-Jacobi equation (3). We assume here that the quantum evaporation/transmission
 92 process induced by the quantum potential (5) is balanced by a purely classical accretion/particle-entry
 93 process to maintain a steady state (Fig. 1). The change in boundary conditions needed to describe the
 94 added classical process will not affect the quantum potential.

95 2.2. Classical higher-derivative gravity

96 In this paper, we seek a generally covariant extension of Einstein's equations that gives a quantum
 97 potential of the form (5) in the corresponding generalization of the Wheeler-Dewitt equation for a
 98 radiating black hole in a steady state.. A simple candidate is constructed by adding terms containing
 99 higher derivatives of the metric to the standard second-order equations. While such higher-derivative
 100 extensions of general relativity have become familiar in the context of quantum corrections to general
 101 relativity, here we regard the resulting theory simply as an alternative to the standard classical theory.

A textbook derivation of Einstein's equations, e.g. [18], relies not only on general covariance, but on an explicit assumption that the equations are second order, or equivalently, that they are scale invariant. General relativity can in fact be extended to theories of the form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}\Lambda + \sum_{n>2} c_n L^{n-2} R_{\mu\nu}^{(n)} = 8\pi T_{\mu\nu} \quad (6)$$

102 where $R_{\mu\nu}^{(n)}$ is a quantity involving a total of n derivatives of the metric, L is a fundamental length
 103 scale, the c_n are dimensionless constants and we have included a cosmological constant Λ for full
 104 generality. If $L = L_p$, the Planck length, then the new terms in the extended theory (6) are negligible on

105 macroscopic scales. They need only be considered if curvature is significant at the Planck length scale.
 106 It might be hoped that no macroscopic effects would ensue in that case, but this is not guaranteed
 107 because of the nonlinearities.

108 A fourth-order classical theory was introduced by Stelle [10], in the hope that the quantized
 109 version of the theory would be renormalizable. A 4th-order classical theory of the same type was
 110 studied by Ruzmaikina and Ruzmaikin [11] for application to cosmology. More recently, a thorough
 111 study of 4th order gravity by Lu *et al.* [12], which we shall rely on heavily, was enabled by the advent
 112 of algebraic calculation tools like Mathematica. Though Lu *et al.* stated that the principal intended
 113 application was to compute quantum corrections, their results are applicable in a purely classical
 114 domain.

115 3. Equivalent of the quantum potential in 4th order gravity

116 A generally covariant description of the quantum potential term must exist to satisfy the principle
 117 of relativity. The same geometrodynamics and the same quantum potential will describe both standard
 118 black-hole evaporation and a steady state of the black hole or bridge. So we seek extensions of
 119 Einstein's equations that would produce the quantum potential term classically. As with a potential
 120 barrier in simple quantum mechanics, the only way that the resulting classical equations would
 121 permit "tunneling" through a modified ER bridge is for the modifications to make the bridge formally
 122 traversable. That is indeed what we will find. (The impossibility of super-luminal signaling will be
 123 discussed briefly in the concluding section.)

The most general 4th order extension of Einsteinian general relativity [12] is given by a Lagrangian density of the form $\mathcal{L} = \sqrt{-g}[\gamma R - \alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta R^2]$ where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, formed by removing non-vanishing contractions from the curvature tensor $R_{\mu\nu\rho\sigma}$. This Lagrangian density can also be written:

$$\mathcal{L} = \sqrt{-g} [\gamma R - 2\alpha R_{\mu\nu}R^{\mu\nu} + (\beta + \frac{2\alpha}{3})R^2] \quad (7)$$

124 by using the topological invariance of a quantity specified in the Gauss-Bonnet theorem [12].

