1 *Article*

2 # Confidence-based participatory forest management

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13 **Abstract:** This paper formulates a new strategy for participatory forest management consisting of
14 encouraging public participation as long as it increases empathy among participants.

15 The strategy requires the homogeneous representation of the opinion of a participant   (i.e. to
16 determine how they assess a forest plan and identify the best one). Utility assessments are prepared
17 for participants through pair-comparisons between meaningful points in the territory and from
18 value functions based on forest indicators. The best plan is designed by applying combinatorial
19 optimization algorithms to the utility of a participant.

20 The calculating of empathy -of one participant relative to another - is based on the equivalence
21 of their respective utilities when the current forest plan is modified. This involves calculating the
22 opinions that are due to systematic changes in the collective plan for those participants that each
23 participant supposes will affect the utility of the other participants. Calculating empathy also
24 requires knowing the interactions among participants, which have been incorporated through
25 agent-based simulation models.

26 Application of the above methodology has confirmed the association between increases in
27 empathy and convergence of opinions in different scenarios: well and medium-informed
28 participants and with and without interaction among them, which verifies the proposed strategy. In
29 addition, this strategy is easily integrated into available information systems and its outcomes show
30 advantages over current participatory applications.

31 **Keywords:** public participation; decision-making; empathetic utility functions; assessment of
32 sustainability

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1. Introduction

35 What should happen if we trust in others to make our own decisions?
36 Virtually all experiments show that the presence of a large number of decision makers improves
37 individual decisions [1]. The internet certainly allows the processing of information from many
38 participants, which mitigates the negative biases of individual decisions [2].
39 In addition, opinions often converge when working in trusted environments. In such
40 environments, each person takes the reasons of others into consideration. To do this, all reasons
41 must be known (which requires them to be explicitly described). In addition, if someone considers
42 other reasons be reasonable, they ought to incorporate them into their personal judgments.
43 Consequently, opinions will converge as empathy increases among the people involved in

44 participative processes (participants). Empathy is reinforced using a common language of
45 communication (ICTs) and when participants have been educated regarding shared ethical and
46 justice criteria.

47   But it is more important to consider empathy as a dynamic process than to calculate it, because
48 it is the increase or decrease of empathy that governs the convergence of opinions. This paper shows
49 that it is possible to objectively measure the evolution of empathy throughout participatory
50 processes, and to do it automatically. This inspires a strategy for collective decision-making based on
51 confidence scenarios: to encourage participatory processes while confidence is increasing, and to
52 conclude them as soon as it ceases to increase. Decisions must then be made by applying the
53 available techniques for aggregation of opinions, but first, it is necessary to have an objective
54 representation of the opinions in order to compare and aggregate them.

55   *1.1. Representation of opinions.*

56   All opinions have two aspects: the way in which a person assesses a solution and the
57 identification of the best solution for a person. The following notation will be adopted in the rest of
58 this paper: X is a solution to a problem; A is an individual who participates in the collective
59 decision-making; $u_A(X)$ is the assessment of X made by A; and X_A is the best solution for A.
60 Consequently, A´s opinion will be represented by the following pair: $o_A = [u_A(X), X_A]$.

61   Assessment is not easy because multiple variables are usually required to describe solutions
62 (for example, a solution has a cost and a rate of return; generates a specific number of jobs, etc.). Not
63 everyone will have the same answer to questions of the type: Would you prefer a project costing €
64 2M and returning € 5M or another project costing € 0.5M and returning € 2M? Assessment is usually
65 individual for each person, and it is deduced from personal patterns of comparison. Assessment
66 transforms each person's preferences into a numerical value that determines how a person assesses a
67 situation. When assessment is homogeneous, it allows third parties to make comparisons between
68 individual preferences: all they need to do is compare their respective numerical representations.

69   From a methodological point of view, there are three procedures for assessment: pair-wise
70 comparison of alternatives –which leads to preference modeling- (see, for example, [3]; the building
71 of a value (or utility) function (first formalized by Debreau [4]) and analysis of past decisions
72 (developed from the works of Arrow [5]). The outcomes arising from the application of the three
73 procedures are interchangeable (see [6], which makes it possible to exchange the concepts of
74 preference, utility, and choice (see, among other authors, Suzumura [7], and Sen [8], for the
75 integration of the available procedures)). It is also possible to apply the calculating capabilities of
76 any of these methodologies to determine the outcomes in the others.

77   Assessment also helps obtain the solution that best fits the preferences of a participant.
78 However, the number of solutions to forest management is normally so high that it is not operative
79 to generate all feasible solutions and then assesses all of them to identify the best one. Instead,
80 combinatorial optimization tools are applied to obtain acceptable solutions. In this context, the best
81 solution is obtained under risk conditions, which means adopting the solution with the highest
82 probability of satisfying the preferences of a specific participant. The building of this subjective
83 likelihood is also based on the assessment procedures.

84   The design of the "best solution" is solved methodologically (see, for example, [9]) by applying
85 different algorithms: recursive algorithms, heuristic optimization algorithms (mathematical
86 programming, Monte Carlo methods, genetic algorithms, simulations of natural systems,
87 tabu-search, etc.), and non-heuristic algorithms (basically neural network analysis), or a combination
88 of all three.

89   *1.2. Opinion aggregation techniques.*

90   Opinion aggregation plays a central role in the operation of collective decision making. But can
91 a group choose between different options?

92   It is clear that when a large number of people make their judgments to assess a solution, there
93 are as many judgments that overestimate the solution as underestimate it. The result is that the

94   average of assessments tends to approach the real value. However, despite efforts to reach
95   consensus, not all members of a group have to accept the aggregated solution. Some advances on
96   these topics are discussed below.
97   The work by Arrow [10] –inspired by the work of Borda [11] and Condorcet [12] – relates
98   "social choices" to individual preferences through social welfare functions. Unfortunately, results in
99   this area are pessimistic: even some very mild conditions of reasonableness could not be
100 simultaneously satisfied by a social choice. In consequence, aggregation of opinions will usually
101 involve circumventing Arrow's impossibility theorem by relaxing some of its applicability
102 conditions (see, for example, [13,14]).
103   Social choice is focused on voting, which is the natural way that decisions are made in a
104 democracy. However, voting is not harmless: methods expressing preferences can be manipulated to
105 guarantee a choice of certain alternatives a priori [15]. Also, majority voting systems do not consider
106 the choice of alternatives representing minorities. Currently there are other social-choice
107 mechanisms based on Web2.0 tools: document ranking, folksonomy, recommender systems, open
108 software, electronic prediction markets and wikis. Sadly, these methods also need to relax even the
109 softest reasonableness conditions to achieve collective decisions.
110   Parallel to social choice, the utilitarian approach [16] and [17] – inspired by Bentham [18] - used
111 the aggregation of individual utilities to obtain evaluations of social interest. They were concerned
112 with the total utility of the community, but they did not incorporate how utility is distributed or
113 concentrated [19]. However, the arising of the logical positivism [20] allowed the identification of
114 individual gains and losses, which made it possible to progress toward the "interpersonal
115 comparison of utilities". This enables the use of many different types of welfare rules
116 (egalitarianism, envy-freeness, etc.), which differ in that way that fairness and efficiency are
117 handled.
118   But is it possible to make comparisons between the utilities of two or more people? There has
119 been progress in this regard (see, for example, [21]) interpersonal utility comparisons have been
120 developed from different postulates (as those proposed by [22]), and very general conditions have
121 been defined for its implementation [23]. At present, the use of computational rules [24] allows
122 aggregation of opinions based on Harsanyi's theory [25] and [26]. This theory adds the notion of
123 empathetic preferences [27-29] to Von Neuman-Morgenster's hypotheses on utility, and configures
124 the paradigm of aggregation described in this paper.

1.3. Available computer applications for collective decision-making.

126   The progress in collective decision-making (see, for example, [30] has led to the implementation
127 of operative computer applications. All of them have important areas that require improvement, but
128 the existing developments are already part of the solution rather than the problem. The applications
129 presented below are grouped based on the decision-making model on which they are based:
130   Type-A applications.
131   These models were based on formalizing strategies for games on a network and require the
132 current advances in computer technology and the methodology of information-decision systems. In
133 these models, each participant tries to maximize their personal gain, which leads to a reduction in
134 the earnings of others. There are two fundamental procedures: Group Decision Support Systems
135 (GDSS) [31]which aim to reach an overall negotiated decision among the participants through
136 face-to-face meetings; and the Social Decisions Support Systems (SDS) [32], which visualize the flow
137 of discussion through a network of statements, opinions, arguments and comments, which helps to
138 gain consensus before voting on an issue.
139   • A-1 : Group Decision Support Systems –GDSS-.
140   The largest number of applications are in this group: PLEXSYS from the University of Arizona
141 [33]; Colab, designed for Xerox at the Palo Alto Research Center, [34]; Shell GDSS, developed at the
142 University of Minnesota [31]; DECAID (Decision Aids for Groups), designed by [35]; LADN (Local
143 Area Decision Network) [36] and SMU Decision Room Project [37-39]. But the most widely used is
144 Loomio, which manages the decision-making in some emerging political parties in Europe [40].

