Article

A method for measuring the real part of the weak value of spin for metastable atoms

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Abstract: A method for measuring the real part of the weak value of spin for non-zero rest mass atoms is presented using a variant on the original Stern-Gerlach apparatus. The experiment utilises helium in the metastable $2^{3}S_{1}$ state. A full simulation for observing the real part of the weak value using the impulsive approximation has been carried out and it predicts a displacement of the beam, Δ_{w} , that is within the resolution of our detector. It also indicates how this shift might be increased. The full analysis also indicated that there is a limit, *L*, to the applicability of the weak value approximation and has been evaluated for our apparatus. This experiment has the possibility to be expanded to utilise other nobal gas species such as neon and argon in the ³P₂ metastable state, but we shall restrict

this paper to metastable helium only.

Keywords: weak measurement; transition probability amplitude; atomic metastable states

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11 0. Introduction

Weak values are, in fact, "Transition Probability Amplitudes", (TPA), which played a significant 12 role in the formulation of the quantum field theory by Dirac [1] and Schwinger [2]. Moreover the 13 weak value of momentum defines a local velocity which is the Bohm momentum [3,4]. This notion 14 was first used by Landau [5] and London [6] in connection with superfluids. As these values were 15 not eigenvalues of the system and could not be measured in the usual way, they were not pursued. 16 However Hirschfelder [7] later realised their importance and discussed them in terms of what he 17 called "subobservables". He also noted their connection with Bohm's proposals. 18 The subject returned to prominence when Aharonov, Albert and Vaidman (AAV) [8,9] suggested 19 an experimental procedure to measure the weak value of spin for a particle. While "TPA and 20 subobservable" were the original names for these variables and are still used by some authors [10], 21

"weak" has become popular in this context. Even though we believe "weak" is misleading and can
cause confusion with the electroweak force, we will continue to use it in the rest of this paper.

Weak values are complex numbers, in contrast to eigenvalues that are only real. It must be clearly stated they cannot be identified as an eigenvalue. The experiment described here will show how the real part of the weak value of spin may be observed. Since weak values are TPAs, weak measurement can reveal more subtle details of quantum processes.

Measuring an eigenvalue uses a von Neumann (strong) measurement [11]. This is a single stage
 process whereby the wave function is said to "collapse". In contrast, the weak measurement process
 has three stages; pre-selection, the weak stage followed by a strong measurement of a post-selected

31 variable.

The real parts of the weak values for the polarisation and momentum of photons [12–14] have

already been observed and measured. It should be noted that the theory of weak measurement was

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originally cast in the non-relativistic regime using Schrödinger's equation (Schrödinger particles),

- whereas photons obey quantised Maxwell's equations and are relativistic. In addition to the photon 35 case, the real and imaginary parts of the weak value of spin for non-relativistic neutrons have been 36 measured [15]; the purpose of this paper is to show how weak measurements can be made using 37 non-relativistic atoms. We are following a scheme outlined in AAV and in Duck, Stevenson and 38 Sudarshan [16] which is a variant of the original Stern-Gerlach (S-G) apparatus [17]. A simulation has 39 been carried out giving firm predictions of what should be observed within the scope of the parameters 40 set by our experiment. 41 We will first present an overview of the experiment giving the main features of the method 42 used. Then we will present the simulation in full and show there is a limit to the weak measurement 43 approximation. Finally we will give a more detailed description of our method and report the
- approximation. Finally we will give a more detailed description of our method and report the
 parameters, such as the speed of the particles, that are required to be able to produce a final design
 and realisation of the experiment
- ⁴⁶ and realisation of the experiment.

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1. Simulation using the impulsive approximation

48 1.1. Weak measurement of spin overview

The weak measurement process allows for the detection of very small phase shifts. By preparing the system in a particular pre- and post-selected quantum state, it is possible to amplify these phase shifts, and from this amplified signal, it is possible to extract the desired observable of interest. As a consequence of this effect, the phrase "weak value amplification" is commonly used in the literature. The amplified shifts are relatively small therefore great care has to be taken in designing and constructing the experiment.

