

1 Article

2 Non-destructive System Based on Electrical 3 Tomography and Machine Learning to Analyze 4 Moisture of Buildings

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11 **Abstract:** The article presents the results of research on a new method of spatial analysis of walls
12 and buildings moisture. Due to the fact that destructive methods are not suitable for historical
13 buildings of great architectural significance, a non-destructive method based on electrical
14 tomography has been adopted. A hybrid tomograph with special sensors was developed for the
15 measurements. This device enables the acquisition of data, which are then reconstructed by
16 appropriately developed methods enabling spatial analysis of wet buildings. Special electrodes that
17 ensure good contact with the surface of porous building materials such as bricks and cement were
18 introduced. During the research, a group of algorithms enabling supervised machine learning was
19 analyzed. They have been used in the process of converting input electrical values into conductance
20 depicted by the output image pixels. The conductance values of individual pixels of the output
21 vector made it possible to obtain images of the interior of building walls, both flat intersections (2D)
22 and spatial (3D) images. The presented group of algorithms has a high application value. The main
23 advantages of the new methods are: high accuracy of imaging, low costs, high processing speed,
24 ease of application to walls of various thickness and irregular surface. By comparing the results of
25 tomographic reconstructions, the most efficient algorithms were identified.

26 **Keywords:** inverse problem; electrical tomography; moisture inspection; dampness analysis;
27 machine learning, nondestructive evaluation

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29 1. Introduction

30 This article presents the results of research on the development of an effective and non-invasive
31 method for the detection of moisture walls and historical buildings. Humidity is one of the basic, and
32 at the same time undesirable, physical characteristics of building materials. Detection of water inside
33 the walls of buildings and structures made of bricks or lightweight concrete and brick blocks is one
34 of the most frequently performed tests.

35 All buildings are exposed to various factors that erode the material which they are made. Such
36 factors include water. Many buildings have damp walls and foundations [1]. This is evidenced by
37 often appearing molds and fungi, dark spots, detachment of plasters or paint coatings. This is
38 especially factual for older buildings. One of many reasons for this is the technology and materials
39 that were formerly used in construction, for example, lack of insulation. Moisture in the walls
40 significantly reduces their durability. The bricks and mortar that contain water are significantly
41 weakened and are less resistant to compression, which is particularly true for lime mortar. This
42 results in both deteriorations of the building's operational conditions and safety. In addition, the
43 water accumulated in the walls significantly worsens the thermal insulation properties of the walls
44 and contributes to the gradual erosion. Another important problem related to moisture in walls is

45 their harmful impact on inhabitant's health. Microclimate, which arises in rooms and walls with high
46 humidity often causes the formation of mold fungi that can cause respiratory diseases and
47 intoxication.

48 It can be argued that the main factor causing the destruction of the walls is just moisture. In
49 combination with daily temperature changes, moisture has the greatest impact on the overall strength
50 and durability of building structures. The water in the outer wall has a negative effect on the walls
51 regardless of the state of aggregation, i.e. in solid form (ice, snow), as a liquid (rain) and gas (water
52 vapor). Most structural defects, e.g. brick movements, cracking, molds, fungi, chemical reaction, are
53 initiated and compounded by the presence of moisture.

54 In order to reduce the risks associated with excessive humidity, moisture evaluations are
55 necessary for buildings. Such tests can be helpful in determining the impact of rainwater and
56 groundwater, leakage and moisture from water supply and sewage systems as well as condensation
57 of water vapor.

58 Permeation of moisture in the walls of old buildings that are in direct contact with the soil, due
59 to the lack of a horizontal or vertical barrier separating the walls from water in the soil, leads to
60 migration of moisture (Figure 1). This leads to the migration of dissolved in water salt, which is
61 responsible for many construction problems. Building materials, both natural and man-made (e.g.,
62 the brick or concrete) are porous. Moisture from bricks and masonries can be drawn by gravity using
63 the capillary effect [2].

64 The condition of effective prevention of moisture in the building walls is its proper identification.
65 There are many methods that can generally be divided into two groups - destructive and non-
66 destructive methods. For obvious reasons, non-destructive methods are more desirable and have
67 bigger applied value [3, 4]. This feature is gaining importance when walls' humidity needs to be
68 measured in buildings of historical importance. Among the destructive methods can be
69 distinguished, among others, the "drying-weight" or "carbide" method. Unfortunately, this is an
70 invasive examination. The destructive methods consist in taking samples of the material being
71 examined, which is not always possible, especially in the case of historic buildings. In such cases,
72 these types of methods are not recommended, because they involve a violation of the structure of the
73 examined object. Therefore, non-invasive tests may be a better solution in such situations.

74 Non-destructive methods include, for example, thermovision or ultrasound methods. The
75 disadvantage of thermovision is its exterior, the impossibility of penetrating under the surface of the
76 investigated structure. The ultrasound approach is of little use due to the high porosity of materials
77 containing cells (pores) filled with air or water.

78 Non-destructive methods also include electrical impedance tomography (EIT), in which
79 electrical measurements are made [5]. This method, thanks to the measuring device and the
80 implemented algorithms, allows for non-invasive spatial determination of the degree of moisture. In
81 the case of impedance tomography, it is a technique for imaging the spatial distribution of
82 conductivity [6].

83 Previous studies show that electrical resistance can be used to measure the humidity of concrete
84 and masonry walls. There is a known relationship between moisture inside a porous building
85 material and its electrical resistivity (Figure 2). Similar relationships can be observed for a brick wall
86 and a lightweight concrete blocks wall. The electrical resistance increases as the moisture content
87 decreases. It can be seen that the smallest change in resistance occurs in the highest range of moisture
88 content. Although current techniques for measuring moisture in concrete and brick walls are
89 accurate, the use of electrical resistance has the advantage of being a relatively simple procedure that
90 can be used with inexpensive equipment [7].

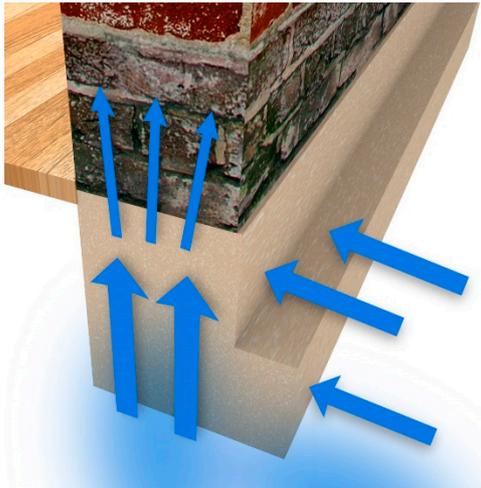


Figure 1. The rising moisture resulting from the direct connection of soil with masonry [8].

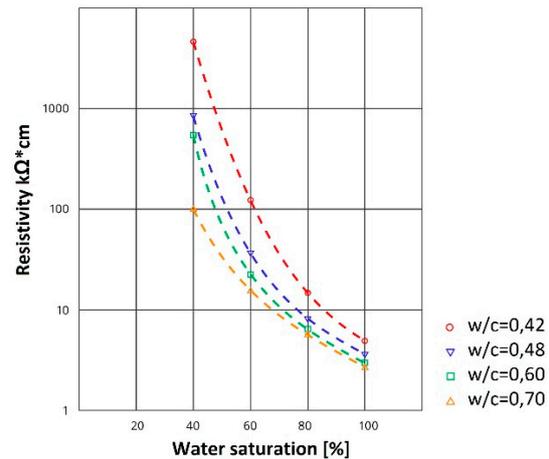
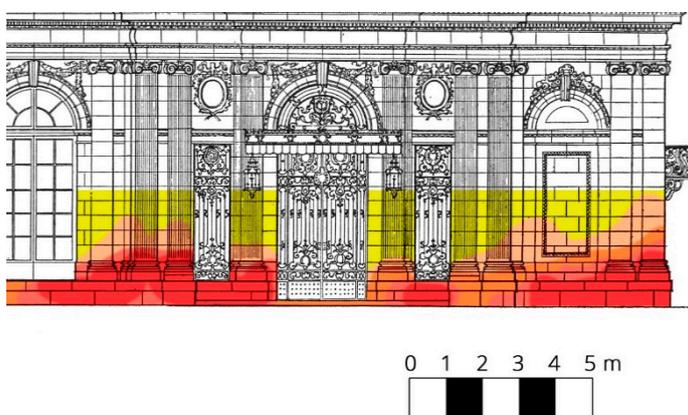


Figure 2. Relationship between saturation and resistivity of concrete [9].

91 Each measuring technique has its own conditions, advantages, and disadvantages. Thanks to
 92 this it can be used only in special circumstances. Currently, the main problem in research on the
 93 concentration of moisture in the walls is the lack of a method that provides spatial imaging of its
 94 distribution inside the wall without the need to take samples. Most of the available research methods
 95 allow only a spot evaluation of moisture, which makes it possible to obtain only a discrete model. In
 96 most cases, methods based on real data are invasive.

97 This fact is a basic problem with regard to the analysis of thick walls because the moisture inside
 98 each wall is usually a few percent higher than at its surface [10]. The destructive nature of currently
 99 used techniques requiring sample collection is unacceptable, especially in historical buildings. In such
 100 cases, only non-invasive methods may be used.

