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Article

Trajectory Tracking between Josephson Junction and Classical Chaotic System Via Iterative Learning Control

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Abstract:

This article addresses the trajectory tracking between two non-identical systems with chaotic properties. We employ the Rossler chaotic and RCL-shunted Josephson junctions model in similar phase space to study trajectory tracking. In order to achieve the goal tracking, we afford two stages to approximate the target tracking. The first stage utilizes the active control technique to transfer the output signal from the RCLs-J system into the quasi-Rossler system. Then next, the RCLs-J system employs the proposed the iterative learning control scheme and the control signal from the drive system to trace the trajectory of Rossler system. The numerical results demonstrate the proposed method and the tracking system is asymptotically stable.

Keywords: Trajectory; Chaos; Josephson Junction; RCL-shunted; Iterative Learning Control (ILC).

1. Introduction

Chaotic phenomenon was found in the rf-base resistive-capacitive shunted Josephson Junction (RCs-JJ) and the numerical study in three system parameters have been described in [1]. Many studies exhibit the chaotic behavior in superconducting resistive-capacitive-inductance Josephson Junction (RCLs-JJ) [2-4]. The homoclinic, heteroclinic, and super-harmonic bifurcations are respectively excited by parameters has investigated in [5]. The damped pendulum equation can describe the junction behavior and demonstrate the chaotic strange attractor in phase space [6]. Synchronization is a significant topic in nonlinear science as the trajectory tracking is essential in studying the chaotic synchronization. A non-linear controller utilized backstepping technique to control bifurcation in the RCLs-J junction has investigated in [7]. The chaos synchronization between two identical systems of RCLs-J junctions investigated in [8-14] in which employ a number of different techniques to design controller, such as using active control in [8], by a common chaos to drive RCLs-J junctions approaching synchronization [9], applying the backstepping in [10, 13], and using time-delay feedback control in [14], respectively. In others, the controller design or controlled rule is directly determined by Lyapunov function in [11-12] and the RCLs-J junctions array synchronization in [12]. In most studies, the synchronization systems were described in identical RCLs-J junction systems. In the classical systems, synchronization is not concerned about a superconducting system. Accordingly, in the trajectory tracking study is rarely based on the combination of RCLs-J and classical chaotic systems.

This article regards the trajectory tracking between the Rossler chaotic and the RCLs-J systems as the classical chaotic system and the mesoscopic system in the Josephson junctions model. They are almost two different systems to trace trajectory. This paper affords two stages to approximate the goal of trajectory tracking. The first stage utilizes the active control technique [15] to transfer RCLs-J system into the quasi-Rossler system. Next, we propose the iterative learning control law which is the purpose to approach the signals from the identical system by correcting repetition the tolerance according to preceding output information [12]. The RCLs-J system employs the iterative learning control procedure and control signal from the drive system to trace Rossler system. Although, most research of the ILC are designed linear ILC law, few applications are available on two different systems synchronizing.

The organization of this article follows: next section is the description of Rossler Chaotic and RCL-Shunted Josephson Junction System. The third section is to demonstrate the simulation results in figures by designing example and investigating to employ the proposed learning control law into the RCLs-J system to trace the path of the Rossler system. Finally, this paper points out the applications in the future and conclusion.

2. The description of Rossler Chaotic and RCL-Shunted Josephson Junction System

2.1. System Description and Transformation

The Rossler chaotic system has initial condition X_0 is drive system in general form as

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{b} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ 0 & 0 & x_1 - c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} \quad (1).$$

The variable $X = [x_1 \quad x_2 \quad x_3]^T$ is the state vector. The RCL-shunted Josephson Junction can be presented by eq. (2) with initial conditions $Y_0 = [0\ 0\ 0]^T$ as

$$\dot{\boldsymbol{Y}} = \boldsymbol{BY} + \boldsymbol{b}\psi(y_1) + \boldsymbol{U}^{(k)}, \quad (2).$$

The parameters in the eq. (2) defined $\boldsymbol{Y} = [y_1 \quad y_2 \quad y_3]^T$, and

$$\boldsymbol{B} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{g(y_2)}{\beta_C} & \frac{1}{\beta_C} \\ 0 & \frac{1}{\beta_L} & -\frac{1}{\beta_L} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ \frac{1}{\beta_C} \\ 0 \end{bmatrix}, \quad \psi(y_1(t)) = i_N - \sin(y_1). \quad (3)$$

There is a function $g(y_2)$ in eq. (3), and given by

$$g(y_2) = \begin{cases} 0.366 & \text{as} \quad |y_2| > 2.9 \\ 0.061 & \text{as} \quad |y_2| \le 2.9, \end{cases} \quad (4).$$

The iterative number k in system (2) is the number to employee the iterative learning control law (ILC) $U^{(k)} = [u_1^{(k)} \quad u_2^{(k)} \quad u_3^{(k)}]^{\mathrm{T}}$. Really, ILC rule is a sequence of control input signal for response system as $\{U^{(k)}\}_{k=1,2,\cdots}$.

The system (1) and system (2) are almost not identical nonlinear systems from the trajectory of them in Figure 1. The nonlinear system (2) should be transferred to the quasi-Rossler system to track the trajectory of the system (1). Therefore, the active control technique [17-18] will be utilized into the system (2).

According to the active control technique, the description of the system (2) and dynamical transformation between the drive (1) and response system (2) show respectively as

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$$\boldsymbol{z} = \begin{bmatrix} z_a \\ z_b \\ z_c \end{bmatrix} = \begin{bmatrix} y_1 - x_1 \\ y_2 - x_2 \\ y_3 - x_3 \end{bmatrix} \tag{5}$$

$$\dot{\boldsymbol{z}} = \begin{bmatrix} \dot{z}_a \\ \dot{z}_b \\ \dot{z}_c \end{bmatrix} = \begin{bmatrix} \dot{y}_1 - \dot{x}_1 \\ \dot{y}_2 - \dot{x}_2 \\ \dot{y}_3 - \dot{x}_3 \end{bmatrix} = \begin{bmatrix} z_b + 2x_2 + x_3 \\ \frac{-1}{\beta_C} g(y_2) z_b + \frac{1}{\beta_C} z_c + \frac{1}{\beta_C}[i_N - \sin(y_1)] - x_1 - \left(a + \frac{g(y_2)}{\beta_C}\right) x_2 - \frac{x_3}{\beta_C} \\ \frac{1}{\beta_L} z_b - \frac{1}{\beta_L} z_c + \frac{1}{\beta_L}(x_2 - x_3) - x_1 x_3 - c x_3 - b \end{bmatrix} + \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{6}$$

