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Article

Analysis of Failure Criteria in Laminas Reinforced with Unidirectional Curauá Fiber Fabric

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Abstract: Natural fibers are being increasingly used in different areas of engineering, including as composite reinforcement. Among these fibers, carauá stands out for its good mechanical properties and adherence to resin. Nevertheless, little is known about the behavior of this material in the manufacture of a composite or whether classic failure theory can be used in this case. In this context, the present study assesses the mechanical properties of two laminas made of unidirectional curauá fiber with volumetric fiber percentages of 30 % and 22 %, and compares the results with the values obtained for four failure criteria reported in the literature, using analysis of variance (ANOVA). To that end, tensile tests were conducted in the direction of the fiber and at other loading angles, in addition to iosipescu shear tests. The results show that the maximum stress criterion does not represent the failure behavior of these materials and that the best was the Hashin criterion.

Keywords: Failure criteria, Curauá fibers, Reinforcement direction, ANOVA.

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1. Introduction

The use of natural fibers has gained popularity in engineering applications, largely due to properties such as low density and high mechanical strength, as well as their low cost and for being a renewable source.

As such, natural fibers are being studied as an option in the manufacture of composites, in an attempt to avoid the high cost of widely used synthetic materials, substituting them with a renewable raw material. Indeed, natural fibers have partially and even completely replaced synthetic fibers in many applications [1-2].

Among the plant fibers in Brazil, which include cotton, jute, ramie and coconut, curauá has attracted considerable attention. It originates in the Amazon region and has demonstrated excellent mechanical properities [3], with potential applications in the automobile industry due to its strength, rigidity and light weight. One of the areas in which it has stood out is as reinforcement in polymer composites, because its mechanical properties perform better than other natural fibers such as jute and cotton [4-7].

However, for a natural fiber-reinforced composite to be applied structurally, its predicted failure and characterization of its engineering constants must be determined, because structures are normally submitted to biaxial or even triaxial stresses. As such, failure theories previously developed for orthotropic materials must be applied and assessed for this particular case [8]. To better compare failure theories, analysis of variance (ANOVA) was conducted to determine which is most suitable.

Thus, the present study investigates two unidirectional curauá/polyester laminas with different volumetric fractions, analyzing failure for several fiber directions based on four failure criteria: maximum stress, Tsai-Wu, Tsai-Hill and Hashin. Different test specimens were submitted to tensile and shear tests and the behavior of the material was examined based on data from the tested samples.

2. Failure criteria studied

2.1. Maximum stress criterion

The maximum stress criterion [9] assumes that the stresses applied in the main directions of the material must be lower than those in the load directions to prevent the material from rupturing [10]. The safety limits for this criterion are presented in equation (1):

$$-X_C \leq \sigma_1 \leq X_T$$

$$-Y_C \leq \sigma_2 \leq Y_T \quad (1)$$

$$|\tau_{12}| \leq S$$

Where X_T, X_C, Y_T, Y_C and S are the strength values obtained in experimental tests, as follows: X_T is longitudinal tensile strength; X_{Ca} longitudinal compressive strength; Y_T transverse tensile strength; Y_C transverse compressive strength; S in-plane shear strength and σ_1, σ_2 and τ_{12} are normal σ_1, σ_2 and shear stresses τ_{12} applied to the material (Figure 1).

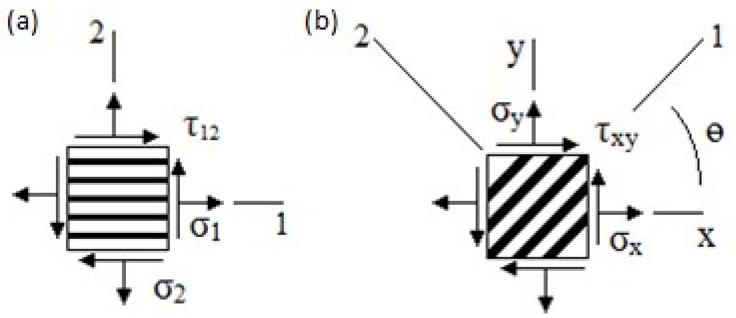

Figure 1. (a) Stresses in direction 1 and 2 (parallel and perpendicular to the fiber). (b) Stresses in direction x and y (at an angle in relation to the fiber).

