A Shear Reynolds Number Based Method to Assess the Locally Dominant Sediment Transport Modes in Large Rivers with Complex Morphodynamics

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Abstract: The aim of this study is to introduce a method which can suggest if sand or gravel dominated bed load transport presents in rivers with mixed-size bed material. As a conventional way, the Shields-Parker diagram could be used for such purposes, however, the method has certain applicability limits, due to the fact that it is based on uniform bed material and provides information rather on river-scale, instead of reach or local scale. When dealing with large rivers with spatially complex hydrodynamics and morphodynamics the bed load transport modes can also indicate spatially strong variation even locally, which calls for a more suitable approach to estimate the locally unique behavior of the sediment transport. Here, we suggest that the decision criteria utilizes the shear Reynolds number (Re), which can be estimated based on the local characteristic bed grain size (e.g. $d_{50}$) and local bed shear stress ($\tau$). The method was verified against field and laboratory measurement data, both performed at non-uniform bed material compositions. The comparative assessment of the results by the novel method and the one suggested by the Shields-Parker diagram highlights that the Shields-Parker diagram might be applied with care due to its high uncertainty when the hydro-morphological conditions are complex. In contrast, the results show that the shear Reynolds number based method is expected to predict the sand or gravel transport domination with a <5% uncertainty. The introduced results can greatly contribute to the improvement of numerical sediment transport modeling as well as to the field implementation of bed load transport measurements.

Keywords: bed load transport, shear Reynolds number, mixed-size bed material, complex morphodynamics

1. Introduction

In terms of typical geomorphological features, rivers can be divided into three main section types: upper, middle and lower course. One of the decisive differences among these river features is the erosion capacity of the flow, which constantly decreases along the river. In fact, it can be stated that the erosion prevails along the upper course rivers, yielding coarser gravel bed material and significant bed load transport. As to the middle course, all the three characteristic sediment transport processes, such as the local erosion of the bed, the transport of coarse and fine particles together with river bed aggradation take place, resulting in a dynamic equilibrium of the river. Finally, the lower course type rivers can be generally characterized with the deposition of fine sediments transported from upstream [1]. Due to the spatially and temporally varying erosion capacity of the flow and the changing river planforms (straight, meandering or braided river channel patterns [2]), the morphological features also show significant variability.
Many classification methods can be found in the literature to describe the morphological properties of river sections (e.g. [3]–[7]). The main goal of these tools is to typify and predict the dominant morphological properties and processes, such as to define the predominant sediment transport mode (bed load, or suspended load) [8], the channel pattern type [1], [9], the specific bed material grain size [7], the erosion capacity [10], or the bed armor measure [6] etc. According to these methods, the morphological properties and processes can be estimated as the function of few, easily definable hydro-morphological variables, e.g. the water depth, the channel width, the mean flow velocity, the mean grain size of the bed material, or the longitudinal bed slope.

For instance, there are classification methods, suggesting that the dominant grain size of the bed load (sand or gravel) can be obtained based on the river course type. In most cases, such a classification is straightforward due to the fact that a dominant bed material fraction can be easily chosen, considering e.g. sand-bed \((d_{50} < 2 \text{ mm})\) or gravel-bed \((d_{50} > 2 \text{ mm})\) streams [8]. However, there are several situations, when such a clear distinction cannot be made as both sand and gravel dominated zones appear even in shorter river reaches. These sections are typical to middle course rivers, at the transition sections in terms of bed material. Here, the flow pattern can easily result in spatially strongly varying bed material composition (e.g. coarse river bed surface in the main stream, sand dominated zones in shallower parts) and distinct characteristic grain sizes of the bed load transport.