145   • A-2: Social Decisions Support Systems –SDSS-.
146   Two applications can be highlighted in this field: Deliberative Opinion Poll (from Stanford
147   University) [41].

148   Type-B applications.

149   In general, conflict resolution requires social collaboration, and this is where the generation of
150   confidence becomes relevant for collective decisions. Applications in this area usually implement
151   models that simulate the propagation of opinions, as is done with infection or contagion processes.
152   In fact, social and environmental agents have always made an effort to know the propagation of
153   their opinions among the components of the society.
154   The software available to simulate these models is not strictly for collective decision-making,
155   but also for the analysis and processing of information (structured and unstructured) with multiple
156   receivers. However, some applications have evolved to provide an aggregate solution by focusing
157   on different aspects. We can distinguish between applications [42] that prioritize,
158   o   organization of information
159   - Paramount [43] Decisions
160   -Analytica [44]
161   o   assessment made by multiple agents
162   - 1000Minds of IBM [45] and
163   o   application of procedures to predict the evolution of systems (analysis of dynamic systems
164   and agent based simulation models) [46].

165   Type-C applications.

166   In the end, although the emphasis may be at one end of the spectrum, systems that take into
167   account both personal preferences and social interactions usually fit the population's view better
168   than systems that consider just one of these circumstances. Applications in this field lead to effective
169   solutions, although they are not always efficient. Two programs are the most used:
170   LIQUIDFEEDBACK. This application, [47] developed at the Massachusetts Institute of
171   Technology (MIT), incorporates social networks of confidence so that the decision can be delegated
172   to others (liquid democracy). It does not ask direct questions about the result, but rather encourages
173   participants to propose alternatives. It has a sophisticated voting system to allow participants to
174   express their opinions without allowing tactical considerations.
175   ALLOURIDEAS. This application uses a pair-comparison method between alternatives, which
176   each participant must select. The outcome is the sorting of a list of proposals (to which new ones can
177   be added) by the whole group. This application was developed at Princeton University [48].

178   **2. Methodology for confidence-based collective decision-making based.**

179   When decisions are reported through digital systems (or the computers capture decisions
180   automatically) the behavior of each person is recorded. The behavioral analysis can be automated
181   and is aimed at representing opinions, which means –as explained on section 1.1- knowing how a
182   person assesses a solution and which solution is best for such a person.
183   But the possibilities of digital management go beyond representing opinions: as Harsany
184   [25,26] proposed, we can also use empathy among opinions.
185   For our purposes, empathy is the ability of a person to see the world through the eyes of
186   another. If A chooses the best solution for B rather than himself, A reveals a preference for B's
187   opinion over his own opinion. If A is consistent in this behavior, we can say A empathizes with B.
188   Butit is a bit more complex than this. Binmore [49] describes properly the additional assumptions of
189   Harsany´s theory, arguing:
190   Besides our personal preferences -which may include sympathetic concerns for others-, we also
191   have empathetic preferences that reflect our ethical concerns. This means that empathy can be
192   graduated. However, some empathetic identification is crucial to the survival of human societies

193 because society needs commonly understood coordinating conventions that select a particular
194 equilibrium when many options are available.
195   When a person empathizes with someone, they do so entirely successfully (the best known
196 historical example of incomplete empathy is quoted from Longinus: Were I Alexander," said
197 Parmenio, "I would accept of these offers made by Darius." "So would I too," replied Alexander,
198 "were I Parmenio."). The empathetic utility function of a person simply expresses the fact that the
199 person in question thinks that A and B personal utils can be traded off at so that e of A's personal
200 utils count the same as a of B's personal utils. In other words, holding an empathetic preference is
201 exactly the same thing as subscribing to a standard for making interpersonal comparisons between
202 A and B.
203   Under the above assumptions, the Harsanyi's theory can be described using a Von Neumann
204 and Morgenstern utility function.
205   The power of empathy comes from its association with convergence of opinions of opinions. It
206 could be accepted that putting yourself in the place of another favors reaching a common opinion.
207 All that is needed to incorporate empathy into manage participatory processes, is to verify that
208 increases in empathy lead to convergence of opinions of opinions. In this case, participation should
209 be encouraged as long as empathy among participants improves, and if not, the process should end.
210   If the overall amount of empathy is high at the end of the participatory process, then the
211 aggregate opinion is representative of the preferences of the people involved. Here, the aggregate
212 opinion will be adopted as the group's joint decision. Otherwise, the group of participants will need
213 to be broken down into homogeneous subgroups (with common opinions within the subgroups)
214 and other aggregation procedures applied to the preferences of the homogeneous communities.

215 *2.1. Calculating empathy.*

216 The methodology described below is applicable to the calculation of empathy in a non-competitive
217 environment. A deeper explanation and the primitive formulation of it can be found in [21]. It is
218 based on calculating empathy for participants connected on a social networks [50,51] and in
219 incorporating Adler's developments [52].

220   To determine the empathy of one person (*A*) towards another (*B*), we must consider the
221 opinions of all those who make up this empathy. At least, *A* and *B* opinions (o_A and o_B), an
222 aggregation of the opinions *A* considers to be closest to *B* in the confidence networks ($o_{n(B)A}$), the
223 history or, at least, the aggregated opinion of the whole group when empathy is calculated (o_g), and
224 an explicit objective opinion formulated by experts (o_{Ob}).
225   The above opinions can be used to determine the utility that a person believes that a solution
226 has for another person, but the weight of all opinions cannot be the same. Therefore, the utility that
227 *A* believes that solution *X* has for *B* can be calculated as:

$$U_{A \to B}(X) = \sum_{i \in \{A, B, n(B)_A, \mathbf{g}, \mathbf{Ob}\}} (1 - D_{i \leftrightarrow B}) \times (1 - \|X - \mathbf{X}_i\|) \times u_i(X), \qquad (1)$$

228 where: $\mathbf{D}_{i \leftrightarrow B}$ is the reciprocal distance between i and *B*'s opinions (calculated using the distance
229 between i and *B* in the confidence networks) it is the complementary of the probability that the
230 opinion of the first person will align with the opinion of the second person); $\|X - \mathbf{X}_i\|$ is the
231 distance between the best solution for i ($\mathbf{X}_i$) [9] and the solution for which we are calculating the
232 utility (X); $u_i(X)$ is the assessment of *X* made by i; and i refers to all the involved opinions($o_i =$
233 $[\mathbf{X}_i, u_i(X)]$), $i \in \{A, B, n(B)_A, \mathbf{g}, \mathbf{Ob}\}$.
234   But the equation 1 does not yet indicate the empathy of *A* relative to *B*. As mentioned, the
235 empathetic function expresses the equivalence in the utilities of *A* and *B* when the aggregate
236 assessment of the group ($\mathbf{X}_g$) is modified. To calculate this, we force *L* systematic changes in the
237 values of the descriptive variables for the group's opinion (for example, increasing and/or
238 decreasing the cost and performance of the $\mathbf{X}_g$ solution) and we determine the changes that these
239 modifications produce in both *A* and *B*'s utilities. If each of these changes is represented by $\mathbf{X}_{g<l>}$ (for

240 $k = 1, ..., L$) then, the modification it induces in the way *A* believes that *B* makes their assessments will
241 be represented as:

$$e_{A \to B, k} = U_{A \to B}(\mathbf{X}_{g<k>}) - U_{A \to B}(\mathbf{X}_g), \tag{2}$$

243 For $k=1, ..., L$ we get a L coordinates vector representing the empathy function of *A* relative to *B*:

$$euf_{AB} = \begin{bmatrix} e_{A \to B,1} \\ e_{A \to B,2} \\ \vdots \\ e_{A \to B,L} \end{bmatrix}, \tag{3}$$

245 The set of empathies of *A* for all participants (N) will be the empathetic utility matrix of *A*
246 (which will be notated as **em**$_A$):

$$\mathbf{em}_A = \begin{bmatrix} euf_{AA_1}, ..., euf_{AA_N} \end{bmatrix} = \begin{bmatrix} e_{A \to A_1,1} & \cdots & e_{A \to A_N,1} \\ \vdots & \ddots & \vdots \\ e_{A \to A_1,L} & \cdots & e_{A \to A_N,L} \end{bmatrix} \tag{4}$$

248 This matrix collects all the information referring to the empathy of a person (*A*) with the other
249 participants. The matrix shows the variations of the utility that *A* believes that certain changes in the
250 aggregate solution (in the *L* rows of **em**$_A$) induce in the rest of the participants (in the N columns of
251 **em**$_A$).

