Article

On eccentricity-based topological indices and polynomials of phosphorus containing dendrimers

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Abstract: In the study of QSAR/QSPR, due to high degree of predictability of pharmaceutical properties, the eccentric-connectivity index has very important place among the other topological descriptors, In this paper, we compute the exact formulas of eccentric-connectivity index and its corresponding polynomial, total eccentric-connectivity index and its corresponding polynomial, first Zagreb eccentricity index, augmented eccentric-connectivity index, modified eccentric-connectivity index and its corresponding polynomial for a class of phosphorus containing dendrimers.

Keywords: Eccentric-connectivity index, augmented eccentric-connectivity index, molecular graph, phosphorus containing dendrimers.

MSC: 05C90.

0. Introduction and preliminary results

Dendrimers are synthetic polymers with a highly branched structure, consisting of multiple branched monomers radiating from a central core. Layers of monomers are attached stepwise during synthesis, with the number of branch points defining the generation of dendrimer [1]. Different kinds of experiments have proved that these polymers with well-defined dimensional structures and topological architectures exhibited array of applications in medicine [12]. Nowadays, dendrimers are currently attracting the interest of a great number of scientists because of their unusual physical and chemical properties and the wide range of potential application in different fields such as physics, biology, chemistry, engineering, and medicine [14]. A topological index sometimes known as a graph theoretic index, is a numerical invariant of a chemical graph. Topological indices are the mathematical measures associated with molecular graph structure that correlate the chemical structure with various physical properties, biological activity or chemical reactivity. A topological index is an invariant of a graph G_1, that is if $Top(G_1)$ denotes a topological index of a graph G_1 and if G_2 is another graph such that $G_1 \cong G_2$, then $Top(G_1) = Top(G_2)$. In chemistry, biochemistry and nanotechnology, distance-based topological indices of a graph are found to be useful in isomer discrimination, structure-property relationship and structure-activity relationship. In this paper, G is considered to be connected and simple molecular graph with vertex set $V(G)$ and edge set $E(G)$. The vertices of G correspond to atoms and an edge between two vertices corresponds to the chemical bond between these vertices. In graph G, two vertices u and v are adjacent if and only if they are end vertices of an edge $e \in E(G)$ and we write $e = uv$ or $e = vu$. For a vertex u, the set of neighbor vertices is denoted by N_u and is defined as $N_u = \{v \in V(G) : uv \in E(G)\}$. The degree of a vertex

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$u \in V(G)$ is denoted by d_u and is defined as $d_u = |N_u|$. Let S_u denotes the sum of the degrees of all neighbors of vertex u that is $S_u = \sum_{v \in N_u} d_v$. A (u_1, u_n)-path on n vertices is defined as a graph with vertex set $\{u_i : 1 \leq i \leq n\}$ and edge set $\{u_i u_{i+1} : 1 \leq i \leq n-1\}$. The distance $d(u, v)$ between two vertices $u, v \in V(G)$ is defined as the length of the shortest (u, v)-path in G. For a given vertex $v \in V(G)$, the eccentricity $\varepsilon(v)$ is defined as the largest distance between v and any other vertex u in G. In 1947, Harold Wiener published a paper entitled "Structural Determination of Paraffin Boiling Points" [15]. In this work the quantity W_e, eventually named Wiener index or Wiener number was introduced for the first time and he showed that there are excellent correlations between W_e and a variety of physico-chemical properties of organic compounds. Another distance-based topological index of the graph G is the eccentric-connectivity index $\xi(G)$ which is defined as [13]

$$\xi(G) = \sum_{u \in V(G)} \varepsilon(u) d_u \tag{1}$$

Different applications and mathematical properties of this index are discussed in [7,10,11,16]. For a graph G, the eccentric-connectivity polynomial is defined as [3]

$$ECP(G, y) = \sum_{u \in V(G)} d_u y^{\varepsilon(u)} \tag{2}$$

The total-eccentricity index of a graph G is expressed as follows

$$\varsigma(G) = \sum_{u \in V(G)} \varepsilon(u) \tag{3}$$

The total eccentric-connectivity polynomial of a graph G is defined as [3]