125 An ADM-type formulation of 4th order gravity is derived as for ordinary general relativity.
 126 The Gauss-Codazzi relations [13,14] are used to express the Riemann tensor, the Ricci tensor and
 127 the Ricci scalar in terms of the corresponding objects for 3-dimensional space-like hypersurfaces,
 128 denoted ${}^{(3)}R_{\mu\nu\rho\sigma}$ etc., and the extrinsic curvature K_{ab} of those hypersurfaces, together with lapse
 129 and shift variables that describe the flow of time and the point-to-point connections, respectively,
 130 between hypersurfaces. The canonical momentum p_{ab} that is conjugate to h_{ab} is $p_{ab} = \partial\mathcal{L}/\partial\dot{h}_{ab}$.
 131 The Hamiltonian density is constructed from $\mathcal{H}(\mathbf{p}, \mathbf{q}) = \mathbf{p} \cdot \dot{\mathbf{q}} - \mathcal{L}(\mathbf{q}, \dot{\mathbf{q}})$, with $\mathbf{q} = \mathbf{h}$, by writing the
 132 kinematic term $\mathbf{p} \cdot \dot{\mathbf{h}}$ also in terms of the extrinsic curvature as with ordinary gravity, and expressing the
 133 extrinsic curvature in terms of the canonical momenta [14]. The lapse appears as a Lagrange multiplier
 134 of the hamiltonian constraint, as in the usual ADM approach. Let us write the hamiltonian constraint
 135 in the form:

$$F(\mathbf{p}, \dot{\mathbf{p}}, \ddot{\mathbf{p}}) - \sqrt{h} [\gamma {}^{(3)}R - 2\alpha {}^{(3)}R_{ab}{}^{(3)}R^{ab} + (\beta + \frac{2\alpha}{3}) ({}^{(3)}R)^2] + \mathcal{H}_{matter} = 0 \quad (8)$$

136 where the function $F(\mathbf{p}, \dot{\mathbf{p}}, \ddot{\mathbf{p}})$ depends also on curvature, to isolate the \mathbf{p} -independent terms. (Greek
 137 indices assume values in $\{0, 1, 2, 3\}$ while latin indices are restricted to $\{1, 2, 3\}$.)¹

¹ The function $F(\mathbf{p}, \dot{\mathbf{p}}, \ddot{\mathbf{p}})$ for the case $\alpha = 0$ is given explicitly in terms of extrinsic curvature in Ref. [15].

This constraint is to be compared to the one for ordinary gravity, with quantum potential added, which is

$$G_{ijkl} p^{ij} p^{kl} - \sqrt{h} \gamma {}^{(3)}\mathcal{R} - \sqrt{h} \frac{k^2}{2r_0^2} + \mathcal{H}_{matter} = 0 \quad (9)$$

138 from which equation (3) is obtained by substituting $p_{ab} \rightarrow \partial S / \partial h_{ab}$. Eq. (8) is generally 4th-order in
 139 the canonical momenta while the corresponding equation for ordinary gravity (9) is 2nd order, but
 140 we claim that we can ignore the terms in p . That is because for static metrics such as we will consider
 141 here, hypersurfaces of constant time have zero extrinsic curvature $K_{ab} \equiv \nabla_a n_b = 0$, where n is a
 142 time-like unit vector field normal to the surface, and ∇ is the projection of the covariant derivative
 143 onto the hypersurface. For such metrics we can take $n(x) = (n_t, 0, 0, 0)$. The Lagrangian density \mathcal{L}
 144 can be written as a sum of terms with factors of the intrinsic curvature ${}^{(3)}\mathcal{R}_{ab}$ or its contraction ${}^{(3)}\mathcal{R}$,
 145 and terms with factors that are quadratic in the extrinsic curvature K_{ab} or that are time derivatives of
 146 K_{ab} , since the metric is Gaussian normal [14] as restricted to the horizon. Using $\partial {}^{(3)}\mathcal{R}_{ij} / \partial \dot{h}_{ab} = 0$ and
 147 $\partial K_{ab} / \partial \dot{h}_{ab} = 1/2l$, where l is the time-independent lapse, and $K_{ab} = 0$, we find that $p_{ab} = \partial \mathcal{L} / \partial \dot{h}_{ab} = 0$.
 148 So the kinematic term $p \cdot \dot{h}$ vanishes and since $K_{ab} = 0$, the Lagrangian contributes only two new terms,
 149 in ${}^{(3)}\mathcal{R}_{ab} {}^{(3)}\mathcal{R}^{ab}$ and $({}^{(3)}\mathcal{R})^2$, and so $F = 0$. The steady states of the dynamics defined by the Hamiltonian
 150 (including the matter terms), as would include ER bridges or wormholes prior to collapse, are the
 151 same in quantized ordinary general relativity and in classical 4th order gravity, if near the horizon:

$$\begin{aligned} (\gamma - \gamma_{GR}) {}^{(3)}\mathcal{R} - 2\alpha {}^{(3)}\mathcal{R}_{ab} {}^{(3)}\mathcal{R}^{ab} \\ + (\beta + 2\alpha/3) ({}^{(3)}\mathcal{R})^2 = -\frac{k^2}{2r_0^2} \end{aligned} \quad (10)$$

