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Article

Quantum Nonlocality from Higher-Derivative Gravity

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Abstract: A classical origin for the quantum potential, as that potential term arises in the quantum mechanical treatment of black holes and Einstein-Rosen (ER) bridges, can be based on 4th-order extensions of Einstein's equations. The 4th-order extension of general relativity required to generate a Bohmian quantum potential is given by adding quadratic curvature terms with coefficients that maintain a fixed ratio, as their magnitudes approach zero. Black hole radiation, and the analogous process of quantum transmission through an ER bridge, can then be described classically. Quantum transmission through the classically non-traversable bridge is replaced by classical transmission through a traversable wormhole. If entangled particles are connected by a Planck-width ER bridge, as conjectured by Maldacena and Susskind, then the classical wormhole transmission effect gives the ontological nonlocal connection between the particles posited in Bohm's interpretation of their entanglement. It is hypothesized that higher-derivative extensions of classical gravity can account for the nonlocal part of the quantum potential generally.

Keywords: 4th order gravity, quantum potential, ER=EPR, wormholes

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1. Motivation and Introduction

Nonlocality is a prominent feature of quantum mechanics both in the standard, probabilistic Copenhagen interpretation and in the deterministic re-interpretation by Bohm [1]. In the former case, the spin of each member of an EPR pair, along any measurement axis, is determined at the time of measurement of either member's spin along that axis, despite any space-like separation, however large, between the measurement apparatus and the remote spin. Bell's Theorem [2] precludes any understanding of this result in terms of spin values that exist at the time of creation of the pair, independently of observation. In Bohm's interpretation, the effect of measurement on the remote spin is taken to be a real physical effect, mediated by a quantum potential.

With either interpretation, one is left to ask if any sort of generalization of the classical, and relativistic, notion of locality can be articulated that does not depend on the full apparatus of standard quantum theory. If observations trigger the collapse of a globally defined wave packet, as in the Copenhagen interpretation, can one be more specific about the aspect of the observational process that is responsible for the collapse? If one adopts Bohm's quantum potential interpretation, can such a non-local potential be linked to interactions that are familiar in classical physics? One must account for the impossibility of non-local transmission of information either through the alleged effect of observation or through the quantum potential.

A picture of quantum nonlocality emerges from the Maldacena-Susskind "ER=EPR" conjecture that was originally put forward to resolve the black-hole information loss paradox [3]. We do not

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review the original motivation for ER=EPR, which involved entangled macroscopic black holes, but simply note that what emerged was a suggestion that any pair of entangled particles is connected by an Einstein-Rosen (ER) bridge of Planck-scale width. Non-locality, in the proposed view, is mediated by ER bridges, which intimately link space-like separated regions. Indeed, the members of an EPR pair have zero separation, as computed along a path through the bridge and thus are, in effect, the same particle. An Einstein-Rosen bridge is an unfolding of a black hole in a multiply-connected space time topology, so information that enters the bridge is not transmitted for the same reason that it cannot escape a black hole. That would explain why the non-local connections cannot be used for signaling.

It is not clear how Einstein-Rosen bridges, while offering a more specific and ontological picture, could lead to the kind of nonlocality that would be induced by observations Copenhagen-style. But it is possible to compare the physical effects of such bridges with those of the quantum potential in the Bohm interpretation. Effects like those of the quantum potential would require that the bridges be traversable, in order for the measurement outcome for one member of an EPR pair to feel the orientation of the measurement apparatus for its partner, as required by Bell's Theorem. Classical ER bridges are not traversable. However we note that just as quantum black holes emit radiation, quantum ER bridges are crudely traversable. There are severe restrictions on the kind of information that can be re-emitted in one case, or transmitted, in the other, but neither effect requires essentially new physics that is beyond general relativity or quantum mechanics.

Further, if the Bohm interpretation can be applied to field theory and to gravity in particular, then the crude traversability of quantum ER bridges should itself be explicable in terms of a quantum potential added to classical general relativity. Thus an extension of general relativity could account for quantum entanglement. Here, we assume that general relativity describes phenomena down to well below the Planck scale L_P. In bootstrap fashion, the quantum behavior of black holes and ER bridges would provide an explanation for quantum nonlocality, at least as it appears strikingly in EPR correlations. A classical re-interpretation of the quantum ER bridge behavior would then also explain nonlocal quantum entanglement generally. But what kind of geometrodynamics would give rise to the quantum potential required in the specific case of an ER bridge? The purpose of this short paper is to examine an extension of Einstein's equations that will account for the extra term and will thus allow us to interpret the quantum ER transmission effect classically.