$$TECP(G, y) = \sum_{u \in V(G)} y^{\varepsilon(u)} \tag{4}$$

The first Zagreb index of a graph G in terms of eccentricity was given by Ghorbani and Hosseinzadeh [8] as follows:

$$M_1^{**}(G) = \sum_{u \in V(G)} (\varepsilon(u))^2 \tag{5}$$

Gupta and his co-authors [9] introduced the augmented eccentric-connectivity index of a graph G and it is defined as

$$^{A}\varepsilon(G) = \sum_{u \in V(G)} \frac{M(u)}{\varepsilon(u)}, \tag{6}$$

where $M(u)$ denotes the product of degrees of all neighbors of vertex u. Various properties of this index have been studied in [5,6]. For a graph G, the modified versions of eccentric-connectivity index and polynomial are defined as follows

$$\Lambda(G) = \sum_{u \in V(G)} S_u \varepsilon(u) \tag{7}$$

$$MECP(G, y) = \sum_{u \in V(G)} S_u y^{\varepsilon(u)} \tag{8}$$

Some mathematical and chemical properties of modified eccentric-connectivity index and polynomial have been studied in [2,3]. In this paper, we will study different topological indices and polynomials of the molecular graph of phosphorus containing dendrimer Cyclotriphosphazene (N_3P_3) which have stable end groups and these are studied by EPR temperature spectrum [4].

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1. The eccentricity-based indices and polynomials for the molecular graph

Let the molecular graph of this dendrimer be $D(n)$, where the generation stage of $D(n)$ is represented by n. The first and second generations are shown in Figures 1 and 2 respectively.

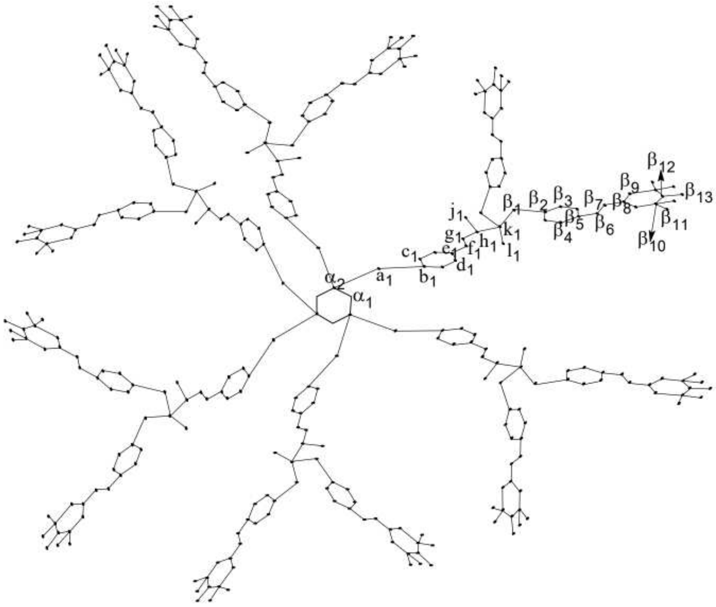

Figure 1. First generation.

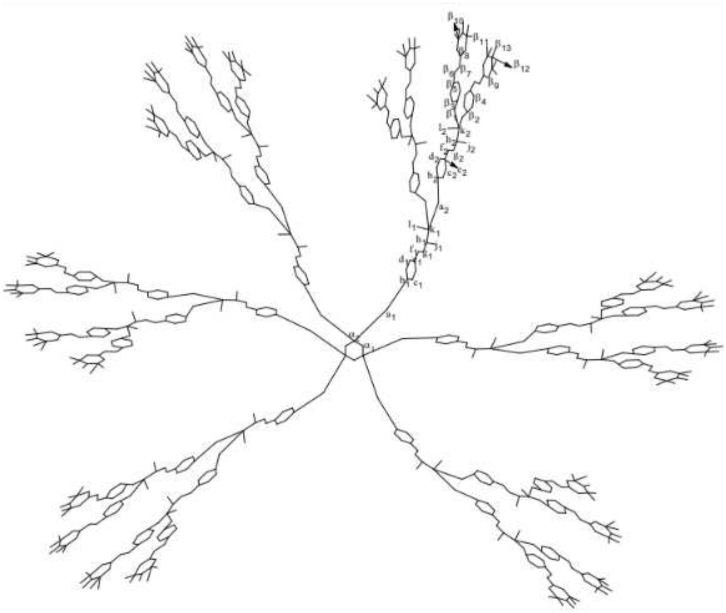

Figure 2. Second generation.