252 A measure of the separation of empathy between a pair of individuals can be obtained from
253 these matrices. Thus, from general operations with matrices:

$$S_{AB} = 1 - (\|\mathbf{em}_A, \mathbf{em}_B\|/N) \tag{5}$$

255 where: N is the total number of evaluators,

$$\|\mathbf{em}_A, \mathbf{em}_B\| = \sqrt{\mathrm{tr}[(\mathbf{em}_A - \mathbf{em}_B)^\circ(\mathbf{em}_A - \mathbf{em}_B)^T]} \tag{6}$$

257 ∘ denotes the usual multiplication of matrices and tr() represents the trace of a matrix
258 The average of equation 5 for all possible participant pairs has been used as a measure of the
259 percentage of empathy among participants:

$$S_T = \frac{1}{N(N-1)/2} \sum_{I=1}^{N-1} \sum_{J=I}^{N} S_{IJ} \tag{7}$$

261 Obviously, the average of similarities of empathy between pairs of participants does not
262 describe the distribution of empathy among participants. For that, other centralization and
263 dispersion measures would need to be applied to the similarities of empathy. But our objective is to
264 measure the evolution of confidence and variation of average empathy over time, to detect the
265 increase or decrease in total empathy and evolution over time represented by equation 7.

266 *2.2. Simulating interactions among participants.*

267 Interactions among opinions occur naturally in participatory processes and can be promoted
268 through digital forums and other tools offered by the Internet. Effects of interactions on collective
269 decisions can also be simulated as described below.
270 The simulation is based on the works of Sayama [51]. Next, let us show the simulation frame
271 with the mathematical notation and simplifications adopted in this paper:
272 • Let $o_A = [u_A(X), \mathbf{X}_A]$ be (as in the rest of the paper) the opinion of participant *A*.

273 • Participants do not know the final aggregated assessment (u_{gF}). The goal is to find a joint
274   evaluation, which should be accepted by most participants at the end of the discussion
275   process. However, the current aggregated value (u_g) is known at any given time.
276 • Each participant has a memory in which they store their personal comprehension of the
277   problem and the history of group discussion. The memory is defined as a list of opinions
278   either they have (or have had) or that other participants have expressed during the
279   discussion. The memory of each participant (M_A) can store up to p of their own opinions (the
280   initial opinion plus the modifications they have accepted during the discussion) and up to q
281   opinions expressed by each of the other participants ($M_A = \{o_{A,k}\}$, $k=1, …, p, p+1, …, p+q$). The
282   total memory of one participant is limited to a certain number of opinions, thus $p + q \leq l$. If
283   the number of opinions participating in the personal understanding of the problem exceeds
284   this number, then some of the opinions in the memory must be removed.
285 • At least, the following opinions always belong to the memory of a participant (A): o_A, o_g, and
286   o_{Ob}.
287   Interaction does not simply accept changes in view of other opinions. It requires discussion:
288 each participant tries to impose their own solution on the group, but at the same time, the
289 participant accepts changes in their solution if convinced. Discussion is an iterative process with the
290 following phases on each one of the iteration:
291   1. Each participant examines whether there is an easy way to improve the utility of their
292 current solution.
293   Every participant examines their neighbors' solution and incorporates the *opinion* with the
294 highest utility for all the participants belonging to their neighborhood into their memory. We shall
295 denote this plan as $u_{n(A)}$.
296   2. A randomly selected speaker is chosen.
297   We shall denote the speaker as S.
298   3. The speaker suggests the revision of the group plan.
299   The speaker (S) identifies which aspect of their individual plan has the most significant impact
300 if incorporated into the group's plan. To do this, we built **em**$_s$ matrix (equation 4), which is now
301 calculated by adding the speaker's opinion to that in the memory of all participants.
302   But, first, we have to calculate the utility that the speaker supposes the solution has for another
303 participant (A) as:

$$U_{S\to A}(X) = \sum_{i \in S \cup M_A}(1 - D_{i\leftrightarrow A}) \times (1 - \|X - \mathbf{X}_i\|) \times v_i^*(X), \qquad (8)$$

304

305 where S refers to the speaker's assessments; M_A is the memory of the A-evaluator; $D_{i\leftrightarrow A}$, X and $\mathbf{X}_i$ are
306 as in equation 1; and v_i^* is not the assessment of a solution (u_i), it is referred to the expert's opinion
307 expressed as:

$$v_i^* = 1 - |u_i - u_{0b}|, \qquad (9)$$

308

309   Then u_g, which belongs to the memory of all participants, is systematically modified to calculate
310 the **em**$_s$ matrix (equations 2 to 4). The row of this matrix with the highest sum of its components (j)
311 determines the suggestion of change proposed by the speaker.
312   The new opinion that the speaker suggests to the group is expressed as $o_{g<j>}=(u_{g<j>}, \mathbf{X}_{g<j>})$.
313   4. The speaker's suggestion is evaluated by all other participants, who respond to the
314 suggestion at the individual level.

315   Each participant then studies the utility of the speaker's suggestion, which is done by applying
316   the next equation:

$$E_A(o_{g<j>}) = \sum_{p \in M_A} \left((1 - D_{p \leftrightarrow A}) \times (1 - \|\mathbf{X}_{g<j>} - \mathbf{X}_p\|) \times u_p \right), \qquad (10)$$

318   where p is any opinion in the memory and M_A is the memory of A.
319   If $E_A(o_{g<j>}) > E_A(o_g)$, then the A-evaluator expresses support for the suggestion, and $o_{g<j>}$ is
320   incorporated to M_A. Otherwise, i.e., $E_A(o_{g<j>}) \leq E_A(o_g)$, then A's individual plan is not affected by the
321   speaker's suggestion. However, $o_{g<j>}$ may still be randomly incorporated into the memory of A with
322   a probability given by:

$$P(o_{g<j>}, o_g) = \exp(- \|E_A(o_{g<j>}) - E_A(o_g)\| / T), \qquad (11)$$

324   where: T is the ''temperature'' of the participant's cognition (i.e., to what degree participants can
325   tolerate low-utility suggestions).
326   If incorporating a new opinion exceeds the maximum number of opinions allowed in the
327   memory, then the previous one has to be removed from memory.
328   5.   Response to suggestion at group level.
329   The group determines if the speaker's suggestion should be adopted as the new group solution
330   by counting the number of participants that have incorporated the speaker's suggestion into their
331   memories. If this is more than 50%, then the old group opinion will be replaced by the suggestion
332   ($o_g \leftarrow o_{g<j>}$).

333   *2.3. The strategy for confidence-based collective decision-making.*

334   From an operational point of view, the proposed strategy consists of the following stages:
335   a.   Opening of an online information and participation process for anyone to formulate their
336   preferences and to incorporate these preferences into the available information systems.
337   b.   Encouraging participation, through the media or by community managers. Motivation
338   should provide and incentive for both the recurrent participation of individuals already involved
339   (they tend to modify their preferences as a result of interaction with other participants), and the
340   incorporation of new participants.
341   c.   Automatic representation of participant opinions. This requires determining how a
342   participant assesses a solution (based on their personal preferences) and designing the solution that
343   best fits a preference.
344   d.   On-line calculation of the empathy of each participant with the others and of the total
345   empathy for the opinions among all participants.
346   e.   Supporting the participatory process if total empathy increases and terminating the
347   process when there is no significant increase in empathy among participants.
348   f.   Application, at the time of termination, of Web2.0 tools and/or building of interpersonal
349   functions to choose the best aggregate decision. In case of a low convergence of preferences, another
350   social or individual selection process should be used to choose representative and socially acceptable
351   solutions.
352   It is evident that the confidence-based strategy satisfies the general requirements for
353   participation to a greater degree than the public-participation applications that are currently
354   available (see Table 1).

355   **Table 1.** Comparing available public-participation applications.

Capabilities of the available application of publics participation to manage collective decision-making		Type-A systems	Type-B systems	Type-C systems (see Section 1.3)	Confidence-based strategy
General requirements	To be transparent and allow rebuttal by third parties in all its decisions	Yes	Partially	Yes	Yes
	Adaptive to the available information and to the knowledge that the evaluator has in the participation process.	Partially	Partially	Partially	Yes
	Friendly (use is not affected by the technological, cultural or generational gap).	No	Yes	Partially	Yes
	Optimizer of individual utility and socially intelligent.	No	No	Yes	Yes
	To serve as an instrument for accountability to society.	Yes	Yes	Yes	Yes
Outcome requirements	To provide assessments based on the representation of personal preferences.	Yes	No	Yes	Yes
	Collective decision-making through aggregation of individual preferences	Yes	Yes	Yes	Yes
	Design of the alternative that best fits to any system of preferences (individual or aggregated).	Yes	No	No	Yes
Capacity to solve large combinatorial problems		Not available in current applications			Yes

356

357   **3. Case study and results.**

358   The case study focused on verifying the strategy for confidence-based collective decision-making. To
359   do this, we simulated a participative process to design the most sustainable forest management plan
360   at a real forest.