The proposed ontological account of quantum nonlocality tends to resolve the contradiction between quantum mechanics and general relativity in a way that gives primacy to the latter. It can be compared to Penrose's suggestion that the collapse of the wavefunction is a gravitational effect [4] in this regard. After reviewing both the quantum potential interpretation of black hole radiation and classical higher derivative gravity in the next section, we compare the two in the following section. We derive a result that only infinitesimal higher-derivative corrections are required (so that the standard second-order theory is a singular limit). Implications for a re-interpretation of quantum theory generally are discussed in the final section.

2. Background

2.1. Quantum potential interpretation of black hole radiation

If black hole radiation, like other quantum processes, can be described by an objective deterministic theory, it should be possible to augment Einstein's classical equations with terms that give such radiation, which can indeed be characterized as a form of tunneling. A quantum potential for black hole radiation was in fact derived by deBarros *et al.* [6,7] in work that has not been widely cited. Recall that for a simple quantum mechanical system with a wave-function Ψ written in polar form as $\Psi = \mathcal{R}\exp(i\mathcal{S})$, the quantum potential is $Q = -(\nabla^2\mathcal{R})/2m\mathcal{R}$, meaning that the Schrodinger

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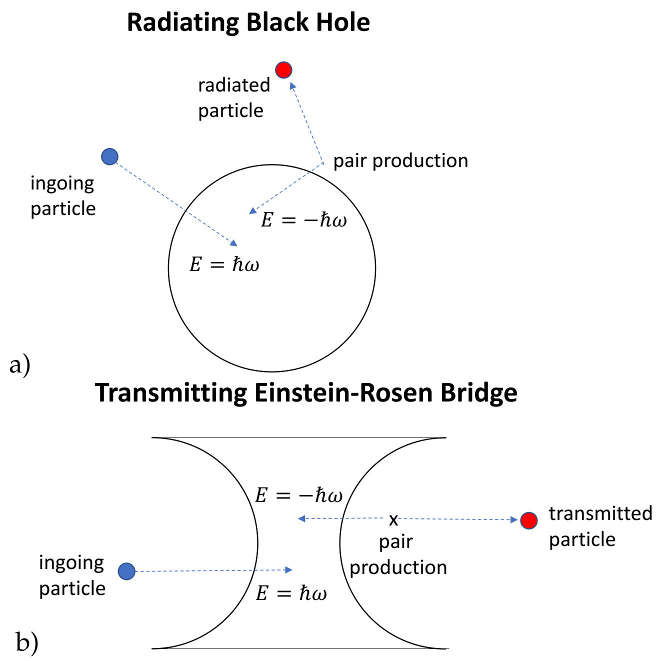

Figure 1. Steady states of a) a radiating black hole, and b) a transmitting Einstein-Rosen bridge, result from pair production just outside the horizon and ongoing classical ingestion.

equation for Ψ implies that particle motion is governed by the Hamilton-Jacobi equation with an extra potential term:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{1}{2m}\frac{\nabla^2 \mathcal{R}}{\mathcal{R}} = 0 \tag{1}$$

In the current context, the quantum potential is gleaned from the Wheeler-Dewitt equation $\hat{H}\Psi = 0$, obtained by quantizing the hamiltonian constraint in the ADM formulation of general relativity, in which spacetime is foliated into a temporal sequence of spacelike hypersurfaces. For a general wave-functional Ψ of the spatial metric on a hypersurface and of the matter fields, this equation is:

$$[G_{ijkl}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}} - \sqrt{h}\ {}^{(3)}R(\boldsymbol{h}) + H_{\text{matter}}]\Psi(\boldsymbol{h}, \text{matter}) = 0 \tag{2}$$

where h is the determinant of the space metric h_{ij}, $G_{ijkl} \equiv \frac{1}{2}h^{-1/2}(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl})$ and ${}^{(3)}R$ is the intrinsic curvature of the evolving space-like hypersurface. Inserting the polar decomposition $\Psi = \mathcal{R}\exp(i\mathcal{S})$ in (2), we get:

$$\begin{aligned} G_{ijkl}\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}} \quad &- \quad \sqrt{h}\ {}^{(3)}R(\boldsymbol{h}) + \sqrt{h}\, Q \\ &+ \quad \text{matter terms} = 0 \end{aligned} \tag{3}$$

for a quantum potential Q that is defined in terms of derivatives of $\mathcal{R}$ both with respect to the metric and with respect to the matter fields. For $Q = 0$, Eq. (3) is the Hamilton-Jacobi equation for the usual action S in the ADM formalism. For $Q \neq 0$, the dynamics are modified, but all fields still follow well-defined trajectories, as for a single particle in the Bohmian interpretation.