The size and order of the graph $D(n)$ are $6(9 \times 2^{n+2} - 13)$ and $9(-8 + 11 \times 2^n)$, respectively. For computing the eccentricity-based indices and polynomials of $D(n)$, it is enough to compute the required information for a set of representatives of $V(D(n))$. We will compute the required information by using computational arguments. We make three sets of representatives of $V(D(n))$, say $A = \{\alpha_1, \alpha_2\}$,

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$B = \{\beta_1, \beta_2, \ldots, \beta_{13}\}$ and $C = \{a_i, b_i, c_i, d_i, e_i, f_i, g_i, h_i, j_i, k_i, l_i\}$ where $1 \le i \le n$, as shown in Figures 1 and 2. The degree, S_u, $M(u)$ and eccentricity for each u for the sets A, B and C are shown in Table 1 and Table 2. For simplicity, we assume $\gamma = 9n + 9i$ throughout the paper. By using the Tables 1 and 2, we calculate the different eccentricity-based indices and their corresponding polynomials. In the following theorem, we determine the eccentric-connectivity index of $D(n)$.

Table 1. Sets A and B with their degrees, S_u, $M(u)$, eccentricities and frequencies.

Representative	Degree	S_u	$M(u)$	Eccentricity	Frequency
α_1	2	8	16	$9n+15$	3
α_2	4	8	16	$9n+14$	3
β_1	2	7	12	$9n+15$	$3\times 2^{n+1}$
β_2	3	6	8	$9n+16$	$3\times 2^{n+1}$
β_3	2	5	6	$9n+17$	$3\times 2^{n+2}$
β_4	2	5	6	$9n+18$	$3\times 2^{n+2}$
β_5	3	6	8	$9n+19$	$3\times 2^{n+1}$
β_6	2	5	6	$9n+20$	$3\times 2^{n+1}$
β_7	2	5	6	$9n+21$	$3\times 2^{n+1}$
β_8	3	6	8	$9n+22$	$3\times 2^{n+1}$
β_9	2	7	12	$9n+23$	$3\times 2^{n+2}$
β_{10}	4	7	6	$9n+24$	$3\times 2^{n+2}$
β_{11}	1	4	4	$9n+25$	$3\times 2^{n+3}$
β_{12}	3	9	16	$9n+25$	$3\times 2^{n+1}$
β_{13}	1	3	3	$9n+26$	$3\times 2^{n+1}$

Table 2. Set C with degrees, S_u, $M(u)$, eccentricities and frequencies.

Representative	Degree	S_u	$M(u)$	Eccentricity	Frequency
a_i	2	7	12	$9n+9i+6=\gamma+6$	3×2^i
b_i	3	6	8	$\gamma+7$	3×2^i
c_i	2	5	6	$\gamma+8$	$3\times 2^{i+1}$
d_i	2	5	6	$\gamma+9$	$3\times 2^{i+1}$
e_i	3	6	8	$\gamma+10$	3×2^i
f_i	2	5	6	$\gamma+11$	3×2^i
g_i	2	5	6	$\gamma+12$	3×2^i
h_i	3	7	8	$\gamma+13$	3×2^i
j_i	1	3	3	$\gamma+14$	3×2^i
k_i	4	8	12	$\gamma+14$	3×2^i
l_i	1	4	4	$\gamma+15$	3×2^i