101 The humidity of the walls can be directly measured with electric meters. Electrical moisture
 102 meters, in particular, conductivity meters, are, however, sensitive to very low amounts of moisture
 103 and/or some types of contaminants with soluble salt. For example, a free moisture content lower than
 104 0.1% can cause high meter readings. Due to the influence of salinity causing a change in electrical
 105 resistance and a small depth of measurement, evaluations made with the use of electric humidity
 106 meters should be considered as not very accurate [2, 10]. They can give an approximate image of the
 107 dampness inside the walls (Figure 3).



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Figure 3. An image of building walls' dampness developed on the basis of the results of an electrical test. Higher moisture concentrations are depicted by a color of higher intensity [11].

111 The main purpose of this study is to present and compare non-destructive algorithms based on
112 electrical tomography, which allow estimation of humidity not only on the wall surface but also
113 inside the masonry wall.

114 With reference to the tomography of building walls, the most commonly used methods are
115 impedance tomography (EIT), capacitance tomography (ECT) and resistive tomography (ERT). All
116 the above methods belong to the group of electrical tomography methods, including many
117 tomographic techniques showing the distribution of electrical parameters in the tested object [12, 13,
118 14] while the EIT shows the spatial distribution of conductivity γ [15, 16]. Authors such as Holder or
119 Karhunen et al. [17, 18] in the monograph give the general principles of the EIT, its instruments,
120 procedures, and challenges.

121 The proposed tomographic method in which the building materials humidity evaluation is an
122 indirect assessment based on a different physical characteristic, such as resistivity, allows for many
123 measurements (tomographic approach in [19, 20]) without the need to damage the tested object.

124 The tomographic approach allows archiving the moisture distribution inside the wall in a digital
125 form and comparing it with the next results in the future (moisture monitoring) when it is necessary.
126 This action is extremely useful in buildings requiring the use of a high imaging efficiency method, in
127 particular: constant monitoring of wall humidity, control of the effectiveness of the methods used for
128 drying walls and assessment of the moisture condition of load-bearing walls, in particular, thick ones.

129 The main advantages of the proposed measurement system are the non-invasive and non-
130 destructive measurement of the tested object thanks to specially designed electrodes (Figure 5), the
131 possibility of imaging the moisture distribution not only on the surface but also inside the
132 investigated object. The described research uses simulation tools based on the Matlab scientific
133 software and scripts in the Python and R programming languages. A special role was played by the
134 toolbox called EIDORS dedicated to the Matlab software [12]. It has been used for modeling domain
135 and topological algorithms using the finite element method (FEM) to solve the inverse problem (IP).

136 The structure of the article was divided into 5 parts including the introduction. Chapter 2
137 presents the hardware of the measurement system and algorithms for solving forward and inverse
138 problems. Section 3 presents the results of tests both in relation to simulation experiments and to the
139 reconstruction of the real object. The analysis covered 3 types of algorithms applicable in EIT: LARS,
140 ElasticNET and ANN. The real object was also reconstructed. Chapter 4 includes the comparison of
141 three selected algorithms and discussions in the perspective of previous studies. It also refers to other,
142 known methods within the studied issues. The possible improvements were suggested, too. Finally,
143 section 5 summarizes this article.

144 2. Materials and Methods

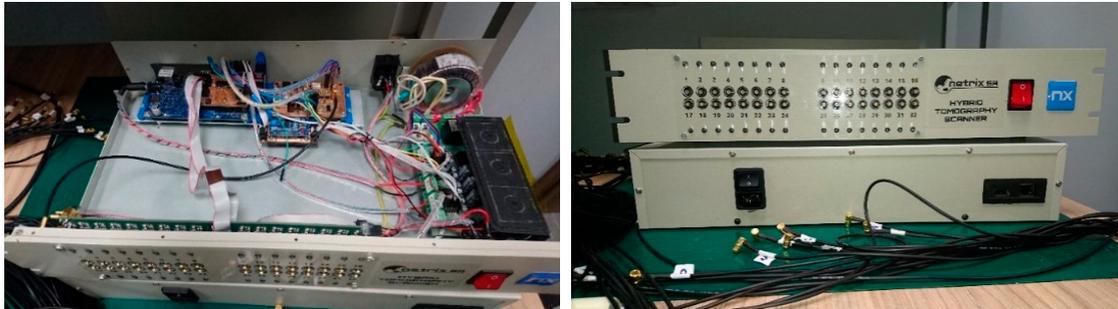
145 This chapter presents a measurement system that enabled the collection of electrical data used
146 subsequently to solve forward and inverse problems. Then there are short descriptions of four
147 selected algorithms: Least Angle Regression (LARS), ElasticNet, Artificial Neural Networks (ANN)
148 and Gauss-Newton. The first three methods were used in the experiments on tomographic imaging
149 and compared with each other. All tested algorithms are classified as machine learning and artificial
150 intelligence methods. Thanks to the above algorithms, it was possible to use the supervised machine
151 learning method, which in combination with the parallel computing (multi-core CPU, GPU) allowed
152 to quickly reach effective solutions in the field of building models of reconstructed objects.

153 2.1 Selected hardware issues, sample models and reconstructions

154 Electrical tomography is a technique for imaging the distribution of conductivity or permeability
155 inside an object under investigation based on measurements of the potential distribution on the
156 object's surface. Numerous different techniques can be used in the process of optimization of
157 tomographic methods.

158 The data collection system collects the measured voltage from the electrodes and then processes
159 the data. Conventional data acquisition systems require voltage, filtering, demodulation and
160 converting equipment to be digitized and a signal processor to transfer data to a computer. Our

161 device for measurement in electrical tomography uses two methods: electrical tomography
 162 capacitance and electrical impedance tomography. It allows you to take measurements up to 32
 163 channels (Fig. 4). These devices provide a non-invasive way to test the spatial distribution of
 164 moisture. The system includes an additional software solutions. The advantage of the system is the
 165 simultaneous measurement of voltage and capacity.



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167 **Figure 4.** The measurement device

168 Ensuring the proper contact of the electrodes with the wall is particularly important in testing
 169 objects with an uneven as well as a rough surface. An example of the use of this type of electrodes is
 170 the moisture condition investigation inside the masonry. The development of effective and efficient
 171 measuring electrodes for impedance tomography has proved to be a serious challenge. In order to
 172 ensure optimal contact between the electrode and the wall, an electrode with a flexible contact surface
 173 and articulated mounting was designed (Figure 5).
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187 **Figure 5.** The concept of the electrodes

188 A complete electrode consists of three modules: a specific electrode, a PCB with a contact socket
 189 and a fastening system. Mechanically, the modules are connected to each other by means of two
 190 sleeves placed one inside the other. Before separating, they are secured by a collar placed on the
 191 upper sleeve. The PCB is made of double-sided 1.54 mm thick laminate with an SMB1251B1-3GT30G-
 192 50 socket.

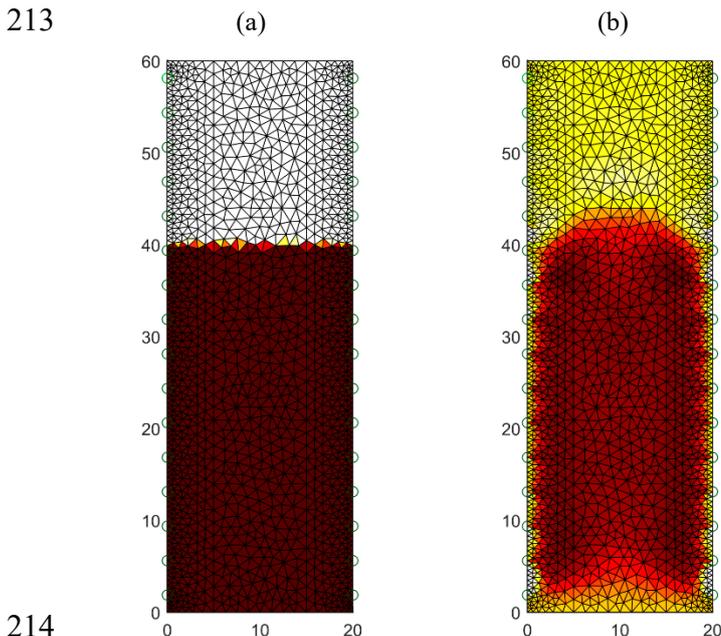
193 From the active surface, the galvanic plate is connected by means of four leads with a specific
 194 electrode made of electro-conducting silicone coated on the one hand in the galvanization process
 195 with a copper layer. Pins and the copper layer allow contact between the PCB and the electrically
 196 conductive silicone.

197 The specific electrode made of flexible electrically conductive silicone improves contact with the
 198 surface of the tested object. This feature is particularly useful in examining objects with increased
 199 porosity.

200 The third module is the tripod electrode mounting system. It is made of ASB in 3D printing
 201 technology. The holder has two parallel channels that allow quick mounting on tripod profiles. The
 202 element responsible for the elasticity of the mount is the rubber ring, which task is to adjust the
 203 position of the electrode to the tested object surface, which aims to eliminate the unevenness and
 204 pressing the electrode to the wall. The post-retrofit version is equipped with an additional 10 mm
 205 thick shock absorber and a flexible connection between the conductive rubber and the PCB. As a
 206 result, the electrodes adhere much better to uneven surfaces of the tested object. Newly designed
 207 electrode systems have great potential in practical applications.

208 2.2. Wall moisture tests with the use of the Gauss-Newton (GNM) method

209 Figure 6 presents the results of tomographic imaging using the Gauss-Newton (GNM) method.
 210 The presented reconstruction (b) deviates somewhat from the pattern image (a). The differences,
 211 however, concern only the details of the contour of the moistened area. So you can use this method
 212 to roughly estimate the moisture level and area.