The $[v_a \quad v_b \quad v_c]^T$ in eq. (6) is the active control function to eliminate the terms in which have no z_i for the $i = a, b, c$. As a result, the active control function can be determined as

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} -2x_2 - x_3 \\ \frac{-1}{\beta_C}[i_N - \sin(y_1)] + x_1 + \left(a + \frac{g(y_2)}{\beta_C}\right) x_2 + \frac{x_3}{\beta_C} \\ \frac{-1}{\beta_L}(x_2 - x_3) + x_1 x_3 + c x_3 + b \end{bmatrix} + \begin{bmatrix} w_a \\ w_b \\ w_c \end{bmatrix} \tag{7}.$$

The $[w_a \quad w_b \quad w_c]^T$ is the error term in active control procedure. Substituting eq. (7) into (6), the eq. (6) became as

$$\dot{\boldsymbol{z}} = \begin{bmatrix} \dot{z}_a \\ \dot{z}_b \\ \dot{z}_c \end{bmatrix} = \begin{bmatrix} z_b \\ \frac{-1}{\beta_C} g(y_2) z_b - \frac{1}{\beta_C} z_c \\ \frac{1}{\beta_L} z_b - \frac{1}{\beta_L} z_c \end{bmatrix} + \begin{bmatrix} w_a \\ w_b \\ w_c \end{bmatrix} \tag{8}$$

The matrix form of eq. (8) is rewritten as

$$\dot{\boldsymbol{z}} = \begin{bmatrix} \dot{z}_a \\ \dot{z}_b \\ \dot{z}_c \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} z_a \\ z_b \\ z_c \end{bmatrix} + \begin{bmatrix} w_a \\ w_b \\ w_c \end{bmatrix} \tag{9}$$

Suppose the matrix $\boldsymbol{A}$ has eigenvalues $(\lambda_a, \lambda_b, \lambda_c) = (-1, -1, -1)$, the characteristic equations of $\boldsymbol{A}$ are demonstrated as

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 + \frac{1}{\beta_C} g(y_2) & \frac{1}{\beta_C} \\ 0 & \frac{-1}{\beta_L} & -1 + \frac{1}{\beta_L} \end{bmatrix} \begin{bmatrix} z_a \\ z_b \\ z_c \end{bmatrix} = \begin{bmatrix} w_a \\ w_b \\ w_c \end{bmatrix} \tag{10}$$

The solution of $[w_a \quad w_b \quad w_c]^T$ is

$$\begin{bmatrix} w_a \\ w_b \\ w_c \end{bmatrix} = \begin{bmatrix} -z_a & -z_b & 0 \\ 0 & -(1 - \frac{1}{\beta_C} g(y_2)) z_b & \frac{1}{\beta_C} z_c \\ 0 & \frac{-1}{\beta_L} z_b & (1 + \frac{1}{\beta_L}) z_c \end{bmatrix} \tag{11}$$

The equation (8) employed eq. (11) and became $[\dot{z}_a \quad \dot{z}_b \quad \dot{z}_c]^T = [-z_a \quad -z_b \quad -z_c]^T$. Substituting eq. (11) and (7) into the RCLs-Josephson Junctions eq. (2) with iterative learning control rule and changing the variable $\boldsymbol{x}$ to $\boldsymbol{y}$, the system became as

$$\dot{\boldsymbol{Y}} = \begin{bmatrix} -y_2 - y_3 - z_a \\ y_1 + a y_2 - z_b \\ y_1 x_3 - c y_3 - z_c \end{bmatrix} + \boldsymbol{U}^{(k)} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ 0 & 0 & y_1 - c \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} - \begin{bmatrix} z_a \\ z_b \\ z_c \end{bmatrix} + \begin{bmatrix} u_1^{(k)} \\ u_2^{(k)} \\ u_3^{(k)} \end{bmatrix} \tag{12}$$

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After the active control procedure, the RCLs-Josephson Junctions system became a quasi-Rossler chaotic system such that the trace of trajectory between different systems became identical systems.

2.2. Trajectory tracking between of Systems via Iterative Learning Control

The RCLs-Josephson Junctions system has now been transferred to the quasi-Rossler chaotical system. The ILC procedure and controll signal from the drive system will be utilized into the response system to track the drive system. When an appropriated $\{U^{(k)}\}_{k=1,2,\cdots}$ is found, and the iteration number k is enough, the tracked error dynamical system should be equal to zero, that is $\dot{e}^{(k)}(t) = \lim_{k\to\infty}|\dot{X}(t) - \dot{Y}(t)| = 0$. The situation of tracking trajectory has changed to two similar systems. In many studies, the synchronization between identical systems employ the control signal from drive system has been studied in [19-21]. Accordingly, The RCLsJ system in eq. (2) utilizing the controlled signals from drive system, x_1 and x_3, is rewritten as

$$\dot{\boldsymbol{Y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{\beta_C}[i_N - g(y_2)x_2 - \sin(y_1) - x_3] \\ \frac{1}{\beta_L}(x_2 - x_3) \end{bmatrix} + \boldsymbol{U}^{(k)} \quad (13).$$

The controlled signals from the Rossler system in eq. (13) is similar to the quasi-Ross system in (12) and the iterative learning control law $\boldsymbol{U}^{(k)}$ which is defined by the error dynamics. The dynamical error system between the Rossler system in (1) and the quasi-Rossler system in (12) exhibit as

$$\dot{\boldsymbol{e}} = \begin{bmatrix} -e_1 - e_2 \\ e_1 + e_2 \\ x_3 e_1 - c e_3 \end{bmatrix} - \begin{bmatrix} z_a \\ z_b \\ z_c \end{bmatrix} + \begin{bmatrix} u_1^{(k)} \\ u_2^{(k)} \\ u_3^{(k)} \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ 0 & 0 & -c \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \boldsymbol{M}(x_i, y_j)\boldsymbol{G}(\boldsymbol{e}) - \begin{bmatrix} z_a \\ z_b \\ z_c \end{bmatrix} + \begin{bmatrix} u_1^{(k)} \\ u_2^{(k)} \\ u_3^{(k)} \end{bmatrix} \quad (14)$$