Theorem 1. *For the graph $D(n)$, the eccentric-connectivity index is given by*

$$\xi(D(n)) = 18(2^{n+2} \times 79 - 78n + 2^n \times 303n + 1).$$

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Proof. By putting the values of Tables 1 and 2 in equation (1), the eccentric-connectivity index of $D(n)$ can be written as follows:

$$\begin{aligned}\xi(D(n)) = \xi(A)+\xi(B)+\xi(C) &= \sum_{u\in A}\varepsilon(u)d_u+\sum_{u\in B}\varepsilon(u)d_u+\sum_{u\in C}\varepsilon(u)d_u\\ &= (2\times 3)(9n+15)+(3\times 4)(9n+14)+(3\times 2^{n+1}\times 2)(9n+15)\\ &+(3\times 2^{n+1}\times 3)(9n+16)+(2\times 2^{n+2}\times 3)(9n+17)+(2\times 2^{n+2}\times 3)(9n+18)\\ &+(3\times 2^{n+1}\times 3)(9n+19)+(2\times 2^{n+1}\times 3)(9n+20)+(2\times 2^{n+1}\times 3)(9n+21)\\ &+(2\times 2^{n+2}\times 3)(9n+23)+(4\times 2^{n+2}\times 3)(9n+24)+(1\times 2^{n+3}\times 3)(9n+25)\\ &+(3\times 2^{n+1}\times 3)(9n+22)+(3\times 2^{n+1}\times 3)(9n+25)+(1\times 2^{n+1}\times 3)(9n+26)\end{aligned}$$

$$\begin{aligned}&+\sum_{i=1}^{n}\Big((2\times 2^i\times 3)(\gamma+6)+(3\times 2^i\times 3)(\gamma+7)+(2\times 2^{i+1}\times 3)(\gamma+8)\\ &+(2^{i+2}\times 3)(\gamma+9)+(3\times 2^i\times 3)(\gamma+10)+(3\times 2^{i+1})(\gamma+11)+(2^{i+1}\times 3)(\gamma+12)\\ &+(3\times 2^i\times 3)(\gamma+13)+(2^i\times 3)(\gamma+14)+(4\times 2^i\times 3)(\gamma+14)+(2^i\times 3)(\gamma+15)\Big).\end{aligned}$$

After some calculations, we get

$$\xi(D(n)) = 18(2^{n+2}\times 79-78n+2^n\times 303n+1),$$

which completes the theorem. □

When the degrees of vertices are not taken into account, then by using the values of Tables 1 and 2 in (3), we have the following result.

Corollary 1. *For the graph $D(n)$, the total eccentric-connectivity index is given by*

$$\varsigma(D(n)) = 9(2^{n+2}\times 69n+2^{n+1}\times 149-72n-3).$$

In the next theorem, the eccentric-connectivity polynomial for the molecular graph has been derived.

Theorem 2. *For the graph $D(n)$, the eccentric-connectivity polynomial is given by*

$$\begin{aligned}ECP(D(n),y) &= 6y^{9n+14}(y+2)+3\times 2^{n+1}y^{9n+15}(y^{11}+7y^{10}+8y^9+4y^8+3y^7+2y^6+2y^5\\ &+3y^4+4y^3+4y^2+3y+2)+\frac{6(y^3+5y^2+3y+2)\times y^{9n+21}(2^ny^{9n}-1)}{2y^9-1}\\ &+\frac{6(2y^5+3y^4+4y^3+4y^2+3y+2)\times y^{9n+15}(2^ny^{9n}-1)}{2y^9-1}.\end{aligned}$$

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Proof. By using Tables 1 and 2 in (2), we have

