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Article

One Universe, Many Spaces: A Non-Local, Relativistic Quantum Spacetime

Jonathan C. Sharp

University of Alberta; jcsharp@ualberta.ca

Abstract: The nonlocality of entangled quantum mechanical systems is incompatible with the standard interpretation of special relativity as a single 4D Minkowskian metric spacetime. The difficulty is that the definition of a spacetime interval between any pair of events precludes any form of nonlocal interaction, even the relatively benign non-signaling correlations. By an application of the relativity principle, and the use of the space ⟵⟶ time symmetry of the Lorentz boost I propose here a reinterpretation of special relativistic spacetime. This new ontology consists of a set of coexisting 3+1D spaces ('framespaces'), each containing unique content in the form of a complex density. These spaces are related by the Lorentz boost, and coupled pairwise in a manner dictated by the Lorentz transformation. The inter-space coupling acting on the spacetime content gives rise to a nonlocal wave phenomenon, which is identified as quantum wave mechanics. The interspace coupling strength is then inversely proportional to Planck's constant. The coexistence of multiple spaces is interpreted as momentum superposition, implying that momentum is the fundamental physical basis of quantum superposition. This new spacetime interpretation of quantum mechanics has many consequences, including explanations of quantum non-locality, the spacetime role of Planck's constant, quantum measurement as a symmetry-breaking process and the redundancy of description of gauge theory.

Keywords: Special Relativity; Quantum Mechanics; Non-Locality; Planck's constant; EPR.

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1. Introduction

There is famously no consensus on the spacetime meaning of quantum mechanics [1]. From the Einstein-Podolsky-Rosen ('EPR') paper in 1935 [2], to Bell's theorem in the 1960s [3], to Aspect's EPR experiment in the 1980s [4] and its recent refinements demonstrating kilometer range correlations [5], it has become increasingly clear that entangled quantum mechanical systems are inherently nonlocal [6,7]. Troublingly, this quantum non-locality is incompatible with the standard interpretation of Minkowskian spacetime as a single 4D metric space, in which a spacetime interval is definable between pairs of events [8]. Moreover, without a theory of quantum spacetime, the fundamental quantum phenomenon of superposition has remained unreconciled with classical 4D relativistic spacetime [9]. Here by appeal to spacetime symmetry we reinterpret relativistic spacetime as a set of coexisting and coupled 3+1D spaces. Assigning a complex density to each space gives rise to a nonlocal inter-space wave phenomenon, identifiable as quantum waves with Planck's constant playing a critical spacetime role governing inter-space coupling. The relativistically necessary coexistence of multiple spaces is identified as the origin of quantum superposition, and offers a physical explanation for the redundancy of description found in gauge theories. One key consequence, impacting the infamous quantum measurement problem, is that momentum is the fundamental physical basis of quantum superposition. This 'many-spaces' proposal represents a unified quantum-spacetime applicable to all atomic-scale quantum phenomena, without any need to appeal to the inaccessible Planck scale. This proposal is both a reinterpretation and, due to the spacetime superposition postulate, a modification of quantum theory.

It is sometimes suggested that relativistic quantum field theory (QFT) [10] adequately integrates special relativity (SR) with quantum mechanics (QM). However, we argue that despite the successes of QFT, a deeper understanding of quantum mechanics within flat spacetime is still needed. Two issues highlight the friction between QFT and spacetime. Firstly, the nonlocality inherent in the path integral formulation [11] illustrates that QFT has merely inherited, rather than resolved, the quantum conflict with metric-based spacetime locality. Secondly, lacking a spacetime role for Planck's constant, QFT has failed to lead to a theory of quantum gravity. Although semiclassical gravity [12] does incorporate both QFT and gravity, and despite successes in black hole thermodynamics [13], the semi-classical approach is of limited validity because expectation values, rather than the full quantum state, act as the source of spacetime curvature.