The three stages of the weak measurement regime for spin are as follows. Atoms are first pre-selected in a desired spin state with the spin axis set at a pre-selected angle θ in the x-z plane, see

⁵⁷ Figure 1. The atoms then propagate through the weak stage which in our case is a S-G magnet with a

field gradient that is very small along the z-axis.



Figure 1. Schematic view of the experimental technique. Helium atoms in the $m_S = +1$ metastable state enter from the left, with spin vector angle θ . The atoms pass through the weak and strong S-G magnets before reaching the detector. The displacement due to the weak measurement process is Δ_w , which is a function of the chosen pre-selected spin state. For simplicity the $m_S = 0$ spin state is not shown.

The strong stage consists of a second S-G magnet, with its inhomogeneous magnetic field aligned along the x-axis. Note the axes of the weak and strong stage magnets are at right angles with each other. The field of the strong stage magnet is large enough to clearly separate the spin eigenstates on this axis. It is this separation that enables us to detect the small phase shift, proportional to Δ_w , eer-reviewed version available at *Entropy* **2018**, 20, 56<u>6; doi:10.3390/e2008056</u>

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- induced by the weak stage as shown in the Figure 1. The size of Δ_w depends on various features of the apparatus. Furthermore, since this shift is still relatively small, we must maximise it by suitably adjusting the experimental parameters as will be discussed below.
- We have chosen to work initially with helium, excited into a metastable $2^{3}S_{1}$ triplet state, $m_{S} =$
- $\pm 1,0$. In the excitation process a metastable 2^1S_0 singlet state is also produced. This state passes
- though the experiment unaffected and is useful as a fiducial showing the mid-point of the distributions.
- ⁶⁹ We also plan to extend the experiment and use other gases such as neon and argon both in the ${}^{3}P_{2}$
- ⁷⁰ metastable state ($m_I = \pm 2, \pm 1, 0$). To confirm production of these metastable states and observe their
- ⁷¹ spin and angular momentum eigenstates experimentally we used a strong S-G magnet with a field
- $_{72}$ gradient of 100 T/m, see Figure 2 and Figure 3.



Figure 2. Distribution of the helium atoms in the metastable triplet state 2^3S_1 with $m_S = \pm 1, 0$, (He*). The central peak is larger because of the double contribution from the $m_S = 0$ and the singlet state 2^1S_0 .



Figure 3. The left hand picture shows the five angular momentum states, $m_J = \pm 2, \pm 1, 0$, of the metastable form of argon, (Ar*), and the right hand picture the identical states of the metastable form of neon (Ne*). The states are clearly delineated indicating that they would be good candidates for measuring weak values of angular momentum.

- ⁷³ Metastable helium in the $2^{3}S_{1}$ state has several advantages:
- 1. Its magnetic dipole moment, μ , has a magnitude of two Bohr magnetons, $\mu = 2\mu_B$, [18,19]. This maximises the displacements produced by the S-G magnets.
- 2. It has a lifetime of approximately 8000 s [20], being unable to decay via electric dipole transitions
 and the Pauli exclusion principle i.e. its decay is doubly forbidden. This lifetime is clearly large enough
- ⁷⁸ for the atoms to pass through all the stages of the apparatus before decaying.

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3. Metastable helium atoms have an internal energy of 19.6 eV, the highest of any metastable noble
gas species. Upon collision with any surface it will easily ionise and the emitted electron is observed
with a multichannel plate detector (MCP).

- All of these characteristics will enhance the overall signal strength and sensitivity of the
- experiment. The simulation given below is based on using this form of metastable helium and preselecting the $m_S = +1$ state.

85 1.2. Simulation

The simulation is divided into three parts; the initial conditions, the application of the interaction Hamiltonian in the weak stage using the impulsive approximation [21] and finally the action of the strong Stern-Gerlach magnet. This approximation neglects the free evolution of the atoms in the weak magnet, only the interaction Hamiltonian is considered. It is also important to note that the inhomogeneous magnetic field produced by the S-G magnet in the weak stage is maximal along the z-axis, but negligible along the other two axes. The analysis follows the scheme outlined in [16] but in our case we are using the spin-1 rather than spin-1/2.