Although this criterion is simple to apply, it does not consider the combined effects of load, a common occurrence during the use of any structural element.

2.2. Tsai-Hill failure criterion

This criterion is based on a generalization of the maximum distortion theory (Von-Mises criterion) applied to isotropic materials, where it is considered that the material has different boundary conditions (strength values), depending on the type of load application. Considering plane stress in a unidirectional lamina, the material does not rupture if the stresses of the material are equal to equation (2) [10].

$$\frac{\sigma_1^2}{X^2} + \frac{\sigma_2^2}{Y^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\tau_{12}^2}{S^2} \leq 1 \quad (2)$$

For this equation, the behavior of the material is equal both to tension and compression ($X_T = X_c$ and $Y_T = Y_C$).

2.3. Tsai-Wu failure criterion

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Tsai and Wu11 proposed a method that includes 2nd, 4th and 9th order tensors (where a constant was previously used in the Tsai-Hill criterion), moving the equation even closer to the experimental data. The problem with this solution was that the analysis procedure and complexity of the model became unfeasible, and as a result, Tsai and Wu11 simplified the model. Thus, for the case of plane stress where $X_T = X_c$ and $Y_T = Y_C$, this model is reduced to equation (3).

$$\frac{\sigma_1^{\,2}}{X^2} - \frac{\sigma_1 \sigma_2}{XY} + \frac{\sigma_2^{\,2}}{Y^2} + \frac{\tau_{12}^{\,2}}{S^2} \leq 1 \tag{3}$$

In the format shown in equation (3), the Tsai-Wu and Tsai-Hill criteria are similar, where only the second term of the two equations are different.

2.4. Hashin failure criterion

Unlike the previous criteria, this criterion considers the failure method in its analysis, differentiating between failure in the fiber and the matrix, or if it occurs by tension or compression, and although its development is lengthy, it can be simplified into two equations (equation 4 and 5) for the case of plane stress under tension, where equation 4 refers to fiber-based failure and equation 5 failure in the matrix [9].

$$\left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \leq 1 \tag{4}$$

$$\left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \leq 1 \tag{5}$$

2.5. Transformation of the stress vector

The values obtained from the mechanical properties of an orthotropic lamina are directly associated with the direction of the fiber in the composite [10-14]. As such, it is important to reiterate the equations that relate the stress applied to the material (σ_x, σ_y, τ_{xy}) and its direction in relation to the fiber (θ) (Figure 1), with stresses in the direction of the fiber (σ_1), perpendicular to it (σ_2), and shear stress (τ_{12}), using equation (6).

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Cos^2\,\theta & Sin^2\,\theta & 2\,Sin\,\theta\,Cos\,\theta \\ Sin^2\,\theta & Cos^2\,\theta & -2Sin\,\theta\,Cos\,\theta \\ -Sin\,\theta\,Cos\,\theta & Sin\,\theta\,\cos\theta & Cos^2\theta - Sin^2\,\theta \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{6}$$

Considering that all the tests in this article were uniaxial ($\sigma_y = 0$ and $\tau_{xy} = 0$), this equation can be incorporated into the equations of each of the failure criteria previously demonstrated for each angle studied, comparing the ultimate tensile strength of the material in off-axis loading with the value obtained by failure criteria.

3. Materials and methods

3.1. Materials used

To manufacture the lamina (composite with a reinforcing layer) fresh curauá (Ananás Erectifolious) was used as fiber reinforcement and unsaturated orthophthalic polyester resin as polymer matrix, with Novapol - L20 technical data sheets. Methyl ethyl ketone (MEKP) was used as an agent of the catalytic system (curing at room temperature).

Fibers with an average length of 700 mm were selected. After the fibers were washed, strands were obtained to manufacture unidirectional fabrics, with an area of 400 x 400 mm. Figure 2(a) shows the loom used and the unidirectional fabric made (Figure 2 b). The hand-lay-up process was used to manufacture the composite.