For instance, such a typical river section is the upper Hungarian Danube reach between rkm 1798 and rkm 1795 (Fig. 1). At this section, the middle course resulted in a meandering pattern type, with non-uniform bed material \((d_{50} \text{ range of } 0.32 - 70.5 \text{ mm})\) [9], [11]. Furthermore, conventional river regulation measures (e.g. groin fields) were installed, which further enhanced the diversity of the morphological properties and processes [12]–[16]. The following parameters characterize the river: the main channel width at mean water regime ranges between 150 m and 350 m; the average water surface gradient is around 0.0002-0.00025 [15], whereas the characteristic flow discharges are \(Q_{m} = 2000 \text{ m}^3/\text{s} \) (mean flow), \(Q_{bf} = 4300 – 4500 \text{ m}^3/\text{s} \) (range of bankfull discharge) and \(Q_{100} = 10400 \text{ m}^3/\text{s} \) (100-year flood event) [17]. Here, the dominant fraction in the bed material and hence in the bed load shows a strongly varying spatial distribution. That is, bed armoring and mainly gravel bed load takes place in the main channel, while the aggradation and erosion of the fine particles can be observed in the near-bank zones and in the groin fields. And finally, the accumulation of the gravel is detectable at some places, resulting in the formation of gravel bars (Fig. 1).

![Figure 1. The sketch of the investigated section of the Danube River.](image-url)
2. Problem statement

The analysis and prediction of reach scale morphodynamic processes play a crucial role in several river engineering activities, e.g. when planning restoration measures. The reliable description of the interaction between river hydrodynamics and the morphological features is a very challenging task in rivers with uniform bed material composition, but shows even higher difficulties in case of mix-sized bed material conditions (e.g. [18]–[21]). For instance, when uniform bed material characterizes the study reach, the potential erosion results also in uniform grain sizes of the bed load transport. For such conditions, many sediment transport models have already been developed and validated [22]–[25]. On the other hand, in case of inhomogeneous river bed material, the characteristic grains in the bed load transport can also show inhomogeneity, and the so called selective erosion phenomenon takes place [26]. Depending on the local flow features and local bed material composition, the dominant fractions in the bed load can easily vary from sand to gravel even in shorter sections [15], [27]. When performing morphodynamic analysis of river reaches, it is a relevant question, how to predict if the sand or rather the gravel transport dominates the sediment transport locally.

As probably one of the most widely used methods, the Shields-Parker river sedimentation diagram can be applied as a comprehensive bed-material and sediment transport nature classification tool for a particular river reach, with uniform bed content. The diagram indicates the particle Froude number (Eq. 1) based on the mean grain size (d50) versus the so called explicit particle Reynolds number (Rep, Eq. 2). Parker stated [8], that alluvial rivers can usually be divided into two types as the function of Rep, (which can, in fact, be considered as the dimensionless substitution of the grain size): sand-bed and gravel-bed rivers. According to his research, a distinction can be defined exactly based on Rep, which actually depends only on d50. That is, considering that d = 0.002 is the threshold value between the sand and gravel fractions, the critical value for Rep also can be calculated according to Eq. 2, where d = 0.002 m results in Rep = 360 (Rep > 360 → gravel bed, Rep < 360 → sand-bed).

\[ \tau^* = \frac{\tau}{g(\rho_s - \rho_w)d} \]  

(1)

where \( \tau^* \) is the particle Froude number, \( \tau \) is the bed shear stress, \( g \) is the acceleration due to gravity, \( \rho_s \) is the sediment density, \( \rho_w \) is the water density and \( d \) is the grain diameter.

\[ Rep = \frac{\sqrt{gd\tau}}{\nu} \]  

(2)

where \( R = (\rho_s - \rho_w)/\rho_w \), \( \rho_w \) is the water, \( \rho_s \) is the sediment density and \( \nu \) is the kinematic viscosity of the fluid. Consequently, the method basically yields a grain size based classification method for near uniform bed materials. This simplified approach is not capable to classify characteristic local morphodynamic processes for complex situations in terms of river bed morphology and inhomogeneous bed material composition though [1]. Again, it has to be enhanced that a different approach is needed to identify locally dominant sediment transport nature when dealing with spatially strongly varying flow and morphological features.