$$\begin{aligned}
ECP(D(n),y) &= ECP(A,y)+ECP(B,y)+ECP(C,y)\\
&= \sum_{u\in A} d_u y^{\varepsilon(u)} + \sum_{u\in B} d_u y^{\varepsilon(u)} + \sum_{u\in C} d_u y^{\varepsilon(u)}\\
&= (2\times 3)y^{9n+15} + (4\times 3)y^{9n+14} + (3\times 2^{n+2})y^{9n+15} + (3\times 3\times 2^{n+1})y^{9n+16}\\
&+ (2\times 3\times 2^{n+2})y^{9n+17} + (2\times 3\times 2^{n+2})y^{9n+18} + (3\times 3\times 2^{n+1})y^{9n+19}\\
&+ (2\times 3\times 2^{n+1})y^{9n+20} + (2\times 3\times 2^{n+1})y^{9n+21} + (3\times 3\times 2^{n+1})y^{9n+22}\\
&+ (2\times 3\times 2^{n+2})y^{9n+23} + (4\times 3\times 2^{n+2})y^{9n+24} + (1\times 3\times 2^{n+3})y^{9n+25}\\
&+ (3\times 3\times 2^{n+1})y^{9n+25} + (1\times 3\times 2^{n+1})y^{9n+26} + \sum_{i=1}^{n}\Big((2\times 3\times 2^{i})y^{\gamma+6}\\
&+ (2\times 3\times 2^{i+1})y^{\gamma+8} + (2\times 3\times 2^{i+1})y^{\gamma+9} + (3\times 3\times 2^{i})y^{\gamma+10}\\
&+ (2\times 3\times 2^{i})y^{\gamma+11} + (3\times 3\times 2^{i})y^{\gamma+7} + (2\times 3\times 2^{i})y^{\gamma+12}\\
&+ (3\times 3\times 2^{i})y^{\gamma+13} + (3\times 2^{i})y^{\gamma+14} + (4\times 3\times 2^{i})y^{\gamma+14} + (3\times 2^{i})y^{\gamma+15}\Big).
\end{aligned}$$

After some calculations, we get the required result. □

By putting the values of Tables 1 and 2 in (4), we have the following result.

Corollary 2. *For the graph $D(n)$, the total eccentric-connectivity polynomial is given by*

$$\begin{aligned}
TECP(D(n),y) &= 3y^{9n+14}(y+1) + 3\times 2^{n+1}y^{9n+15}(y^{11}+5y^{10}+2y^{9}+2y^{8}+y^{7}+y^{6}+y^{5}\\
&+y^{4}+2y^{3}+2y^{2}+y+1) + \frac{6(y^{3}+2y^{2}+y+1)\times y^{9n+21}(2^{n}y^{9n}-1)}{2y^{9}-1}\\
&+\frac{6(y+1)(y^{2}+1)^{2}\times y^{9n+15}(2^{n}y^{9n}-1)}{2y^{9}-1}.
\end{aligned}$$

In the next theorem, we compute the closed formula for the first Zagreb eccentricity index.

Theorem 3. *For the graph $D(n)$, the first Zagreb eccentricity index is given by*

$$M_1^{**}(D(n)) = 3(2^{n+4}\times 7295n^{2} + 2^{n+3}\times 2097n - 1944n^{2} - 162n + 2^{n+1}\times 11641 - 4053).$$

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Proof. By using the values of Tables 1 and 2 in (5), we compute the first Zagreb eccentricity index of $D(n)$ as follows:

$$\begin{aligned}
M_1^{**}(D(n)) &= M_1^{**}(A) + M_1^{**}(B) + M_1^{**}(C) = \sum_{v\in A}[\varepsilon(v)]^2 + \sum_{v\in B}[\varepsilon(v)]^2 + \sum_{v\in C}[\varepsilon(v)]^2 \\
&= 3(9n+15)^2 + 3(9n+14)^2 + (3\times 2^{n+1})(9n+15)^2 + (3\times 2^{n+1})(9n+16)^2 \\
&+ (3\times 2^{n+2})(9n+17)^2 + (3\times 2^{n+2})(9n+18)^2 + (3\times 2^{n+1})(9n+19)^2 \\
&+ (3\times 2^{n+1})(9n+20)^2 + (3\times 2^{n+1})(9n+21)^2 + (3\times 2^{n+1})(9n+22)^2 \\
&+ (3\times 2^{n+2})(9n+23)^2 + (3\times 2^{n+2})(9n+24)^2 + (3\times 2^{n+3})(9n+25)^2 \\
&+ (3\times 2^{n+1})(9n+25)^2 + (3\times 2^{n+1})(9n+26)^2 + \sum_{i=1}^{n}\Big((3\times 2^i)(\gamma+6)^2 \\
&+ (3\times 2^i)(\gamma+7)^2 + (3\times 2^{i+1})(\gamma+8)^2 + (3\times 2^{i+1})(\gamma+9)^2 + (3\times 2^i)(\gamma+10)^2 \\
&+ (3\times 2^i)(\gamma+11)^2 + (3\times 2^i)(\gamma+12)^2 + (3\times 2^i)(\gamma+13)^2 + (3\times 2^i)(\gamma+14)^2 \\
&+ (3\times 2^i)(\gamma+14)^2 + (3\times 2^i)(\gamma+15)^2\Big).
\end{aligned}$$