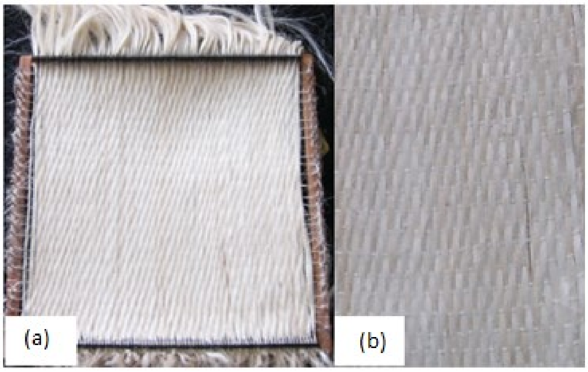

Figure 2. (a) Manual loom, (b) Curauá fabric.

A DERSA-LASER 35 30 W laser cutting machine was used to cut the test specimens for the mechanical tests.The test specimens were cut with reinforcement direction at 0°, 30°, 45°, 60° and 90° for the first lamina (22% fiber volume) and at 0°, 10°, 20°, 75° and 90° for the second lamina (30% fiber volume). Test specimens for iosipescu shear tests were also obtained and all were cut at 90^0 (the fiber is at an angle of 90º in relation to the width of the test specimen). At least 5 test specimens were prepared, as per the guideline, for each case.

3.2. Tensile and Iosipescu tests

The tensile test measures the mechanical properties of ultimate tensile strength, modulus of elasticity and rupture strain of composites. The standard test specimens used for tensile tests in continuous fiber composites were manufactured according to ASTM guideline D303915. Figures 3(a) and 3(b) show the dimensions of the test specimens at 0^0 and 90^0 used in uniaxial tensile tests.

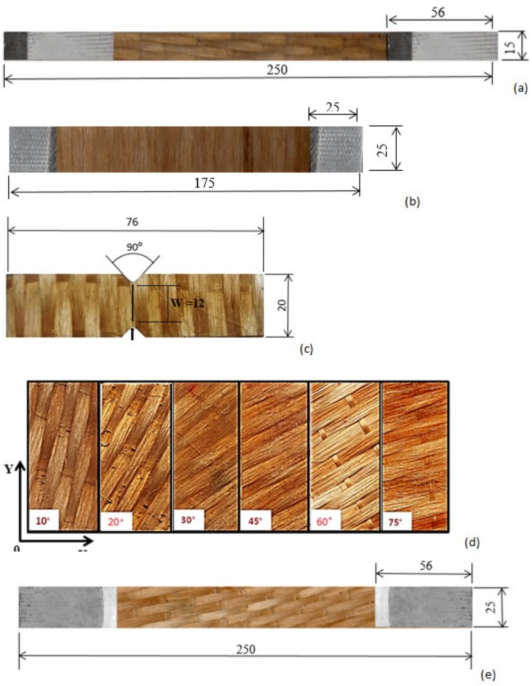

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Figure 3. Test specimen at 0° (a); Test specimen at 90°(b); Test specimen dimensions (mm) for the iosipescu test at 90° (c); Test specimens with the fiber angle used in the two laminas (d); Off-axis test specimen.(e).

The iosipescu test consists of a V-notch test specimen that forms a 90° angle, as illustrated in Figure 3(c), which is submitted to force P with the use of an appropriate grip system16, using ASTM guideline D537917 as reference.

The off-axis tensile test consists of applying uniaxial tension to a sample (test specimen) of a unidirectional sample such that the fibers (reinforcement) are inclined in relation to the main fiber axis.

For this type of test, specimens were manufactured using ASTM guideline D303915, applying fiber angles of 30^0, 45^0 and 60^0 for the lamina with a volumetric fraction of 22 % fiber (C-22) and 10^0, 20^0, and 75^0 for the lamina with volumetric fraction of 30 % fiber (C-30). These directions were obtained using the cut of each lamina and their respective reinforcement angles. Figures 3(d) and 3(e) exhibit test specimens with the fiber angle used in the two plates. Analysis of variance (ANOVA) was conducted between the experimental variables exhibited here and the the most common failure criteria applied to laminas and demonstrated in item 2.