3. Materials and methods

The basic idea here is to keep the simplicity of the classification method shown by Parker, but at the same time to involve a new indicator, which accounts for the local hydrodynamic effects that characterize the interaction between the water flow and the river bed, i.e. the erosion capacity. The well-known Shields diagram provides information on the grain stability classification in function of the so called shear Reynolds number (Re\( \star \), Eq. 3) [25]. The curve shows the critical particle Froude number (\( \tau^*_e \), critical dimensionless bed shear stress, Eq. 1) as the function of the Re\( \star \). Based on
experimental results, Shields pointed out that the stability of the grain and the inception of motion depends not on the grain size, but also on the hydraulic flow regime around the grain, which can be expressed by $Re^*$.  

$$Re^* = \frac{u_* D}{\nu},$$  \hspace{2cm} (3) 

where $u_*$ is the shear velocity ($u_* = \left( \frac{1}{\rho} \right)^{0.5}$, where $\tau$ is the local bed shear stress).

The paper also emphasizes that the benchmark measurements were elaborated for hydraulic smoother ($Re^* < 500$) regime. As one of the most widely tested and applied sediment transport formulas for river engineering applications, the van Rijn bed load transport equation [28] also considers the application of the Shields curve. The transport model was developed and validated for sand fractions, and so the application limit was defined accordingly ($d < 2 \text{ mm}$) [7]. However, Parker pointed out [7] that at higher hydraulic rough regime the motion of the sand fractions is rather suspended like rolling. Therefore, a more accurate calculation of the sand motion is expected by a suspended sediment transport formula instead of a bed load formula, in hydraulic rough regime. That is to say, the grain size is not able to determine correctly the application limits of the formula. However, the $Re^*$ values regarding to the calibration ($Re^* < ~ 100$) and validation ($Re^* < ~ 400$) measurements [28]–[30] suggest that the van Rijn bed load model is valid for hydraulic smoother regime. Although it was not exactly defined, van Rijn mentioned [28] that the model is valid only for lower $Re^*$.

The laboratory experiments, where non-uniform bed material conditions were investigated by Wilcock et al. [19] (for which the Wilcock and Crowe [31] bed load transport formula was developed and validated) is also taken into consideration here. Fig. 1 A shows the rate of the gravel load to the sand load as the function of the $Re^*$. The coloring of the points refers to the sand content of the initial mixture. Within any mixtures the higher the $Re^*$, the coarser the composition of the bed load. A clear behavior can be noticed: in case of the same mixtures, at lower $Re^*$ only the finer part of the bed material is being transported, but at higher $Re^*$ gravel load dominates. It is also visible that the gravel load is equal to the sand load when the $Re^*$ is between 300 and 400, around 350 (Fig. 1 A). Thus, $Re^* = 350$ seems a critical value; above this the gravel transport is dominant. In turn, below 350 the sand transport prevails, which underlines the earlier assumption that the fine sediment bed load transport happens and dominates in the hydraulic smoother range.

Based on the above findings, the following hypothesis was stated. The $Re^*$ parameter is a more suitable parameter for the prediction and distinction of the sand or gravel dominated bed load transport, than either simply a characteristic grain size ($d_{90}$ or $d_{50}$), or the $Re_\tau$, when spatially strongly varying morphological features are present.

4. Results

In order to confirm the assumption made in the previous point, an assessment, considering characteristic local bed grain sizes, local bed shear stress values as well as calculated Reynolds numbers, was performed, using datasets from recent laboratory and field experiments of the authors. First, the results from the laboratory experiments of Török et al. [33] were used. In this experiment, morphological processes (scouring, bed armoring and aggradation) were investigated around a single groin, using mixed size bed material with an initial $d_{50}$ of 5.16 mm. The local grain size distributions were determined by an automated image based grain detection software tool BASEGRAIN [34]. Local bed shear stress values were estimated according to the TKE-Method [35] using the near-bed point-velocity measurements, carried out with a 3D Acoustic Doppler Velocimeter (ADV) [33].

Second, field data from a section of a large river was also assessed. The field experiments were carried out in the Hungarian section of the Danube River at mean flow regime, where the main characteristic morphological parameters were already introduced in the Introduction point. 47 bed material samples were taken by a drag-bucket sampler, for which, the grain-size distributions of the samples were determined by sieving analysis. Parallel to the bed material sapling, the velocity
profiles were measured by an acoustic Doppler current profiler (ADCP) (WorkHorse Rio Grande 1200 kHz) in fixed boat mode. Based on the long-term, fixed boat ADCP measurements, the local bed shear stress values could be estimated using the turbulent wall law [35].

