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Technical Note

Simulation of Bell Correlations in Walker Systems

Marc Fleury [1]

3344 Peachtree #4301, Atlanta, GA 30326, USA; marcf999@gmail.com

Abstract: We describe results from a Monte-Carlo simulation of Bell-CHSH type correlations in hydrodynamic walkers. We study feasibility of a real life walker test with relevant hydrodynamic parametric ranges. We observe the generic formation of pairs of walkers strongly anti-correlated both in position and momentum. With this source of entangled walkers, we model the insertion of 2 pins in the bath as a notion of measure, akin to the polarizers of photonic Bell tests. This insertion of pins, either static or dynamic, introduces 2 weak field signals. Each field has the physical form of a standing wave Bessel hat, representing the non-local (field mediated) influences of the measure on the walkers. With this representation of the measure, we develop protocol for a Bell game with actual hydrodynamic walkers. We model both static and dynamic insertion of pins in the walker bath. Static pins result in numerical $S > 2$, as a permissible Bell violation for a non-local (field based) effect. Dynamic insertion of the pins, however, leads to causal space separation of the two arms. We observe the again expected $S \leq 2$. We argue for the hydrodynamic implementation and observation of these effects as a walker visualization of Bell inequalities.

Keywords: entanglement, hydrodynamic analogs of quantum systems, Bell's inequalities, Bohmian mechanics

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1. Introduction

The hydrodynamic walkers of Prof. Walker, Bush and Couder [1,2] have given us a rich 3-dimensional visual ontology with which to understand, in mechanical terms, the formalism of Quantum Mechanics. The walkers introduce, as a distinct category, the association of a point particle with an extended self-generated field. We observe the orbits of this dynamic duo which give us analogs of dynamic behaviors previously only observed in the quantum domain. Amongst the behaviors thought to exclusively belong to the quantum, Bell's theorem establishes the statistical impossibility to reproduce entanglement with local realistic constructs. Here we develop a protocol for a simulation of entanglement in a walker system.

2. Conceptual Scheme of Bell-CHSH type correlations for walkers

We use as starting point a scheme recently suggested by Vervoort[3]. Conceptually, we have a source in the center (the black box of figure 1) emitting anti-correlated walkers, one propagating to the left and the other to the right. We insert two pins placed in the bath at positions represented by vectors $\mathbf{x} = (a, a')$ and $\mathbf{y} = (b, b')$.

These vectors model the notion of a measure by intruding on the intrinsic dynamics of walkers. We model the pins by static Bessel functions in the bath with the same Faraday wavelength as the walkers and with similar amplitude.

We visualize the lateral left and right sides of the box as in figure 1, and from that top view if $y > 0$ then we assign value $\sigma_1 = +1$ for the left walker and similarly $\sigma_2 = +1$ for the right walker; -1 otherwise for both walkers in case $y_1 < 0$, $y_2 < 0$. We compute over many samples of the walkers the following correlation between σ_1 and σ_2:

$$M(x, y) = \sum P(\sigma_1, \sigma_2)\sigma_1\sigma_2 \tag{1}$$

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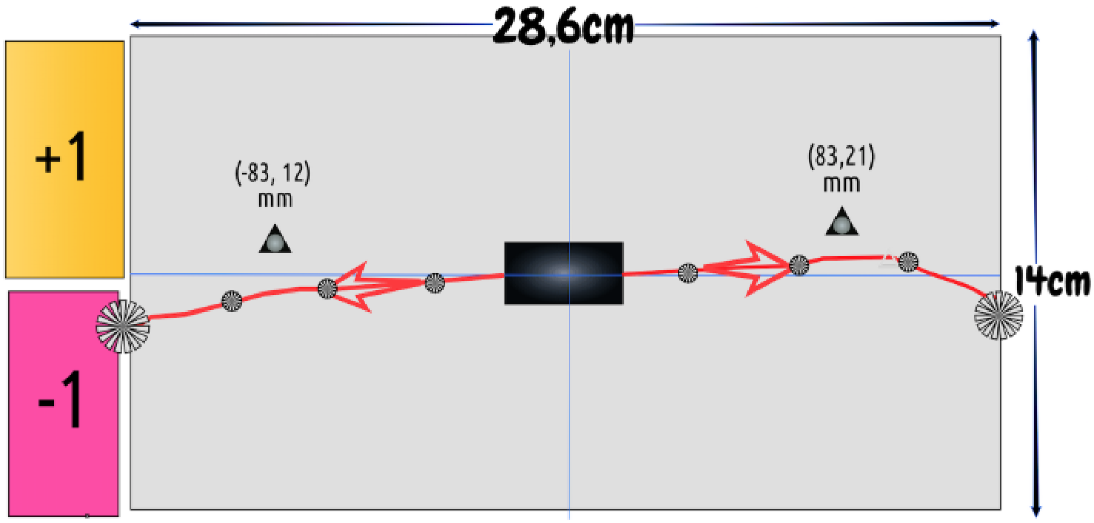