We argue here that the de facto adoption of a classical-style spacetime ontology, consisting of a single objective, unobservable invariant reality [14], has critically hindered the conceptual unification of spacetime and quantum mechanics. By contrast, a spacetime ontology consisting of multiple subjective observable realities would be much closer aligned to that of the quantum world. Such an approach is not without foundation, bearing in mind that in addition to invariants, relativity theory also encompasses observer-dependent covariant quantities. Accordingly we propose a relativistic spacetime ontology in which the direct observational experience of simple inertial observers is treated as primary. This emphasis on observables over invariants might be captured by the phrase: 'reality is relative'. There are some communalities in approach between this and relational quantum mechanics [15] and also with a 'relationalist' interpretation of spacetime [16].

2. A Many-Spaces, Non-Local Spacetime

We introduce here a 'many-spaces' interpretation of spacetime, illustrated in Fig.1. Although there is a great deal that might be said in critique of conventional single-space 4D spacetime concepts and in support of this approach, in the interests of brevity the bulk of that discussion will be deferred, but we will outline the arguments in this section.

Consider the direct observations made by a set of inertial observers, all in relative motion. Following the non-detection of the ether by Michelson and Morley, each observer - unable to detect any self-motion - directly observes itself at rest within a 3+1D space. Each observer *Alice* will assert that its own directly experienced 3+1D space is physically real. No other observer is in a position to deny that, for Alice at least, Alice's space is real. In this paper we will focus on the interpretation of these multiple realities or views.

It is true that Minkowski successfully showed that within the confines of classical physics a 4-dimensional manifold can be used to encompass and relate all the differing views. We will take the position here that despite all the benefits and insights of the 4-dimensional scheme, it does not invalidate Alice's direct experience. The direct experiences of all observers A,B,C and D do not dissolve into a 4-dimensional mist; they must persist. In other words, spacetime ontology cannot be simply a 4-dimensional manifold, but must respect the existence of multiple observed realities.

The other important caveat of Minkowski's classical scheme is that the move to a 4-dimensional metric space, with a meaningful concept of locality, has unfortunately failed for quantum systems. Quantum nonlocality violates the locality so prized by Minkowski. In retrospect we can see Minkowski's contribution was a regressive move, tethering Einstein's theory to 19th Century absolutists notions, and blinding it to the multiplicity that would become so central to the quantum world.

We will take the position here that for a set of coexisting observers, there must be a corresponding set of coexisting physical 3+1D spaces, with each pair related by a relative velocity.

In this picture, the Lorentz boost represents a transformation between physically distinct 3+1D spaces, and not, as commonly supposed, a mere relabeling exercise of a 4D spacetime [14]. The entirety of this spacetime consists of many coexisting spaces, each containing unique content. These spaces (`framespaces') can be thought of as reference frames made real: pairs are related by the

Lorentz boost (like reference frames), however each contains unique content (like spaces). This expanded spacetime ontology does not conflict with established experimental or theoretical results of classical relativity, but we will show that it does provide a much more coherent framework for quantum systems.