The iterative learning control rule (ILC) in [16, 21] $\boldsymbol{U}^{(k)}$ is defined as

$$\boldsymbol{U}^{(k)} = \boldsymbol{B_2}\Delta^{(k)} + \boldsymbol{B_1}\boldsymbol{U}^{(k-1)} \quad (15)$$

where the matrix $\boldsymbol{B_1} = (\boldsymbol{M})^m * (realeig(\boldsymbol{B_2}))^{-n}$ with appropriated $0 \leq m \leq 1$, and $1 \leq n < k$. The $\boldsymbol{B_2}$ is the coefficient matrix of $\Delta^{(k)} = [e_1 \quad e_2 \quad e_3]^T$ in (14) and $realeig(\boldsymbol{B_2})$ is the real part of eigenvalue of $\boldsymbol{B_2}$. When the $z_{i=a,b,c} = e_{j=1,2,3}$, the term $\boldsymbol{M}(x_i, y_j)\boldsymbol{G}(\boldsymbol{e})$ in eq.14 can absorb the $[z_a \quad z_b \quad z_c]^T$ to choose the appropriate matrix $\boldsymbol{M}$

By induction, the expansion of $\boldsymbol{U}^{(k)}$ in eq. (15) wrote as

$$\boldsymbol{U}^{(k)} = (\boldsymbol{B_1})^k \boldsymbol{U}^{(0)} + (\boldsymbol{B_1})^{k-1}\boldsymbol{B_2}\Delta^{(1)} + (\boldsymbol{B_1})^{k-2}\boldsymbol{B_2}\Delta^{(2)} + \cdots + \boldsymbol{B_1}\boldsymbol{B_2}\Delta^{(k-1)} + \boldsymbol{B_2}\Delta^{(k)} \quad (16).$$

2.3. Lyapunov Stability of Systems

The equation (6) can be the dynamical error system in the active control procedure. Hence, the Lyapunov function is defined as

$$\boldsymbol{V} = \frac{1}{2}(\boldsymbol{s_1 z_a}^2 + \boldsymbol{s_2 z_b}^2 + \boldsymbol{s_3 z_c}^2). \quad (17)$$

The $s_{j=1,2,3}$ are constant such that $\dot{V} < 0$.

Theorem 1. The Lyapunov function in active control procedure to transfer the RCLs-J system (2) to the quasi-Rossler system (12) can be defined as in eq. (17).

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Proof:
The equation (17) should be proved the first derivative is negative and the dynamical system is stable at the equilibrium (0, 0, 0). The first derivative of the Lyapunov function is

$$\dot{\boldsymbol{V}} = (\boldsymbol{s_1}\dot{\boldsymbol{z}}_{\boldsymbol{a}}\boldsymbol{z_a} + \boldsymbol{s_2}\dot{\boldsymbol{z}}_{\boldsymbol{b}}\boldsymbol{z_b} + \boldsymbol{s_3}\dot{\boldsymbol{z}}_{\boldsymbol{c}}\boldsymbol{z_c}) \tag{18}$$

Substituting eq. (11) into eq. (8) and taking $s_1 = s_2 = s_3 = 1$, it is easy to show that

$$\dot{\boldsymbol{V}} = -(\boldsymbol{z_a}^2 + \boldsymbol{z_b}^2 + \boldsymbol{z_c}^2) \leq \boldsymbol{0} \tag{19}$$

□

Theorem 2. Let the $\boldsymbol{U}^{(k)}$ is in the eq. (15), the Lyapunov function is defined in the iterative control stage to trace the trajectory of Rossler system as

$$\boldsymbol{V} = \frac{1}{2}(\boldsymbol{r_1}\boldsymbol{e_1}^2 + \boldsymbol{r_2}\boldsymbol{e_2}^2 + \boldsymbol{r_3}\boldsymbol{e_3}^2) \tag{20}$$

Proof:
Let $\boldsymbol{U}^{(\boldsymbol{k}-\boldsymbol{1})}$ be defined as

$$\boldsymbol{U}^{(k-1)} = \boldsymbol{B_2}\Delta^{(k-1)} + \boldsymbol{M}(x_i, y_j)\boldsymbol{G}(\boldsymbol{e}) \tag{21}$$

Applying $-\boldsymbol{U}^{(k-1)}$ to the eq. (14), we can obtain the error dynamics as $\dot{\boldsymbol{e}} = [-z_a \quad -z_b \quad -z_c]^T$. Let $z_{i=a,b,c} = e_{j=1,2,3}$ and $r_{j=1,2,3} = 1$. The Lyapunov function should be as

$$\dot{\boldsymbol{V}} = -(e_1{}^2 + e_2{}^2 + e_3{}^2) \leq \boldsymbol{0} \tag{22}$$

which implies eq. (14) employs the iterative learning control law is asymptotical stable at equilibrium.

□

3. Demonstrating Results by example and discussion

To verify the proposed the iterative learning control law, we utilize an example to demonstrate the tracing error and trajectory between the Rossler dynamical system as in (1) with initial state (x_{10}, x_{20}, x_{30}) = (0.2, 0.4, 0.1) and the RCLSJ system in (2) with initial state (y_{10}, y_{20}, y_{30}) = (0, 0, 0), respectively.

3.1. Deciding Iterative Control Learning Law by Example

The Rossler system in (1) is given as:

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{a} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & x_3 - 5.7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}, \qquad x_0 = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.1 \end{bmatrix}. \tag{23}$$

The RCL-shunted Josephson Junction model in (2) is given by:

$$\dot{\boldsymbol{Y}} = \boldsymbol{BY} + \boldsymbol{b}\psi(y_1) + \boldsymbol{U}^{(k)}, \qquad \boldsymbol{Y_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

where the parameters in eq. (24) have defined in eq. (3) and (4) in which the values of entries in matrix $\boldsymbol{B}$ are $\beta_L = 2.6$, $\beta_c = 0.707$ and the $i_N = 1.132$ is in function $\psi(y_1) = 1.132 - \sin(y_1)$, respectively.