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Tomimatsu [5] solved the Wheeler-Dewitt equation (2) for a spherically symmetric case, using boundary conditions that describe a scalar field in an evaporating black-hole metric, limiting attention to the region near the apparent horizon. He found:

$$\Psi(r_o, \Phi) = C \exp[i\left(\frac{r_o}{4} + \frac{k^2}{2r_o}\right) - |k\Phi|] \quad (4)$$

giving the wave-functional $\Psi(r_o, \Phi)$ as a function of the instantaneous apparent black-hole radius r_o and the scalar field Φ at the apparent horizon, with k an eigenvalue to be determined. We take the wave functional to have the same form for a transmitting ER bridge, since only the topology is different from that of a black hole. Traversability can indeed be determined at the horizon. For the wave function (4), we have $\mathcal{R} = C \exp(-|k\Phi|)$, and so the quantum potential depends only on derivatives with respect to the scalar field and not with respect to the metric. As found by deBarros *et al.* [7], the term in the matter sector of the hamiltonian, $(1/2r_o^2)\hat{P}_\Phi^2 = -(1/2r_o^2)\delta^2/\delta\Phi^2$, contributes a quantum potential

$$Q = -\frac{k^2}{2r_o^2} \quad (5)$$

to the Hamilton-Jacobi equation (3). We assume here that the quantum evaporation/transmission process induced by the quantum potential (5) is balanced by a purely classical accretion/particle-entry process to maintain a steady state (Fig. 1). The change in boundary conditions needed to describe the added classical process will not affect the quantum potential.

2.2. Classical higher-derivative gravity

In this paper, we seek a generally covariant extension of Einstein's equations that gives a quantum potential of the form (5) in the corresponding generalization of the Wheeler-Dewitt equation for a radiating black hole in a steady state.. A simple candidate is constructed by adding terms containing higher derivatives of the metric to the standard second-order equations. While such higher-derivative extensions of general relativity have become familiar in the context of quantum corrections to general relativity, here we regard the resulting theory simply as an alternative to the standard classical theory.

A textbook derivation of Einstein's equations, e.g. [16], relies not only on general covariance, but on an explicit assumption that the equations are second order, or equivalently, that they are scale invariant. General relativity can in fact be extended to theories of the form:

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + g_{\mu\nu}\Lambda + \sum_{n>2} c_n L^{n-2} R_{\mu\nu}^{(n)} = 8\pi T_{\mu\nu} \quad (6)$$

where $R_{\mu\nu}^{(n)}$ is a quantity involving a total of n derivatives of the metric, L is a fundamental length scale, the c_n are dimensionless constants and we have included a cosmological constant Λ for full generality. If $L = L_P$, the Planck length, then the new terms in the extended theory (6) are negligible on macroscopic scales. They need only be considered if curvature is significant at the Planck length scale. It might be hoped that no macroscopic effects would ensue in that case, but this is not guaranteed because of the nonlinearities.

A fourth-order classical theory was introduced by Stelle [8], in the hope that the quantized version of the theory would be renormalizable. A 4th-order classical theory of the same type was studied by Ruzmaikina and Ruzmaikin [9] for application to cosmology. More recently, a thorough study of 4th order gravity by Lu *et al.* [10], which we shall rely on heavily, was enabled by the advent of algebraic calculation tools like Mathematica. Though Lu *et al.* stated that the principal intended application was to compute quantum corrections, their results are applicable in a purely classical domain.

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3. Equivalent of the quantum potential in 4th order gravity

A generally covariant description of the quantum potential term must exist to satisfy the principle of relativity. The same geometrodynamics and the same quantum potential will describe both standard black-hole evaporation and a steady state of the black hole or bridge. So we seek extensions of Einstein's equations that would produce the quantum potential term classically.

The most general 4th order extension of Einsteinian general relativity [10] is given by a Lagrangian density of the form $\mathcal{L} = \sqrt{-g}[\gamma R - \alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta R^2]$ where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, formed by removing non-vanishing contractions from the curvature tensor $R_{\mu\nu\rho\sigma}$. This Lagrangian density can also be written:

$$\mathcal{L} = \sqrt{-g}\,[\gamma R - 2\alpha R_{\mu\nu}R^{\mu\nu} + (\beta + \frac{2\alpha}{3})R^2] \tag{7}$$

by using the topological invariance of a quantity specified in the Gauss-Bonnet theorem [10].