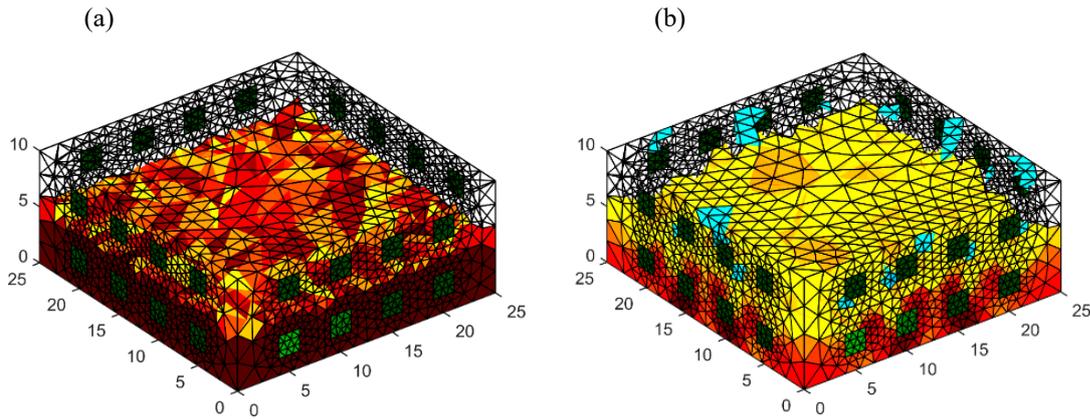
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215 **Figure 6.** The geometrical model of the tested wet wall with 32 electrodes: (a) the pattern image, (b)
 216 the image reconstructed by Gauss-Newton method

217 Figure 7 presents the spatial reconstruction of a wall fragment using the GNM method by means
 218 of 32 measurement electrodes located around the object. Figure (a) is a reference image. Figure (b) is
 219 the result generated by the use of GNM. Comparing both images, you can see differences in the
 220 intensity of the color. Brighter colors of the reconstructed image indicate less intense moisture inside
 221 the wall compared to the reference image.

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Figure 7. The geometrical model 3D with 4x8 electrodes – the image reconstruction: (a) pattern model, (b) Gauss-Newton method with Laplace regularization

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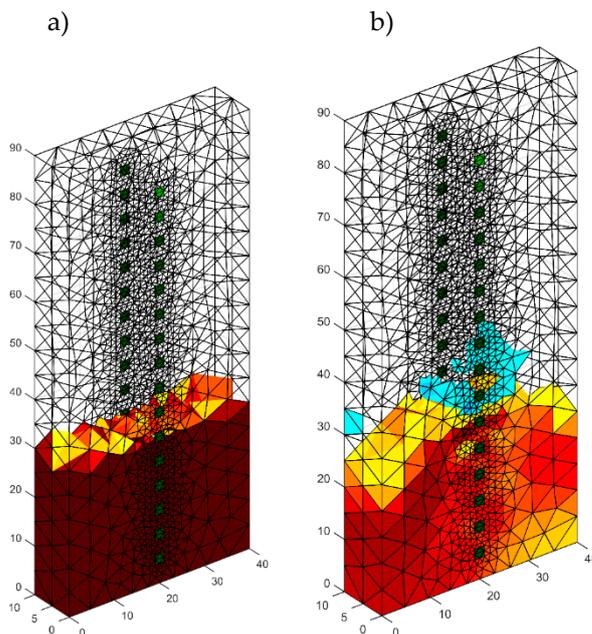
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In Figure 8, we can see an example of a reconstruction of a damp concrete block using 32 electrodes located on both sides of the tested object. In order to solve the problem of the three-dimensional finite element mesh was prepared. It can be noticed that surfaces of finite elements which are localized near electrodes are small. Hence, the solution of the forward problem is precise. The results obtained are similar to those obtained by placing 32 electrodes around the lightweight concrete block with dimensions 10x40x90 cm. The reconstructed image deviates from the pattern with the too low intensity of colors.



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Figure 8. The geometrical model 3D with 2 x 16 electrodes – the image reconstruction: (a) pattern model, (b) Gauss-Newton method with Laplace regularization

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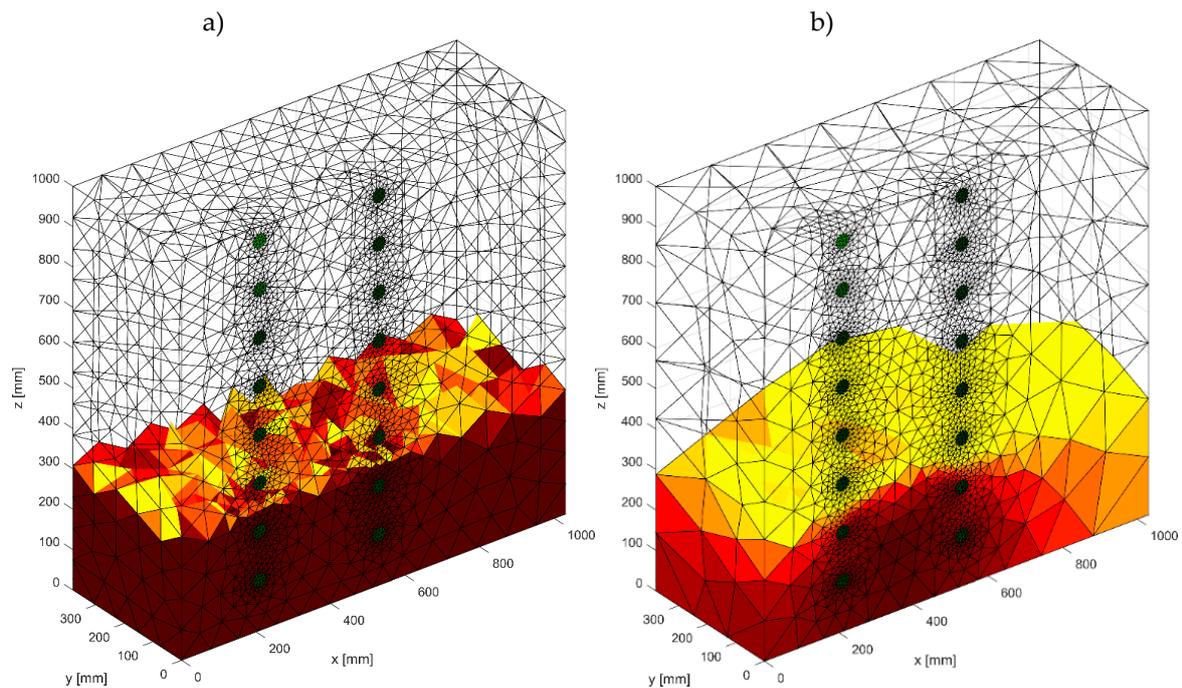
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In Figure 9 two special models of the brick cube “wet” and “moist” with 2x8 electrodes are presented. The image was reconstructed by Gauss-Newton method with Laplace regularization or Tikhonov regularization.

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Figure 9. The geometrical model 3D with 2 × 8 electrodes – the image reconstruction: (a) pattern model, (b) Gauss-Newton method with Laplace regularization

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2.3. Measuring system description

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The measurements were made using the EIT Polar GND method, at 1kHz frequency and 100uA excitation current. An object whose internal electrical properties are unknown is surrounded by electrodes disposed on its edge and electrically excited in various combinations. Measurements are made for all possible ways of connecting the power source to the area, to increase the number of information about the object and to improve the signal-to-noise ratio. After the first series of measurements, the excitation system switches to the neighboring electrodes. This process is repeated sequentially for all possible power source connection systems. In this way, multiple "x-rays" of the tested object are made. Due to the symmetry of the system for $n = 16$ electrodes, $n / 2 = 8$ independent current distributions can be obtained. Let each configuration of the stimulus source be called projection angle, in which case the total number of these angles at the above assumptions is 8.

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Input data for the image construction algorithm are voltage measurements made between adjacent electrodes. Measurements made on electrodes with an attached excitation source are omitted due to unknown voltage drop between these electrodes and the tested area. For a system of $n = 16$ electrodes and any projection angle, $n - 3 = 12$ independent measurements can be obtained. Thus, the full number of voltages measurable between neighboring voltage electrodes at $n / 2 = 8$ angles is: $(n - 3) (n / 2) = 12 \cdot 8 = 96$. The method of measuring the inter-electrode voltages shown in Fig. 10 corresponds to the first and the second projection angle. For subsequent angles, sequential switching of the power supply and measurement circuit to the neighboring electrodes takes place.