361   The forest is in the Fuenfría Valley (Madrid, Spain). It covers an area of 96.04 ha (40°45′N,
362   4°5′W), and has altitudes ranging from 1310 to 1790 m, an average annual temperature of 9.4°C, and
363   average annual rainfall of 1180 mm. Its forest vegetation is Scots pine (*Pinus sylvestris* L.). Figure 1
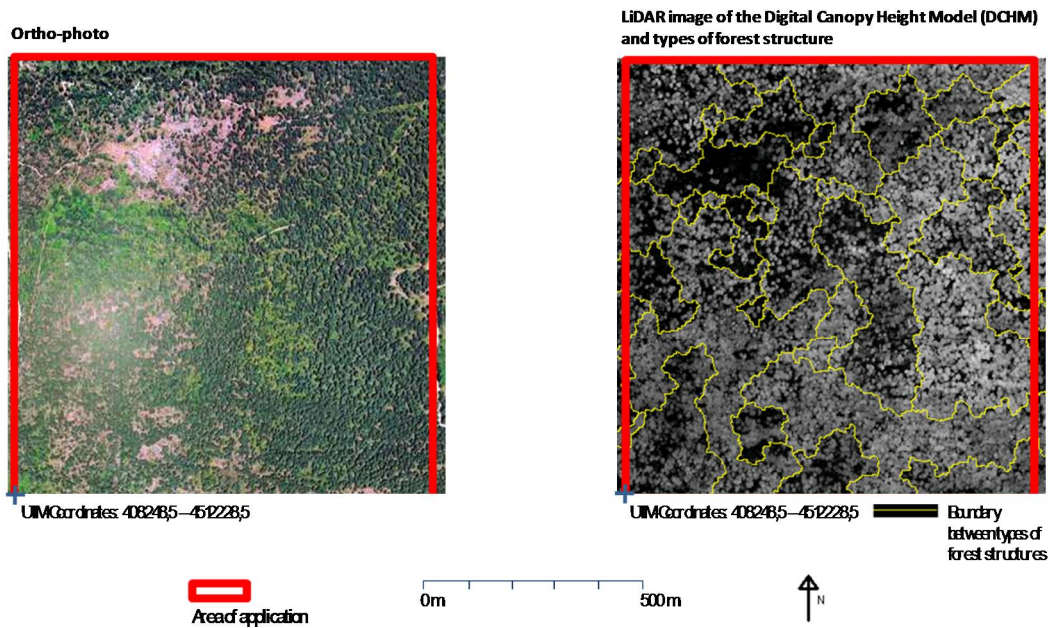**364**   shows the area selected for this test case; further information of the territory can be found in [53].

365

366   **Figure 1.** Area selected for the test case.

367   The strategy consisted of encouraging participation as long as empathy among participants
368   increased. To verify this strategy we needed to compare the evolutions of empathy with the
369   convergence of opinions throughout the participative process (Section 3.3). To do that, we needed to
370   measure both empathy and convergence of opinions of opinion (Section 3.2). In turn, this calculation
371   requires the representation of the opinions of each of the participants in the process (Section 3.1).

372   *3.1. Representation of opinions.*

373   Sustainability is a complex, multidimensional idea that does not allow an overall method of
374   assessment. It is possible to measure partial aspects –indicators-, but each person has their own way
375   of aggregating those indicators for assessing sustainability. In other words, each person has an
376   opinion on sustainability.

377   For this case study, representing an opinion means:

378   • Assessing the sustainability on each one of the points of the territory according to personal
379   preferences on sustainability.
380   • Designing the forest management plan that best suits personal preferences on sustainability.

381   3.1.1. Personal assessment of sustainability at each point of the territory.

382   Sustainability is calculated at each one of the points of a grid superimposed on the study area
383   with a pixel width of one meter.

384   The methodology for the assessment of sustainability was the one that incorporates *SiLVANET*
385   [54], a computer application for participatory forest management.

386   The information required came from three different sources: *physical and management parameters*
387   *of the area* (mainly: height distribution of trees within a 50 meter diameter around each point on the
388   grid, and sustainability indicators such as structural diversity, timber yield or the biomass at each
389   point); *personal information* on the individual whose opinion was going to be represented (gender,
390   age, level of education, occupation, type of residence –urban or rural-, and stakeholder category);
391   and the *personal sustainability preferences* of each participant.

392   Personal preferences are captured through a pair-wise comparison process (see [30] on a set of
393   meaningful points in the territory. Comparison requires the selection of a set of meaningful points in
394   the territory and, after determining all feasible pairs of meaningful points, to ask the participants for
395   their preferences within a pair (they must indicate which of the two points they consider to be more
396   sustainable). The answers are recorded in a preference matrix (Table 2). As can be deduced, the
397   number of meaningful points must be small (no more than six) to be able to make the comparison.

398   Next, the *SiLVANET* methodology [55,56] was applied to achieve the following outcomes:

399   1.   To get the personal sustainability value at each one of the compared points (Table 2).

400   2.   To extend the assessment to the remaining points in the area (Figure 2*a*).

401   3.   To obtain the analytical equation of sustainability as a function of sustainability indicators
402   at each point of the territory (Figure 2*b*).

403   **Table 2.** Data entered (comparison points and personal preferences) and sustainability assessment
404   outcomes through a pair-wise comparison process.

Comparison points			INDIVIDUAL PREFERENCES (Data entered by a participant)[1]						Preferences after sequel and transitive closure [57][2]						Sustainability at compared points	
Id	UTM X-coord.	UTM Y-coord.	1	2	3	4	5	6	1	2	3	4	5	6	Original value	[0,1] Rank
1	408916.5	4512672.5	3	3	3	3	1	2	0	1	1	0	1	1	4.3	**0.86**
2	408776.5	4513172.5	3	3	1	3	3	3	1	0	1	1	1	1	5	**1**
3	408916.5	4512272.5	3	2	3	2	2	1	0	1	0	0	1	1	3	**0.6**
4	408976.5	4512852.5	3	3	1	3	1	2	1	1	1	0	1	0	4.1	**0.82**
5	408936.5	4512252.5	2	3	1	2	3	1	1	1	1	0	0	1	3.9	**0.78**
6	408516.5	4513012.5	1	3	2	1	2	3	1	1	1	1	0	0	3.7	**0.74**

405   [1]Row more sustainable than column =1; Column more sustainable than row =2; No one preferred to another =3

406   [2] Row preferred to column =1; Column preferred to row =0

407

(a)   **(b)**
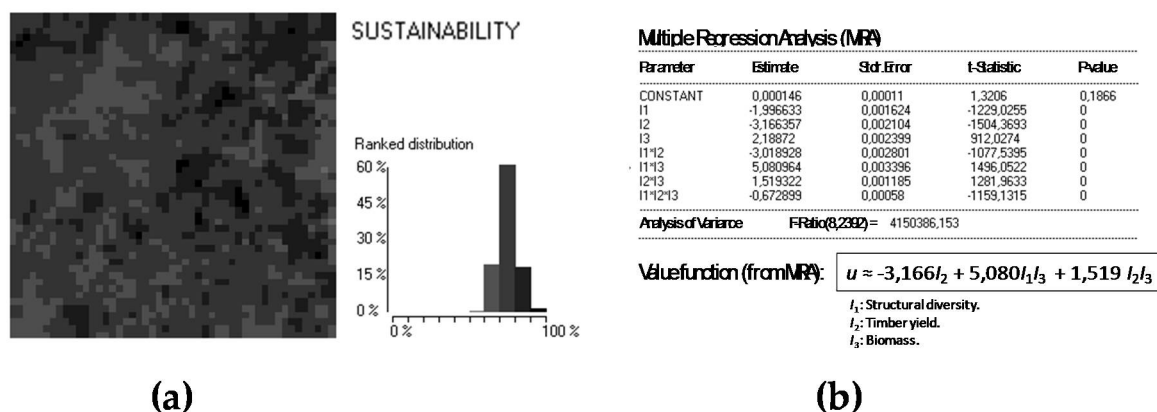
Figure 2. Sustainability assessment throughout the entire the Study Area; (**a**) Extension from the pair-wise comparison assessment described in Table 2 (the size of the displayed grid has been increased to 10 meters wide for easier comprehension); (**b**) Analytical equation of sustainability calculated through multiple regression analysis between the value in Figure 2 (**a**) and the value of the main sustainability indicators at all of the points of the territory.