93 1.3. Initial conditions

The helium gas is initially prepared as a pulsed beam and is described by the normalised Gaussian wave packet at time t = 0

$$\psi(z,0) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} \exp\left(-\frac{z^2}{4\sigma^2}\right),$$
(1)

where σ is the width in position space. The width of the atomic beam is set by passing it through an orifice/skimmer at the entrance of the weak stage. We describe the spinor in terms of polar angles θ and ϕ in the following form [22],

$$\xi_{i}(\theta,\phi,0) = \begin{bmatrix} \frac{1}{2}(1+\sin(\theta))e^{-i\phi} \\ \frac{1}{\sqrt{2}}\cos(\theta) \\ \frac{1}{2}(1-\sin(\theta))e^{i\phi} \end{bmatrix} = \begin{bmatrix} c_{+} \\ c_{0} \\ c_{-} \end{bmatrix}.$$
(2)

The initial orientation of the spin vector angle θ can be seen in Figure 1, where the azimuthal angle ϕ (not shown), is the corresponding angle in the x-y plane. We set $\phi = 0$ and only consider variations of the angle θ . Therefore the initial wave function prior to entering the weak stage is

$$\Psi_{i}(z,0) = \psi(z,0)\xi_{i}(\theta).$$
(3)

102 1.4. Simulation of the weak stage process

The atoms then traverse the weak stage magnet, where the wave function evolves under the
 interaction Hamiltonian, weakly coupling the spin to the centre-of-mass wave function. The interaction
 Hamiltonian is given by

$$H_I = \mu\left(\mathbf{\hat{s}}.\mathbf{B}\right),\tag{4}$$

where μ is the magnetic moment, \hat{s} are the spin-1 matrices $\hat{s} = [\hat{s}_x, \hat{s}_y, \hat{s}_z]$, and the magnetic field **B** = $[B_x, B_y, B_z]$. The inhomogeneous field in the z-direction is maximal $B_z = \frac{\partial B}{\partial z} z$. Explicitly the interaction Hamiltonian is then,

$$H_I = \mu \begin{pmatrix} B_z & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -B_z \end{pmatrix}.$$
 (5)

At this point Schrödinger's equation is used to calculate the state of the system at a later time Δt , which is the time that the atom spends in the weak field. The resultant wave function is now given by

$$\Psi_w(z,\Delta t) = \exp\left(-\frac{i}{\hbar}\int_0^{\Delta t} H_I dt\right)\psi(z,0)\xi_i(\theta).$$
(6)

Following the process of the weak measurement regime as described in [23], the pre-selected wave function is then post-selected via the strong stage into the spin-up, m = +1 state in the x-basis $\xi_{f}^{\dagger} = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}$. Giving the final wave function

$$\Psi_{\rm f}(z,\Delta t) = \xi_{\rm f}^{\dagger} \exp\left(-i\frac{\mu\Delta t\frac{\partial B}{\partial z}z\hat{s}_z}{\hbar}\right)\psi(z,0)\xi_{\rm i}(\theta). \tag{7}$$

The explicit wave function at the exit of the weak stage is now

$$\Psi_{\rm f}(z,\Delta t) = \psi(z,0) \left(\frac{1}{2} \exp\left(-i\frac{\mu\Delta t \frac{\partial B}{\partial z}z}{\hbar} \right) c_+ + \frac{1}{\sqrt{2}} c_0 + \frac{1}{2} \exp\left(i\frac{\mu\Delta t \frac{\partial B}{\partial z}z}{\hbar} \right) c_- \right). \tag{8}$$