4. Results and Discussion

4.1. Mechanical properties of laminas C-30 and C-22

Analysis of the stress-strain curves of the two laminas (Figure 4 and 5) shows approximately linear behavior for all the fiber angles. This characteristic is common for on-axis loaded laminas (with load at 0º or 90º in relation to the fibers) [18]. However, this may not occur for off-axis loading [19].

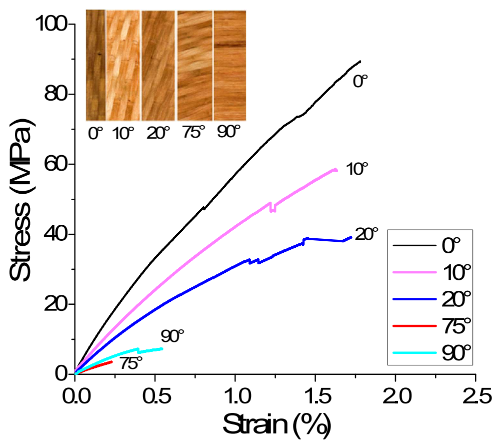

Figure 4. Typical stress-strains varying the angle of reinforcement of lamina *C-30* test specimens.

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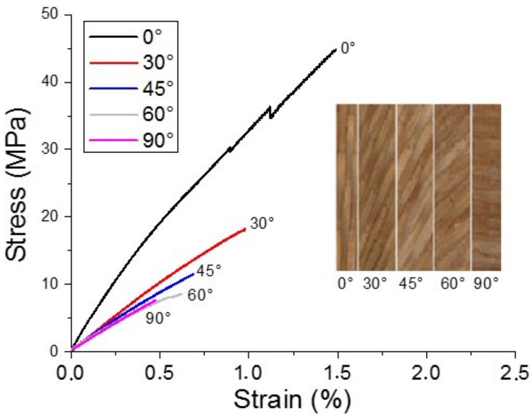

Figure 5. Typical stress-strain curves varying the angle of reinforcement of lamina *C-22* test specimens.

Another important point assessed was the existence of discontinuities on the curves due to sharp drops in loading during the test, caused by transverse cracks in the matrix of the composite.

The discontinuous aspects of the stress-strain curves were also reported by [20], who worked with a unidirectional natural fiber fabric, demonstrating that this behavior is related to localized ruptures of one or more fibers and redistribution of strain in fibers without their breaking.

Figures 4 and 5 shows that with an increase in the angle, the strength and stiffness of the laminas decline, initially occurring more intensely. Furthermore, lamina C-30 is stronger than C-22.

Tables 1 and 2 can be used to better demonstrate the mechanical properties of these two laminas, since they contain the strength values and engineering constants of these laminas in the longitudinal and transverse directions of the fiber (Table 1), represented by E_1, E_2, υ_{12}, υ_{21}, G_{12}, X, Y and S for off-axis loadings (Table 2), depicted by ultimate tensile strength (σult), module of elasticity (E) and Poisson's coefficient (υ).

Table 1. Experimentally measured mechanical properties of unidirectional curauá fabric laminas.

Lamina	X (MPa)	Y (MPa)	S	E_1 (GPa)	E_2 (GPa)	G_{12}	ν_{12}	ν_{21}
C-30	101.56	7.83	16.21	5.31	1.71	0.75	0.34	0.12
C-22	50.81	10.32	10.97	4.31	2.73	1.50	0.35	0.16

Table 2. Off-axis curauá lamina properties.

Lamina	Angle	σ_{ult}	E	ν	Lamina	Angle	σ_{ult}	E	ν
C-30	10º	54.17	3.55	0.37	C-22	30º	19.00	3.35	0.33
	20º	37.48	2.91	0.44		45º	13.25	2.66	0.38
	75º	3.9	1.68	0.43		60º	8.88	2.62	0.43

As expected, due to the higher percentage of fibers, C-30 exhibits greater tensile strength and stiffness than C-22, but for higher angles (above 60°) these values are closer together. This occurs because the matrix acts as an element of tensile strength in the material.