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The $\boldsymbol{U}^{(k)} = \begin{bmatrix} u_1^{(k)} & u_2^{(k)} & u_3^{(k)} \end{bmatrix}^{\mathrm{T}}$ in the system (24) is ILC rule and defined in eq. (15) to achieve enough small tracking error between the Rossler system and RCLs-J system. The matrices $\boldsymbol{M}(x_i, y_{\mathrm{j}})$ of eq. (14) and $\boldsymbol{B_2}$ of eq. (15), respectively alternate as

$$\boldsymbol{M}(x_i, y_{\mathrm{j}}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_1 & 0 & 1 \end{bmatrix} \text{ and } \boldsymbol{B_2} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & -0.2 & 0 \\ -1 & 0 & 5.7 \end{bmatrix} \tag{25}$$

where the x_1 is from Rossler system and matrix $\boldsymbol{B_2}$ is the decomposition of matrix $\boldsymbol{A}$ in eq. (1). The time interval of simulation is from 0 to 300 sec and the minimum time step is 0.01sec. The results and figures in this article utilize the MATLAB to investigate trajectory tracking by the ILC law in eq. (16) in which used program of the Euler method. In the active control procedure, transferring the RCLs-J system to Rossler system employs the Simulink in MATLAB.

3.2. Exhibiting Simulation Results and Discussion

The fig. 1 and fig. 2 show the time response of state and phase portrait of two distinct systems in which are Rossler and the RCL-shunted Josephson Junctions systems with different initial states, respectively. In fig. 1, the trajectory error between them should be enormous in each state. The fig. 2 displays two non-identical phase portraits of two systems and the chaotic behavior of RCLs-J shows in fig. 2(d). To overcome the non-identical trajectory between two systems, the first stage employs the active control to change RCLs-J systems into the quasi-Rossler system form Eq. (5) to Eq. (12). After utilizing the active control technique, the phase portraits of two systems show in the fig. 3 (a), (b), and (c). The new phase portraits of RCLs-J are almost not belonged to original phase portraits and closed to the Rossler system; therefore, we call the new system is the quasi-Rossler system. The time response of each component in the two systems indicated in the fig. 3 (d), (e), (f) in which the paths of Rossler and RCLs_J systems, respectively, are not close to each other.

The fig.4 is the tracking error between the Rossler and the quasi-Rossler system which transfers from RCLs-J systems. The vibration of the tracking error has many large amplitudes in the second (y_2-x_2) and third (y_3-x_3) components at the specific moment.

The fig. 5 demonstrates the phase portrait of the x_1 (y_1) and x_2 (y_2) by utilizing the ILC to track the trajectory. Two trajectories are almost overlapping in fig. 5 in which the phenomena of tracking errors in fig. 6 are also validated. The tracking errors oscillation in fig. 6 are between 0.1408 and -0.1959 for the first tracking error, second one among 0.1434 and -0.4217, and the third tracking error between 0.4784 and -0.8344, respectively.

The fig. 7 exhibits the tracking error which is the most different ingredient after using the ILC law and compares the fig. 1 to each other. The lager vibration will always happen at a particular moment such as t_i because that the tracking error between two systems became larger at some moments t_i. Comparing between fig. 4 and fig. 6, the tracking errors are successful to be suppressed between 0.2 and -0.2 for the first two components and the error of the third component between 0.5 and -0.8 by proposing ILC law, and the error is asymptotically stable.

4. Conclusions

This article has proposed a learning control law to trace the trajectory between two non-identical nonlinear systems and successfully utilized a two-stage approach of combining active control technique and iterative learning control law significantly to inhibit and improve the tracking errors in the numerical results. The simulation example helps to infer the trajectory tracking process and assert the proposed ILC rule. The critical

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work of ILC in the future would be error convergent between multiple non-identical systems and employed encryption and decryption, signal tracking of bio-system, and AI arm.