115 1.5. Obtaining the weak value of spin

The exponential (phase shift) in Equation 7 can be Taylor expanded

$$\Psi_{\rm f}(z,\Delta t) = \langle S_{\rm f} | \left[1 - i \frac{\mu \Delta t \frac{\partial B}{\partial z} z \hat{s}_z}{\hbar} - \frac{1}{2} \left(\frac{\mu \Delta t \frac{\partial B}{\partial z} z \hat{s}_z}{\hbar} \right)^2 + \dots \right] |S_{\rm i}\rangle \psi(z,0), \tag{9}$$

where for convenience we have written $|S_i\rangle$ for ξ_i and $\langle S_f|$ for ξ_f^{\dagger} . Hence

$$\Psi_{\rm f}(z,\Delta t) = \left[\langle S_{\rm f} | S_{\rm i} \rangle - i \frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} \langle S_{\rm f} | \hat{s}_z | S_{\rm i} \rangle - \frac{1}{2} \left(\frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} \right)^2 \langle S_{\rm f} | \hat{s}_z^2 | S_{\rm i} \rangle + \dots \right] \psi(z,0).$$
(10)

If the phase shift in Equation 10 is sufficiently small such that the inequalities

$$\left| \left(\frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} \right)^n \langle S_f | \hat{s}_z^n | S_i \rangle \right| < < \left| \langle S_f | S_i \rangle \right|$$
(11)

119 and

$$\left| \left(\frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} \right)^n \langle S_{\mathbf{f}} | \hat{s}_z^n | S_{\mathbf{i}} \rangle \right| < < \left| \left(\frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} \right) \langle S_{\mathbf{f}} | \hat{s}_z | S_{\mathbf{i}} \rangle \right|$$
(12)

hold true for $n \ge 2$ [16,23], then Equation 10 can be expanded to first order

$$\Psi_{\rm f}(z,\Delta t) = \left(\langle S_{\rm f} | S_{\rm i} \rangle - i \frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} \langle S_{\rm f} | \hat{s}_z | S_{\rm i} \rangle \right) \psi(z,0), \tag{13}$$

and the transition probability amplitude $\langle S_{\rm f}|S_{\rm i}
angle$ can be factored out

$$\Psi_{\rm f}(z,\Delta t) = \langle S_{\rm f} | S_{\rm i} \rangle \left(1 - i \frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} \frac{\langle S_{\rm f} | \hat{s}_z | S_{\rm i} \rangle}{\langle S_{\rm f} | S_{\rm i} \rangle} \right) \psi(z,0).$$
(14)

The weak value of spin is defined as $W = \frac{\langle S_f | \hat{S}_2 | S_i \rangle}{\langle S_f | S_i \rangle}$. Note *W* is in general a complex number with real and imaginary parts. When $\phi = 0$ the imaginary part goes to zero and only the real part, W_{Re} , contributes and thus becomes, Peer-reviewed version available at Entropy 2018, 20, 566; doi:10.3390/e20080

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$$\Psi_{\rm f}(z,\Delta t) = \langle S_{\rm f} | S_{\rm i} \rangle \left(1 - i \frac{\mu \Delta t \frac{\partial B}{\partial z} z}{\hbar} W_{Re} \right) \psi(z,0).$$
(15)

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Using the pre- and post-selected states of the system, the real part of the weak value becomes,

$$W_{Re} = \tan\left(\frac{\theta}{2}\right). \tag{16}$$

In order to cast Equation 15 into an exponential form the following inequality must be met,

$$L = \left| \frac{\mu \Delta t \frac{\partial b}{\partial z} z}{\hbar} W_{Re} \right| << 1 \tag{17}$$

where *L* is a limit to be determined [16,23].

As the spread in z-axis is related experimentally to the width of the atomic beam in question [16], it can be replaced by σ , therefore the inequality becomes,

$$L = \left| \frac{\mu \Delta t \frac{\partial b}{\partial z} \sigma}{\hbar} \tan\left(\frac{\theta}{2}\right) \right| << 1.$$
(18)

The final wave function after the Gaussian wave packet has trasversed both the weak and strongmagnets is,

$$\Psi_{\rm f}(z,\Delta t) = \langle S_{\rm f} | S_{\rm i} \rangle \exp\left(-i\frac{\mu\Delta t\frac{\partial b}{\partial z}z}{\hbar}\tan\left(\frac{\theta}{2}\right)\right)\psi(z,0). \tag{19}$$

In this experiment, the real part of the weak value of spin will be measured by setting $\phi = 0$ and varying the angle θ between 0 and π .