152 where $\gamma_{GR} = 1/16\pi G$ is the canonical value of γ . (The usual ‘‘supermomentum’’ part of the total
 153 Hamiltonian vanishes for $p = 0$.) It is noted that the negative sign of the quantum potential Q , viewed
 154 as a contribution to the energy density at the horizon, is what is required of the ‘‘exotic matter’’ that
 155 could serve to violate the averaged weak energy condition and to keep a traversable wormhole open
 156 [16].

For black holes and Einstein-Rosen bridges, we generally have $R_{\mu\nu}, R = O(1/r_0^2)$. The requirement of Eq. (10) is thus that the $1/r_0^4$ terms in the expressions that are quadratic in curvature, ${}^{(3)}\mathcal{R}_{ab} {}^{(3)}\mathcal{R}^{ab}$ and $({}^{(3)}\mathcal{R})^2$ cancel, and that these quadratic curvature expressions, taken together, are $O(1/r_0^2)$. We consider spherically symmetric, time-independent metrics of the form:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \quad (11)$$

157 with the spatial metric on hypersurfaces of constant t given trivially by: $h_{ij} = g_{ij}$. For a metric of this
 158 form, the indices 0, 1, 2, 3 are more specifically written as t, r, θ, ϕ , respectively. The Ricci tensor on the
 159 hypersurfaces is diagonal with three non-vanishing elements:

$$\begin{aligned} {}^{(3)}\mathcal{R}_{rr} &= -\frac{1}{r} \frac{A'}{A} \\ {}^{(3)}\mathcal{R}_{\theta\theta} &= -1 - \frac{A'}{2A^2} + \frac{1}{A} \\ {}^{(3)}\mathcal{R}_{\phi\phi} &= \sin^2\theta \left(-1 - \frac{A'}{2A^2} + \frac{1}{A} \right) \end{aligned} \quad (12)$$

where primes denote derivatives with respect to r . Denoting $A^{-1}(r_0) {}^{(3)}\mathcal{R}_{rr}(r_0) \equiv a$, $r_0^{-2} {}^{(3)}\mathcal{R}_{\theta\theta}(r_0) \equiv b$, and $r_0^{-2} \sin^2\theta {}^{(3)}\mathcal{R}_{\phi\phi}(r_0) \equiv c$, we have at the horizon:

$${}^{(3)}\mathcal{R} = a + b + c \quad (13)$$

$${}^{(3)}\mathcal{R}_{ij} {}^{(3)}\mathcal{R}^{ij} = a^2 + b^2 + c^2 \quad (14)$$

160 Assuming $a, b, c \sim 1/r_o^2$, the requirement that the quadratic curvature terms mimic the quantum
161 potential is that the terms of leading order in $1/r_o$ vanish, and that:

$$\begin{aligned} -2\alpha {}^{(3)}R_{ab} {}^{(3)}R^{ab} + \left(\beta + \frac{2\alpha}{3}\right) ({}^{(3)}R)^2 &= \\ -2\alpha(a^2 + b^2 + c^2) + (\beta + 2\alpha/3)(a + b + c)^2 &= \\ -\left(\frac{4\alpha}{3} - \beta\right)a^2 - 4\left(\beta - \frac{2\alpha}{3}\right)b^2 + 4\left(\beta + \frac{2\alpha}{3}\right)ab &= \\ &= \frac{-k^2}{2r_o^2} \end{aligned} \quad (15)$$

162 having noticed that $b = c$, and having taken $\gamma = \gamma_{GR}$, as we will henceforth for agreement with general
163 relativity at large scales.