The above outcomes are sequential: first, the sustainability on the significant points was obtained (outcome number 1). Then outcomes 2 and 3 were obtained by adding the physical and management parameters of the territory to outcome 1. Consequently, for specified forest exploitation, the relevant information came from the sustainability in the six points subjected to comparison.

3.1.2. The management plan that best suits individual preferences.

A management plan involves formulating a set of temporally-located actions to be carried out at a point in the territory. However, the number of viable plans is so high that it is not feasible to generate all of the possible options to choose the best one. For instance, if there were ten activities per tree and year (cutting, pruning, labor, pest treatment,…, and combinations of these) and the number of trees in our study area was 500,000, then the number of potential management plans would be $100^{500,000}$ (for a planning horizon of ten years). To get around this problem, we determined the best management plan by applying combinatorial algorithms for optimization ([56] and [58]).

Specifically, we applied the Simulated Annealing algorithm [59] to optimize the following function:

$$U_A(X) = -\left(\sum_{i=1,\dots,C}\sum_{j=1,\dots,L} \mathbf{1}_{P_i=y_j} u_A\left(P_i = y_j\right) - \beta V(X)\right) \tag{12}$$

where: *A* is a participant in the decision process.

P_i is a point in the territory and its neighborhood (a circle of 50-meter diameter around each point);

Y_j is a specified set of activities (cut trees, prune lower branches, styling, eliminate feedback, etc.) carried out on P_i;

L is the number of feasible activities that can be carried out at any point and in its neighborhood;

436　　　C is the number of points in the territory (usually the points are pixels, but they can also be
437　　　　trees or homogeneous territorial units);
438　　　**1** is the indicator function (1 if $P_i = y_j$ and 0 otherwise);
439　　　$\beta V(X)$ is a regularization term;
440　　　$u_A(P_i = y_j) = k_1 v_A(P_i = y_j) + k_2 B_i(P_i = y_j)$.
441　　　where:　$v_A(P_i = y_j)$ is the A assessment of sustainability for $P_i = y_j$ and $B_i(P_i = y_j)$ is the economic
442　　　　　balance for changing the current land use to $P_i = y_j$.

443　　Optimizing expression (12) leads to the most sustainable forest management plan for a specific
444　participant. Table 3 describes an example of the best management forest plan in the opinion of the
445　participant from Fig. 2 (a detailed explanation of the spatial-temporal location of activities for this
446　management plan can be found in the [54].

447　　　**Table 3.** Current stock, trees to be cut in the next 10 years and description of the current and of the
448　　　　proposed forest plan for the preferences of the participant in Table 2, for the total study area.

Stock variables for height distribution			
Tree height (meters)	**Timber to harvest in 10 years (stems/ha)**	**Trees after the Management Plan (stems/ha)**	**Standing trees (stems/ha)**
0-3	30879	632880	230142
3-6	3600	46407	24599
6-9	2458	13106	14741
9-12	630	7176	7508
12-15	210	6155	6774
15-18	165	5032	4869
18-21	64	3515	3171
21-24	264	1459	1493
Economic variables after the Plan (in euros for the entire area of 96.04 ha)			
Cut & Harvest wood COST			39549.01
Pruning & Debris elimination COST			54088.35
Gross Timber INCOME			54399.77
Bio-fuel & Public subsidies INCOME			41130.92
Sustainability (as % of the IDEAL FOREST sustainability)			
Current overall sustainability value			74.99
Overall sustainability after Plan			86.51

449　*3.2. Measuring empathy and convergence of opinions.*

450　　Our approach to collective decisions requires recurrent calculating of two items at any point in
451　the process: the convergence of the expressed opinions, and the overall empathy among
452　participants. This section describes how to calculate both items for a group of twenty participants
453　(the analysis of larger groups is similar, but showing results is more complicated).
454　　The information required to calculate these items is the representation of opinions for all
455　participants, which is calculated from the sustainability at six meaningful points of the territory.
456　Table 4 shows the sustainability for all members of the group at the most meaningful spatial points.

457   **Table 4.** Assesments of sustainability for the group of 20 participants.

Participant-ID	Sustainability on the comparison points and for all the territory						
	P-1	P-2	P-3	P-4	P-5	P-6	*Study area v(X)*
E-1	0.49	0.41	0.12	0.07	0.12	0.42	0.272
E-2	0.89	0.16	0.37	0.27	0.07	0.42	0.363
E-3	0.37	0.07	0.52	0.57	0.52	0.32	0.395
E-4	0.42	0.34	0.87	0.46	0.42	0	0.418
E-5	1	0	0.22	0.5	0.22	0.59	0.422
E-6	0.59	0.46	0.97	0.27	0.07	0.27	0.438
E-7	0	1	0.62	0.51	0.07	0.02	0.37
E-8	0.85	0.42	0.82	0.62	0	0	0.452
E-9	1	0.62	0.22	0	0	0.59	0.405
E-10	0.75	0.07	0.52	0.47	0.02	0.87	0.45
E-11	0.83	0.62	0.12	0.55	0	0.74	0.477
E-12	0.45	0.62	1	0.47	0.32	0.22	0.513
E-13	0.83	0.67	0.07	0.25	0	0.94	0.46
E-14	0.32	0.22	0.72	0.67	0.92	0.22	0.512
E-15	0.92	0.07	0.72	0.52	0.77	0.07	0.512
E-16	0.57	0.07	0.67	0.52	0.97	0.02	0.47
E-17	0.54	0.66	0.97	0.92	0	0.42	0.585
E-18	1	0.72	0.32	0.7	0.32	0.64	0.617
E-19	0.42	0.07	1	0.72	0.77	0.22	0.533
E-20	1	0.87	0.07	0.8	0.27	0.64	0.608
Group (g)	0.414	0.254	0.341	0.308	0.183	0.238	0.29
Expert (Ob)	0.92	0.75	0.54	0.42	0.26	0.07	0.493

458

459   3.2.1. Convergence of opinions.

460   Calculation of convergence requires knowing the aggregated solution over the entire time. For
461 this case study, the aggregated assessment is the average of the values for all the participants at the
462 six meaningful points. Convergence of opinions (*CoO*) has been calculated as the sum of the
463 variation coefficients at each of the six meaningful points:

464                 $$CoO = 100\left\{\sum_{i=1}^{6}(s_i/\bar{x}_i)\right\}/6 \tag{13}$$

465 where $\bar{x}_i$ and s_i are, respectively, the average and the standard deviation of the sustainability
466 assessments in the *i*-meaningful point and for all the participants in the group.

467   3.2.2. Overall empathy of the group.

468   All the information necessary to calculate empathy is derived from the data provided in Table
469 4. Following the methodology described in Section 2.1, the first task to calculate is the utility that a
470 participant (*A*) believes that a solution (*X*) has for another participant (*B*), which involves
471 application of the equation 1. The variables we have used to calculate equation 1 are described
472 below.

473   • $D_{i \leftrightarrow K}$, which is the reciprocal distance between *i* and *B*'s opinions, is calculated as:

474          $$D_{i \leftrightarrow B} = \|\boldsymbol{u}_i(\mathbf{X}_{\text{Real}}), \boldsymbol{u}_B(\mathbf{X}_{\text{Real}})\| = \frac{\sqrt{(\boldsymbol{u}_i - \boldsymbol{u}_B)\Sigma^{-1}(\boldsymbol{u}_i - \boldsymbol{u}_B)^T}}{D_{\text{max}}} \tag{14}$$

475   where: $D_{i \leftrightarrow B}$ is the reciprocal similarity in the way participants *i* and *B* make their
476   sustainability assessments; $u_i(\mathbf{X}_{Real}) = [v_i(\mathbf{X}_{Real})_{P1}, \ldots, v_i(\mathbf{X}_{Real})_{P6}]$ are the sustainability values
477   that *i* assigns to the six points being compared (P1, …, P6) in the current land use ($\mathbf{X}_{Real}$); Σ^{-1}
478   is the inverse covariance matrix of the sustainability assessments at the most representative
479   territorial points for the data of all the active participants (Table 5); and D_{max} is the
480   maximum value of the numerator in equation 14 for all pairs of active participants.

481   **Table 5.** Sample covariance matrix and its inverse for the simulated opinions of 100 participants.
482   Both are calculated for all the pairs of meaningful points (P1 to P6).