134 1.6. Free evolution of the Gaussian wave packet at the detector

After the strong stage, the problem is treated as the free evolution of a Gaussian wave packet by solving the Pauli equation using well-known methods [21]. The probability density can now be computed, giving the form of the wave function as seen by the detector

$$|\Psi_{\rm D}(z,t)|^2 = |\langle S_{\rm f}| S_{\rm i}\rangle|^2 \left[2\pi\sigma^2 \left(1 + \frac{\hbar^2 t^2}{4m^2\sigma^4}\right)\right]^{-\frac{1}{2}} \exp\left[\frac{-(z + utW_{Re})^2}{2\sigma^2 \left(1 + \frac{\hbar^2 t^2}{4m^2\sigma^4}\right)}\right],\tag{20}$$

¹³⁸ Where *t* is the time of flight from the exit of the strong magnet to the detector, the mean of the ¹³⁹ post-selected wave function shifts by the value $utW_{Re} = \left(\frac{\mu}{m}\frac{\partial B}{\partial z}\Delta t\right)t \tan\left(\frac{\theta}{2}\right)$, where *u* is the transverse ¹⁴⁰ velocity of the helium atoms. We will denote this as Δ_w , being the displacement of the wave packet ¹⁴¹ due to the weak value of spin. This is in contrast to the standard S-G experiment where the shift is ¹⁴² only *ut*.

As the pre- and post-selected spin states approach orthogonality, θ tends to π , Δ_w increases but the transition probability decreases. This reduces the number of post-selected events of interest, leading to the need for longer experimental runs.

Again it is important to understand that this effect only arises when the phase shift acquired at the first stage is sufficiently small, see Equation 18. The centre-of-mass wave function is displaced but its overall shape is maintained after exiting the weak stage.

149 1.7. The limit and its validity

In literature the real part of the weak value is given as $\tan\left(\frac{\theta}{2}\right)$. This functional dependance is for an ideal case when the limit in Equation 18 is equal to, or smaller than, an optimal value which we will eer-reviewed version available at *Entropy* 2018, 20, 566; doi:10.3390/e2008050

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call L_o . For this experiment it is crucial to know L_o in order to successfully measure the well known tan $\begin{pmatrix} \frac{\theta}{2} \end{pmatrix}$ dependance. If L exceeds L_o , then this leads to alterations to the weak value as higher order terms begin to dominate.

In this case L_o can be determined by analysing the weak measurement process for two Gaussian wave packets, one describing the first order approximation, Equation (20), and the other, an exact case where no approximation is considered, using Equation (7).

 L_o is calculated by increasing the inhomogeneous magnetic field in the weak stage only, thus 158 increasing the limit, all other variables are held constant. Fig. 4 illustrates the behaviour of the the 159 two Gaussians. For small values of L the two curves strongly overlap, the point just before the two 160 wave packets deviate is the optimal limit, L_o . By finding this value the experiment can be tailored in 161 order to maximise the displacement measured, this is important as the amplified displacement is still 162 relatively small and on the limits of our detectors resolution. Past L_0 the first order approximation 163 continues to move to the left, while the full order approximation slowly reverts to that of a standard 164 S-G measurement. 165

> Limit >> L_o Limit < L Limit = L Limit > L 1st order 3.5 3.5 Probability Density 2.5 2 4 0. 0.5 -30 0 15 30 45 -45 -30 -15 0 15 30 45 -45 -30 -15 0 15 30 45 60 -60 -40 -20 0 20 40 60 80 z-axis (μ m)

Figure 4. A series of plots showing how the displacement, Δ_w , of the Gaussian wave packet is constrained by various limits. The red curve is the first order approximation which is dominated by $\tan(\frac{\theta}{2})$. The blue curve is the exact treatment of the system taking into account the higher order terms. The red and blue curves coincide when the limit $L = L_0 = 0.37$; this is the maximum limit for which the first order approximation holds.