164 We have solved for the metric (11) in terms of $A(r)$ and $B(r)$, in series expansions, following the
165 method of Lu *et al.* [12]:

$$\frac{1}{A(r)} \equiv f(r) = f_1(r - r_o) + f_2(r - r_o)^2 + \dots \quad (16a)$$

$$B(r) = b_o + b_1(r - r_o) + b_2(r - r_o)^2 + \dots \quad (16b)$$

166 There are indeed wormhole solutions with $A(r) = \infty$ at the horizon $r = r_o$, as with a Schwarzschild
167 black hole, but $B(r_o) \neq 0$, unlike a black hole in ordinary gravity or in 4th order gravity [17]. Such
168 traversable wormholes were studied by Lu *et al.* [12] in the restricted case $\beta = 0$. Here, $\beta \neq 0$ is needed
169 so that there are non-trivial solutions to (15) with $\alpha \neq 0$ and different from ordinary gravity. The 4th
170 order theory admits traversable wormhole solutions, as in [12], but with more free parameters than for
171 the $\beta = 0$ case. Quantum tunneling is thus accounted for classically.

172 The equations of motion are derived by minimizing the action $I = \int d^4x \sqrt{-g} \mathcal{L}$ formed from the
173 Lagrangian density (7). They are:

$$\begin{aligned} 0 &= \frac{1}{\sqrt{-g}} \frac{\delta I}{\delta g^{\mu\nu}} \equiv H^{\mu\nu} \\ &= -\gamma(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) - \frac{2}{3}(\alpha - 3\beta)R_{;\mu;\nu} \\ &\quad + 2\alpha g^{\rho\sigma}R_{\mu\nu;\rho;\sigma} - \frac{1}{3}(\alpha + 6\beta)g_{\mu\nu}g^{\rho\sigma}R_{;\rho;\sigma} \\ &\quad + 4\alpha R^{\rho\sigma}R_{\mu\rho\nu\sigma} + 2\left(\beta + \frac{2}{3}\alpha\right)RR_{\mu\nu} \\ &\quad + \alpha g_{\mu\nu}R^{\rho\sigma}R_{\rho\sigma} - \frac{1}{2}g_{\mu\nu}\left(\beta + \frac{2}{3}\alpha\right)R^2 \end{aligned} \quad (17)$$

174 where a semicolon denotes covariant differentiation with respect to the index following. One first
175 substitutes the spherically symmetric, static metric form (11) in the usual expressions for the Ricci
176 tensor (see e.g. [18]), to get 4-dimensional expressions for $R_{\mu\nu}$ that are extensions of (12), and inserts
177 these expressions in (17) to derive differential equations for $A(r)$ and $B(r)$. Combining the equations
178 for H^{tt} and H^{rr} (sufficient to determine $H^{\theta\theta}$ and $H^{\phi\phi}$ as well, because of a Bianchi identity) in the
179 manner of Ref. [12], one derives a pair of third-order differential equations for $A(r)$ and $B(r)$ that are

180 found to match those given in the Appendix to [12]. The series expansions (16) were substituted in
181 those equations and solved for the lowest order coefficients in Mathematica. We find:

$$f_1 = \frac{1}{3} \sqrt{\frac{-2(\alpha - 3\beta)(2\alpha - 6\beta - 3\gamma r_0^2)}{3\alpha\beta r_0^2}} \quad (18a)$$

$$b_1/b_0 = \frac{2(\alpha + 6\beta)}{(\alpha - 3\beta)r_0} \quad (18b)$$

182 The condition $b_0 > 0$ implies that the solution is a wormhole, as in [12]. But unlike the $\beta = 0$
183 case, we find that b_2 is unconstrained, and therefore can be tuned for asymptotic flatness. It can be
184 checked that the extra terms in (17), as compared to the usual second-order Einstein equations, give an
185 effective negative energy contribution, when averaged along a null geodesic through the interior of
186 the wormhole, that violates the averaged weak energy condition, as required for traversability.