![Figure 2. A: Calculated rates of the gravel load to the sand load as the function of the $Re^*$, based on the laboratory experiments of Wilcock et al. [19]. B: The bed shear stress as the function of the $d_{50}$ grain size, regarding to field and laboratory measurements [15], [27], [32], [33]. C: Particle Froude number as the function of the shear Reynolds number, based on laboratory and field data [15], [27], [32], [33]. Shields curve is indicated with black line. D: Particle Froude number as the function of the explicit particle Reynolds number, based on laboratory and field data [15], [27], [32], [33]. Shields curve is indicated with black line.]

At the investigated Danube site, the $d_{50}$ grain size in the river bed ranges between 0.32 to 70.5 mm. The estimated local bed shear stress values increase with higher grain sizes, however, quite significant scattering of the points can be seen (Fig. 2 B). Nevertheless, when indicating a separation line, which represents a $Re^*$ of 350, the points from the three well distinguishable regions indeed show different features. First, the points that represent the shallower, sand dominated regions fall below the line, second, the points located along the line represent the gravel bars, and third, the points above the line represent the main stream with coarse grains and clear gravel bed load.

Assessing the relationship between local particle Froude number and the shear Reynolds number for both the laboratory and field experiments (Fig. 2 C), it can be stated that there is a correlation between the $Re^*$ and the local sediment transport nature: the higher the $Re^*$ number the more dominant the coarser grains in the local sediment transport nature. In turn, the lower the $Re^*$ the more prevailing process is the sand motion. Furthermore, it can also be observed that the $Re^* = 300-400$ is a suitable indicator for the identification whether the gravel (coarsening, scouring and bed armoring) or rather the sand (aggradation, erosion) particles dominated processes take place, in case of non-uniform bed material. No clear distinction of the dominating morphodynamic processes can be made if the particle Reynolds number is consider instead of the shear Reynolds number versus $r^*$ (Fig. 2 D). Most of the points, representing all the three different transport modes, range between $1000 < Re^*_p < 10000$, below the critical condition provided by Shields.

In order to perform a quantitative assessment on the performance of the $Re^*$ based approach, a statistical method was applied, which provides information on the representativity of the Reynolds
number ranges for the characteristic bed load transport processes. Here, the log-normal distributions
of the $Re^*$ and $Re_p$ values were calculated, separately for the three above distinguished groups. The
reason for using the log-normal distribution was the fact that asymmetric distributions were expected
(based on the scatters of the points), where Re numbers cannot take negative values [36]. The
probability density for the log-normal distribution is expressed as:

$$P(x) = \frac{1}{(x-\gamma)\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\ln(x-\gamma)-\mu)^2},$$  \hspace{1cm} (4)

where the parameters are $0 \leq \gamma < x$ (location parameter), $-\infty < \mu < \infty$ (scale parameter) and $\sigma > 0$
(shape parameter), can be obtained by the fitting of the function to the known points. The integration
of the log-normal distribution curves results in the cumulative distribution function, which is:

$$F(x) = \Phi \left( \frac{\ln(x-\gamma)-\mu}{\sigma} \right),$$  \hspace{1cm} (5)

where $\Phi$ is the Laplace integral [37], [38].

As a summary of the calculations, the probability data was summarized for the six log-normal
distributions, considering $Re^* = 300$, $Re^* = 400$, and $Re_p = 360$ as critical values.

Table 1. Probability values of the shear Reynolds number and the particle Reynolds number,
calculated based on the fitted log-normal distribution for grouped data.