As the optimal limit is now fixed, we can rearrange the wave packets deviation Δ_W with respects to this fixed limit

$$\Delta_{\rm w} = \frac{\mu \frac{\partial B}{\partial z} (\Delta t) t}{m} \tan\left(\frac{\theta}{2}\right) = \frac{\hbar t}{\sigma m} L.$$
(21)

We now can see the maximum deviation of the wave packet is only dependent on t and σ . By changing θ and adjusting other experimental parameters so that $L = L_o$, for all values of $\theta > \frac{\pi}{2}$ we will measure the same displacement, a maximal displacement, and from this the functional dependance $\tan\left(\frac{\theta}{2}\right)$ can be observed. Using parameters from our proposed experiment, of which the most important are the atomic velocity of our beam, 1717 m/s, our free flight distance, 2.4 m, the optimal limit, $L_o = 0.37$, and the width of the beam, $\sigma = 0.5 \,\mu m$, our expected displacement, Δ_w , is of the order of 20 μm .

Note: this optimal limit is only valid if $\theta > \frac{\pi}{2}$.

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176 2. Method for the weak measurement of spin for atomic systems: experimental realisation



Figure 5. The pulsed helium gas enters from the left. Preparation of the metastable atoms occurs in the first two chambers. In the next chamber the hexapole magnet (HM) preselects the $m_S = +1$ state which moves onto the weak measuring process. This comprises a weak stage magnet (WS) and a strong stage magnet (SS). Finally the atoms are detected using a micro-channel plate detector (MCP).

A schematic of our experimental arrangement is shown in Figure 5. The first step is to produce a beam of the metastable states of helium. Helium gas at high pressure enters the apparatus from the left and is pulsed using an electromagnetic valve producing a supersonic beam. The atomic beam is excited via an electron seeded discharge, where the atoms collide with a stream of energetic electrons in a 300 V/cm electric field [19]. The excited gas then passes through a 2 mm diameter skimmer and travels between two electrically charged plates to remove residual ionised atoms and free electrons.

The next step is to select the correct spin state, $m_S = +1$, and set its spin axis at an angle θ . A hexapole magnet focuses the $m_S = +1$ state to a point along the axis of propagation, defocusing the $m_S = -1$ state. The $m_S = 0$ state is left untouched. A 50 μm slit is then used to select the atoms with a particular spin vector, θ , within the x-z plane, see Figure 1. The $m_S = +1$ spin state is focused onto a second slit, producing an atomic beam with a width between $0.5 - 1.0 \,\mu m$ before entering the weak stage.

¹⁹⁹ Upon exiting the weak stage, the atomic beam enters the strong stage. Subsequently the atoms ¹⁹⁰ propagate freely onto a detector that consists of two micro-channel plates in a chevron configuration, ¹⁹¹ coupled to a phosphor screen and CCD camera. The measured deflection Δ_w will be proportional to ¹⁹² the weak value of the atomic spin.

3. Conclusion

The experiment described in this paper is designed to measure the real part of the weak value of spin for an atomic system. A full simulation of the process has been carried out giving a prediction of the displacement, Δ_w . A limit, L_o , has been determined defining the range over which the first order approximation holds. We have analysed and optimised the experimental parameters to achieve the largest possible displacement.

¹⁰⁹ Using the parameters of our experiment a shift, Δ_w , of the order 20 μm is predicted which is ²⁰⁰ within our experimental resolution. There is also scope to increase Δ_w by cooling the atomic beam and ²⁰¹ thus reducing the velocity of the atoms and by reducing the width of the beam. These refinements ²⁰² can increase Δ_w to 20 – 40 μm . Our experiment is designed to vary the angle θ and thereby show its ²⁰³ relationship with Δ_w , i.e. $\tan\left(\frac{\theta}{2}\right)$.

The authors would like to thank the Fetzer Franklin Fund of the John E. Fetzer Memorial Trust for their continued generous support.

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