An ADM-type formulation of 4th order gravity is derived as for ordinary general relativity. The Gauss-Codazzi relations [11,12] are used to express the Riemann tensor, the Ricci tensor and the Ricci scalar in terms of the corresponding objects for 3-dimensional space-like hypersurfaces, denoted ${}^{(3)}R_{\mu\nu\sigma\rho}$ etc., and the extrinsic curvature K_{ab} of those hypersurfaces, together with lapse and shift variables that describe the flow of time and the point-to-point connections, respectively, between hypersurfaces. The canonical momentum p_{ab} that is conjugate to h_{ab} is $p_{ab} = \partial\mathcal{L}/\partial\dot{h}_{ab}$. The Hamiltonian density is constructed from $\mathcal{H}(\boldsymbol{p},\boldsymbol{q}) = \boldsymbol{p}\cdot\dot{\boldsymbol{q}} - \mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}})$, with $\boldsymbol{q} = \boldsymbol{h}$, by writing the kinematic term $\boldsymbol{p}\cdot\dot{\boldsymbol{h}}$ also in terms of the extrinsic curvature as with ordinary gravity, and expressing the extrinsic curvature in terms of the canonical momenta [12]. The lapse appears as a Lagrange multiplier of the hamiltonian constraint, as in the usual ADM approach. Let us write the hamiltonian constraint in the form:

$$\begin{aligned} F(\boldsymbol{p},\dot{\boldsymbol{p}},\ddot{\boldsymbol{p}}) - \sqrt{h}\,[\gamma\,{}^{(3)}R \quad &- \quad 2\alpha\,{}^{(3)}R_{ab}{}^{(3)}R^{ab} \\ &+ \quad (\beta + \frac{2\alpha}{3})\,({}^{(3)}R)^2] + \mathcal{H}_{matter} = 0 \end{aligned} \tag{8}$$

where the function $F(\boldsymbol{p},\dot{\boldsymbol{p}},\ddot{\boldsymbol{p}})$ depends also on curvature, to isolate the $\boldsymbol{p}$-independent terms. (Greek indices assume values in $\{0,1,2,3\}$ while latin indices are restricted to $\{1,2,3\}$.) [1]

This constraint is to be compared to the one for ordinary gravity, with quantum potential added, which is

$$G_{ijkl}\,p^{ij}p^{kl} - \sqrt{h}\,\gamma\,{}^{(3)}R - \sqrt{h}\,\frac{k^2}{2r_o^2} + \mathcal{H}_{matter} = 0 \tag{9}$$

from which equation (3) is obtained by substituting $p_{ab} \to \partial S/\partial h_{ab}$. Eq. (8) is generally 4th-order in the canonical momenta while the corresponding equation for ordinary gravity (9) is 2nd order, but we claim that we can ignore the terms in $\boldsymbol{p}$. That is because for static metrics such as we will consider here, hypersurfaces of constant time have zero extrinsic curvature $K_{ab} \equiv \nabla_a n_b = 0$, where $\boldsymbol{n}$ is a time-like unit vector field normal to the surface, and ∇ is the projection of the covariant derivative onto the hypersurface. For such metrics we can take $\boldsymbol{n}(\boldsymbol{x}) = (n_t, 0, 0, 0)$. The Lagrangian density $\mathcal{L}$ can be written as a sum of terms with factors of the intrinsic curvature ${}^{(3)}R_{ab}$ or its contraction ${}^{(3)}R$, and terms with factors that are quadratic in the extrinsic curvature K_{ab} or that are time derivatives of K_{ab}, since the metric is Gaussian normal [12] as restricted to the horizon. Using $\partial\,{}^{(3)}R_{ij}/\partial\dot{h}_{ab} = 0$ and $\partial K_{ab}/\partial\dot{h}_{ab} = 1/2l$, where l is the time-independent lapse, and $K_{ab} = 0$, we find that $p_{ab} = \partial\mathcal{L}/\partial\dot{h}_{ab} = 0$. So the kinematic term $\boldsymbol{p}\cdot\dot{\boldsymbol{h}}$ vanishes and since $K_{ab} = 0$, the Lagrangian contributes only two new terms, in ${}^{(3)}R_{ab}{}^{(3)}R^{ab}$ and $({}^{(3)}R)^2$, and so $F = 0$. The steady states of the dynamics defined by the Hamiltonian

[1] The function $F(\boldsymbol{p},\dot{\boldsymbol{p}},\ddot{\boldsymbol{p}})$ for the case $\alpha = 0$ is given explicitly in terms of extrinsic curvature in Ref. [13].