Figure 1. Anti-correlated walkers in dots seen with their red path as filmed from above. Bath dimensions are given. Pin inserted at position signified by triangles. Outcomes σ =+1 or -1 are said to occur when the walker walks into a respective bucket on respective side. Source of anticorrelated walkers is in the middle.

where $P(\sigma_1, \sigma_2)$ is the joint probability of outcome (σ_1, σ_2). We do this for four values a, a', b and b' following the rules of the Bell game, and we calculate the scalar value:

$$S = M(a,b) + M(a,b') + M(a',b) - M(a',b') \tag{2}$$

The Bell-CHSH theorem states that for a model where spatial regions can be causally separated as prescribed by special relativity, the joint probability $P(\sigma_1, \sigma_2)$ factorizes in $P(\sigma_1)P(\sigma_2)$, and we immediately derive:

$$|S| \leq 2 \tag{3}$$

aka Bell-CHSH inequality under Clauser separability.

3. Source of anti-correlated walkers

We now detail the implementation of the source of entangled walkers sitting in the middle of the bath in figure 1. A source needs to emit highly anti-correlated walkers both in position and momentum. We seed the simulation with 2 walkers created close to each other so they interact with each other's wakefield inside the black box. We observed the formation of pairs in a highly anti correlated fashion. The effect arose from walker to walker field mediation with high reproducibility. Entangled walkers appeared generically in our simulation with real hydrodynamic ranges.

3.1. Random Initial Conditions (IC)

For the purposes of a Bell-CHSH type test, we created and initialized pairs of walkers in the center at the same time. To run a Monte-Carlo simulation, we initialized both walkers with random positions and random velocities with pre-established parametric ranges. This way, each walker got a narrowly defined, but random, initial position and momentum. This randomness of the initial state represents the only stochastic element of our model.

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3.2. Observed anticorrelation from random ICs

First we studied the source dynamics without any interference from the pins. We only considered walker to walker interactions, via their respective fields. We enforced this separation numerically in the dynamic equations by zeroing out the force contributions from pin elements in the differential equations of Newton (as if not inserted). With random initial variables in position and momentum, we obtained almost exclusively anti-correlated pairs. This genericity gives us some hope that we could see such anticorrelated walkers in real life in the near future. In our numerical parameter range, in which we observed the real hydrodynamic behaviors, walkers got quickly and generically anti-correlated in both position and momentum. This entanglement appeared solely from field mediation between the walkers in the source box. We mostly saw perfectly anti-correlated pairs sometimes with numerical value matching up to 6-10 digits. We sometimes see captured circular orbits, in what people call the "ouroboros" formation (reminiscent of the snake biting its tail), and sometimes a "promenade", where the two walkers move side by side.

3.3. No real life observation

To the best of our knowledge, we do not yet observe these anti-correlated walkers in real life hydrodynamics systems. In our simulations however they seemed rather generic, almost the default statistical outcome, and in a wide parameter range without further appeal to fine-tuning.

3.4. On the importance of cavity resonances, emergence of chaos

Walkers exhibit behaviors that belong to the class of deterministic chaotic integrators [4] where small differences lead, by time integration along the path, to macroscopically differentiated and sometimes quantized outcomes. This quantization usually follows the standing wave patterns or cavity resonances which the walker will explore statistically, Statistical confinement and sometimes a clear quantization in orbit emerge naturally. We observe this quantized statistical position confinement in 1D hydrodynamic cavities [5] or 2D spherical hydrodynamic corrals [6]. If we put a cavity in our simulation, we lose the anti-propagating walkers and recover the captured orbits.