187 To satisfy the equivalence condition (15), we first note that the curvature variables $a =$
188 $A^{-1}(r_0) {}^{(3)}R_{rr}(r_0)$ and $b = c = r_0^{-2} {}^{(3)}R_{\theta\theta}(r_0)$, where ${}^{(3)}R_{ij}$ is given by (12), depend only on the leading
189 coefficient f_1 :

$$a = \frac{f_1}{r_0}$$

$$b = c = -\frac{1}{r_0^2} + \frac{f_1}{2r_0} \quad (19)$$

190 We will use the solution (18a) for f_1 in terms of α and β to compute the quadratic curvature contribution
191 (15), which we now denote $V(r_0)$.

$$V(r_0) \equiv -\left(\frac{4\alpha}{3} - \beta\right) (a(\alpha, \beta))^2 - 4\left(\beta - \frac{2\alpha}{3}\right) (b(\alpha, \beta))^2$$

$$+ 4\left(\beta + \frac{2\alpha}{3}\right) a(\alpha, \beta)b(\alpha, \beta) \quad (20)$$

192 Substituting from (19) and (18), the condition for equivalence of $V(r_0)$ to the de Barros *et al.* quantum
193 potential becomes:

$$V(r_0) \equiv \frac{2}{81r_0^4\omega} \left\{ 3\gamma_{GR}r_0^2(36 - 15\omega + \omega^2) \right.$$

$$+ \beta \left[216 - 2\omega^3 \right.$$

$$+ 36\omega \left(-9 + \sqrt{\frac{6(-3 + \omega)(6\beta + 3\gamma_{GR}r_0^2 - 2\beta\omega)}{\beta\omega}} \right)$$

$$\left. \left. + 6\omega^2 \left(15 + \sqrt{\frac{6(-3 + \omega)(6\beta + 3\gamma_{GR}r_0^2 - 2\beta\omega)}{\beta\omega}} \right) \right] \right\}$$

$$= -\frac{k^2}{(2r_0^2)} \quad (21)$$

194 where we have expressed V in terms of β and $\omega \equiv \alpha/\beta$ for convenience.

The coefficient of the second term in the braced expression, β , must vanish to avoid a $1/r_0^4$ contribution for small r_0 . Then the first term gives

$$2\gamma_{GR}(36 - 15\omega + \omega^2)/(27\omega) = -\frac{k^2}{2} \quad (22)$$

195 a quadratic equation that can be solved for ω , for given k . Since the same geometrodynamics
 196 must describe black-hole evaporation, a value of the eigenvalue k can be obtained by matching
 197 the black-hole evaporation rate as computed by deBarros *et al.* [7], $k^2/4M^2$, to the Hawking rate
 198 [19] $P = \hbar c^6 / (15360\pi G^2 M^2)$. This gives $k = \sqrt{\hbar c^6 / 3840\pi G^2}$, or in units for which $\hbar = c = G = 1$,
 199 $k \approx 0.01$. In the limit $\beta \rightarrow 0$, the desired value of the ratio of the coefficients $\omega = \alpha/\beta$ solves (22) and is
 200 found to be $\alpha/\beta = \omega = 3.009$ or $\alpha/\beta = 11.962$. For α/β fixed at these values, the contribution of the
 201 quadratic curvature terms is plotted vs. r_o in Fig. 2 as the two coefficients each approach zero. As we
 202 approach the limit, the quadratic curvature V contribution matches the quantum potential $-k^2/(2r_o^2)$
 203 down to smaller and smaller radii r_o , but very large values of V are found for a narrowing range
 204 around $r_o = 0$. We note that the large potentials could conceivably help in avoiding divergences due to
 205 an arbitrarily large tendency to form arbitrarily narrow wormholes. Otherwise the quantum potential
 206 can be matched as closely as desired for sufficiently small coefficients.

207 4. Discussion

208 The extension of ER=EPR that is required here is only slight. The authors of that conjecture, who
 209 started with macroscopic black holes and bridges, did not consider quantum tunneling. Alice, at one
 210 end of such a bridge, can send any kind of object or signal through the bridge (including a firewall)
 211 that Bob will encounter as soon as he crosses the horizon from the other end. But macroscopic objects
 212 cannot reach him before that time, either classically or through tunneling, just as they cannot tunnel
 213 through a potential barrier in simple quantum mechanics. The suggested impact of tunneling is for
 214 Planck-scale configurations, where it is enough to take us from the Copenhagen interpretation to the
 215 ontological one, in one sense a slight change.