Sample covariance matrix						
Participants	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**
P1	0.0865	0.01127	-0.02471	-0.01023	-0.01914	0.03407
P2	0.001127	0.10156	-0.00428	0.00334	0.0266	-0.01522
P3	0.02471	-0.00428	0.08395	0.02746	0.01118	-0.02735
P4	-0.01023	0.00334	0.02746	0.0673	0.01217	-0.01564
P5	-0.01914	-0.0266	0.01118	0.01217	0.0692	-0.01385
P6	0.03407	-0.01522	-0.02735	-0.01564	-0.01385	0.09163
Inverse of sample covariance						
Participants	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**
P1	14.816	-1.6899	2.4984	-0.172	2.1223	-4.7533
P2	-1.6899	11.9061	1.0144	-1.3497	4.8655	3.4124
P3	2.4984	1.0144	15.3689	-5.2802	0.1151	2.9422
P4	-0.172	-1.3497	-5.2802	17.7403	-2.6566	0.8895
P5	2.1223	4.8655	0.1151	-2.6566	17.8146	2.2913
P6	-4.7533	3.4124	2.9422	0.8895	2.2913	14.6224

483

484   • $\|X - \mathbf{X}_i\|$ is the distance between the best solution for *i* ($\mathbf{X}_i$) and the solution to which we are
485   calculating the utility (X), it is calculated as:

$$\|X - \mathbf{X}_i\| = |U_i(X) - U_i(\mathbf{X}_i)| \qquad (15)$$

486
487   where $U_i(X)$ is the utility of X solution for the *i* participant, calculated like in equation 12.
488   • $v_i(X)$ is the assessment of X made by *i*.
489   It is calculated as the average sustainability at all the points of the study area (see Section
490   3.1.i)
491   • *i* refers to all the involved opinions($o_i = [\mathbf{X}_i, v_i(X)]$), $i \in \{A, B, n(B)_A, g, Ob\}$. *A* and *B* are the
492   mentioned participants, and:
493   – $n(B)_A$ are the opinions that a participant (*A*) considers to be close to the opinion of each
494   of the other participants (*B*). These opinions satisfy both: the reciprocal similarities
495   between their sustainability assessments and the *A* assessment of sustainability are
496   bounded to a fixed value ($D_{A \leftrightarrow K} < K_0$ for K=1, …, N participants; with $D_{A \leftrightarrow K}$ calculated as
497   shown in the equation 14 and, simultaneously, their opinions belong to stakeholders of
498   the same type as *B*.
499   – o_{Ob} is the expert opinion on sustainability (which is supposed to be more objective than
500   other opinions).
501   – o_g is the aggregate opinion, calculated as an average of the assessment for all
502   participants.

503   The next task is to determine the equivalence of the utilities of *A* and *B* when the aggregate
504   assessment of the group (X_g) is modified. The changes we adopted represented a variation of ± 5% in
505   the values of all the components in $u_g(X_g)$, which is the equivalent of 729 changes. We then carried
506   out the process described in Section 2.1 up to calculate **emf**$_A$ for a *A*-participant (equation 4). Each
507   one of these matrices has 729 rows and 20 columns.

508   The reciprocal similarities of empathy between participants (equation 5) are shown in Table 6.

509   **Table 6.** Matrix of similarity of reciprocal empathy (%) between participant opinions for the group of
510   20 participants.

GROUP	E-1	E-2	E-3	E-4	E-5	E-6	E-7	E-8	E-9	E-10	E-11	E-12	E-13	E-14	E-15	E-16	E-17	E-18	E-19	E-20
E-1	100	83.27	75.83	72.95	73.79	72.35	73.79	72.37	75.57	71.71	63.08	60.82	52.67	63.2	63.44	59.35	56.78	49.97	55.62	64.3
E-2	83.27	100	88.93	85.39	88.9	87.2	63.2	87.29	71.87	85.57	62.58	75.83	46.9	65.55	74.38	54.14	72.15	66.05	60.56	74.46
E-3	75.83	88.93	100	93.58	88.11	89.97	54.91	89.58	61.35	83.5	52.73	82.4	36.57	72.14	83.11	57.42	75.87	69.08	67.63	68.11
E-4	72.95	85.39	93.58	100	86.34	91.29	54.39	91.33	59.71	82.7	51.6	85.32	35.23	74.45	86.95	57.82	78.19	69.42	71.28	67.35
E-5	73.79	88.9	88.11	86.34	100	90.7	56.55	90.32	66.37	93.83	59.56	82.21	42.41	64.09	79.04	50.24	79.57	73.19	62.53	76.04
E-6	72.35	87.2	89.97	91.29	90.7	100	56.2	97.55	63.9	88.9	57.17	87.9	39.84	66.98	81.22	50.86	83.54	74.08	65.37	74.18
E-7	73.79	63.2	54.91	54.39	56.55	56.2	100	56.36	79.17	57.44	71.43	45.57	68.62	45.21	45.47	43.33	44.42	36.26	40.15	58.97
E-8	72.37	87.29	89.58	91.33	90.32	97.55	56.36	100	64.15	88.18	57.34	87.39	40.03	67.02	81.32	50.98	83.2	73.92	65.28	74.41
E-9	75.57	71.87	61.35	59.71	66.37	63.9	79.17	64.15	100	67.75	86.42	53.09	74.46	44.22	49.74	38.29	53.62	48.19	39.89	73.91
E-10	71.71	85.57	83.5	82.7	93.83	88.9	57.44	88.18	67.75	100	62.35	81.21	44.86	61.06	75.94	47.06	80.96	73.11	60.73	78.33
E-11	63.08	62.58	52.73	51.6	59.56	57.17	71.43	57.34	86.42	62.35	100	48.09	80.3	34.16	41.94	26.53	50.91	46.59	31.29	74.54
E-12	60.82	75.83	82.4	85.32	82.21	87.9	45.57	87.39	53.09	81.21	48.09	100	29.86	63.65	81.8	44.88	90.67	80.79	64.54	68.82
E-13	52.67	46.9	36.57	35.23	42.41	39.84	68.62	40.03	74.46	44.86	80.3	29.86	100	21.63	25.83	18.1	32.06	27.26	17.98	55.26
E-14	63.2	65.55	72.14	74.45	64.09	66.98	45.21	67.02	44.22	61.06	34.16	63.65	21.63	100	77.97	78.02	56.08	46.39	86.73	44.2
E-15	63.44	74.38	83.11	86.95	79.04	81.22	45.47	81.32	49.74	75.94	41.94	81.8	25.83	77.97	100	60.02	74.4	65.74	79.18	58.31
E-16	59.35	54.14	57.42	57.82	50.24	50.86	43.33	50.98	38.29	47.06	26.53	44.88	18.1	78.02	60.02	100	37.49	28.38	69.75	31.03
E-17	56.78	72.15	75.87	78.19	79.57	83.54	44.42	83.2	53.62	80.96	50.91	90.67	32.06	56.08	74.4	37.49	100	85.05	57.97	72.77
E-18	49.97	66.05	69.08	69.42	73.19	74.08	36.26	73.92	48.19	73.11	46.59	80.79	27.26	46.39	65.74	28.38	85.05	100	47.02	71.44
E-19	55.62	60.56	67.63	71.28	62.53	65.37	40.15	65.28	39.89	60.73	31.29	64.54	17.98	86.73	79.18	69.75	57.97	47.02	100	42.45
E-20	64.3	74.46	68.11	67.35	76.04	74.18	58.97	74.41	73.91	78.33	74.54	68.82	55.26	44.2	58.31	31.03	72.77	71.44	42.45	100

511   Equation 6 is then applied to the data on Table 6 to calculate the average of the empathetic
512   similarities. The overall similarity for the twenty analyzed participants is S_T = 80.99%.

513   *3.3. Verification of the strategy for confidence-based collective decision-making.*

514   We used simulated data instead of real opinions to verify the strategy because we wanted to
515   analyze the influence of the two main factors affecting collective decisions: the number of
516   well-informed participants and the interactions among them. It is not possible to separate a person's
517   own opinion from the changes due to interactions when dealing with human decision-makers, but it
518   is easier to isolate the effects of these factors with simulated data.

519   As shown in sections 3.1 and 3.2, all the relevant information for calculating empathy and
520   convergence comes from the sustainability at the six most meaningful points in the territory. We
521   have therefore simulated the opinions of one hundred participants by assigning a sustainability
522   value to these points, under the following conditions:

523   –   The sustainability for each meaningful point and each stakeholder came from sampling a
524   normal distribution with the mean and standard deviation shown in Table 7. The mean
525   corresponds to the sustainability assessments of a well-informed participant on each type of
526   stakeholder. Standard deviations show variations in data from real participative processes
527   [30,58].
528   –   Table 7 also shows the number of participants for each type of stakeholder.
529   –   The opinions for the simulated participants (one hundred) are summarized in Fig. 3.
530   –   To measure the separation between opinions (equation 4), we used the inverse of the covariance
531   matrix (for the one hundred simulated participants) between all the possible pairs of significant
532   points. The covariance matrix - and its inverse - is shown in Table 6.

533
534
535
536

Table 7. Parameters for generating sustainability of 100 participants, grouped into 5 stakeholder types (S1 to S5 in Fig.4) and at 6 meaningful points (P1 to P6 in Table 2). Sustainability assessments are simulated from sampling an $N(\mu; \sigma)$ distribution with the parameters showed in the table. The number of simulations carried out for each type of stakeholder is also shown.