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(including the matter terms), as would include ER bridges or wormholes prior to collapse, are the same in quantized ordinary general relativity and in classical 4th order gravity, if near the horizon:

$$\begin{aligned}(\gamma - \gamma_{GR})\,{}^{(3)}R \quad &- \quad 2\alpha\,{}^{(3)}R_{ab}\,{}^{(3)}R^{ab} \\ &+ \quad (\beta + 2\alpha/3)({}^{(3)}R)^2 = -\frac{k^2}{2r_o^2} \end{aligned} \tag{10}$$

where $\gamma_{GR} = 1/16\pi G$ is the canonical value of γ. (The usual "supermomentum" part of the total Hamiltonian vanishes for $\boldsymbol{p} = 0$.) It is noted that the negative sign of the quantum potential Q, viewed as a contribution to the energy density at the horizon, is what is required of the "exotic matter" that could serve to violate the averaged weak energy condition and to keep a traversable wormhole open [14].

For black holes and Einstein-Rosen bridges, we generally have $R_{\mu\nu}, R = O(1/r_o^2)$. The requirement of Eq. (10) is thus that that the $1/r_o^4$ terms in the expressions that are quadratic in curvature, ${}^{(3)}R_{ab}\,{}^{(3)}R^{ab}$ and $({}^{(3)}R)^2$ cancel, and that these quadratic curvature expressions, taken together, are $O(1/r_o^2)$. We consider spherically symmetric, time-independent metrics of the form:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \tag{11}$$

with the spatial metric on hypersurfaces of constant t given trivially by: $h_{ij} = g_{ij}$. For a metric of this form, the indices $0, 1, 2, 3$ are more specifically written as t, r, θ, ϕ, respectively. The Ricci tensor on the hypersurfaces is diagonal with three non-vanishing elements:

$$\begin{aligned} {}^{(3)}R_{rr} &= -\frac{1}{r}\frac{A'}{A} \\ {}^{(3)}R_{\theta\theta} &= -1 - \frac{A'}{2A^2} + \frac{1}{A} \\ {}^{(3)}R_{\phi\phi} &= \sin^2\theta \left(-1 - \frac{A'}{2A^2} + \frac{1}{A}\right) \end{aligned} \tag{12}$$

where primes denote derivatives with respect to r. Denoting $A^{-1}(r_o)\,{}^{(3)}R_{rr}(r_o) \equiv a$, $r_o^{-2}\,{}^{(3)}R_{\theta\theta}(r_o) \equiv b$, and $r_o^{-2}\sin^{-2}\theta\ {}^{(3)}R_{\phi\phi}(r_o) \equiv c$, we have at the horizon:

$${}^{(3)}R = a + b + c \tag{13}$$

$${}^{(3)}R_{ij}\,{}^{(3)}R^{ij} = a^2 + b^2 + c^2 \tag{14}$$

Assuming $a, b, c \sim 1/r_o^2$, the requirement that the quadratic curvature terms mimic the quantum potential is that the terms of leading order in $1/r_o$ vanish, and that:

$$\begin{aligned} -2\alpha\,{}^{(3)}R_{ab}\,{}^{(3)}R^{ab} + (\beta + \frac{2\alpha}{3})\,({}^{(3)}R)^2 &= \\ -2\alpha(a^2 + b^2 + c^2) + (\beta + 2\alpha/3)(a + b + c)^2 &= \\ -\left(\frac{4\alpha}{3} - \beta\right) a^2 - 4\left(\beta - \frac{2\alpha}{3}\right) b^2 + 4\left(\beta + \frac{2\alpha}{3}\right) ab &= \\ \frac{-k^2}{2r_o^2} \end{aligned} \tag{15}$$

having noticed that $b = c$, and having taken $\gamma = \gamma_{GR}$, as we will henceforth for agreement with general relativity at large scales.