Another point that stands out in Table 1 concerns Poisson's coefficient υ_{21}, obtained experimentally. The literature reports [21-22] that Poisson's coefficient υ_{21} can be obtained from Equation 7. In the case of lamina C-30, the result obtained by Equation 7 (υ_{21} = 0.12) is close to that obtained experimentally (υ_{21} = 0.11), but for lamina C-22, the difference between these values was significantly higher (υ_{21} = 0.22 obtained by Equation 7 and υ_{21} = 0.16), with a difference of 27%.

$$\upsilon_{21} = \frac{E_2}{E_1}\upsilon_{12} \quad (7)$$

Considering the values obtained by the shear test (S and G_{12}) demonstrated in Table 1, once again the lamina with 30% fiber displays greater tensile strength (16.21 Mpa compared to 10.97 MPa), but this did not occur with the modulus, where C-22 exhibits a shear modulus (1.5 GPa) that is twice as high as the value obtained by C-30 (0.75 GPa).

4.2. Analysis of failure criteria for the curauá fiber lamina

In order to determine whether failure criteria satisfactorily represent the experimental data, the graphs of Figures 6 and 7 were constructed for materials C-30 and C-22, repectively. These figures reveal that the four failure criteria under study provided a better model of the mechanical behavior of lamina C-30 (Figure 6) than C-22 (Figure 7), since not all the failure criteria are considered conservative for C-22 (Figure 7). This also occurred for the angle of 75° of lamina C-30, where all the failure criteria displayed higher values than those obtained experimentally, with an increase of 50.2%.

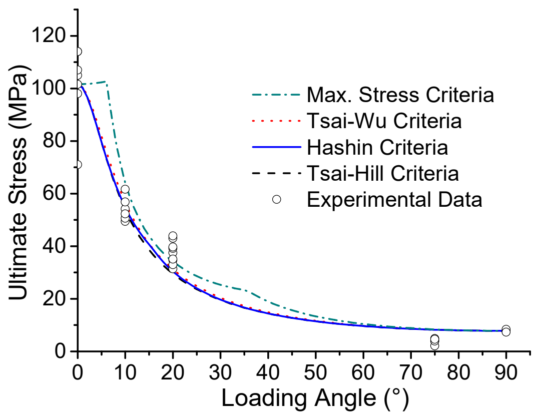

Figure 6. Ultimate tensile strength versus the angle of the fiber for lamina *C-30*, comparing the different failure criteria.

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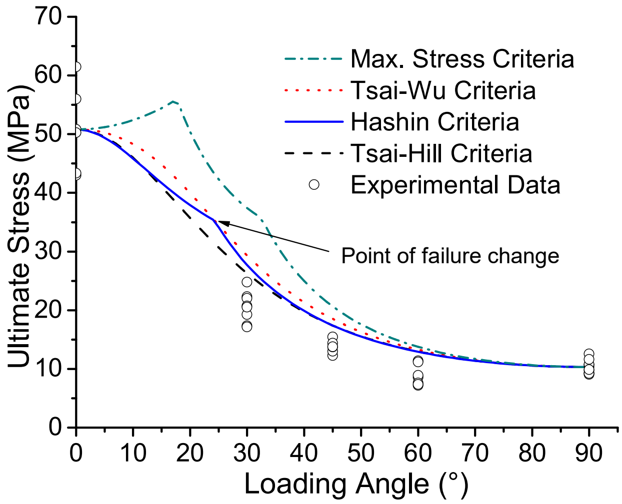

Figure 7. Ultimate tensile strength versus the angle of the fiber for lamina *C-22*, comparing the different failure criteria.

Figure 7 shows that the Tsai-Hill and Hashin criteria are the closest to the experimental values.

To better evaluate each of the failure criteria, analysis of variance (ANOVA) was conducted between the values obtained for each model and experimental data, considering a 95% confidence interval. The main data produced by this method are depicted in Tables 3 and 4, for materials C-30 and C-22, respectively.

Table 3. ANOVA of failure criteria for lamina *C-30*.

Criteria	R^2 (%)	R^2_{max} (%)	$QM = QM_R/QM_r$	$F_{3,22}$	Representativeness (QM/F)
Tsai-Wu	93.72		111.62		36.61
Hashin	93.20		108.83		35.69
Tsai-Hill	93.54	95.09	104.06	3.05	34.12
Maximum Stress	92.92		99.35		32.58

Table 4. ANOVA of failure criteria for lamina *C-22*.