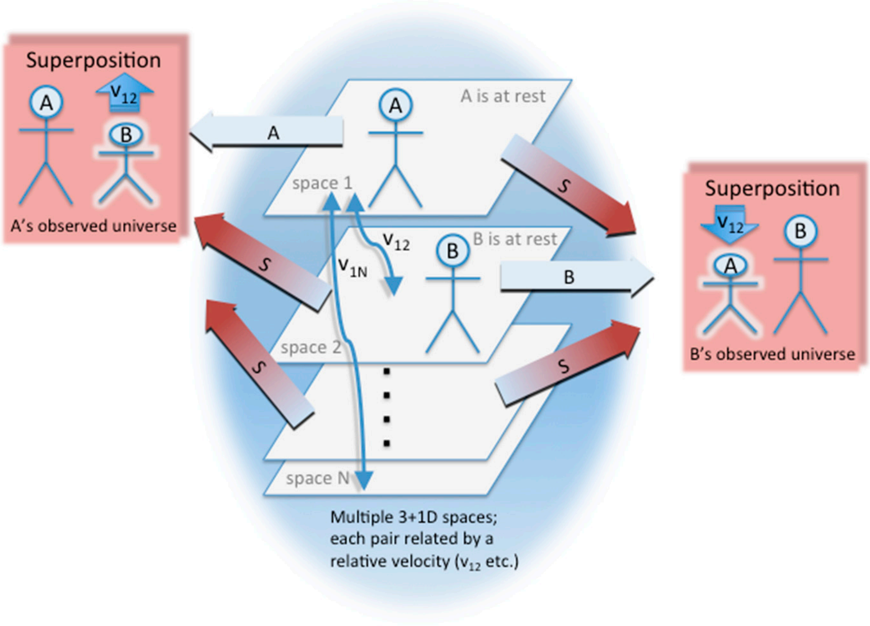

Figure 1. Diagrammatic representation of the many-spaces spacetime for classical systems. The many 3+1D spaces, each pair related by a relative velocity (e.g. v_{12}), are represented diagrammatically as a stack of 2D planes. There is no motion within any one 3+1D space, and classical relatively moving inertial observers A and B each reside at rest within one of these spaces. As shown to left and right, and as indicated by the arrows, each observer's reality is constructed from a superposition of many 3+1D spaces. However, A and B each experience a different version of the superposition of the underlying multiple spaces. For the classical relativistic case shown, each observes the other as in motion and Lorentz contracted. Although for classical systems, the familiar special relativistic 4-vector calculus may still be applied, the corresponding ontology is no longer that of a single 4D spacetime.

Non-Locality

This many-spaces ontology is able to very naturally accommodate both local classical and nonlocal quantum systems without any conflict or paradox, as follows. A classical system consists of a set of objects (inertial observers), each pair possessing a definite relative velocity (Fig.1). This is interpreted as each object residing (at rest) wholly within its own space. There is no motion within any one space, however relatively moving objects enter into the universe of an observer by the superposition of all spaces. Therefore even in classical systems superposition plays a role.

Transit of content between spaces corresponds to acceleration, so an inter-space coupling mechanism is required. It is important to realize that in special relativity, the concept of locality (i.e. the ability to define a spacetime interval) is completely dependent upon the operation of the Lorentz boost velocity transformation. Locality is therefore contingent upon the existence of definite relative velocities. Because classically relative velocities are indeed well defined, classical systems behave locally.

However, the situation is quite different for quantum systems. A quantum system in momentum superposition lacks definite relative velocities, i.e., coexists within multiple spaces, see Fig.2. Lacking well-defined relative velocities, such a system behaves non-locally. To allow unitary quantum evolution, again, an inter-space coupling will be required. Next we show that this picture is more than mere metaphor because the mechanics of a system coexisting within multiple spaces is indeed quantum.

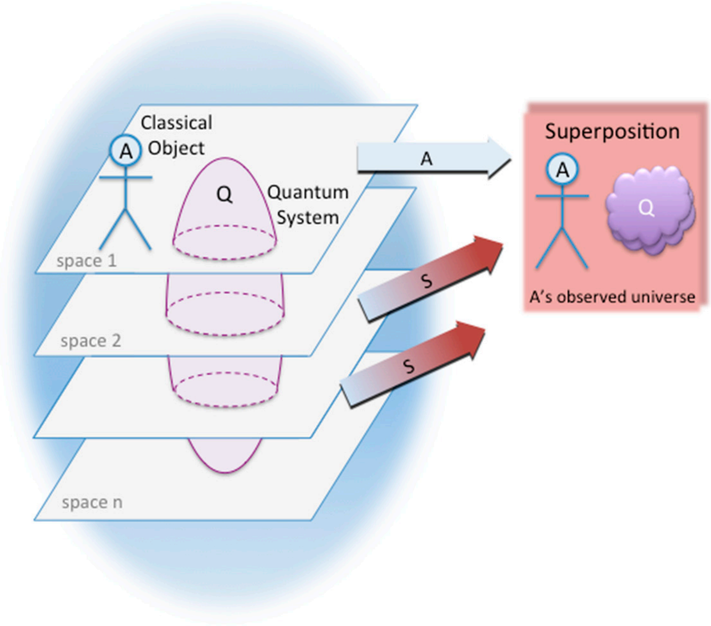

Figure 2. Diagrammatic representation of the many-spaces spacetime for a quantum system. Here a classical object A is shown together with a quantum system Q which is depicted as penetrating multiple planes, representing its coexistence within multiple 3+1D spaces. On the right hand side, the observed universe of classical object A is created from the superposition of the content of the multiple spaces, as indicated by the arrows labeled 'S'. In this superposition, the quantum system Q is represented as a cloud, indicating quantum properties such as non-locality.