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We have solved for the metric (11) in terms of $A(r)$ and $B(r)$, in series expansions, following the method of Lu *et al.* [10]:

$$\frac{1}{A(r)} \equiv f(r) = f_1(r - r_o) + f_2(r - r_o)^2 + \ldots \quad (16a)$$

$$B(r) = b_o + b_1(r - r_o) + b_2(r - r_o)^2 + \ldots \quad (16b)$$

There are indeed wormhole solutions with $A(r) = \infty$ at the horizon $r = r_o$, as with a Schwarzchild black hole, but $B(r_o) \neq 0$, unlike a black hole in ordinary gravity or in 4th order gravity [15]. Such traversable wormholes were studied by Lu *et al.* [10] in the restricted case $\beta = 0$. Here, $\beta \neq 0$ is needed so that there are non-trivial solutions to (15) with $\alpha \neq 0$ and different from ordinary gravity. The 4th order theory admits traversable wormhole solutions, as in [10], but with more free parameters than for the $\beta = 0$ case. The equations of motion are derived by minimizing the action $I = \int d^4x\sqrt{-g}\,\mathcal{L}$ formed from the Lagrangian density (7). They are:

$$\begin{aligned} 0 &= \frac{1}{\sqrt{-g}}\frac{\delta I}{\delta g^{\mu\nu}} \equiv H^{\mu\nu} \\ &= -\gamma(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) - \frac{2}{3}(\alpha - 3\beta)\,R_{;\mu;\nu} \\ &\quad +2\alpha g^{\rho\sigma}R_{\mu\nu;\rho;\sigma} - \frac{1}{3}(\alpha + 6\beta)\,g_{\mu\nu}g^{\rho\sigma}R_{;\rho;\sigma} \\ &\quad +4\alpha R^{\rho\sigma}R_{\mu\rho\nu\sigma} + 2\left(\beta + \frac{2}{3}\alpha\right)RR_{\mu\nu} \\ &\quad +\alpha g_{\mu\nu}R^{\rho\sigma}R_{\rho\sigma} - \frac{1}{2}g_{\mu\nu}\left(\beta + \frac{2}{3}\alpha\right)R^2 \end{aligned} \quad (17)$$

where a semicolon denotes covariant differentiation with respect to the index following. One first substitutes the spherically symmetric, static metric form (11) in the usual expressions for the Ricci tensor (see e.g. [16]), to get 4-dimensional expressions for $R_{\mu\nu}$ that are extensions of (12), and inserts these expressions in (17) to derive differential equations for $A(r)$ and $B(r)$. Combining the equations for H^{tt} and H^{rr} (sufficient to determine $H^{\theta\theta}$ and $H^{\phi\phi}$ as well, because of a Bianchi identity) in the manner of Ref. [10], one derives a pair of third-order differential equations for $A(r)$ and $B(r)$ that are found to match those given in the Appendix to [10]. The series expansions (16) were substituted in those equations and solved for the lowest order coefficients in Mathematica. We find:

$$f_1 = \frac{1}{3}\sqrt{\frac{-2(\alpha - 3\beta)(2\alpha - 6\beta - 3\gamma r_o^2)}{3\alpha\beta r_o^2}} \quad (18a)$$

$$b_1/b_0 = \frac{2(\alpha + 6\beta)}{(\alpha - 3\beta)r_o} \quad (18b)$$

But unlike the $\beta = 0$ case, we find that b_2 is unconstrained, and therefore can be tuned for asymptotic flatness. It can be checked that the extra terms in (17), as compared to the usual second-order Einstein equations, give an effective negative energy contribution, when averaged along a null geodesic through the interior of the wormhole, that violates the averaged weak energy condition, as required for traversability.

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To satisfy the equivalence condition (15), we first note that the curvature variables $a = A^{-1}(r_o)\,{}^{(3)}R_{rr}(r_o)$ and $b = c = r_o^{-2}\,{}^{(3)}R_{\theta\theta}(r_o)$, where ${}^{(3)}R_{ij}$ is given by (12), depend only on the leading coefficient f_1:

$$\begin{aligned} a &= \frac{f_1}{r_o} \\ b = c &= -\frac{1}{r_o^2} + \frac{f_1}{2r_o} \end{aligned} \tag{19}$$

We will use the solution (18a) for f_1 in terms of α and β to compute the quadratic curvature contribution (15) , which we now denote $V(r_o)$.

$$\begin{aligned} V(r_o) &\equiv -\left(\frac{4\alpha}{3} - \beta\right)(a(\alpha,\beta))^2 - 4\left(\beta - \frac{2\alpha}{3}\right)(b(\alpha,\beta))^2 \\ &\quad +4\left(\beta + \frac{2\alpha}{3}\right) a(\alpha,\beta) b(\alpha,\beta) \end{aligned} \tag{20}$$