After some calculations, we obtain

$$M_1^{**}(D(n)) = 3(2^{n+4}\times 7295n^2 + 2^{n+3}\times 2097n - 1944n^2 - 162n + 2^{n+1}\times 11641 - 4053),$$

that finishes the theorem. □

Now, we determine the augmented eccentric-connectivity index in the next theorem.

Theorem 4. *For the graph $D(n)$, the augmented eccentric-connectivity index is given by*

$$\begin{aligned}
{}^{A}\varepsilon(D(n)) &= \frac{48}{9n+15} + \frac{48}{9n+14} + \frac{36\times 2^{n+1}}{9n+15} + \frac{24\times 2^{n+1}}{9n+16} + \frac{18\times 2^{n+2}}{9n+17} + \frac{18\times 2^{n+2}}{9n+18} \\
&+ \frac{24\times 2^{n+1}}{9n+19} + \frac{18\times 2^{n+1}}{9n+20} + \frac{18\times 2^{n+1}}{9n+21} + \frac{24\times 2^{n+1}}{9n+22} + \frac{36\times 2^{n+2}}{9n+23} + \frac{18\times 2^{n+2}}{9n+24} \\
&+ \frac{12\times 2^{n+3}}{9n+25} + \frac{48\times 2^{n+1}}{9n+25} + \frac{9\times 2^{n+1}}{9n+26} + \left(\frac{72}{9n+15} + \cdots + \frac{36\times 2^n}{18n+6}\right) \\
&+ \left(\frac{48}{9n+16} + \cdots + \frac{24\times 2^n}{18n+7}\right) + \left(\frac{72}{9n+17} + \cdots + \frac{18\times 2^{n+1}}{18n+8}\right) \\
&+ \left(\frac{72}{9n+18} + \cdots + \frac{18\times 2^{n+1}}{18n+9}\right) + \left(\frac{48}{9n+19} + \cdots + \frac{24\times 2^n}{18n+10}\right)
\end{aligned}$$

$$\begin{aligned}
&+ \left(\frac{36}{9n+20} + \cdots + \frac{18\times 2^n}{18n+11}\right) + \left(\frac{36}{9n+21} + \cdots + \frac{18\times 2^n}{18n+12}\right) \\
&+ \left(\frac{48}{9n+22} + \cdots + \frac{24\times 2^n}{18n+13}\right) + \left(\frac{18}{9n+23} + \cdots + \frac{9\times 2^n}{18n+14}\right) \\
&+ \left(\frac{72}{9n+23} + \cdots + \frac{36\times 2^n}{18n+14}\right) + \left(\frac{24}{9n+24} + \cdots + \frac{12\times 2^n}{18n+15}\right).
\end{aligned}$$

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Proof. By using the values of Tables 1 and 2 in (6), we compute the augumented eccentric-connectivity index of $D(n)$ in the following way:

$$
\begin{aligned}
{}^{A}\varepsilon(D(n)) &= {}^{A}\varepsilon(A) + {}^{A}\varepsilon(B) + {}^{A}\varepsilon(C) = \sum_{u\in A}\frac{M(u)}{\varepsilon(u)} + \sum_{u\in B}\frac{M(u)}{\varepsilon(u)} + \sum_{u\in C}\frac{M(u)}{\varepsilon(u)} \\
&= \frac{3\times 16}{9n+15} + \frac{3\times 16}{9n+14} + \frac{3\times 2^{n+1}\times 12}{9n+15} + \frac{3\times 2^{n+1}\times 8}{9n+16} + \frac{3\times 2^{n+2}\times 6}{9n+17} \\
&+ \frac{3\times 2^{n+2}\times 6}{9n+18} + \frac{3\times 2^{n+1}\times 8}{9n+19} + \frac{3\times 2^{n+1}\times 6}{9n+20} + \frac{3\times 2^{n+1}\times 6}{9n+21} \\
&+ \frac{3\times 2^{n+1}\times 8}{9n+22} + \frac{3\times 2^{n+2}\times 12}{9n+23} + \frac{3\times 2^{n+2}\times 6}{9n+24} + \frac{3\times 2^{n+3}\times 4}{9n+25} \\
&+ \frac{3\times 2^{n+1}\times 16}{9n+25} + \frac{3\times 2^{n+1}\times 3}{9n+26} + \sum_{i=1}^{n}\Big(\frac{3\times 2^{i}\times 12}{\gamma+6} + \frac{3\times 2^{i}\times 8}{\gamma+7} \\
&+ \frac{3\times 2^{i+1}\times 6}{\gamma+8} + \frac{3\times 2^{i+1}\times 6}{\gamma+9} + \frac{3\times 2^{i}\times 8}{\gamma+10} + \frac{3\times 2^{i}\times 6}{\gamma+11} + \frac{3\times 2^{i}\times 6}{\gamma+12} \\
&+ \frac{3\times 2^{i}\times 8}{\gamma+13} + \frac{3\times 2^{i}\times 3}{\gamma+14} + \frac{3\times 2^{i}\times 12}{\gamma+14} + \frac{3\times 2^{i}\times 4}{\gamma+15}\Big).
\end{aligned}
$$

After some calculations, we obtain the required result. □

Now, we compute the closed formula for the modified eccentric-connectivity index.

Theorem 5. *For the graph $D(n)$, the modified eccentric-connectivity index is given by*

$$\Lambda(D(n)) = 6(2^{n}\times 2277n - 567n + 2^{n+1}\times 1229 + 21).$$

Proof. By using the values of Tables 1 and 2 in (7), we compute the modified eccentric-connectivity index of $D(n)$ in the following way:

$$
\begin{aligned}
\Lambda(D(n)) &= \Lambda(A) + \Lambda(B) + \Lambda(C) = \sum_{u\in A} S_u\varepsilon(u) + \sum_{u\in B} S_u\varepsilon(u) + \sum_{u\in C} S_u\varepsilon(u) \\
&= (8\times 3)(9n+15) + (8\times 3)(9n+14) + (7\times 3\times 2^{n+1})(9n+15) \\
&+ (5\times 3\times 2^{n+2})(9n+17) + (5\times 3\times 2^{n+2})(9n+18) + (6\times 3\times 2^{n+1})(9n+19) \\
&+ (5\times 3\times 2^{n+1})(9n+20) + (5\times 3\times 2^{n+1})(9n+21) + (6\times 3\times 2^{n+1})(9n+22)
\end{aligned}
$$

$$
\begin{aligned}
&+ (7\times 3\times 2^{n+2})(9n+23) + (7\times 3\times 2^{n+2})(9n+24) + (4\times 3\times 2^{n+3})(9n+25) \\
&+ (9\times 3\times 2^{n+1})(9n+25) + (3\times 3\times 2^{n+1})(9n+26) + (6\times 3\times 2^{n+1})(9n+16) \\
&+ \sum_{i=1}^{n}\Big((7\times 3\times 2^{i})(\gamma+6) + (6\times 3\times 2^{i})(\gamma+7) + (5\times 3\times 2^{i+1})(\gamma+8) \\
&+ (5\times 3\times 2^{i+1})(\gamma+9) + (6\times 3\times 2^{i})(\gamma+10) + (5\times 3\times 2^{i})(\gamma+11) \\
&+ (5\times 3\times 2^{i})(\gamma+12) + (7\times 3\times 2^{i})(\gamma+13) + (3\times 3\times 2^{i})(\gamma+14) \\
&+ (8\times 3\times 2^{i})(\gamma+14) + (4\times 3\times 2^{i})(\gamma+15)\Big).
\end{aligned}
$$

After some calculations, we obtain

$$\Lambda(D(n)) = 6(2^{n}\times 2277n - 567n + 2^{n+1}\times 1229 + 21),$$

this completes the proof. □

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Finally, we compute the closed formula for the modified eccentric-connectivity polynomial.