Type of stakeholder	Parameters: $(\mu; \sigma)$						Number of simulations
	P1	P2	P3	P4	P5	P6	
S1	(0.88; 0.2)	(0.6; 0.3)	(0.75; 0,4)	(0.55; 0.2)	(0.2; 0.3)	0.25; 0,4)	18
S2	(0.4; 0.4)	(0.92; 0.4)	(0.75; 0.4)	(0.54; 0.2)	(0.1; 0,2)	(0.2; 0.2)	13
S3	(0.5; 0.2)	(0.2; 0.2)	(0.75; 0.2)	(0.6; 0.2)	(0.7; 0.2)	(0.3; 0.2)	14
S4	(0.96; 0.2)	(0.5; 0.3)	(0.25; 0.2)	(0.33; 0.3)	(0.1; 0.2)	(0.72; 0.2)	23
S5	(0.77; 0.3)	(0.69; 0.4)	(0.75; 0.3)	(0.5; 0.4)	(0.2; 0.3)	(0.4; 0.4)	32

537   The average convergence of opinions (equation 3) for the participants in Table 7 is:
538   *CoO* = 73.67%.
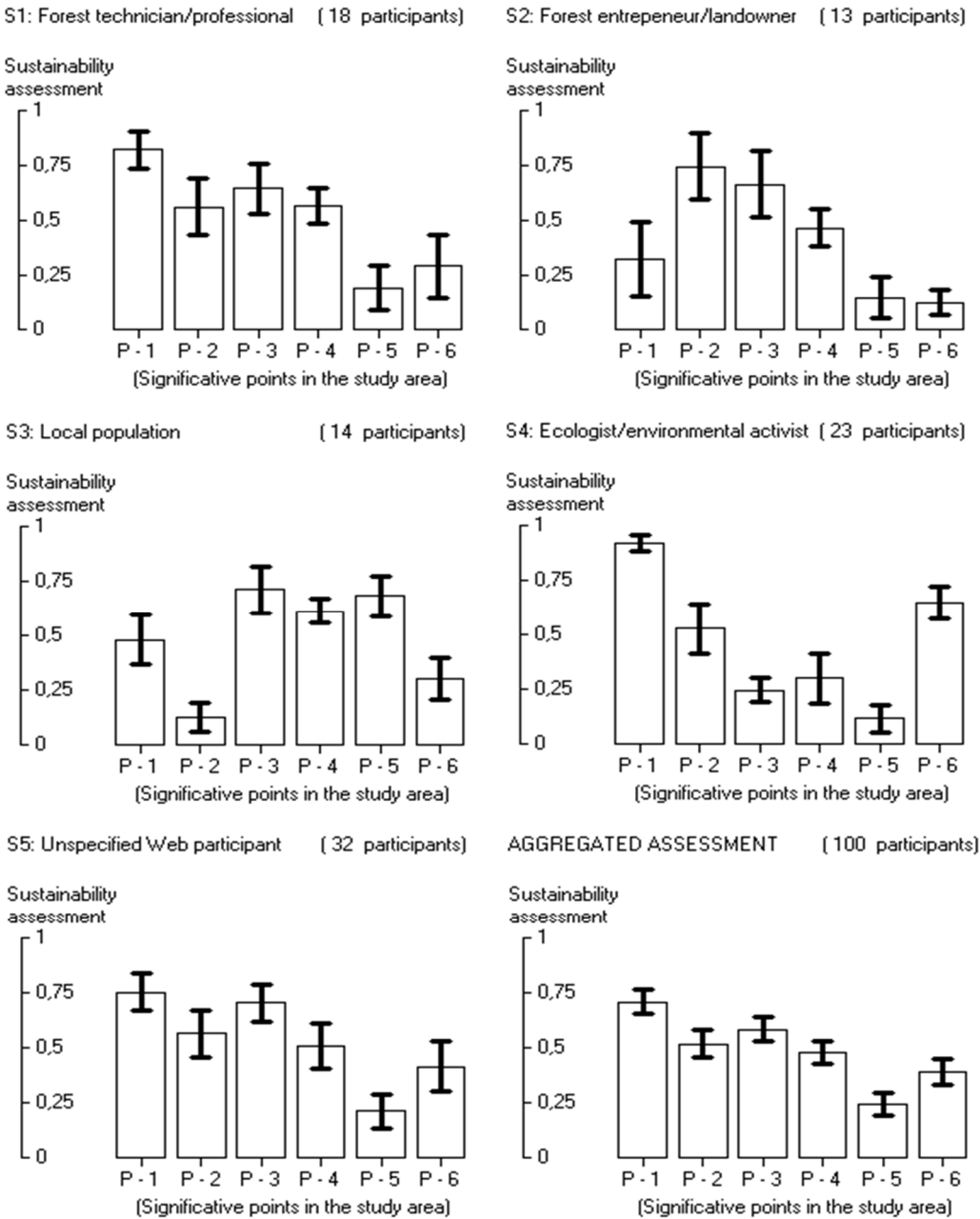
Figure 3. Summary of the sustainability assessments for the participants under the simulation parameters described in Table 4. Top ends of bars indicate observation means of the group. Line segments represent confidence intervals ($1-\alpha$=95%) for the individual assessments.

To analyze the evolution of the participative process (Section 3.3.1), we started with a group of five participants (randomly chosen from the set of one hundred). For the participants in this group, we measured both the aggregated empathy among participants, and the convergence of their opinions. These calculations were repeated in new groups, formed by randomly adding five new participants to the current group. The expansion process continued until one hundred participant groups were reached.

Later, Section 3.3.2 analyzes the effects of interactions among the simulated participants.

3.3.1. Convergence of opinions by adding new participants.

551   Fig. 4 shows the evolution of the measures used to monitor the participative process: the overall
552   empathy between the opinions of the group (S_T, as in equation 6), and the convergence of opinions
553   among the participants (CoO, as in equation 13).
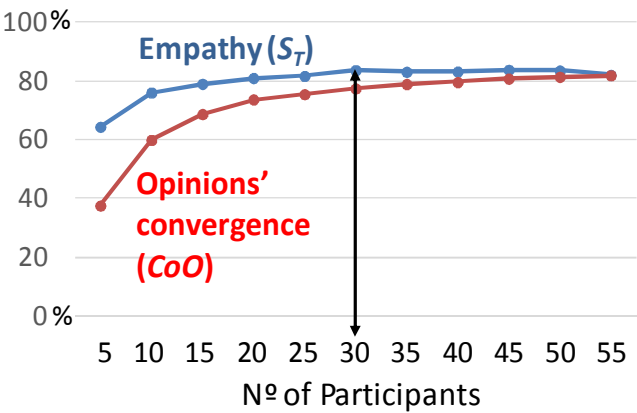
Figure 4. Evolution of the overall empathy and of the convergence of opinions with the number of participants.

557   As shown in the graph, empathy increased with participation until it reached a local maximum
558   for thirty participants ($S_{T(30)}$ = 83.65%). It then shows very small changes that do not depend on the
559   increase in the number of participants. On the other hand, the convergence of opinions grows
560   rapidly from the group of five participants ($CoO_{(5)}$ = 37.59%) to the group of thirty participants
561   ($CoO_{(30)}$ = 77.41%). It then continues to increase, but at a much lower rate ($CoO_{(55)}$ = 81.89%). The
562   growth rate remains constant in the range between thirty and one hundred participants (for a
563   number of participants not shown in the figure: $CoO_{(80)}$ = 84% and $CoO_{(100)}$ = 86.82%).

564   The association between empathy and convergence is evident: while empathy increased (up to
565   30 participants), convergence increased an average of 1.52% for each new participant. Subsequently,
566   it increased at only a rate of 0.13% for each new participant.

567   The previous result could, on its own, validate the strategy of encouraging participation as long
568   as empathy increases. But, aside from the number and quality of participants, there is another
569   feature that plays a fundamental role in the convergence of opinions: the interactions among
570   participants.

571   3.3.2. Influence of the interactions among participants on the collective solution.

572   In participatory processes, the convergence of opinions due to the interactions among the
573   participants would be expected to be at least as important as the convergence due to the number of
574   well-informed participants. We validated this assertion through a computer simulation that
575   analyzed the influence of interactions on the final decision.

576   The mathematical simulation was used to calculate -and compare- the aggregate assessment of
577   sustainability in two scenarios. The first one refers to an aggregate assessment for the participants
578   generated based on the parameters in Table 7. The second scenario incorporates interactions through
579   an agent-based simulation (described on Section 2.2) to explain the interactions among participants
580   for the same participants than in the first case.