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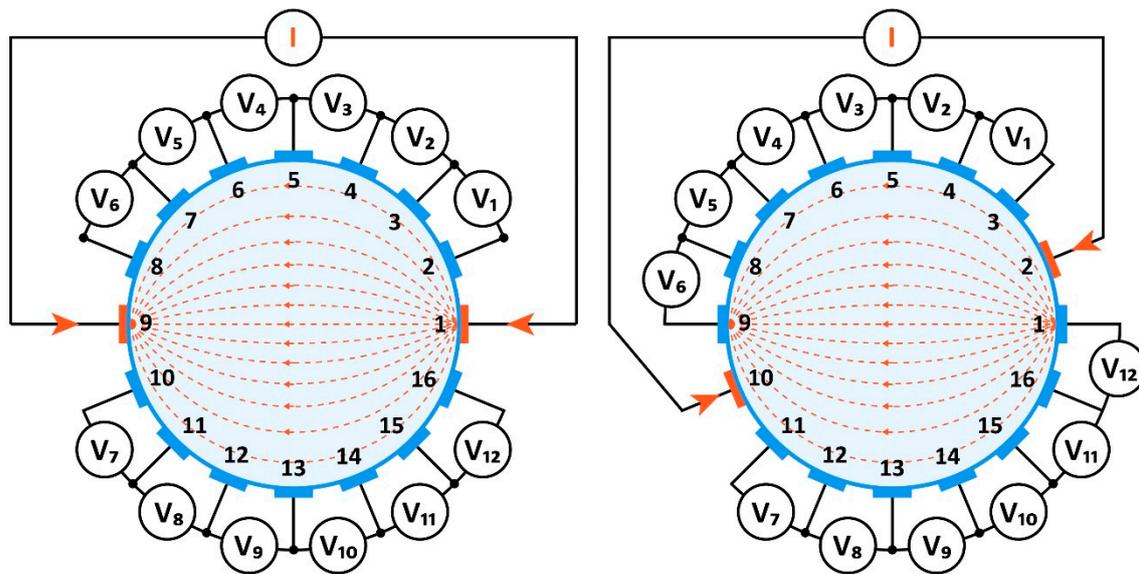
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For a system of $n = 32$ electrodes and any angle of projection, $n - 3 = 28$ independent measurements can be obtained. Hence the full number of possible independent measurements of voltages between neighboring voltage electrodes at $n / 2 = 16$ angles is: $(n - 3) (n / 2) = 28 \cdot 16 = 448$.

Potential values of electrodes depend on the current distribution within the region, and thus also on the distribution of conductivity. The algorithm of computer image reconstruction is looking iteratively for such a distribution of conductivity, for which the calculated values of inter-electrode voltages are as close as possible to the corresponding measurement values.

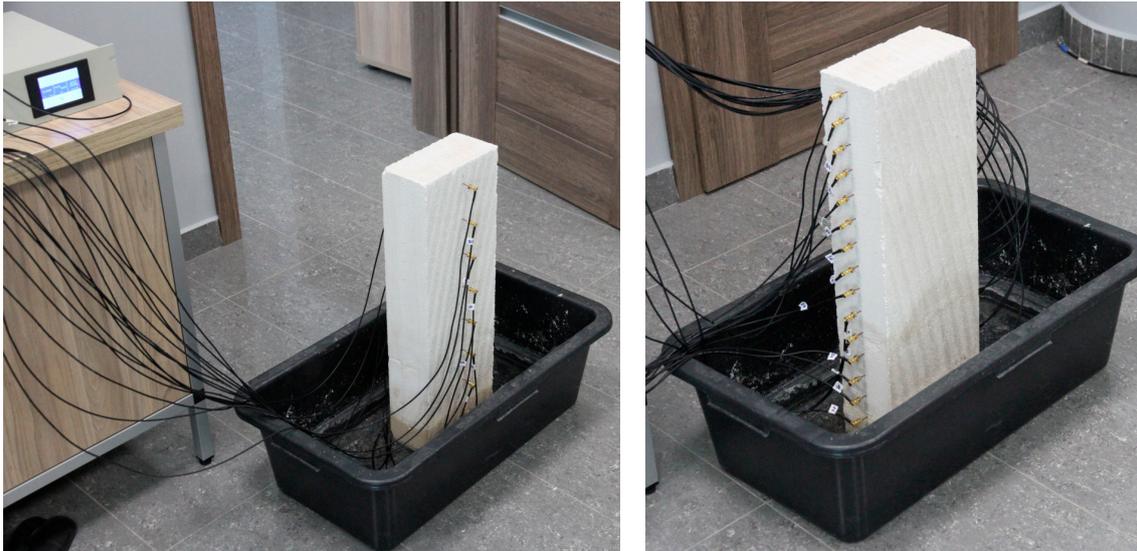


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274 **Figure 10.** Voltage measurement method
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276 The measured voltage is generally a voltage drop on the two impedances of the electrodes and
277 the impedance of the object. The voltage drop on the impedances of the spot electrodes can be omitted
278 due to the high impedance of the measuring system. In the case of surface electrodes, the potential
279 decrease at the electrode is very low, the tested object has low conductivity, while the electrode has
280 a high conductivity. Therefore, the voltage drop is negligible, the surface impedance coefficient of the
281 electrode tends to zero (it is negligibly small), but it is programmatically included in the
282 reconstruction process. The contact impedance is included in the model, but has a limited effect on
283 the measurement.

284 Voltage drops are measured on the surface of the tested object, so it is a non-invasive method.
285 After applying the power source in the wall, current starts flowing, which has a higher value closer
286 to the shore and the power source. The further away from the electrodes, the more the current flow
287 is getting smaller (tends to zero). This is the factor that causes the reconstruction to be more optimal
288 closer to the measuring electrodes, the further away from the electrodes, the detection precision may
289 be lower (the reconstruction may be worse due to the current depth distribution). Therefore,
290 measurements on one edge usually give worse quality compared to measurements on two or more
291 edges. Sometimes, however, only one-sided measurement is possible.

292 The tested physical models of the wall parts contained 16 or 32 electrodes each for measuring
293 the wet wall. The electrodes were placed on both sides of the tested wall sample. Electrical impedance
294 tomography is based on the measurement of the potential difference. The ability to determine the
295 wall condition results from the unique conductivity value of each material. The necessary equipment
296 such as electrodes, meters, alternating current generator, multiplexer and a computer with LabVIEW
297 and EIT modules was used for the measurements. Figure 11 shows the partially immersed
298 lightweight concrete block on which the surface electrodes are placed. It can be seen that in the
299 presented picture the block samples have 2x8 and 2x16 electrodes.
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Figure 11. A tomographic laboratory to study the moisture inside the cellular lightweight concrete block

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As mentioned before, a big problem related to the moisture content testing is the lack of a method allowing to determine its spatial distribution without the need to take samples. The method that enables this is electrical impedance tomography. It consists in evenly distributing the electrodes on the tested object and ensuring a good contact of their surface with the tested surface. Unfortunately, often the wall surfaces have a varied shape. Also, the building materials themselves, such as brick or plaster, have a certain porosity, which makes measurement difficult. To ensure an adequate flow of electric current between the individual electrode pairs, a special multilayer electrode was developed.

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2.4. Masonry humidity testing by the Least Angle Regression (LARS) method

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In order to obtain more accurate and stable reconstruction results in solving the inverse problem in electrical tomography [13, 14, 15], a new solution based on the method of the least angle regression was tested [21]. There are many methods to solve the optimization problem. The statistical methods can be used to reconstruct an image in electrical impedance tomography [19, 20, 22, 23].

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The main objective of the tomography is to perform image reconstruction. During the measurements, we can see that the measured values from some electrodes are strongly correlated (due to the way of measurement). In this case, we have a multicollinearity problem. When the independent variables (predictors) are correlated (collinear), then the matrix tends to a single matrix. By means of the least squares method, we obtain large absolute values of some estimators with unknown parameters. Forecasts based on this model are unstable. The most common approach is to reduce the set of input variables (removing the same predictors that apply to multicollinearity). Then we have a problem with the selection of predictor variables that will be included in the regression model. For example, when comparing the AIC (Akaike Information Criterion) value for linear models with different sets of predictors, we can choose the best model.

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Another possible way to reduce the problem of multicollinearity between predictors depends on the application of the least angle regression algorithm. This algorithm takes into account only causal variables in the linear model (from the set of predictors, you should select the input variables that have a direct impact on the response variable). In this case, the linear model is built by means of step forward regression, where the best variable is added to the model in every step.

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Let the linear system be described by the state equation

$$Y = X\beta + \varepsilon \quad (1)$$

333 where $Y \in R^n, X \in R^{n \times (k+1)}$ denote the observation matrices of response and input variables
 334 respectively, $\beta \in R^{k+1}$ denotes the vector of unknown parameters. When the linear model (1)
 335 contains the intercept, then the first column of matrix X is a column of ones. The object $\varepsilon \in R^n$ in the
 336 linear system (1) presents a sequence of disturbances, which is usually defined as a vector of
 337 independent identically distributed random variables with normal distribution $N(\tilde{0}, \sigma^2 I)$, which,
 338 $\tilde{0} \in R^n$ is a zeros vector but $I \in R^{n \times n}$ is an identity matrix. The classical Least Square Method
 339 depends on identification of unknown parameters $\beta = (\beta_0, \beta_1, \dots, \beta_k)$ in (1) by solution the task

$$\min_{\beta \in R^{k+1}} \|Y - X\beta\|^2 \quad (2)$$

340 If $\det(X^T X) \neq 0$, then the best unbiased linear estimator of unknown parameters β is

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad (3)$$

341 The problem is often when $X^T X$ is singular.

342 The following is a short version of the least angle regression algorithm as the workflow. An extended
 343 version of LAR has been presented in [24].

- 344 1. The predictors should be standardized. The intercept β_0 in expression (1) is equal a mean of
 345 the response variable and we put $\beta_1 = \beta_2 = \dots = \beta_k = 0$. Active set A (set of predictors) is
 346 empty.
 - 347 2. Calculate the residuals $r = Y - \beta_0 - X_{(A)}\beta_{(A)}$ for the linear model with all predictors from
 348 active set A . Determine the predictor X_j (which is not in active set) most correlated with
 349 residuals r and attach to the active set A .
 - 350 3. Move coefficient β_j from 0 towards its least-squares coefficient $\langle X_j, r \rangle$ until some other
 351 competitor X_k has a much correlation with the current residuals as does X_k .
 - 352 4. Move β_j and β_s in the direction defined by their joint least square coefficient of the current
 353 residual on $\langle X_j, X_s \rangle$ until some other competitor X_l has a much correlation with the current
 354 residual.
- 355 Go to step 2 and continue in this way until all k predictors have been entered.