4. Bell-CHSH Test implementation

Assuming we have built and observed a reliable source of pairs of anticorrelated hydrodynamic walkers, we now turn our attention to the Bell game software implementation. We develop a protocol for the usual Bell inequality test based on 4 different values of pin positions. We describe the model. Our simulation is written in Java and we use the default threaded libraries for real-hardware multi-core support in order to implement a Monte-Carlo simulation which runs on commodity hardware (MacBook Pro 2014 with Java 8). We have used the Math CERN java libraries [7] for the mathematical libraries we require, namely Bessel functions of order zero and one.

4.1. Walker model assumptions

We operate in the so-called *stroboscopic approximation*, where we only consider the horizontal point dynamics of a walker. We do not look at the vertical movement of our idealized dot, only the in-plane lateral 2D dynamics and forces. We simplify the picture further by only considering phase locked bouncers, where we assume all walkers bounce at the same time. This last restriction corresponds to a generic phase of the walker system even if more phases exist in real life systems [2]. All the walkers have the same mass, which we set to numerical value of 1.

4.2. Newtonian integrator

We custom built a Monte-Carlo simulation to study the collective hydrodynamic behavior of many-walkers interacting system. We brute force integrate the Newtonian differential equations

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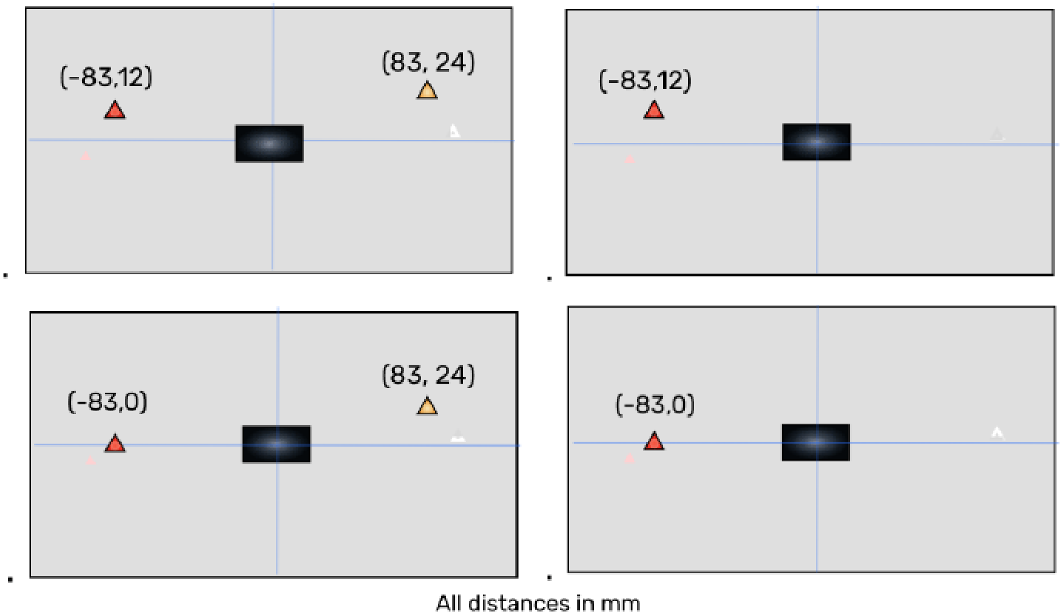

Figure 2. 4-way Bell test via pin settings, and one involves no pin

describing the lateral dynamics of a hydrodynamic droplet bouncing on its own waves. Multi-threaded Runge-Kutta-4 (RK4) integration routine is employed, which was developed previously in collaboration with chaos team at Georgia Tech [8]. We compute hydrodynamic forces at each step of the bounce. We speak about the force imparted by the wake on the walker, a force proportional to the gradient at the point of impact. The wake results from waves emitted by the walkers as well as the measurement apparatus as modeled by Vervoort[3] with static and dynamical pins. The walkers also feel a viscous force, following a hydrodynamic treatment [1].