Theorem 6. *For the graph $D(n)$, the modified eccentric-connectivity polynomial is given by*

$$\begin{aligned} MECP(D(n),y) = {} & 24y^{9n+14}(y+1) + 2^{n+1} \times y^{9n+15}(9y^{11} + 75y^{10} + 42y^{9} + 42y^{8} \\ & + 18y^{7} + 15y^{6} + 15y^{5} + 18y^{4} + 30y^{3} + 30y^{2} + 18y + 21) \\ & + \frac{6(5y^5 + 6y^4 + 10y^3 + 10y^2 + 6y + 7)y^{9n+15}(2^n y^{9n} - 1)}{2y^9 - 1} \\ & + \frac{6(4y^3 + 11y^2 + 7y + 5)y^{9n+21}(2^n y^{9n} - 1)}{2y^9 - 1}. \end{aligned}$$

Proof. By using the values of Tables 1 and 2 in (8), we compute the modified eccentric-connectivity polynomial of $D(n)$ in the following way:

$$\begin{aligned} MECP(D(n),y) &= MECP(A,y) + MECP(B,y) + MECP(C,y) \\ &= \sum_{u \in A} S_u y^{\varepsilon(u)} + \sum_{u \in B} S_u y^{\varepsilon(u)} + \sum_{u \in C} S_u y^{\varepsilon(u)} \\ &= (8 \times 3)y^{9n+15} + (8 \times 3)y^{9n+14} + (7 \times 3 \times 2^{n+1})y^{9n+15} \\ &+ (6 \times 3 \times 2^{n+1})y^{9n+16} + (5 \times 3 \times 2^{n+2})y^{9n+17} + (5 \times 3 \times 2^{n+2})y^{9n+18} \\ &+ (6 \times 3 \times 2^{n+1})y^{9n+19} + (5 \times 3 \times 2^{n+1})y^{9n+20} + (5 \times 3 \times 2^{n+1})y^{9n+21} \\ &+ (6 \times 3 \times 2^{n+1})y^{9n+22} + (7 \times 3 \times 2^{n+2})y^{9n+23} + (7 \times 3 \times 2^{n+2})y^{9n+24} \\ &+ (4 \times 3 \times 2^{n+3})y^{9n+25} + (9 \times 3 \times 2^{n+1})y^{9n+25} + (3 \times 3 \times 2^{n+1})y^{9n+26} \\ &+ \sum_{i=1}^{n} \Big((7 \times 3 \times 2^{i})(y^{\gamma+6}) + (6 \times 3 \times 2^{i})(y^{\gamma+7}) + (5 \times 3 \times 2^{i+1})(y^{\gamma+8}) \\ &+ (5 \times 3 \times 2^{i+1})(y^{\gamma+9}) + (6 \times 3 \times 2^{i})(y^{\gamma+10}) + (5 \times 3 \times 2^{i})(y^{\gamma+11}) \\ &+ (5 \times 3 \times 2^{i})(y^{\gamma+12}) + (7 \times 3 \times 2^{i})(y^{\gamma+13}) + (3 \times 3 \times 2^{i})(y^{\gamma+14}) \end{aligned}$$

$$+ (8 \times 3 \times 2^{i})(y^{\gamma+14}) + (4 \times 3 \times 2^{i})(y^{\gamma+15}) \Big).$$

After some calculations, we obtain the required result. □

2. Conclusion

In this paper, we compute the precise values of eccentric-connectivity index and its corresponding polynomial, total eccentric-connectivity index and its corresponding polynomial, first Zagreb eccentricity index, augmented eccentric-connectivity index, modified eccentric-connectivity index and its corresponding polynomial for a class of phosphorus containing dendrimers.

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