Criteria	R^2 (%)	R^2_{max} (%)	$RQ = QM_R/QM_r$	$F_{3,38}$	Representativeness (QM/F)
Tsai-Hill	81.84		55.22		19.36
Hashin	75.91		44.85	2.85	15.72
Tsai-Wu	70.41	93.48	37.77		13.24
Maximum Stress	24.63		22.03		7.73

These tables show the coefficient of determination (R_2) of each failure criterion, maximum coefficient of determination (R_{2max}) (in this case, if the equation produced this maximum coefficient

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of determination value, it would obtain the mean value for each loading angle), the ratio between the quadratic means (QM) (which is the ratio between the quadratic mean of the failure criterion (QMR) and the quadratic mean of the residue (QMr), defined in Equations 8 and 9).

$$QM_R = \frac{\sum(\hat{y}_i - \bar{y})^2}{P-1} \quad (8)$$

$$QM_r = \frac{\sum(y_i - \hat{y}_i)^2}{n-P} \quad (9)$$

y_i – experimental value, $\hat{y}_i$ – result obtained by the failure criterion, $\bar{Y}_i$ – mean obtained for experimental data, P – number of parameters in the equation and n – number of experimental data.

The QM value should be compared to the F-distribution value with the same number of degrees of freedom, since the closer these values the less significant the model is in representing the experimental data. Thus, the tables are representative of the model, which is the ratio between the QM and F-distribution, and the higher this value the more representative the model (failure criterion) in relation to the experimental data.

These analysis of variance (ANOVA) tables show that the failure criteria models are a better representation of lamina C-30 than C-22. Table 3 demonstrates that the coefficient of determination (R_2) of all the models is close to the maximum value (R_{2max}) and that the ratio between the quadratic means (QM) is 30 times higher than the values obtained by F-distribution, showing that they are representative. Furthermore, the best failure criteria obtained here were those of Tsai-Wu and Hashin, with similar values, and the maximum stress criterion produced the results that differed most from experimental values.

For the case of lamina C-22 (Table 4), the findings were less representative, where the coefficient of determination (R_2) was far from the maximum coefficient of determination (R_{2max}) for all the failure criteria, and representativeness was lower than that obtained in C-30. Nevertheless, the Tsai-Hill and Hashin criteria are sufficiently representative (more than 15 times) for application, and the maximum stress criterion should not be applied in this case, since it has a representativeness value of less than 10 and a coefficient of determination of 24.63%.

It is interesting to observe that the fiber studied here is natural and can have different synthetic fiber characteristics, which may have influenced the behavior of lamina C-22 (primarily in the shear test). When the value obtained in the test coincides with half that of the test at 45°, failure criteria tend to better represent the experimental data and are widely used in the literature when no shear tests are performed [8].

Finally, the Hashin criterion best represents this type of lamina (regardless of fiber percentage), since it always obtains good results, irrespective of the material. Furthermore, it is the only criterion that demonstrates the point of failure change between the fiber and matrix, as shown in Figure 7.

5. Conclusions

The following conclusions can be drawn from the results obtained:

- The tensile tests for C-30 and C-22 obtained tensile strength and modulus of elasticity values that demonstrated strong dependence on the direction of reinforcement. This dependence is more intense for low angles, that is, for test specimens in which fibers are closer to the direction of reinforcement (0°, 10°, 20° and 30°);
- The decline in mechanical strength, due to the angle of inclination of the fiber, is independent of the volumetric fraction of the fiber in the lamina;
- The results obtained for the test specimens with a high fiber angle (60°, 75°, and 90°), show that mechanical strength is less influenced by the direction of reinforcement than the strength of the material, which has a greater effect on the matrix. The higher the angle of inclination of the fiber the lower the ultimate tensile strength and modulus of elasticity;

- According to the behavior of the experimental data of lamina C-30 using the four failure criteria under study, all exhibited satisfactory strength. However, for lamina C-22, only the maximum stress criterion did not display satisfactory behavior.

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