<table>
<thead>
<tr>
<th></th>
<th>$Re^*$</th>
<th>$Re_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt; 300</td>
<td>&gt; 300</td>
</tr>
<tr>
<td>Sand aggradation, near-bank points</td>
<td>95%</td>
<td>5%</td>
</tr>
<tr>
<td>Gravel bar</td>
<td>5%</td>
<td>60%</td>
</tr>
<tr>
<td>Main channel, bed armor</td>
<td>0%</td>
<td>3%</td>
</tr>
</tbody>
</table>

The probability values indicated for the three main morphodynamic processes emphasize the
suitability of $Re^*$ for the determination of the locally dominant grain size range in the bed load
transport. On the other hand, the application of $Re_p$ suggests no clear classification way for the same
categorization, when spatially varied bed material is present, such as at the investigated river reach.
It can be seen that the higher the $Re^*$ the more dominant the gravel rate and less the sand motion.
Note that similar behavior could be observed in Fig. 2 D. Furthermore, an important outcome of the
probability analysis is that the $Re^*$ between 300 and 400 is indeed a critical range: below 300 the sand
transport, above 400 the gravel motion dominates.

5. Discussion

A novel classification method is introduced in this paper, which is expected to predict the locally
dominant sediment transport nature (sand or gravel) in a more accurate manner compared to other
methods found in the literature, such as the Shields-Parker diagram, when investigating river
morphodynamics on reach or local scale. As reported in previous studies, the dominant sediment
transport nature, in rivers with uniform bed material, can be reliably determined as the function of the
so called explicit particle Reynolds number ($Re_p$), but yields less accurate estimations in cases,
when the bed material composition shows strong variability even along shorter reaches. The authors
of this paper believe that introducing the local bed shear stress values in the categorization method
will provide better distinction for follow-up morphodynamic analysis.

Based on 70 bed material samples and related local bed shear stress values gathered both from
recent own laboratory and field experiments, it could be confirmed that, instead of the utilization of
the explicit particle Reynolds number \((Re_p)\), or the application of characteristic grain sizes, e.g. \(d_{50}, d_{90}\), the shear Reynolds number \((Re^*)\), in which the momentum forces at the river bed are represented by the shear velocity, is a more adequate parameter to assess the locally dominant sediment transport nature. Besides the own experimental data, it could also be shown, that the \(Re^*\) based approach works very well for the laboratory and field data based on which the widely used van Rijn [28]–[30] and the Wilcock and Crowe [19] bed load formulas were validated: sand motion takes place at lower (< ~300) \(Re^*\) range, while the coarser gravel dominated transport is indicated at higher (> ~400) \(Re^*\) values. As another example, we assessed the datasets, consisting of 45 coupled bed shear stress – \(d_{so}\) characteristic grain size value pairs from field measurements, published by Mueller et al. [39]. In that study, the median surface grain sizes of the gravel-bed streams and rivers were reported to vary between 0.027 and 0.21 m, which indicates quite coarse bed material and armored bed surface. Accordingly, the calculated \(Re^*\) are consequently high, all estimated \(Re^*\) values exceed 400.

Based on the data assessment introduced above, the following classification could be set up:

- \(Re^* < ~300 \rightarrow\) sand transport dominates,
- \(Re^* > ~400 \rightarrow\) gravel transport dominates,
- \(~300 < Re^* < ~400 \rightarrow\) gravel accumulating and gravel bar formation is expected.

6. Conclusion

Based on the findings of this study, an improved understanding of the locally dominant bed load transport can be gained, which can contribute to the better implementation of different sediment transport investigation methods applied in large rivers. For instance, a well-known issue related to field sediment transport monitoring is the high uncertainty of direct bed load sampling methods. Having information on the local bed material and on the flow field, the \(Re^*\) based approach can suggest where and what sort of sampling techniques would be the most suitable to collect reliable sediment information, eventually yielding a more cost-efficient and more accurate field procedure. The results can also contribute to the development of improved computational modeling tools. For instance, instead of applying one specific empirical sediment transport formula in a simulation to calculate local bed load transport and related morphological changes, the \(Re^*\) based approach can be utilized to distinguish between several formulas, each of them having a certain application range. For instance, as already mentioned above, the van Rijn model could provide better sediment transport estimation in sand dominated zones, whereas the Wilcock and Crowe model suits better to gravel dominated cases. Implementing these two models in a computational tool, the calculated local \(Re^*\) could suggest which formula to apply locally.

Funding

This research was funded by the ÚNKP-17-3 and 17-4 New National Excellence Program of the Ministry of Human Capacities.

Acknowledgements

The authors acknowledge the support of the MTA TKI of the Hungarian Academy of Sciences.

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