356 2.5. Masonry humidity testing by the ElasticNet method

357 Another way to determine the linear regression when the input variables are collinear depends
 358 on the solution of the task

$$\min_{(\beta_0, \beta') \in R^{k+1}} \frac{1}{2n} \sum_{i=1}^n (y_i - \beta_0 - x_i \beta')^2 + \lambda P_\alpha(\beta'), \quad (4)$$

359 where $x_i = (x_{i1}, \dots, x_{ik})$, $\beta' = (\beta_1, \dots, \beta_k)$ for $1 \leq i \leq n$ and P_α is an elastic net penalty given by

$$P_\alpha(\beta') = (1-\alpha) \frac{1}{2} \|\beta'\|_{L_2}^2 + \alpha \|\beta'\|_{L_1} = \sum_{j=1}^k \left(\frac{1-\alpha}{2} \beta_j^2 + \alpha |\beta_j| \right) \quad (5)$$

360 We see that the penalty is a linear combination of norms L_1 and L_2 of unknown parameters β' .
 361 The introduction the penalty function dependent from parameters to the objective function allows to
 362 shrink the estimators of unknown parameters.

363 The parameter λ in the task (4) denotes the coefficient of penalty, but the parameter $0 \leq \alpha \leq 1$
 364 creates the compromise between LASSO (Least Absolute Shrinkage and Selection Operator) and
 365 ridge regression. The ridge regression ($\alpha = 0$) is called Tikhonov regularization [25] and is one of the
 366 most commonly used for regularization of linear models. LASSO ($\alpha = 1$) was introduced by Roberta
 367 Tibshirani [26, 27]. This method performs the variable selection and regularization in linear statistical
 368 models [28, 29]. For the ridge regression, the penalty is calculated in the norm L_1 but for LASSO in
 369 L_2 . Difference between ridge regression and LASSO is symbolic, only the norms are changed. The
 370 ridge regression shrinks coefficients for correlated predictors towards each other. When the
 371 correlated predictors depend on any latent factor, then ridge regression allows to uniformly

372 distribute the strength of latent factor on these predictors. Whereas LASSO is indifferent to correlated
 373 predictors. This method allows to determine the preferred predictor and to ignore the rest. By
 374 applying LASSO method we obtain a model, where the many coefficients to be close to zero, and as
 375 a result, we receive a sparse model. The elastic net is a connection of ridge regression and LASSO [30,
 376 31]. Choosing the appropriate α we may create the compromise between ridge regression and
 377 LASSO.

378 By solution the task (4) for fixed λ and α we estimate the unknown parameters of the linear
 379 system (1), where predictors are correlated. Then the prediction based on model (1) is given by the
 380 formula $\hat{Y} = X\hat{\beta}$, where the vector of estimators of unknown parameters $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_k)$ is
 381 estimated by solution the task (4).

382 2.6. Masonry humidity testing by the Gauss-Newton method

383 In the electrical impedance tomography in the reconstruction of the image, the so-called
 384 Generalized Tikhonov regularization is very often used. In the literature on the subject, this method
 385 is also known as the Gauss-Newton algorithm in a generalized form.

386 The Gauss-Newton method is based on the application of the least squares method in which the
 387 matrix $Z^{(l)}$ fulfills the role of matrix X (first partial derivatives relative to fixed approximations $\beta^{(l)}$
 388 and observed values of independent variables), and the role of the vector y (observation of the
 389 dependent variable) vector $e^{(l)}$. It is a vector of differences between the empirical values of the
 390 dependent variable and the l th of its approximations $f(x_t, \beta^{(l)})$.

391 The Gauss-Newton algorithm is used to estimate the structural parameters of non-linear models.
 392 The general form of the non-linear function is presented below:

$$y_t = f(x_t, \beta) + \varepsilon_t \quad (10)$$

393 where:

394 y_t – observations of the explanatory variable,

395 $x_t = [x_t]$ – P vector of observations for explanatory variables,

396 $\beta_t = [\beta_j]$ – K vector of structural parameters,

397 ε_t – implementations of random elements (we assume that random components are
 398 uncorrelated, have an average of zero and equal, positive and finite variance).

399 In the Gauss-Newton method, the reconstruction of the internal image of the investigated object
 400 is related to the determination of the global minimum of the fitness function. In order to carry out
 401 quantitative considerations, we assume that the tested object is polarized with an alternating low-
 402 frequency current. Then, the electrical material properties can be described by a function with real
 403 values. In this case, in the generalized Laplace equation, we neglect the word proportional to the
 404 frequency, and this function can be equated with the electrical conductivity (real isotropic admittivity
 405 case).

406 2.7. Masonry humidity testing by the neural imaging

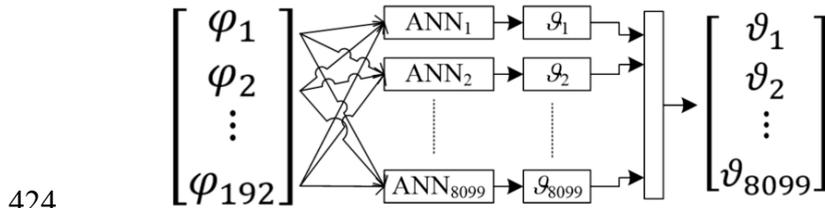
407 In order to solve the problem of non-invasive imaging of the interior of moist walls, the method
 408 of electrical tomography in connection with artificial neural networks was also used. So far,
 409 tomographic and neural networks methods have not been widely disseminated in the assessment of
 410 the wall. The reason is the low resolution of the reconstructed image and the low accuracy of
 411 mappings [4].

412 To increase the resolution of tomographic reconstructions depicting the degree of internal
 413 humidity of walls, a new method was developed based on a set of many separately trained neural
 414 networks. The number of neural networks corresponds to the 3D resolution of the lattice dividing the
 415 inside of the wall into individual pixels. In the presented experiment a lightweight concrete block
 416 with dimensions 10x40x90 cm was used, which was divided into 8099 points.

417 Using a device called a multiplexer, in short intervals, the tomographic system generates 192
 418 values of voltage drops readings between different electrode pairs. These are the input data for the
 419 neural network system. The neural networks are designed in such a way that on the basis of an input

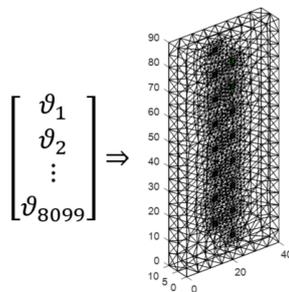
420 vector containing 192 elements, each of the 8099 neural networks generates the value of a single pixel
 421 of the output image.

422 Figure 12 shows the mathematical form of the neural model used during simulation
 423 experiments. At the model input, there are 192 electric signals generated by 16 electrodes.



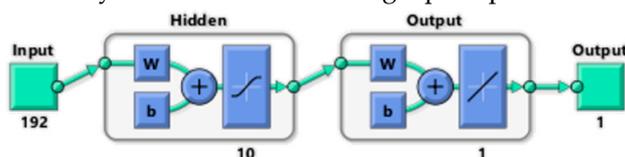
425 **Figure 12.** The scheme of converting electrical signals into the image pixels

426 The same input vector is the basis for training 8099 separate artificial neural networks (ANN).
 427 In this way, from a vector of 192 variables representing electrical values, a set of neural networks
 428 creates a complete lattice of the lightweight concrete block image. The output image is created by
 429 assigning colors to the output values of each pixel. The transformation method is shown in Figure 13.
 430



432 **Figure 13.** The way of converting a real number vector into the lightweight concrete block's spatial
 433 image

434 Each of the 8099 neural networks had a multi-layered perceptron structure with 10 neurons in
 435 the hidden layer. The scheme of a single perceptron is shown in Figure 14.



437 **Figure 14.** Structure of the selected multi-layer perceptron

438 In order to collect data necessary to train the neural network, physical and mathematical models
 439 were developed. The finite element method was used for this. Based on a mathematical model, a data
 440 set was generated. After that, it was used to train the neural network system.

441 To train the mentioned above neural network, a collection of 6,140 historical cases was used (see
 442 Table 1). The main set of data has been divided into 3 separate subsets: a training set, validation set,
 443 and testing set, in the proportions of 70%, 15%, 15%. This method of data preparation has been used
 444 for all 8099 neural networks.

445

446

Table 1. Training results for one of 8099 neural networks

	 Samples	 MSE	 R
 Training:	4298	5.31979e-6	9.99983e-1
 Validation:	921	1.68249e-5	9.99947e-1
 Testing:	921	2.03645e-5	9.99934e-1

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The highest Mean Squared Error (MSE) concerned the testing set and was 0.000020364. In the case of validation set, a slightly smaller error was noted. Mean Squared Error is the average squared difference between outputs and targets. Lower values mean better performance. Zero means no error (excellent performance). The training set was trained with the lowest training error, which is the most common and correct situation. A low MSE error in the training set results from better network adaptation to training cases. Another indicator of the quality of network learning was R (Regression). An R value of 1 means a close relationship between pattern and output, 0 a random relationship. In all three cases of data sets (learning, validation, and testing), R was close to 1. This also applies to the test and validation set, which is particularly valuable. Values close to 1 indicate a good match of the results obtained by the network (output vectors) to the patterns included in the individual sets (training, validation, and testing).