581   To get an aggregated assessment of sustainability without interactions, the joint value was
582   calculated as the average of the individual sustainability for all the participants. We proceeded this

583 way because it was the simplest method of aggregation. In general, there will be a different
584 aggregate assessment, but this does not affect our objective: to see how interactions modify the
585 aggregation of preferences. For this case study: u_g = (0.71, 0.51, 0.58, 0.47, 0.24, 0.39).

586   We then applied the methodology described in Section 2.2 to trigger interactions among the
587 members of the group of 100 participants. The simulation was parameterized as discussed below:

588 •   We were not interested in artificial acceleration of convergence of opinions. Therefore, only
589   low-utility suggestions of changes were accepted in the random changes of opinion needed to
590   simulate interactions. From a mathematical point of view, this meant adopting a low
591   temperature for equation 11 that controls the randomized changes of opinions.

592 •   Since we accepted random and synchronized changes of opinion (all participants change, at the
593   same time, to solutions accepted by more than 50% of them), the convergence of this algorithm
594   is not monotonous [60]. This means that each iteration does not have to improve the previous
595   solution, but after a certain number of iterations (in our case we limited this to 300) there are
596   clear improvements.

597   To measure the convergence for interaction among participants, we used the distance between
598 the last group-opinion accepted by each evaluator into their memory ($o_{g(A)}$=($u_{g(A)}$, $v_{g(A)}$)) and the
599 current aggregated group-opinion (o_g=(u_g,v_g)). Both opinions change in each iteration of the process
600 (i). We will therefore re-write them as $o_{g(A)\text{-}i}$ and $o_{g\text{-}i}$ respectively. So:

$$OpC_i = \frac{1}{N}\sum_{A=1}^{N}\left\| u_{g(A)-i} - u_{g-i} \right\| \tag{16}$$

602 where $\left\| u_{g(A)-i} - u_{g-i} \right\|$ as in equation 14; OpC_i is the measure of the convergence of opinions in the
603 i-iteration; and N is the total number of participants (N=100).

604   In the simulation carried out, the maximum convergence of opinions corresponds to the
605 iteration number 268 (see Fig. 5). In this iteration, the aggregate evaluation corresponds to $u_{g\text{-}268}$ =
606 (0.81, 0.56, 0.53, 0.52, 0.24, 0.44), which represents an improvement of 12.07% with respect to the "no
607 interaction" scenario (percentage of improvement between OpC_{268} and OpC_1).

608
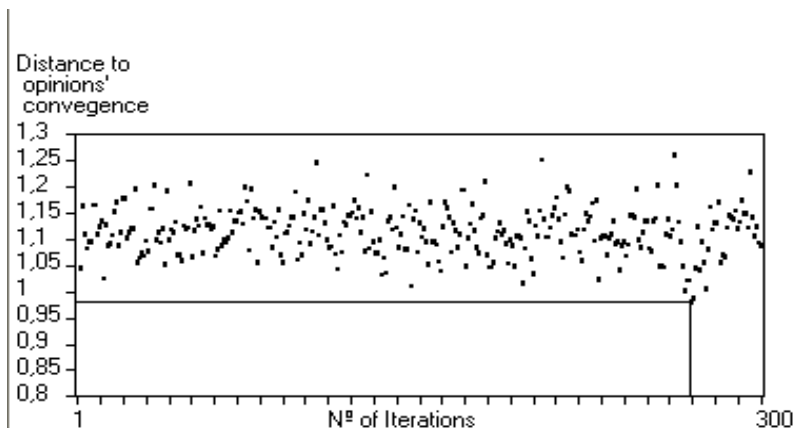
609   **Figure 5.** Convergence of aggregated opinion due to interactions among participants.

610   The reality is that there is no "no interaction" scenario with which to compare. We have
611 simulated opinions around well-informed participants, but no opinion can be formed without some
612 kind of interaction. That means all we have is a scenario of well-informed participants who have not
613 interacted after formulating their opinions. On the other hand, we have a scenario with interaction
614 subsequent to the initial formulation of preferences in which no new participants have entered. In
615 any case, it is notable that:

616 – Both scenarios improve convergence.

617 – The only way we found to simulate participatory processes that do not converge is to avoid the
618 simultaneous emergence of both factors. In other words, to incorporate ill-informed
619 participants (with greater randomness in the opinion of the participants) and not allow
620 post-formulation of opinions.

621 These outcomes reinforce the idea that participation means both things: the incorporation of a
622 large number of participants and interaction between them.

4. Discussion and conclusions

624 The first notable conclusion is that it is possible to measure confidence among participants in
625 collective decisions. This is done through the construction of empathic utility functions.

626 A second conclusion deals with the association between empathy and the convergence of
627 opinions. This relation has been commonly accepted, but this is the first time that it has been
628 verified: the case study shows that increases in empathy are associated to greater convergence of
629 opinions among participants, and when empathy ceases to increase, the rate of convergence of
630 opinions slows.

631 The main conclusion of the paper was drawn by combining the two conclusions mentioned
632 above: empathy variations can be incorporated to manage participatory processes and this can be
633 done by applying a simple strategy. The strategy is to encourage participation while empathy
634 increases and, if not, to stop participation. When the participatory process ends, the results from the
635 *Web*2.0 and from interpersonal utilities are applied to aggregate opinions.

636 The strategy always leads to a solution for collective decision making. The aggregated solution
637 will be socially acceptable in the case of high convergence of the opinions. In that case, the aggregate
638 opinion will represent the preferences of almost all the people involved, and it will be the joint
639 solution for the group. Otherwise, the set of participants must be broken down into homogeneous
640 subgroups (with common opinions within each subgroup) and other aggregation procedures must
641 be applied. These procedures may range from voting to an explained personal decision. Even in the
642 case that the final decision was made by an official, the use of this strategy provides the information
643 required for accountability.

644 It is also notable that incorporation of empathy improves the results obtained so far. When
645 comparing the confidence-based strategy with the available participatory applications, it shows
646 improvements in all the functional requirements required for a participative process.

647 Other conclusions deal with the power of networking. We confirmed that if participants report
648 their judgments in digital systems, then the representation can be automated for both: the opinions
649 of each participant, and the changes of empathy throughout the participative process. But
650 automation also promotes an increase in the number of participants and facilitates interaction
651 among them.

652 The influence of the number of participants is well known. Condorcet [12] was the first to relate
653 the number of participants and the quality of the information they have with the final decision):
654 when the components of a group are well-informed and the number of participants expands to
655 incorporate all possible individuals, and the group will then certainly make correct decisions. The
656 *Web* increases the number of people involved because access is free and it also allows "delegation" of
657 the decisions of non-participants to participants close to them in social networks. But the *Web* also

658 increases the quality of the information, because participatory applications usually explain basic
659 rules for forest management and for understanding natural processes. Besides, this information fits
660 to the characteristics of a participant (for example, *SILVANET*, [54], provides information
661 considering the coherence and depth of everyone's knowledge). All of these factors increase the
662 likelihood that a participant will be well informed, although it does not ensure that everyone to be
663 well-informed (the likelihood of a participant making correct decisions is greater than 50%) as
664 requires Condorcet´s theorem.

665 The consequences of interaction between participants are usually considered under the
666 umbrella of self-organization (in the terminology of complex systems). Self-organization is the
667 process by which coordination arises out of interactions between participants in collective decisions.
668 A computer simulation has verified that self-organization enhances emergence (the whole – the
669 results of the participatory process – is more than the parties –the mere aggregation of participants-)
670 in assessment of sustainability. In fact, the convergence of opinions after simulating interaction was
671 greater than the convergence without self-organization. However, this does not seem to be the result
672 just of the simulation carried out: self-organization has provided great advances in natural
673 evolution, technology and computation. Consequently, digital discussion forums should be favored
674 to increase interactions among participants.

675 The confidence-based strategy also provides operative advantages, such as:

676 • The homogeneous representation of opinions for all participants. This means knowing how a
677 participant assesses a forest plan and which forest management plan is best for them.

678 • The strategy is easily integrated into the most widely-used information systems. In fact, the
679 information required is obtained from available information systems and from the preferences
680 of each participant (which is new). Information is then processed with using the usual big-data
681 and business analytic tools and, after applying the specific participatory algorithms described
682 in this paper, the outcomes can be transferred to most common decision-support systems.

683 • Accountability in collective decision-making.

684 There is another indirect advantage: the proposed strategy counteracts the excesses of new ICT
685 developments that tend to limit universal access to information and propose decisions that are only
686 economically efficient. Specifically – unlike the *Web*, which provides free and global access to results
687 - some of the new ICT developments - such as Artificial Intelligence algorithms or Internet of Things
688 - require large investments and their outcomes are owned by the entities that have made the
689 investments. Most of the time, these entities are for-profit organizations that optimize only their
690 economic returns.

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