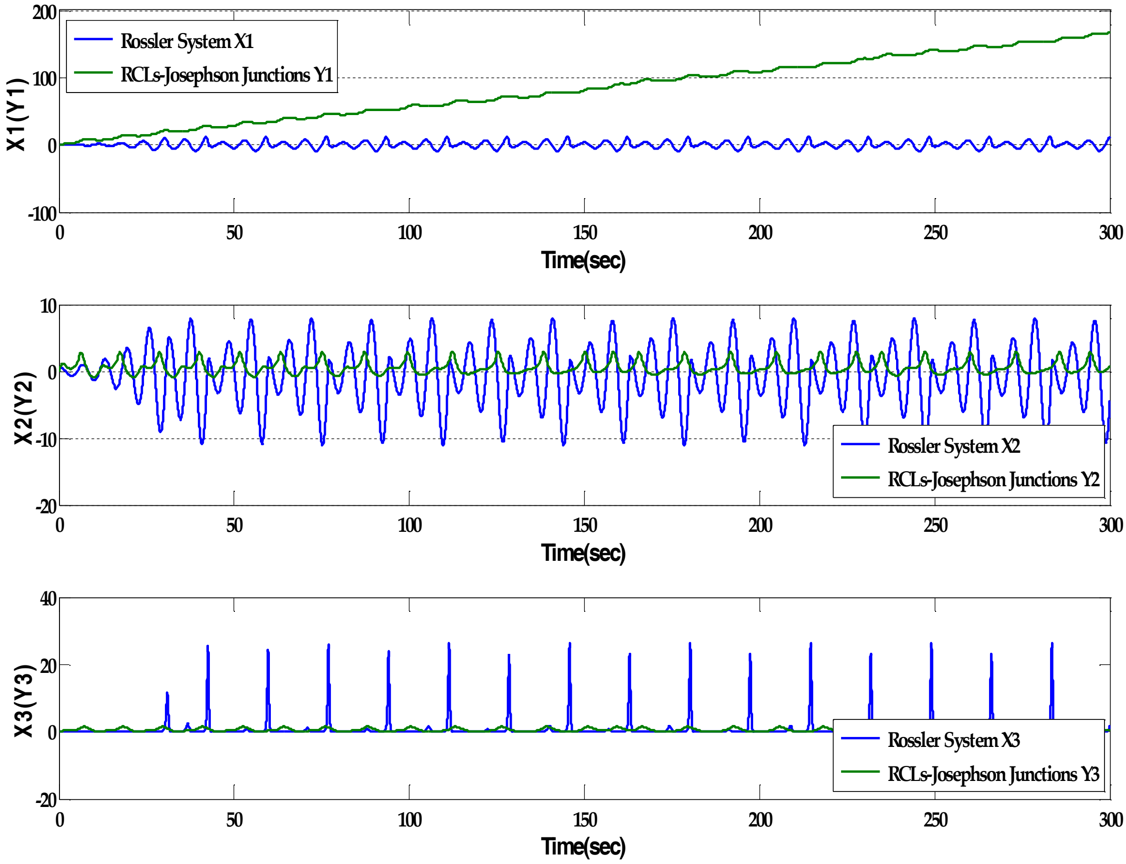
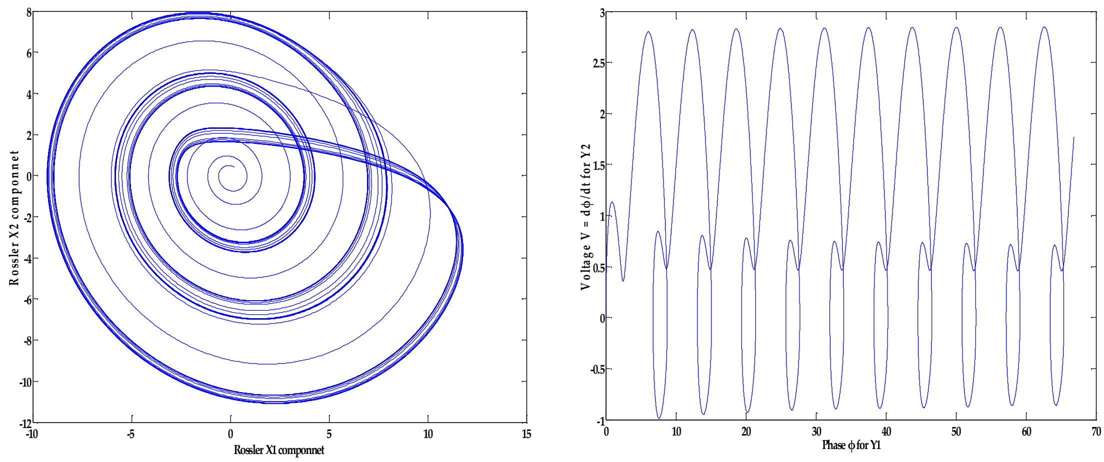
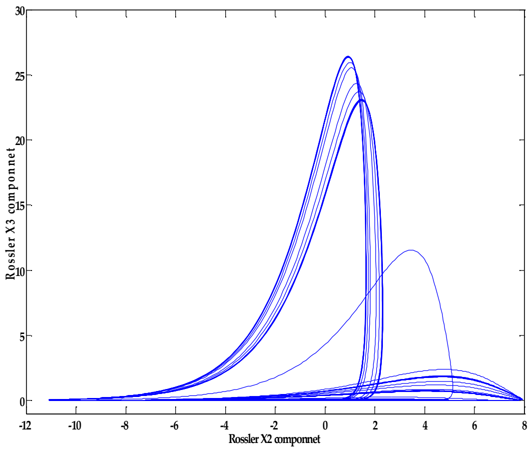


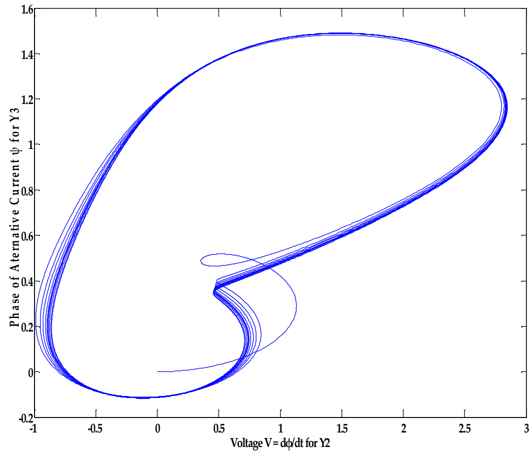
Fig. 1 The time response of Rossler system and RCLs-J system



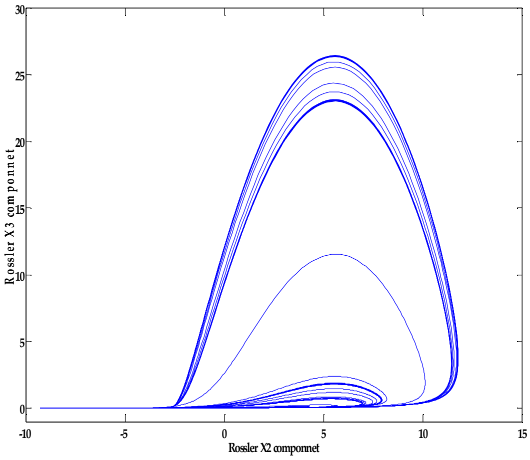
(a) Rossler System x_1, x_2 (d) RCLs-J System y_1, y_2