3. Superposition, Coupling and Wave Behaviour

A radical yet compelling interpretation is to equate *coexistence* with *superposition*, i.e. the many spaces coexist in superposition. This implies that coexistence of spaces is the origin of quantum superposition, and reinterprets spacetime as an inherently quantum mechanical superposed set of 3+1D spaces, rather than a single 4D classical space. This is a major step in the argument: the claim is that superposition is a fundamental ingredient in the structure of spacetime, not merely an expression of `inexplicable quantum weirdness'. This is an attempt to take superposition seriously in a spacetime context. Does this claim stand up? We show below that it does.

Equating framespace-coexistence with quantum-superposition immediately leads to the identification of momentum as the fundamental basis of quantum superposition. These spaces are real, contain unique content, and interact. Specifically, we postulate that each space F_i contains a complex-valued density function: $\rho_i(x_i, ct_i)$. Spaces are related by a relative velocity therefore the inter-space interaction must be pairwise and must conform to the two fundamental relativistic observational conditions [17], namely the simultaneous observation of length ('simultaneous-length'), and the colocal observation of duration ('colocal-duration'). The duality between these conditions is a consequence of the space ← → time symmetry of the Lorentz boost [17].

Observation and Coupling between Spaces

Due to the Relativity of Simultaneity and its dual the Relativity of Colocality [17], observation possesses an inherent directionality in which the observational condition (simultaneity or colocality respectively) – *applying to the observer only* – physically distinguishes between observer and observed. This is a relativistic answer to the question: `What is an observer?'

To be physically meaningful, it is necessary that observation result in physical change for the observer. Accordingly, we propose an active form of inter-space coupling, arising from the Lorentz boost's mixing of spatial and temporal axes, and therefore only in force for non-zero relative velocity. We show below that this results in wave behavior with coupling strength governed by Planck's constant. For brevity, in the following we present one case that illustrates this 'coupling postulate' (see appendix for other cases).

Non-local Waves Behavior

We have postulated that each framespace F_i contains a complex-valued density function: $\rho_i(x_i, ct_i)$. A simultaneous-length observation made from space F_1 results in a change of observer density $\delta\rho_1$ equal to the product of a coupling strength (written as ib), the density ρ_2 in the observed space F_2, and a temporal interval $c\delta t_2$ in the observed space F_2:

$$\delta\rho_1 = ib\rho_2(c\delta t_2) \tag{1}$$

where the temporal interval arises from the relativity of simultaneity $\delta x_1 = -(1/\beta\gamma)c\delta t_2$. Equation (1) in differential form becomes:

$$\frac{\partial\rho_1}{\partial x_1} = -i(b\beta\gamma)\rho_2 \tag{2}$$

Analogously to Eq.(2), a colocal-duration observation from space F_2 yields:

$$\frac{\partial\rho_2}{c\partial t_2} = +i(a\beta\gamma)\rho_1 \tag{3}$$

The parameters a and b are respectively space-to-time and time-to-space inter-space coupling parameters. By writing a complex density as $\rho = \rho^R + i\rho^I$, the following pair of equations for ρ_2^I and ρ_1^R can be derived from Eq.(3) and Eq.(2) respectively:

$$\frac{\partial\rho_2^I}{c\partial t_2} = (a\beta\gamma)\rho_1^R \quad \text{and} \quad \frac{\partial\rho_1^R}{\partial x_1} = (b\beta\gamma)\rho_2^I \tag{4}$$

This equation pair is of the form $\partial U/\partial t_2 = \omega V$ and $\partial V/\partial x_1 = kU$, using the substitutions $U = \rho_2^I$, $V = \rho_1^R$, $\omega = a\beta\gamma c$ and $k = b\beta\gamma$. Following Bohm [18], U and V are the real and imaginary parts of a wave function $\psi = U + iV = e^{i(kx_1 - \omega t_2)}$, so in this case we have:

$$\psi = \rho_2^I + i\rho_1^R = e^{i\beta\gamma(bx_1 - act_2)} \tag{5}$$

This wave function represents a superposition of content within different spaces and so inherently has nonlocal properties. This form of wave function can satisfy either a Schrodinger type wave equation:

$$i\,\partial\psi/\partial t_2 = -(ac/b^2\beta\gamma)\,\partial^2\psi/\partial x_1^2 \tag{6}$$

or a 1D wave equation:

$$(b/ac)^2\,\partial^2\psi/\partial t_2^2 = \partial^2\psi/\partial x_1^2. \tag{7}$$