460

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Good indicators (MSE and R) for the training set show the lack of overtraining effect and the ability of the network to knowledge generalization (i.e., correct conversion of input data to output information not only for learning cases).

463

3. Results

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This chapter presents the results of wall humidity tests by EIT tomography in combination with the following machine learning algorithms: Least Angle Regression (LARS), ElasticNet and Artificial Neural Networks. The root mean square error of prediction (RMSE) indicator was used to quantify the quality of the reconstructions obtained using simulation models.

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Let vector $x = (x_1, \dots, x_n)$ presents the pattern, which should be reconstructed. After reconstruction we obtain the vector, $\hat{x} = (\hat{x}_1, \dots, \hat{x}_n)$ which contains the expected values of reconstruction of explored object. The root mean square error of prediction was determined by the formula (11).

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \hat{x}_i)^2} \quad (11)$$

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In the further part of this paper two variants of images were compared: 2D and 3D. The 2D image lattice consisted of 2908 pixels, while the 3D grid consisted of 8099 pixels. Thus in the first case (2D) $n = 2908$, while for the 3D variant $n = 8099$.

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All results enabling the comparison of LARS, ElasticNET and ANN methods were obtained thanks to the use of computer simulation methods.

477

3.1. Results of wall moisture tests obtained using the Least Angle Regression (LARS) method

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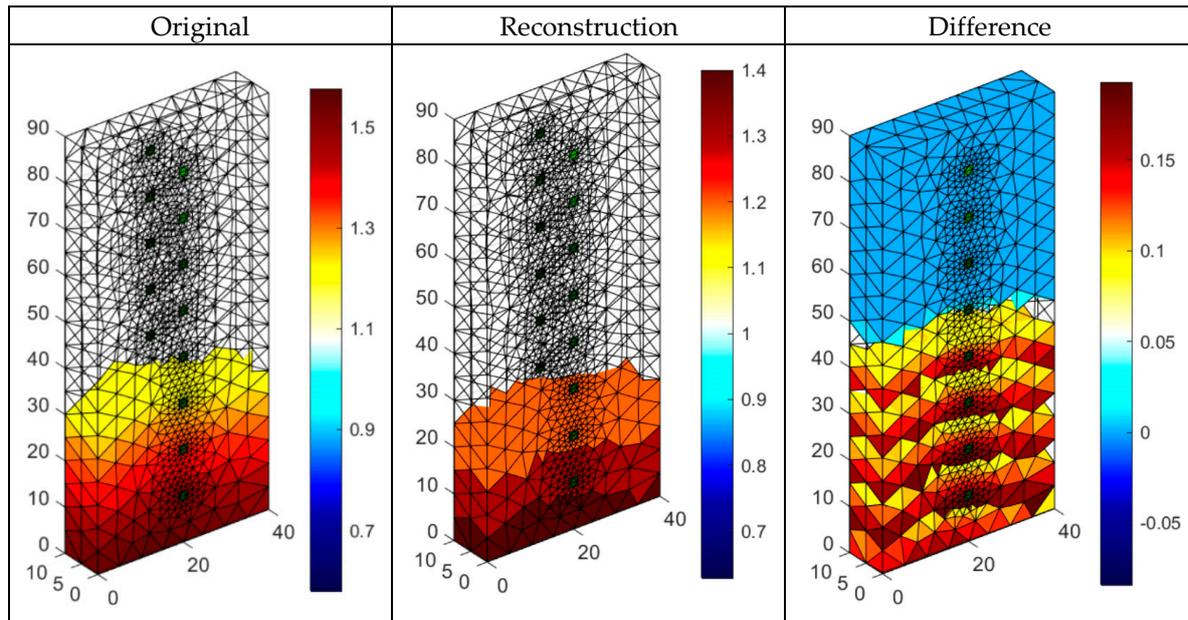
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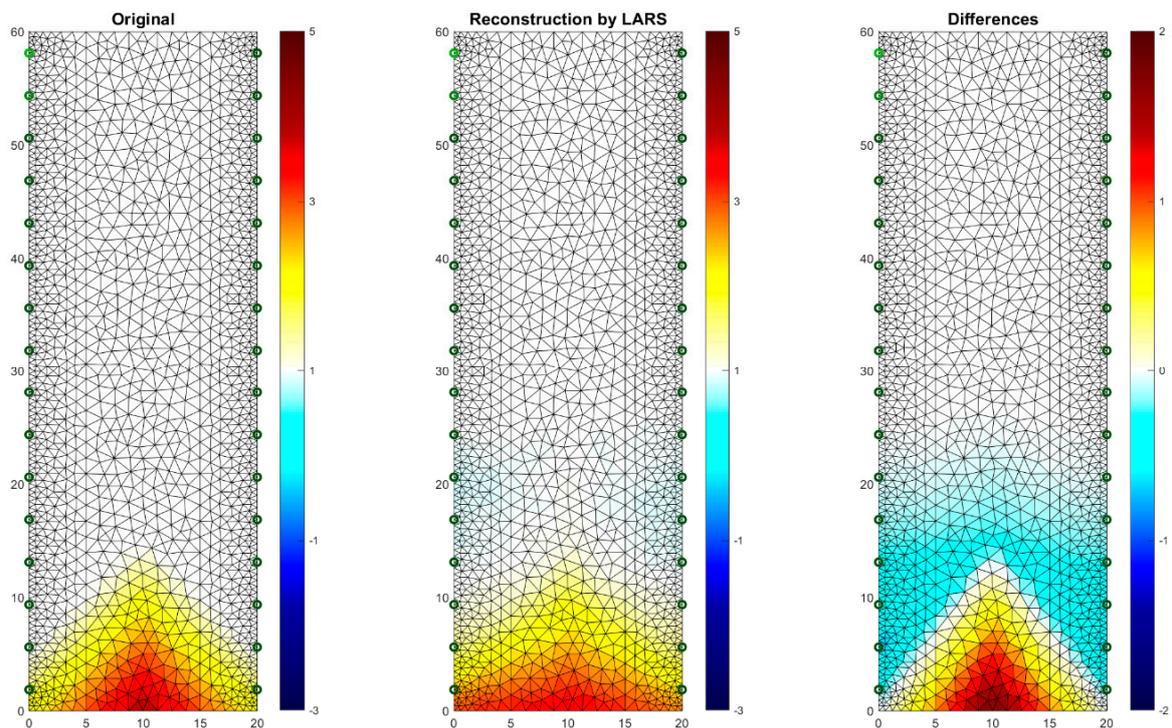
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Figure 15 presents one of the results of tomographic imaging using the LARS method. The input data was obtained thanks to the use of an EIT tomograph equipped with 16 electrodes (2x8). Intense colors indicate areas with higher humidity. It can be seen, the obtained reconstructive image (in the middle) is very close to the reference image (left). The difference image (right) indicates small deviations of the grid points in the reconstructed image from the reference image. The colors in the images reflect the conductance of the individual pixels that each image consists of. The lack of color in the original and reconstruction images testify to the lack of moisture. RMSE for a 3D sample with the use of LARS is 0.033019.



487 **Figure 15.** The result of Least Angle Regression (LARS) moisture testing of the lightweight concrete block
 488 for the case of 2x8 electrodes

489 Figure 16 shows a case analogous to the previous one, which was presented in Figure 15, but
 490 this time a cellular concrete sample with slightly different dimensions (10x20x60 cm) was used. The
 491 reconstruction was carried out in 2D. RMSE for a 2D sample with the use of LARS is 0.122599.

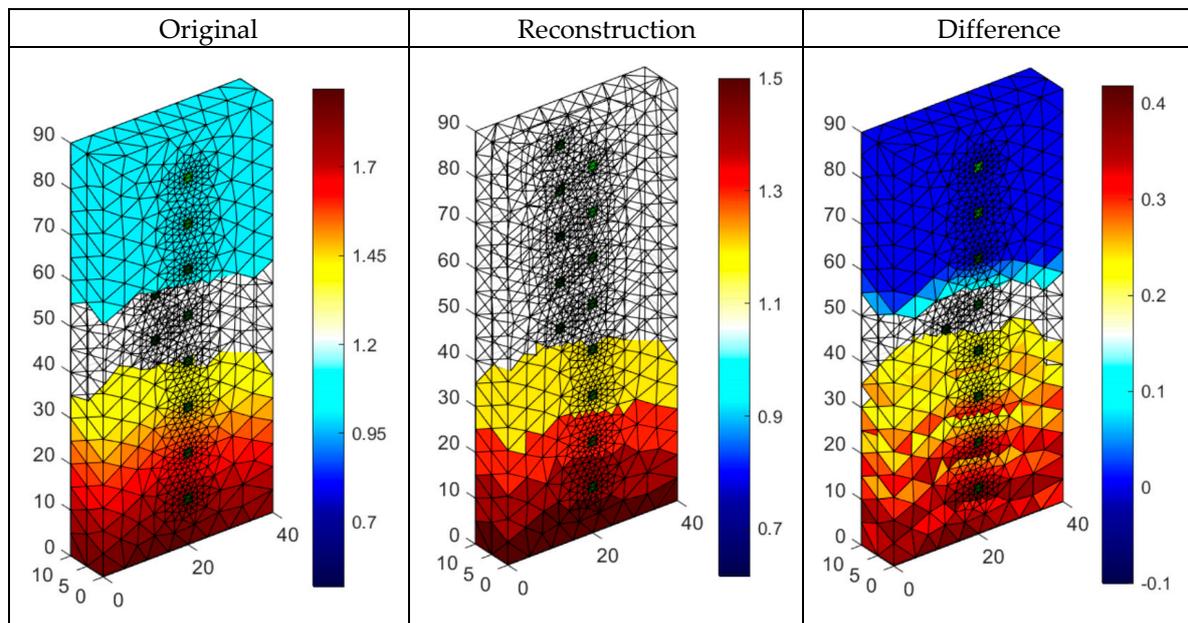


492 **Figure 16.** The result of Least Angle Regression (LARS) moisture testing of the lightweight concrete
 493 block for the case of 2x16 electrodes
 494