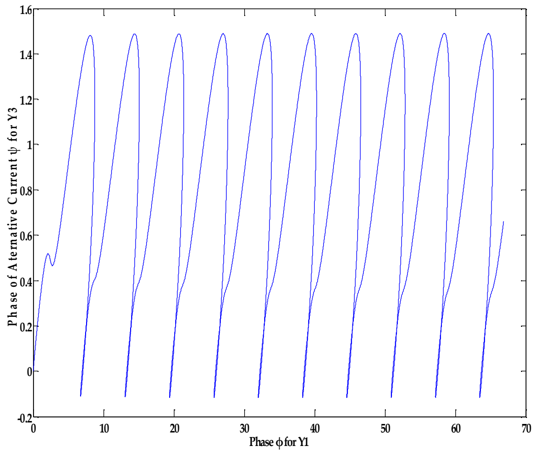
(b) Rossler System x_2, x_3



(e) RCLs-J System y_2, y_3

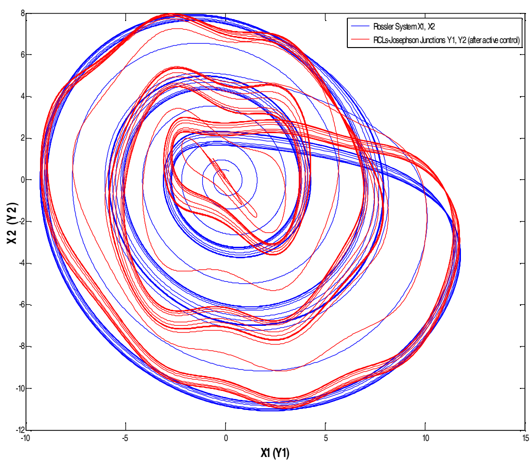


(c) Rossler System x_1, x_3

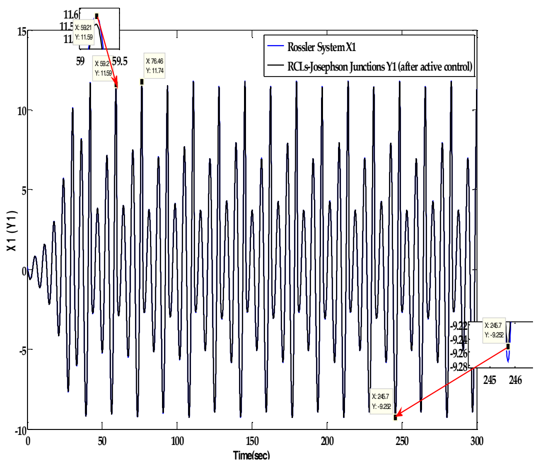


(f) RCLs-J System y_1, y_3

Fig.2, Original trajectories of Rossoler having initial condition with $[x_{10} \quad x_{20} \quad x_{30}]^T = [0.2 \quad 0.4 \quad 0.1]^T$and RCL-shunted Josephson Junctions with initial condition at original.



(a) Phase portrait x_1 (y_1), x_2 (y_2)



(d) Time response of state x_1-blue, y_1-black

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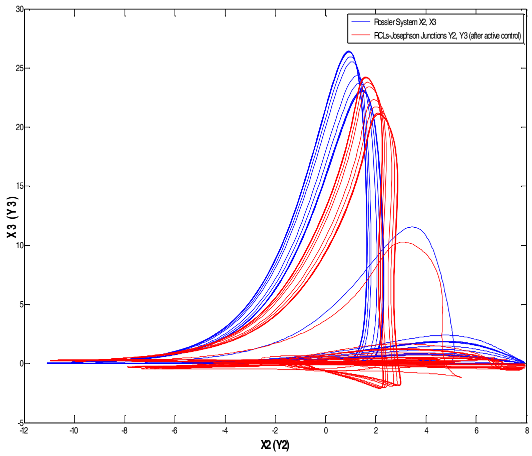
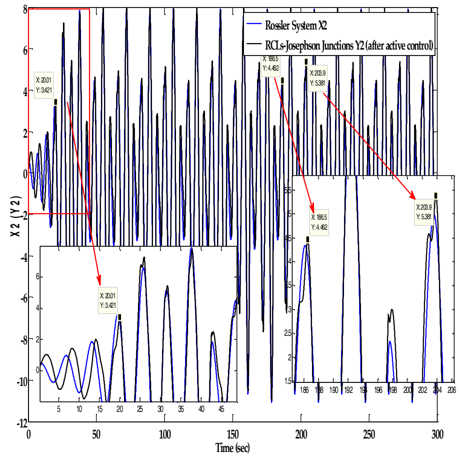
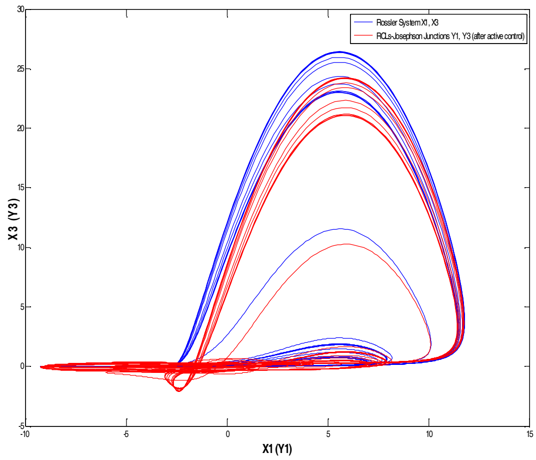


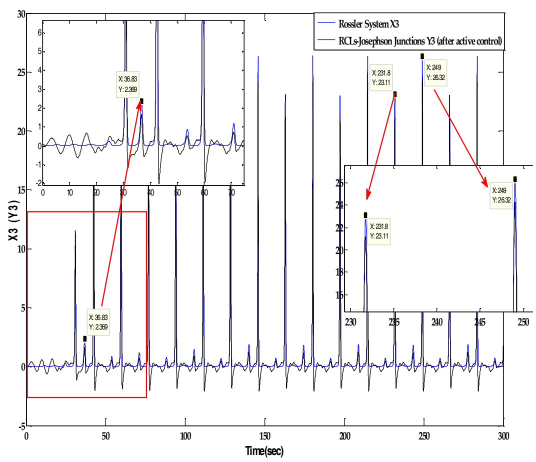
(b) Phase portrait x_2 (y_2), x_3 (y_3)



(e) Time response of state x_2-blue, y_2-black



(c) Phase portrait x_1 (y_1), x_3 (y_3)



(f) Time response of state x_3-blue, y_3-black

Fig3. after active control procedure: (a), (b), (c) the phase portrait of Rossler (x_i-blue) and RCLs-J system (y_j-red) and (c), (d), (f) time response of system state Rossler (x_i-blue) and RCLs-J system (y_j-black)