4.3. Walker field assumptions

Each impact of each walker creates a standing wave centered at said point of impact. We represent these wavelets with standing Bessel waves. The Bessel function has a wavelength parameter that we set equal to Faraday wavelength, providing an amplitude envelope (Bessel form) in the spatial domain. It also has a parametric scaling amplitude mimicking real life parameters, and a parametric exponential decay time representing the concept of memory [1]. In effect, we let the walkers interact via their standing wavelets at creation time, but only up to a distance, after which we numerically zero out the cross-contributions to account for absence of interaction between causally space-like separated events.

4.4. Pin wave assumptions (Static/Dynamic)

Each pin is represented by a Bessel standing wave. Wavelength is assumed to be equal to Faraday wavelength. Amplitude is set equal to 1 (as by default in the CERN libraries). From there on, we parametrically modulate said Bessel functions. Amplitude can be set to zero, as it was in the above study of the walker source.

We model two different protocols. In the first protocol the pins remain statically inserted in the bath, so their wake occupies all of the cavity. In the second protocol the pins get inserted dynamically, at the last minute, to separate regions causally, analogous to causality bounds on pins-walkers cross-arm interaction. This gives us a notion of spatial separability as in the in-flight setting of polarizers in photonic Bell test [9]. We modulate the force of the field depending on position, and zero out the

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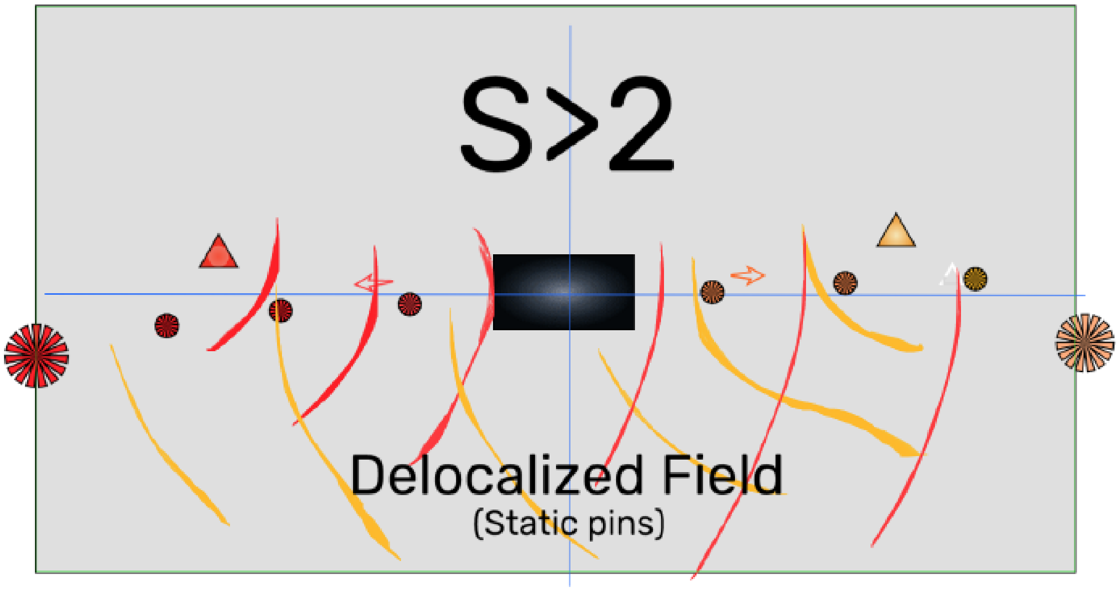

Figure 3. Bell violations with static pins.

field from one pin to the other walker. This represents the Parameter Independence (PI) described by Vervoort[3]. It states that the measure in one arm cannot influence the walker in other arm, we have separability of the walker in one arm to the pin in the other.

4.5. Walkers as spooky action ontology

The velocity of the walker itself and the velocity of the impact wave it each time it bounces may differ. The standing wave signal appears in the wake of the transient "impact" wave. The transient wave travels 10 times faster than the walker that creates them. The separability of the non-local pin cross contributions proves a necessary and sufficient condition for our observations of Bell violations as we have detailed below.