495 3.2. Results of wall moisture tests obtained using the ElasticNet method

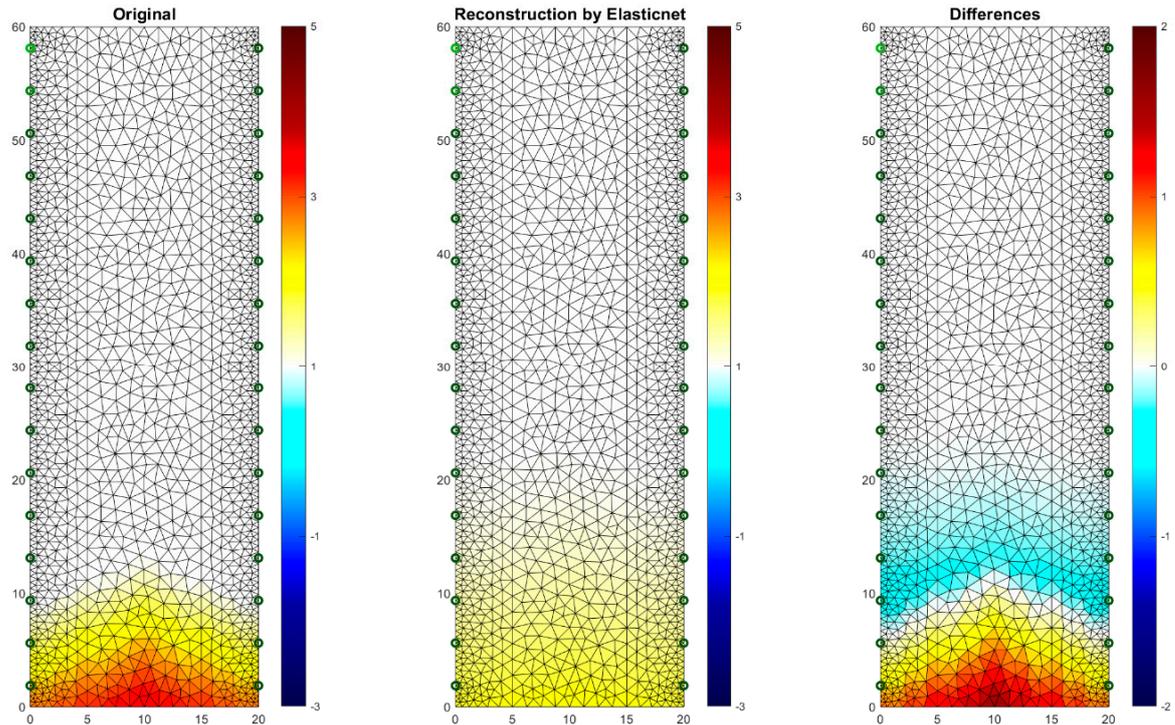
496 Figure 17 presents an example of a tomographic 3D imaging result using the ElasticNet method.
 497 The input data was obtained thanks to the use of an EIT tomograph equipped with 16 electrodes
 498 (2x8). Intense colors indicate higher humidity spots. It can be seen, the obtained reconstructive image
 499 (middle image) is comparable to the reference image (left). The image of residuals (right) indicates

500 the occurrence of deviations of the grid points of the image reconstructed from the reference image.
 501 The lack of color and shades of blue in the original and reconstruction images mean the lack of
 502 moisture. RMSE for a 3D sample with the use of ElasticNet is 0.036184.
 503



504 **Figure 17.** The result of ElasticNet moisture testing of the lightweight concrete block for the case of
 505 2x8 electrodes

506 Figure 18 presents an example of a tomographic imaging result using the ElasticNet method.
 507 The input data was obtained thanks to the use of an EIT tomograph equipped with 32 electrodes
 508 (2x16). Intense colors indicate higher humidity spots. It can be seen that the obtained reconstructive
 509 image (middle image) reproduces the reference image (left) poorly. The image of residuals (right)
 510 indicates the occurrence of significant deviations of the grid points of the image reconstructed from
 511 the reference image. RMSE for a 2D sample with the use of ElasticNet is 0.282520. Compared with
 512 LARS, ElasticNet showed the lower quality (higher RMSE) of reconstruction in this case.
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Figure 18. The result of ElasticNet moisture testing of the lightweight concrete block for the case of 2x16 electrodes

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3.3. Results of wall moisture tests obtained using the neural imaging

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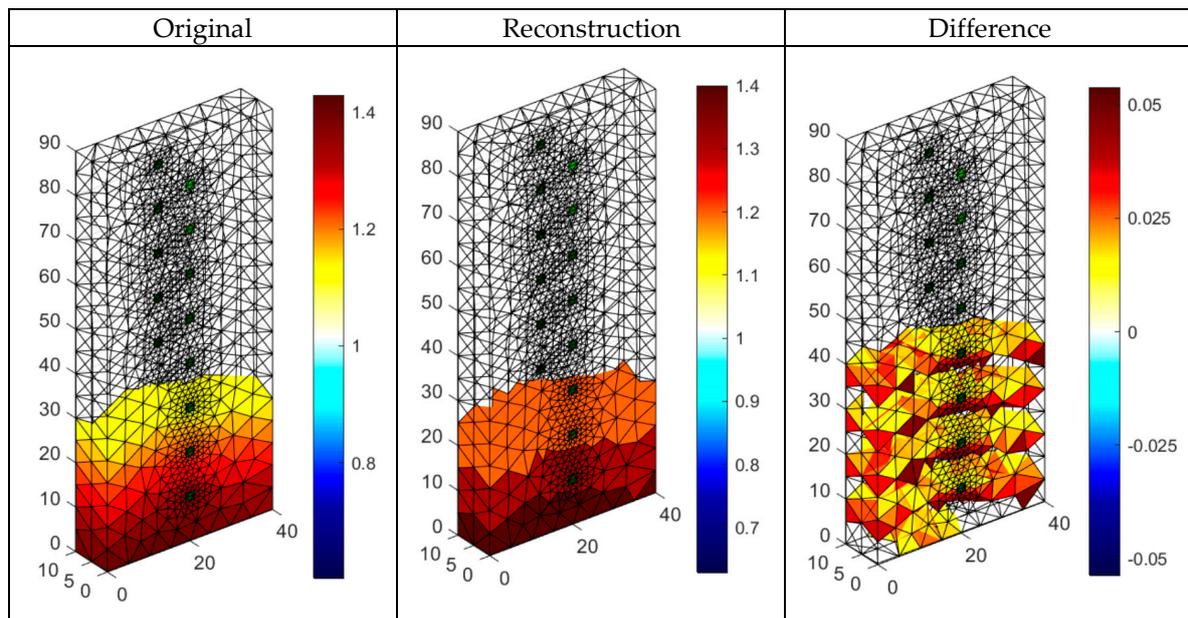
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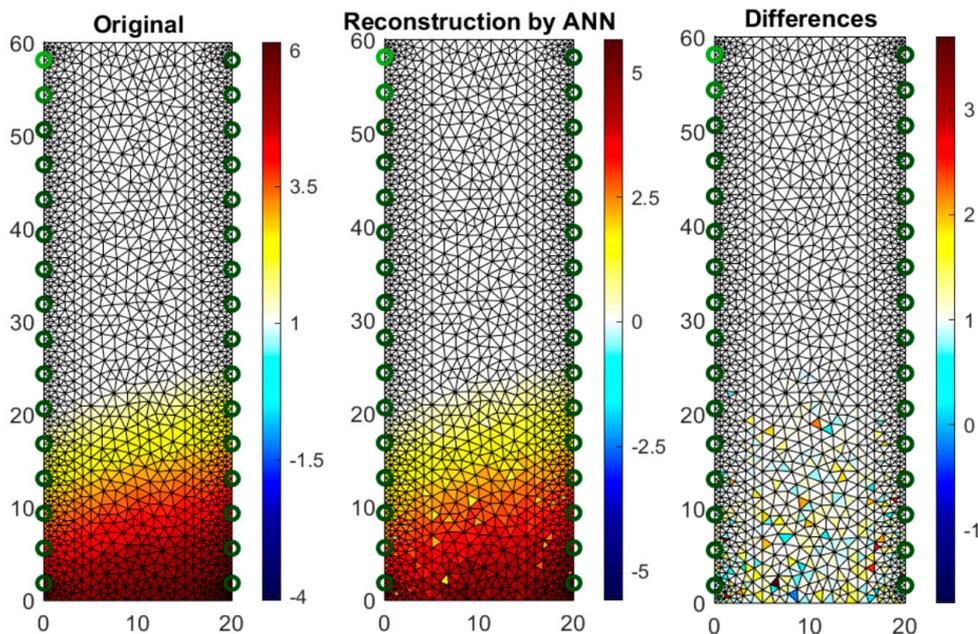
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Figure 19 presents the results of neural imaging in conjunction with the EIT. Conductive (positive) areas are shown in shades of red. Non-conductive (negative) areas are shown in blue or they are transparent. Comparing the pattern of the damp block (original) with the output image, we conclude that the accuracy of the imaging is very high. The right image shows the absolute (numerical) differences in the values of individual pixels are minimal. They do not exceed ± 0.05 . It can be seen that the results obtained by the neural imaging method are comparable to the results obtained by both previous methods. RMSE for a 3D sample with the use of ANN is 0.010819 so the quality is better than LARS and ElasticNET.