216 For black holes or non-traversable ER bridges of macroscopic size, the quadratic curvature terms
 217 in the above analysis have no significant effect on the dynamics that is different from that of the
 218 quantum potential, or therefore, of the usual quantum theory. Indeed, the potential that follows
 219 from the quadratic curvature terms is unique, and does not depend on a choice of eigenstate as
 220 in the treatment of Tomimatsu [9] where classical (non-radiating) black hole behavior occurs for
 221 another wavefunction solution. So the ambiguity reported by deBarros *et al.* [7] regarding black hole
 222 evaporation in a quantum potential treatment, is avoided if the potential comes from extra classical
 223 curvature terms. An absence of macroscopic effect might have been expected if the coefficients α
 224 and β (which have the dimensions [length]²) had Planck-scale values. That these coefficients can be
 225 infinitesimal strengthens the argument that the macroscopic predictions of ordinary general relativity
 226 are preserved.

227 At sub-Planck scales, where $r_o \ll L_p$, on the other hand, the potential arising from the quadratic
 228 curvature terms can deviate strongly from that predicted by quantum theory, for any values of α and β
 229 that remain finite. Differences could have detectable physical consequences.

230 In regard to verifiable consequences, we also note that the quantum potential depends on specific
 231 combinations of the matter fields with gravity. The quantum potential (5) used here arises from a single
 232 scalar field in the matter portion of the hamiltonian, but in a traditional EPR pair, an electromagnetic
 233 field is used in the observation of charged spinor particles. It is noteworthy that wormhole solutions
 234 with electric charge in a modified theory that includes small quadratic curvature terms with a fixed
 235 ratio have been previously reported [20]. In general, allowable combinations of matter fields might be
 236 constrained by a supersymmetry required to maintain the equivalence between the quantum potential
 237 due to the matter fields and contributions from the extra curvature terms. The prescribed combinations
 238 could be compared with the spectrum of particles known to populate the vacuum.

239 Gravity itself has not been quantized in the configuration studied here, since the quantum
 240 potential (5) comes only from the matter portion of the Hamiltonian through Dirac replacement of
 241 the scalar field momenta P_Φ by operators. It is interesting that a previous attempt to quantize gravity,
 242 using the asymptotic safety approach [21] to defining renormalizable equations with higher derivatives
 243 has yielded similar results: 4th order equations involving very small coefficients with fixed ratios and a