These wave equations contain dimensions from different spaces, so are also nonlocal. This represents a wave-like interaction between framespaces. In summary, since multiple spaces are not related by a metric, the wave behavior arising from interaction between these spaces is also non-local. This emergence of nonlocal wave behavior is of course is highly suggestive of quantum behavior.

Matter Waves

This nonlocal complex wave mechanics has arisen purely from relativistic arguments and an inter-space coupling postulate, (i.e. without any specifically wave or quantum postulates). The striking parallels demand the identification of these relativistic waves as indeed quantum waves. Making this identification by invoking the quantum mechanical formulae for energy: $KE = \hbar\omega = mc^2(\gamma - 1)$, and momentum: $p = \hbar k = \gamma m\beta c$, of a quantum particle of rest mass m, allows the evaluation of the coupling parameters a and b as:

$$a = \frac{mc}{\hbar}\frac{(\gamma-1)}{\beta\gamma} \quad \text{and} \quad b = \frac{mc}{\hbar}. \tag{8}$$

A pair of equations describing quantum spacetime wave behavior for matter may then be written:

$$\frac{\partial \rho_2}{c\partial t_2} = \frac{imc(\gamma-1)}{\hbar}\rho_1 \quad \text{and} \quad \frac{\partial \rho_1}{\partial x_1} = -\frac{imc\beta\gamma}{\hbar}\rho_2 \tag{9}$$

To respect the symmetry between spaces there is also a companion pair with the space labels reversed, (see Appendix). The full evolution of a quantum system is described by a set of these equations between all pairs of participating spaces. It is notable that quantum $(\hbar, \rho_i)$ and many-space spacetime (x_i, t_i, β_{ij}) quantities are directly related. This results in quantum evolution being expressed explicitly in non-local terms, in contrast to the classical local single-space spacetime Schrodinger equation formulation.

The two-way nature of the Born probability rule arises from this pairwise interaction between spaces, itself a consequence of the two-way nature of relative velocity (i.e. there is no concept of a three-way relative velocity).

Electromagnetic Waves

As a second case, by postulating equal coupling: $a = b = p/\hbar\beta\gamma$, the non-dispersive waves at velocity $\omega/k = c$, occur and may be identified as electromagnetic waves.

$$\frac{\partial \rho_2}{c\partial t_2} = \frac{ip}{\hbar}\rho_1 \quad \text{and} \quad \frac{\partial \rho_1}{\partial x_1} = -\frac{ip}{\hbar}\rho_2 \tag{10}$$

In this case, the relative velocity between the spaces disappears from the equations. In both cases the coupling between spaces is inversely proportional to Planck's constant. Quantum behavior is therefore completely integral to this spacetime picture.

4. Discussion

To conclude, a new relativistic-quantum unification has been achieved without violation of established principles. Planck's constant plays a fundamental role in an inherently quantum mechanical spacetime, in which relativistic and quantum concepts are interdependent, rather than in conflict. The 'many spaces' spacetime describes the coexistence of a set of (3+1)-dimensional spaces, tightly but not rigidly coupled by a 'quantum glue'. Coexistence of spaces corresponds to superposition of momentum, which therefore emerges as the fundamental physical basis of quantum superposition. This implies that momentum measurement is fundamentally different from the quantum measurement of other observables. The spacetime approach provides general insights into the nature of quantum measurement. Although classical relativistic observers disagree on simultaneity and colocality, such separable observers are free to 'agree to disagree'. However in a non-separable quantum system disagreement becomes unsustainable. Experimental interactions can then demand the answer to an 'unanswerable question' for which the unitarily evolving quantum system itself possesses no unique answer. A response can only be provided by a symmetry-breaking process for which no predictable deterministic outcome is possible. This momentum-based symmetry-breaking description of the reduction of the quantum state is a new proposal, with different experimental predictions from other interpretations.