527 **Figure 19.** The result of Artificial Neural Network system (ANN) moisture testing of the lightweight
 528 concrete block for the case of 2x8 electrodes

529 Figure 20 shows an analog measurement of a 2D sample with two rows of electrodes on both
 530 sides of the block (2x16). Noteworthy is the small amount of colored pixels in the differential image.
 531 This indicates high quality imaging, which is confirmed by the low RMSE index, which in this case
 532 equals 0.106301.



533
 534 **Figure 20.** The result of Artificial Neural Network system (ANN) moisture testing of the lightweight
 535 concrete block for the case of 2x16 electrodes

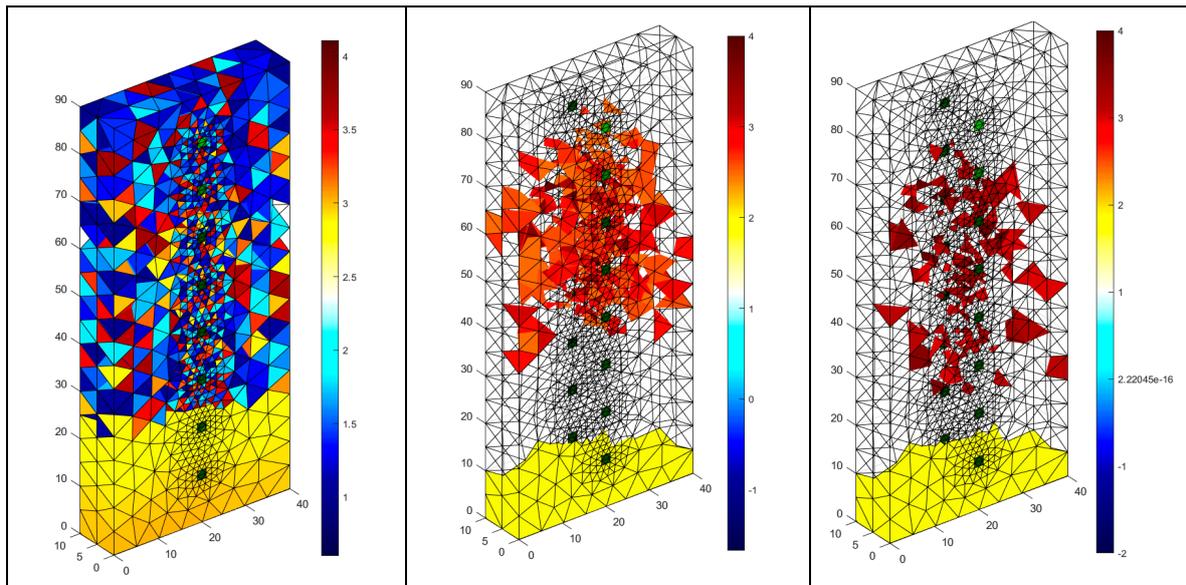
536 3.4. Moisture test of real object

537 Figure 21 shows the results of the reconstruction of a block of cellular concrete immersed in
 538 water, using a system of artificial neural networks. Reconstructions based on data obtained from real
 539 objects, in contrast to simulation experiments based on numerical models are the most difficult type
 540 of test for tomographic systems. The first image on the left is the result of the direct processing of the
 541 tomographic data with the use of ANN. The ambiguous visual effect is caused by noise in the value
 542 of the input vector, which in the case of real objects is basically unavoidable.

543 To show the results in a way that visually identifies the moisture area, the input vector was
 544 subjected to a denoising procedure using denoising stacked autoencoders. The results of denoising
 545 were presented in the middle image entitled "First denoising". Finally, the output image was
 546 subjected to one more processing using a filtering script whose objective was to cut off the output
 547 values, which obviously exceeded the acceptable range. The filtering effects are shown in the image
 548 entitled "Second denoising".

549 Denoising of tomographic data is an important issue because it affects the results of
 550 reconstruction. Due to the complexity of this subject, it may be the subject of separate studies aimed
 551 at improving the quality of tomographic images.
 552

Real object reconstruction	First denoising	Second denoising
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553 **Figure 21.** The result of the real object ANN moisture testing of the lightweight concrete block for the
 554 case of 2x8 electrodes

555 4. Discussion

556 The imaging results presented in the previous chapter show great application possibilities of the
 557 machine learning algorithms combined with EIT. The analysis involved 3 methods and algorithms
 558 converting input vectors (values of voltage drops) into reconstructed images reflecting the
 559 conductance: Least Angle Regression (LARS), ElasticNet and Artificial Neural Networks (ANN). Of
 560 the above methods, the best results were obtained using ANN. However, the LARS method in terms
 561 of fidelity representation is very similar to ANN.

562 Table 2 presents a summary of the RMSE values for cases of 2D and 3D imaging in relation to 3
 563 methods tested: ANN, LARS and ElasticNET. The lowest values of the indicators, demonstrating the
 564 highest imaging quality, were obtained for ANN.

565
 566

Table 2. Comparison of the quality of different imaging methods

Method	RMSE	
	2D samples	3D samples
ANN	0.106301	0.010819
LARS	0.122599	0.033019
ElasticNET	0.282520	0.036184

567
 568 Both LARS and ANN can be successfully used in the EIT tomography dedicated to the
 569 reconstruction of moisture in masonries and building walls. In comparison to other, previously used
 570 algorithms, these methods allow obtaining precise images with sufficient resolution to perform an
 571 effective and error-free analysis of the moisture content of the walls. It is worth noting that taking
 572 into account the possibilities of spatial image creating, the LARS and ANN methods are more reliable
 573 than invasive methods requiring the sampling of masonry.

574 It is also important that the presented algorithms, used in connection with the EIT system and
 575 specially designed electrodes, have large application possibilities. Their basic advantages are
 576 functionality, reliability, measurement accuracy and reasonable price. The method is universal due
 577 to the possibility of applying to masonry and walls of the various structure, thickness and moisture
 578 level. An important role is also played by the speed of the computed tomography scanner. Output
 579 images are obtained in real time.

580 The impedance electrical tomography proposed in this article enables the creation of a new non-
 581 invasive technique for measuring the humidity of building walls. The EIT has been used to determine

582 the conductivity distribution in specially constructed wall models made of light concrete blocks or
583 bricks. The finite element method implemented in the EIDORS environment has been used to solve
584 the problem. The numerous different lattices were used in the presented numerical models. The
585 analyzed measurement systems contained various electrode distributions. Thanks to this, it is sure
586 that the obtained results are not accidental but repeatable while maintaining similar conditions of the
587 measurement environment.

588 The research has provided new and promising results. Future work will be continued thanks to
589 the use of regularization techniques in the optimization process and the hybrid measurement system.
590 These types of hybrid measuring system should be even more reliable in practical applications. It
591 would also be interesting to extend the experimental measurements over time by monitoring the
592 walls at regular intervals. Thanks to this, it would be possible to estimate the speed of spreading the
593 moisture inside walls, as well as its sources and propagation directions.
594

595 5. Conclusions

596 The main goal of the work was to analyze the solution based on electrical tomography to study
597 the moisture of walls. Non-destructive methods and algorithms have been analyzed and compared,
598 which allow estimation of humidity also inside the wall. A new concept of a non-destructive system
599 based on electrical tomography has been presented. For research purposes, specially designed
600 electrodes were used, which were placed on the tested lightweight concrete and brick blocks. Three
601 machine learning algorithms were tested: Least Angle Regression (LARS), ElasticNet and Artificial
602 Neural Networks.

603 It was found that all four methods are suitable for practical applications in EIT tomography
604 dedicated to the detection of moisture in building walls, however, the best results were obtained
605 using the LARS method and the specially designed multi-ANNs system. A characteristic feature of
606 the analyzed solution is the division of the modeled object using a specially developed mesh for a set
607 of elements. The color of each individual mesh element corresponds to the conductance value (in the
608 EIT tomograph). Thanks to this approach the number of information determining the reconstructive
609 picture was large enough to guarantee a sufficient resolution of tomography imaging.

610 The presented research results contain relevant information that may contribute to the
611 acceleration of the development of computational intelligence and machine learning methods in EIT.
612 The research contributes to the improvement of the tomographic imaging efficiency of known
613 methods in the aspect of algorithms for processing input information (electrical quantities) into
614 images. In addition, enriching an input vector with values other than electrical is an easy way to
615 develop new, intelligent tomographic hybrid systems.

616 **Author Contributions:** Tomasz Rymarczyk has developed a research project and methods for testing the
617 moisture of walls using electrical tomography. As an expert in the work of tomography, he was the originator
618 of most machine learning concepts presented in this study. Grzegorz Kłosowski carried out research especially
619 in the field of artificial neural networks and was responsible for the substantive and editorial aspects of the
620 presented paper. Edward Kozłowski has implemented statistical methods and developed mathematical models
621 descriptions.

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