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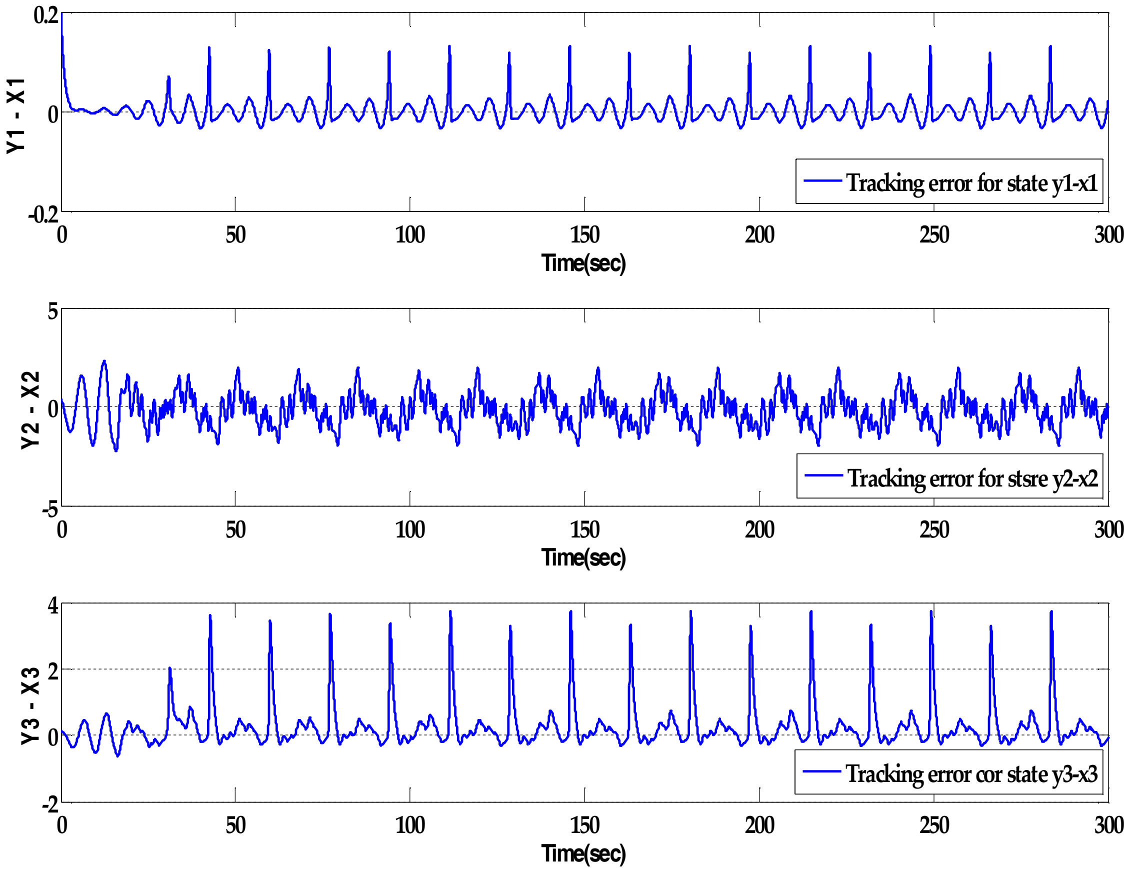

Fig. 4 The tracking error between Rossler and RCLs-J system via active control procedure