To return to issues of locality: EPR non-locality arises because two quantum-entangled particles in relative motion are a single entity residing within multiple different spaces. Coexistence across multiple spaces implies non-locality because the concept of definite relative velocity is lost, and with it the ability of the Lorentz transformation to calculate definite spacetime intervals. A further consequence of the coexistence of multiple observers is a necessary redundancy of physical description, which may be directly related to the origin and ubiquity of gauge invariance. In summary, the recasting of superposition from an abstract principle into a specific physical phenomenon has far reaching consequences for the interpretation of quantum mechanics and for quantum measurement. The integration of quantum and spacetime concepts results in a quantum spacetime providing solutions to many hitherto paradoxical phenomena.

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Appendix

Consider the possible interactions between a pair of spaces. Between the two complex functions $\rho_1(x_1, ct_1)$ and $\rho_2(x_2, ct_2)$, representing content in two coupled spaces (F_1, F_2), there are four possible measurement interactions:

Observer	Observation		
F$_1$	Simultaneous-Length	$\frac{\partial\rho_1}{\partial x_1} = -i(b\beta\gamma)\rho_2$	(A1)
F$_2$	Colocal-Duration	$\frac{\partial\rho_2}{c\partial t_2} = +i(a\beta\gamma)\rho_1$	(A2)
F$_1$	Simultaneous-Length	$\frac{\partial\rho_2}{\partial x_2} = +i(b\beta\gamma)\rho_1$	(A3)
F$_2$	Colocal-Duration	$\frac{\partial\rho_1}{c\partial t_1} = -i(a\beta\gamma)\rho_2$	(A4)

Table A1: Four possible measurement interactions between two spaces. Note that these are symmetric between spaces: swapping space labels $1 \leftrightarrow 2$ implies $\beta \rightarrow -\beta$.

Observational Compatibility

As shown by the four rows in Table A1, there are four measurement modes: F$_1$-SL, F$_1$-CD, F$_2$-SL, F$_2$-CD. However not all of these measurement modes are compatible. In a given space, the simultaneous-length (SL) and colocal-duration (CD) observational conditions are incompatible, so only one may apply. The condition for length measurement (simultaneity) precludes the condition for measurement of duration (colocality). Therefore, the F$_1$-SL and F$_1$-CD modes are incompatible, as are the F$_2$-SL and F$_2$-CD modes. The relativity of simultaneity forbids simultaneity in both spaces. Similarly, the relativity of colocality forbids colocality in both spaces. Therefore F$_1$-SL and F$_2$-SL are incompatible, as are F$_1$-CD and F$_2$-CD. The only two compatible combinations are therefore the pair F$_1$-SL & F$_2$-CD, and the pair F$_1$-CD & F$_2$-SD.

Real Densities

It is useful to express the complex densities as a pair of real densities. Each of Eqns.(A1)-(A4), containing complex density functions, may be split into two equations with real density functions, using $\rho = \rho^R + i\rho^I$. Illustrating this for Eq.(A1) (the F$_1$ observation of simultaneous-length) we have:

$$\frac{\partial\rho_1}{\partial x_1} = -i(b\beta\gamma)\rho_2 \qquad \text{(A1)}$$

$$\frac{\partial(\rho_1^R + i\rho_1^I)}{\partial x_1} = -i(b\beta\gamma)(\rho_2^R + i\rho_2^I) = +(b\beta\gamma)(\rho_2^I - i\rho_2^R)$$

which splits into the independent pair:

$$\partial\rho_1^R/\partial x_1 = +(b\beta\gamma)\rho_2^I \qquad \text{(A1a)}$$

$$\partial\rho_1^I/\partial x_1 = -(b\beta\gamma)\rho_2^R \qquad \text{(A1b)}$$

This procedure generates eight equations with real quantities, as shown:

F_1 Obs SL	$\frac{\partial \rho_1}{\partial x_1} = -i(b\beta\gamma)\rho_2$	(A1)	$\partial\rho_1^R/\partial x_1 = +(b\beta\gamma)\rho_2^I$ $\partial\rho_1^I/\partial x_1 = -(b\beta\gamma)\rho_2^R$	**(A1a)** (A1b)
F_2 Obs CD	$\frac{\partial \rho_2}{c\partial t_2} = +i(a\beta\gamma)\rho_1$	(A2)	$\partial\rho_2^R/c\partial t_2 = -(a\beta\gamma)\rho_1^I$ $\partial\rho_2^I/c\partial t_2 = +(a\beta\gamma)\rho_1^R$	(A2a) **(A2b)**
F_1 Obs SL	$\frac{\partial \rho_2}{\partial x_2} = +i(b\beta\gamma)\rho_1$	(A3)	$\partial\rho_2^R/\partial x_2 = -(b\beta\gamma)\rho_1^I$ $\partial\rho_2^I/\partial x_2 = +(b\beta\gamma)\rho_1^R$	(A3a) (A3b)
F_2 Obs CD	$\frac{\partial \rho_1}{c\partial t_1} = -i(a\beta\gamma)\rho_2$	(A4)	$\partial\rho_1^R/c\partial t_1 = +(a\beta\gamma)\rho_2^I$ $\partial\rho_1^I/c\partial t_1 = -(a\beta\gamma)\rho_2^R$	(A4a) (A4b)

Table A2: As shown in the text, pairs of these equations can be combined to exhibit non-local wave behaviour. One example is the pair of equations (A1a) and (A2b), shown highlighted.

References

1. 1. Schlosshauer, M., Kofler, J., Zeilinger, A.: A snapshot of foundational attitudes toward quantum mechanics. Stud Hist Philos M P **44**(3), 222-230 (2013).
2. 2. Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? Phys Rev **47**(10), 0777-0780 (1935). doi:10.1103/Physrev.47.777
3. 3. Bell, J.S.: On the Einstein Podolsky Rosen Paradox. Physics **1**(3) (1964).
4. 4. Aspect, A., Grangier, P., Roger, G.: Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment - a New Violation of Bell Inequalities. Phys Rev Lett **49**(2), 91-94 (1982). doi:10.1103/PhysRevLett.49.91
5. 5. Tittel, W., Brendel, J., Gisin, B., Herzog, T., Zbinden, H., Gisin, N.: Experimental demonstration of quantum correlations over more than 10 km. Phys Rev A **57**(5), 3229-3232 (1998). doi:DOI 10.1103/PhysRevA.57.3229
6. 6. Brown, H.R.: Physical Relativity. Space-time Structure from a Dynamical Perspective. Oxford University Press, Oxford (2005)
7. 7. Maudlin, T.: Quantum non-locality and relativity : metaphysical intimations of modern physics, 3rd ed. Wiley-Blackwell, Malden, MA (2011)
8. 8. Rindler, W.: Relativity: Special, General, and Cosmological, 2d ed. Oxford University Press, (2006)
9. 9. Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. W. H. Freeman, San Francisco, (1973)
10. 10. Briggs, G.A.D., Butterfield, J.N., Zeilinger, A.: The Oxford Questions on the foundations of quantum physics. P R Soc A **469**(2157) (2013). doi:Artn 20130299 Doi 10.1098/Rspa.2013.0299
11. 11. Feynman, R.P., Hibbs, A.R.: Quantum mechanics and path integrals. International series in pure and applied physics. McGraw-Hill, New York (1965)
12. 12. Giulini, D., Kiefer, C., Laemmerzahl, C.: Quantum gravity : from theory to experimental search. Lecture notes in physics,, vol. 631. Springer, Berlin ; New York (2003)
13. 13. Hawking, S.W.: Black-Hole Explosions. Nature **248**(5443), 30-31 (1974). doi:DOI 10.1038/248030a0
14. 14. Wald, R.M.: General relativity. University of Chicago Press, Chicago (1984)
15. 15. Rovelli, C.: Quantum gravity. Cambridge monographs on mathematical physics. Cambridge University Press, Cambridge, UK ; New York (2004)
16. 16. Dainton, B.: Time and Space, 2nd ed. McGill-Queens University Press, Montreal & Kingston (2010)
17. 17. Sharp, J.C.: Symmetry of the Lorentz boost: the relativity of colocality and Lorentz time contraction. Eur J Phys **37**(5) (2016).

18. Bohm, D.: Quantum Theory. Prentice Hall, New York (1951).