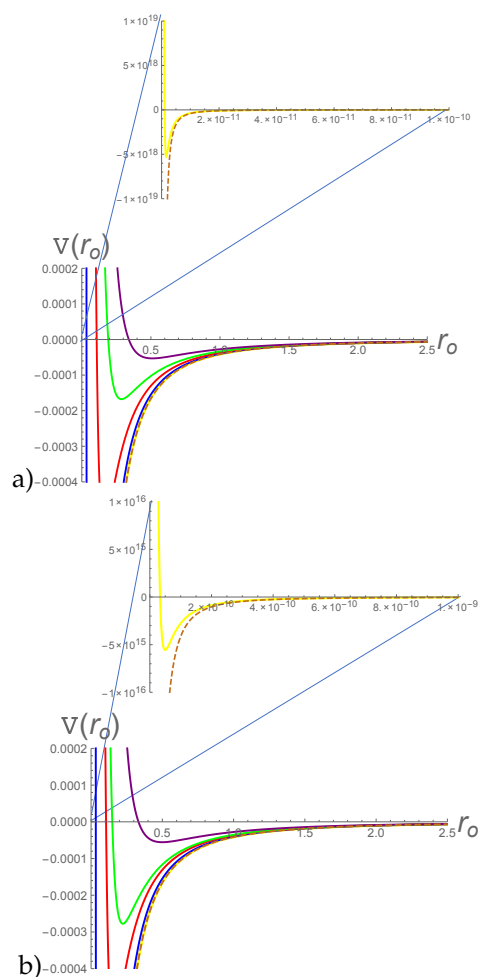


Figure 2. The contribution of the quadratic curvature terms to the hamiltonian constraint, vs. wormhole radius r_o , for a) decreasing values of β : $\beta = 10^{-7}$ (purple), $10^{-7.5}$ (green), 10^{-8} (red), 10^{-9} (blue), and 10^{-30} (yellow), while the ratio of the coefficients is held fixed at the first solution to (22), $\alpha/\beta \approx 3.009$; and for b) values of β : $\beta = 10^{-10}$ (purple), 2×10^{-11} (green), 10^{-11} (red), 10^{-12} (blue), and 10^{-30} (yellow), while the ratio is held at the second solution, $\alpha/\beta \approx 11.962$. The quadratic curvature contribution V converges to the quantum potential $-k^2/(2r_o^2)$ (dashed line) as $\beta \rightarrow 0$ in both cases, except for large positive values for a decreasing range of small r_o , as shown for the smallest value $\beta = 10^{-30}$ (yellow line) in expanded views about the origin.

244 steady state black hole solution [22]. The relationship between such attempts at a full quantum theory
245 of gravity and our treatment of the scalar field in a fixed space-time remains to be studied.

246 The focus of our analysis has been on steady entangled states. Collapse of the wormhole has
247 not been discussed. It is thought that the fourth-order time derivatives in the classical hamiltonian
248 constraint (8), away from the steady state, could describe a physical process of wormhole/wavefunction
249 collapse on very short time scales. Unlike the situation with a macroscopic wormhole, the entry
250 of a single particle would trigger collapse, preventing [23] the problems due to closed time-like
251 curves through macroscopic wormholes and the resulting vacuum polarization divergence that was
252 previously debated between Kim and Thorne [24] and Hawking [25]. Hawking's argument still
253 precludes macroscopic wormholes. For a Planck scale wormhole, the collapse itself (with some
254 sensitivity to the orientation of the remote measuring field, as required by Bell's theorem) could indeed
255 be the only effect transmitted. And in the absence of a Maxwell's Demon, one could not effectively
256 control the collapse so as to transmit information. That is, one does not have knowledge of the detailed
257 state at either end and considers only a statistical ensemble of states. That the statistical properties are
258 unchanged by the nonlocal effects of measurement is also what bars the use of the quantum potential
259 for superluminal signaling in Bohm's original interpretation [8].

260 It is tempting to elevate the proposed role of traversable wormholes in mediating entanglement to
261 a full interpretation of quantum mechanics. Planck's constant would emerge if any physical processes
262 select a preferred width for the wormholes that is $O[1]$ in non-dimensional units. One candidate for
263 such a process is just the weak vacuum polarization divergence [24,25] that precludes macroscopic
264 widths. But in addition to the need for a description of collapse, one would also need to show that all
265 of quantum theory follows from nonlocal entanglement conjoined with some properly constructed,
266 local classical theory.

267 While we have not yet shown that nonlocal entanglement is everything, it is certainly central. It is
268 hypothesized that at least the non-local part of the quantum potential, in the general situation, arises
269 from the new, multiply connected topology. As with an Einstein-Rosen bridge, entangled particles
270 connected by a traversable wormholes with r_0 of Planck scale are not separated within the wormhole
271 and are in a sense the same particle, until the connecting geometry collapses. The resulting negation of
272 the usual concept of the continuum establishes a new order in the physical world, best described in an
273 atomized spacetime together, arguably, with non-standard metrics in state space [26] and an ontology
274 given by nonlocally defined basis states that are not easily specified [27]. The view taken here is that an
275 account of how the new order forms from the intrinsically required breakdown of the classical order,
276 through gravitational instability of the populated vacuum at microscale, could ultimately yield a more
277 complete, more detailed and more predictive version of quantum theory.