5. $S > 2$ Violation of Bell inequality with static extended fields (/static pins)

With no separability and static pins we clearly have seen many violation of the $S > 2$ bound. We deal with a non-local construct in the field mediation. We have observed many such violations across wide ranges of pin positions as detailed in section 4. It should be noted that the height of the Bessel function from the opposite part of the bath is around 1% of the original height (or the local pin) and that this butterfly effect proves enough to give us the dominant statistics and violations up to $S = 3.4$, exceeding the Tsirelson limit of optic equal to $2\sqrt{2} \approx 2.82$.

We have found strong Bell violations in large areas of experimentally accessible configurations. The bath we explored for real life applications is depicted in figure 1, a 30 cm $\times$ 15cm planar system with pins positioned at about $x_{pin} = \pm 8$ cm. With this bath, we have observed S values up to 3.4, well in excess the 2 limit for local realist models as predicted and allowed by Bell's Theorem under no-separability.

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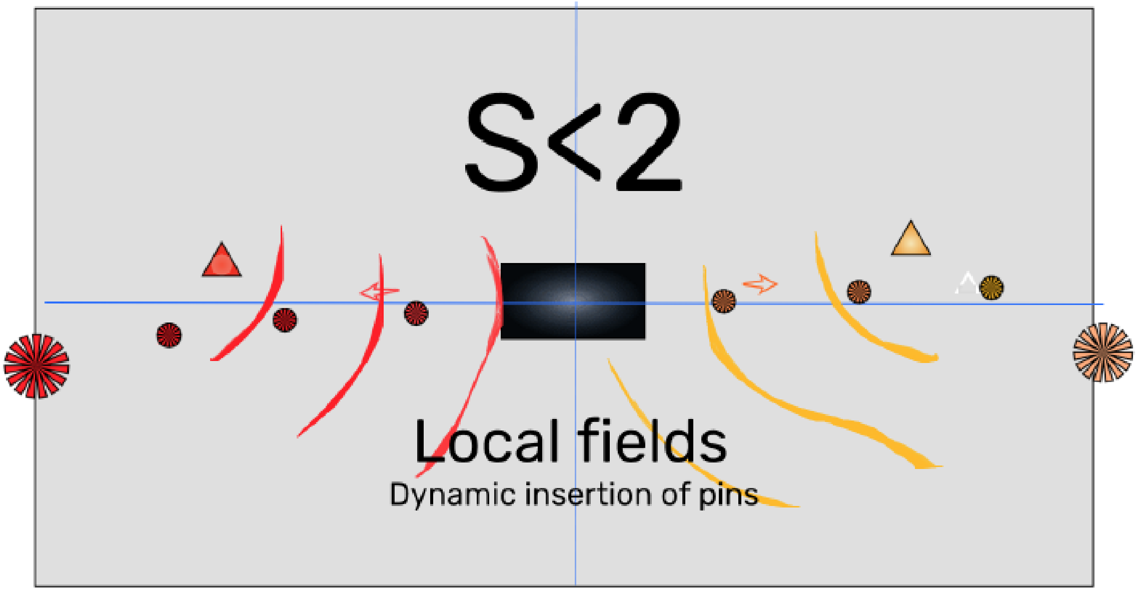

Figure 4. Loss of Bell violation upon enforcing parameter independence, all other parameters remaining unchanged.

6. $S < 2$ Loss of Bell violation with separability (pin insertion)

However if we enforce *parameter independence* (PI) and artificially encode separability then we recover Clauser factorability, and therefore we verify, in theory, the Bell inequalities for our category. Indeed we did not observe Bell violations in our simulations with dynamic pin insertion. We have failed to observe any violation of the 2 limit, again as predicted by the Bell's Theorem.

7. Discussion and Conclusion

We have reported the observation of anticorrelated walkers and have discussed their implementation with real life hydrodynamic walkers. We have modeled an explicitly non-local wave mediation system with static pins, and shown that such a measurement setup, with delocalized fields, should in fact reproduce a type of Bell violation for real life walkers. We have also shown that last minute pin insertion, in order to mimic separability of the walkers to the distant measures, has lead to the loss of said Bell violations in our simulation. Visualization and realization of both effects in real hydrodynamic walkers may prove to be an interesting project.

Conflicts of Interest: The author declares no conflict of interest.

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