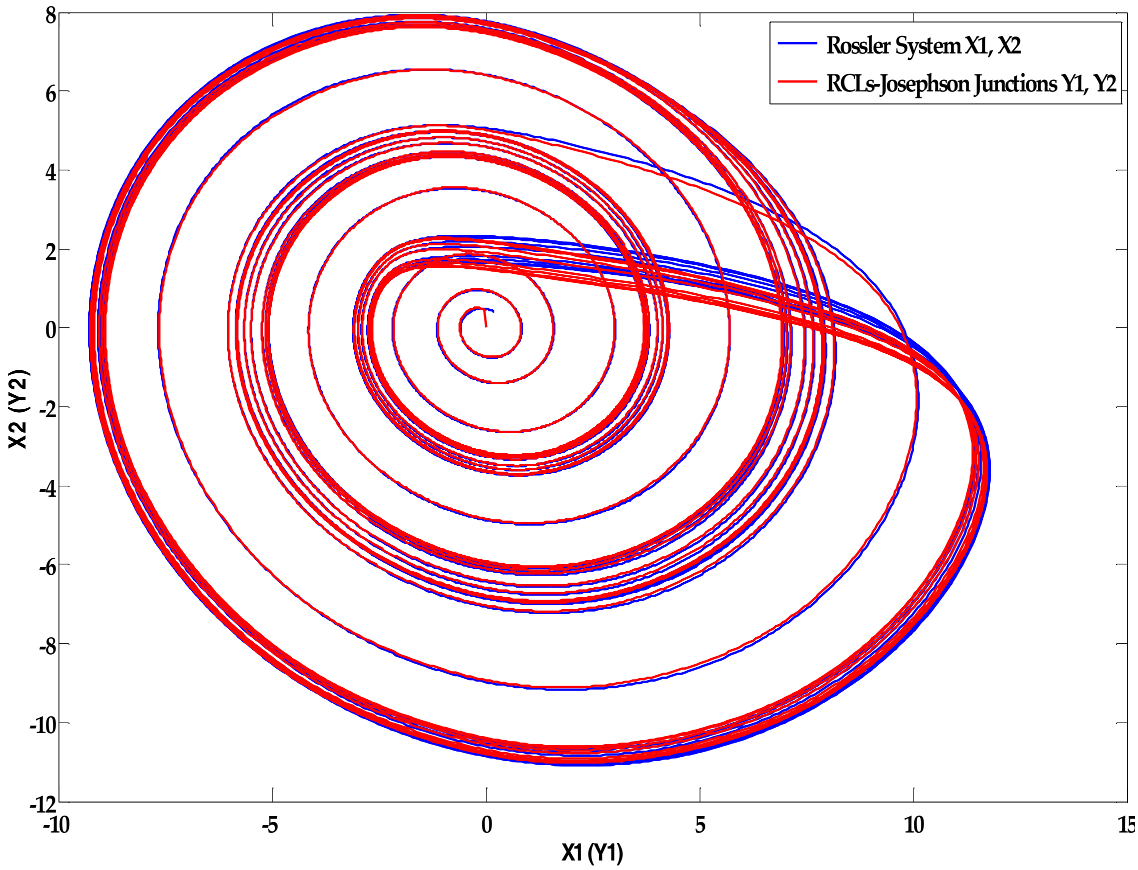

Fig. 5 The phase portrait of x_1 (y_1) and x_2 (y_2) via ILC rule

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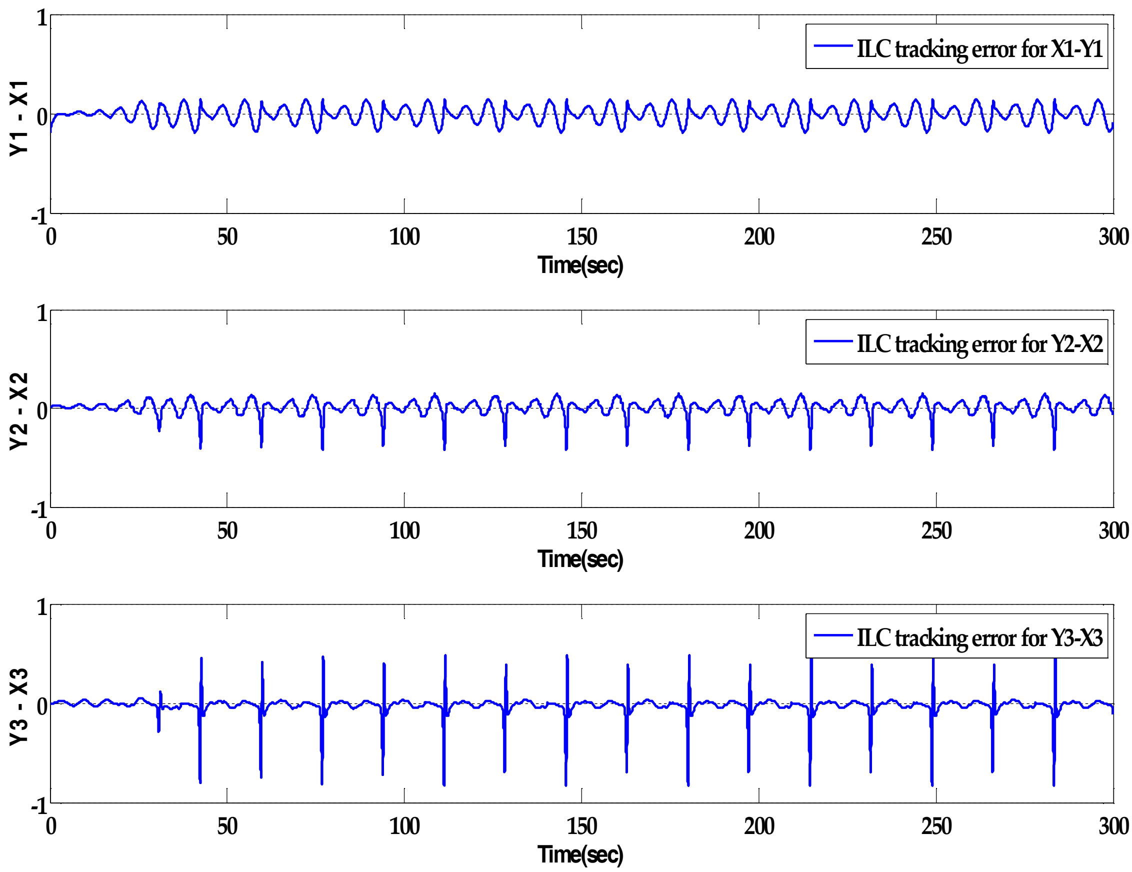

Fig. 6 The tracking error between Rossler and RCLs-J system utilizing ILC rule

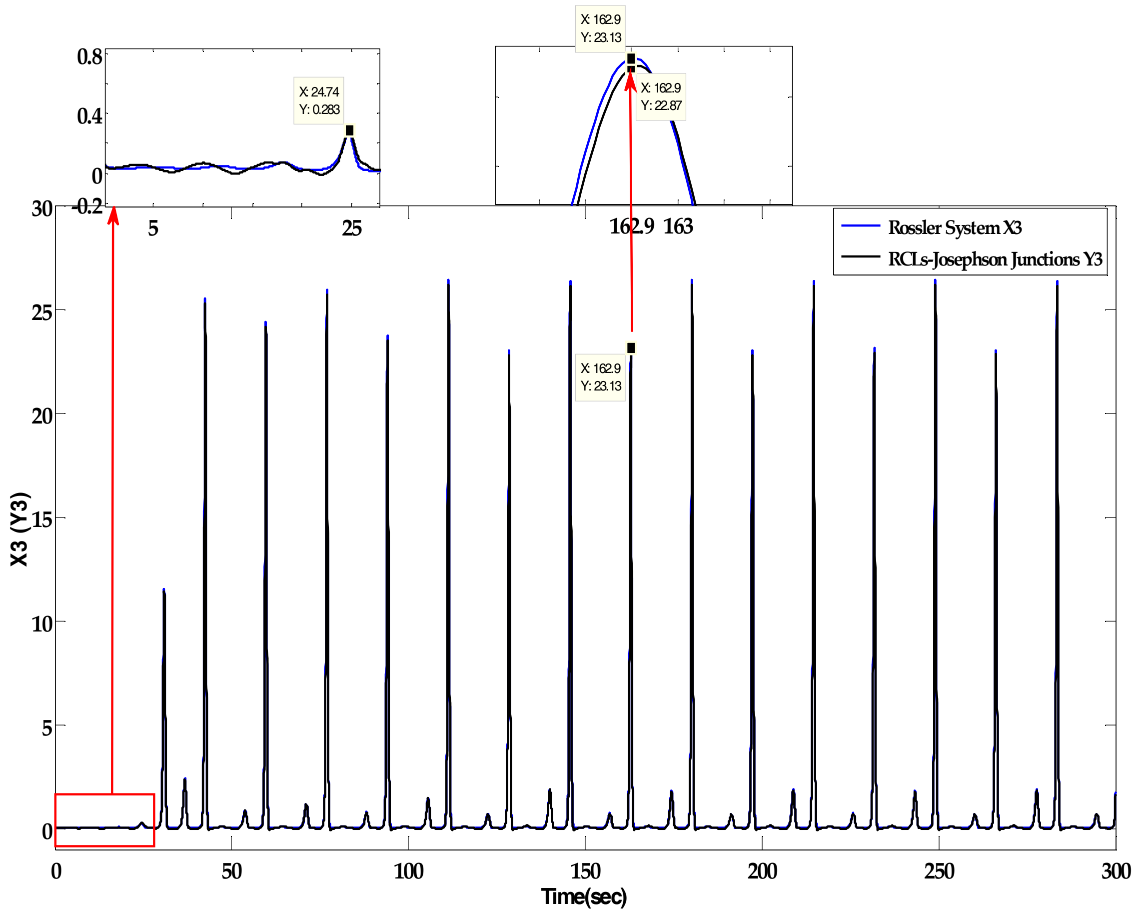

Fig. 7 The time response of x_3 for Rossler system and y_3 for RCLs-J via ILC rule

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