Substituting from (19) and (18), the condition for equivalence of $V(r_o)$ to the de Barros *et al.* quantum potential becomes:

$$\begin{aligned} V(r_o) &\equiv \frac{2}{81 r_o^4 \omega}\Bigg\{ 3\gamma_{GR} r_o^2 (36 - 15\omega + \omega^2) \\ &+ \beta\Bigg[216 - 2\omega^3 \\ &+ 36\omega\left(-9 + \sqrt{\frac{6(-3+\omega)(6\beta + 3\gamma_{GR} r_o^2 - 2\beta\omega)}{\beta\omega}}\right) \\ &+ 6\omega^2\left(15 + \sqrt{\frac{6(-3+\omega)(6\beta + 3\gamma_{GR} r_o^2 - 2\beta\omega)}{\beta\omega}}\right)\Bigg]\Bigg\} \\ &= -\frac{k^2}{(2r_o^2)} \end{aligned} \tag{21}$$

where we have expressed V in terms of β and $\omega \equiv \alpha/\beta$ for convenience.

The coefficient of the second term in the braced expression, β, must vanish to avoid a $1/r_o^4$ contribution for small r_o. Then the first term gives

$$2\gamma_{GR}(36 - 15\omega + \omega^2))/(27\omega) = -\frac{k^2}{2} \tag{22}$$

a quadratic equation that can be solved for ω, for given k. Since the same geometrodynamics must describe black-hole evaporation, a value of the eigenvalue k can be obtained by matching the black-hole evaporation rate as computed by deBarros *et al.* [7], $k^2/4M^2$, to the Hawking rate [17] $P = \hbar c^6/(15360\pi G^2 M^2)$. This gives $k = \sqrt{\hbar c^6/3840\pi G^2}$, or in units for which $\hbar = c = G = 1$, $k \approx 0.01$. In the limit $\beta \to 0$, the desired value of the ratio of the coefficients $\omega = \alpha/\beta$ solves (22) and is found to be $\alpha/\beta = \omega = 3.009$ or $\alpha/\beta = 11.962$. For α/β fixed at these values, the contribution of the quadratic curvature terms is plotted vs. r_o in Fig. 2 as the two coefficients each approach zero. As we approach the limit, the quadratic curvature V contribution matches the quantum potential $-k^2/(2r_o^2)$ down to smaller and smaller radii r_o, but very large values of V are found for a narrowing range around $r_o = 0$. We note that the large potentials could conceivably help in avoiding divergences due to an arbitrarily large tendency to form arbitrarily narrow wormholes. Otherwise the quantum potential can be matched as closely as desired for sufficiently small coefficients.

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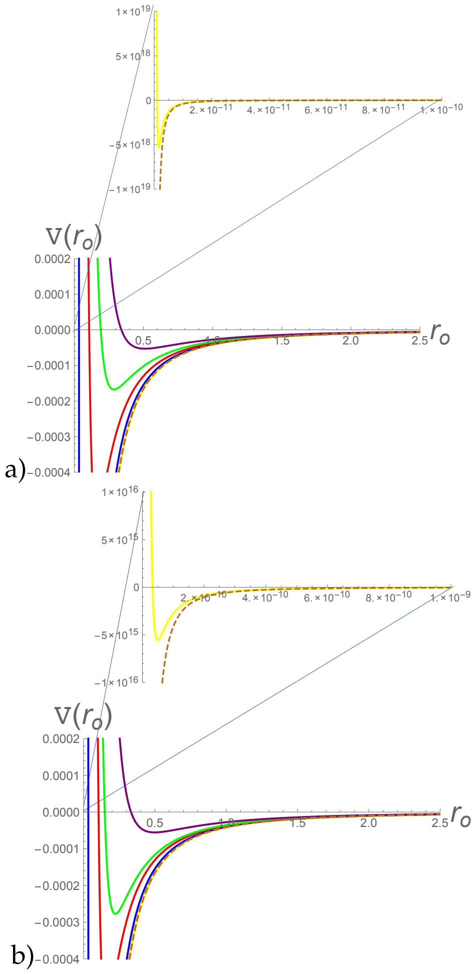

Figure 2. The contribution of the quadratic curvature terms to the hamiltonian constraint, vs. wormhole radius r_o, for a) decreasing values of β: $\beta = 10^{-7}$ (purple), $10^{-7.5}$ (green), 10^{-8} (red), 10^{-9} (blue), and 10^{-30} (yellow), while the the ratio of the coefficients is held fixed at the first solution to (22), $\alpha/\beta \approx 3.009$; and for b) values of β: $\beta = 10^{-10}$ (purple), 2×10^{-11} (green), 10^{-11} (red), 10^{-12} (blue), and 10^{-30} (yellow), while the ratio is held at the second solution, $\alpha/\beta \approx 11.962$. The quadratic curvature contribution V converges to the quantum potential $-k^2/(2r_o^2)$ (dashed line) as $\beta \to 0$ in both cases, except for large positive values for a decreasing range of small r_o, as shown for the smallest value $\beta = 10^{-30}$ (yellow line) in expanded views about the origin.