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280 **Conflicts of Interest:** Conflicts of Interest

281 The author declares no conflict of interest.

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- 283 1. Bohm, D. A suggested interpretation of the quantum theory in terms of "hidden" variables, I and II. *Phys.*
284 *Rev.* **1952**, *85*, 166–193.
- 285 2. Bell, J.S. On the Einstein-Podolsky-Rosen paradox. *Physics* **1964**, *1*, 195–200.
- 286 3. Maldacena, J.; Susskind, L. Cool horizons for entangled black holes. *Fortschr. Phys.* **2013**, *61*, 781–811.
- 287 4. Penrose, R. *The Emperor's New Mind*; Oxford University Press: Oxford, UK, 1989.
- 288 5. Parikh, M.K.; Wilczek, F. Hawking radiation as tunneling. *Phys. Rev. Lett* **2000**, *85*, 5042–5045.
- 289 6. De Barros, J.A.; Pinto-Neto, N. The causal interpretation of quantum mechanics and the singularity problem
290 and time issue in quantum cosmology. *Int. J. Mod. Phys. D* **1998**, *7*, 201–213.
- 291 7. De Barros, J.A.; Oliveira-Neto, G.; Vale, T.B. Bohmian trajectories for an evaporating blackhole. *Phys. Lett.A*
292 **2005**, *336*, 324–330.

- 293 8. Bohm, D.; Hiley, B.J. *The Undivided Univers e*; Routledge: London, UK, 1992.
- 294 9. Tomimatsu, A. Evaporating black holes in quantum gravity. *Phys. Lett. B* **1992**, *289*, 283–286.
- 295 10. Stelle, K.S. Renormalization of higher-derivative quantum gravity. *Phys. Rev. D* **1977**, *16*, 953–969.
- 296 11. Ruzmaikina, T.V. ; Ruzmaikin, A.A. Quadratic corrections to the Lagrangian density of the gravitational field
297 and the singularity. *Sov. Phys. JETP* **1970**, *30*, 372–374.
- 298 12. Lu, H.; Perkins, A.; Pope, C.N.; Stelle, K.S. Spherically symmetric solutions in higher-derivative gravity. *Phys.*
299 *Rev. D* **2015**, *92*, 124019.
- 300 13. Eisenhart, L.P. *Riemannian Geometry*; Princeton Univ. Press, Princeton, New Jersey 1926.
- 301 14. Misner, C.W.; Thorne, K.S.; Wheeler, J.A. *Gravitation*; W.H. Freeman, San Francisco 1973.
- 302 15. Cotsakis, S.; Kadry, S.; Trachilis, D. The regular state in higher-order gravity. *Int. J. Mod. Phys. A* **2016**, *31*,
303 1650130.
- 304 16. Morris, M.S.; Thorne, K.S. Wormholes in spacetime and their use for interstellar travel: A tool for teaching
305 general relativity. *Am. J. Phys.* **1988**, *56*, 395–412..
- 306 17. Lu, H.; Perkins, A.; Pope, C.N.; Stelle, K.S. Black holes in higher derivative gravity. *Phys. Rev. Lett.* **2015**, *114*,
307 171601.
- 308 18. Weinberg, S. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*; Wiley:
309 New York, NY, USA, 1972.
- 310 19. Hawking, S.W. Black-hole explosions. *Nature* **1974**, *248*, 30–31.
- 311 20. Olmo, G.J.; Rubiera-Garcia, D.; Sanchis-Alepuz, H. Geonic black holes and remnants in Eddington-inspired
312 Born-Infeld gravity. *Eur. Phys. J. C* **2014**, *74*, 2804.
- 313 21. Weinberg, S. Critical phenomena for field theorists, in *Understanding the Fundamental Constituents of Matter*, ed.
314 A. Zichichi; Plenum Press: New York, 1977.
- 315 22. Cai, Y.-F.; Easson, D.A. Black holes in an asymptotically safe theory with higher derivatives. *J. Cosmology and*
316 *Astroparticle Phys.* **2010**, *9*, 002.
- 317 23. Duane, G.S. Synchronicity from synchronized chaos. *Entropy* **2015**, *17*, 1701–1733.
- 318 24. Kim, S.-W.; Thorne, K.S. Do vacuum fluctuations prevent the creation of closed timelike curves? *Phys. Rev. D*
319 **1991**, *43*, 3929–3947.
- 320 25. Hawking, S.W. The chronology protection conjecture. *Phys. Rev. D* **1992**, *46*, 603–611.
- 321 26. Palmer, T.N. Experimental non-violation of the Bell inequality. *Entropy* **2018**, *20*, 356.
- 322 27. 't Hooft, G. *The Cellular Automaton Interpretation of Quantum Mechanics*; Springer: Berlin, Germany, 2016.