4. Discussion

For black holes or non-traversable ER bridges of macroscopic size, the quadratic curvature terms in the above analysis have no significant effect on the dynamics that is different from that of the quantum potential, or therefore, of the usual quantum theory. Indeed, the potential that follows from the quadratic curvature terms is unique, and does not depend on a choice of eigenstate as in the treatment of Tomimatsu [5] where classical (non-radiating) black hole behavior occurs for another wavefunction solution. So the ambiguity reported by deBarros *et al.* [7] regarding black hole evaporation in a quantum potential treatment, is avoided if the potential comes from extra classical curvature terms. An absence of macroscopic effect might have been expected if the coefficients α and β (which have the dimensions $[\text{length}]^2$) had Planck-scale values. That these coefficients can be infinitesimal strengthens the argument that the macroscopic predictions of ordinary general relativity are preserved.

At sub-Planck scales, where $r_o << L_P$, on the other hand, the potential arising from the quadratic curvature terms can deviate strongly from that predicted by quantum theory, for any values of α and β that remain finite. Differences could have detectable physical consequences.

In regard to verifiable consequences, we also note that the quantum potential depends on specific combinations of the matter fields with gravity. The quantum potential (5) used here arises from a single scalar field in the matter portion of the hamiltonian, but in a traditional EPR pair, an electromagnetic field is used in the observation of charged spinor particles. It is noteworthy that wormhole solutions with electric charge in a modified theory that includes small quadratic curvature terms with a fixed ratio have been previously reported [18]. In general, allowable combinations of matter fields might be constrained by a supersymmetry required to maintain the equivalence between the quantum potential due to the matter fields and contributions from the extra curvature terms. The prescribed combinations could be compared with the spectrum of particles known to populate the vacuum.

The focus of our analysis has been on steady entangled states. Collapse of the wormhole has not been discussed. It is thought that the fourth-order time derivatives in the classical hamiltonian constraint (8), away from the steady state, could describe a physical process of wormhole/wavefunction collapse on very short time scales. Unlike the situation with a macroscopic wormhole, the entry of a single particle would trigger collapse, preventing [19] the problems due to closed time-like curves through macroscopic wormholes and the resulting vacuum polarization divergence that was previously debated between Kim and Thorne [20] and Hawking [21]. Hawking's argument still precludes macroscopic wormholes. For a Planck scale wormhole, the collapse itself could indeed be the only effect transmitted. And in the absence of a Maxwell's Demon, one could not effectively control the collapse so as to transmit information. That is, one does not have knowledge of the detailed state at either end and considers only a statistical ensemble of states. That the statistical properties are unchanged by the nonlocal effects of measurement is also what bars the use of the quantum potential for superluminal signaling in Bohm's original interpretation [22].

It is tempting to elevate the proposed role of traversable wormholes in mediating entanglement to a full interpretation of quantum mechanics. Planck's constant would emerge if any physical processes select a preferred width for the wormholes that is O[1] in non-dimensional units. One candidate for such a process is just the weak vacuum polarization divergence [20,21] that precludes macroscopic widths. But in addition to the need for a description of collapse, one would also need to show that all of quantum theory follows from nonlocal entanglement conjoined with some properly constructed, local classical theory.

While we have not yet shown that nonlocal entanglement is everything, it is certainly central. It is hypothesized that at least the non-local part of the quantum potential, in the general situation, arises from the new, multiply connected topology. As with an Einstein-Rosen bridge, entangled particles connected by a traversable wormholes with r_o of Planck scale are not separated within the wormhole and are in a sense the same particle, until the connecting geometry collapses. The resulting negation of the usual concept of the continuum establishes a new order in the physical world, best described in an atomized spacetime together, arguably, with non-standard metrics in state space [23] and an ontology given by nonlocally defined basis states that are not easily specified [24]. The view taken here is that an account of how the new order forms from the intrinsically required breakdown of the classical order, through gravitational instability of the populated vacuum at microscale, could ultimately yield a more complete, more detailed and more predictive version of quantum theory.

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Conflicts of Interest: Conflicts of Interest

The author declares no conflict of interest.

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