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Cosmological constant decaying with CMB temperature

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Abstract: Recent observations of the dark energy density have demonstrated the fine-tuning problem, and the challenges faced by theoretical modeling. In this study, we apply the self-similar symmetry (SSS) model, describing the hierarchical structure of the universe based on the Dirac large numbers hypothesis, to Einstein's cosmological term. We introduce a new simi-larity dimension, D_B, in the SSS model. Using the D_B SSS model, the cosmological constant Λ is simply expressed as a function of the cosmic microwave background (CMB) temperature. The result shows that both the gravitational constant G and Λ are coupled with the CMB temper-ature, which simplifies the solution of Einstein's field equations for the variable Λ-G model.

Keywords: dark energy; dark matter; cosmic microwave background; large numbers hypothesis; varying fundamental constants; symmetry

1 Introduction

The cosmological constant problem, i.e., the dark energy problem, poses a formidable challenge in physics. In 1998, observations of distant supernovae provided evidence of the acceleration of the expansion of the universe [1, 2]. Einstein's cosmological term came to be recognized as the simplest candidate to explain the mechanism behind the accelerating universe. However, the inconsistencies between theoretical expectations and observations are extremely problematic, despite many theoretical models being proposed to provide a suitable explanation [3, 4, 5, 6, 7, 8, 9]. To approach this issue from a different aspect, an axiomatic method has been proposed by Beck [10]. Beck formulated a description of the cosmological constant, Λ, using four statistical axioms: fundamentality (Λ depends only on the fundamental constants of nature), boundedness (Λ has a lower bound, $0 < \Lambda$), simplicity (Λ is given by the simplest possible formula, consistent with the other axioms), and invariance (Λ values obtained using potentially different values of the fundamental parameters preserve the scale-invariance of the large-scale physics of the universe). Using these four axioms, Beck showed that Λ is given as follows:

$$\Lambda = \frac{G^2}{\hbar^4}\left(\frac{m_\mathrm{e}}{\alpha_f}\right)^6, \tag{1}$$

where G is the gravitational constant, $\hbar$ is the reduced Planck constant, α_f is the fine-structure constant, and m_e is the electron mass. The same formula

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has been proposed using different approaches [11, 12], and has recently been discussed in several reports [13, 14, 15, 16].

While the standard cosmological model assumes that Λ is invariant, recent observations in particle physics and cosmology indicate that Λ ought to be treated as a dynamical quantity rather than a simple constant [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. In addition, dimensional analysis [29] and the action principle [30] suggest that Λ and G cannot vary independently. Therefore, a number of cosmological models incorporating variable Λ and G have subsequently been studied [31, 32, 33, 34, 35, 36, 37, 38, 39].

The result in our earlier study [40], which explains the hierarchical structure of the universe based on the Dirac large numbers hypothesis (LNH) [41, 42], indicates that both G and m_e are coupled with the cosmic microwave background (CMB) temperature. In the present study, we applied the previous result to Eq. (1). The new result indicates that Λ can be expressed as a function of the CMB temperature. It is well known that Λ and G are both important parameters in Einstein's field equations. Our result indicates that Λ and G are not independent functions, but are coupled with the CMB temperature. This means that we can simplify the solution of Einstein's field equations for the variable Λ-G model, because we can unify the two independent variables Λ and G into one simple function of the CMB temperature.

2 D_B SSS model

The self-similar symmetry (SSS) model [40] describes the CMB with a symmetrical self-similar structure. The model consists of dimensionless values, because a physical constant with a dimension would not have universality [29]. Therefore, we define the fundamental dimensionless mass ratios of the proton mass m_{pr}, electron mass m_{e}, and Planck mass m_{Pl} as follows:

$$A = \log\alpha = \log\left(\frac{m_{\mathrm{Pl}}}{m_{\mathrm{pr}}}\right) \approx 19.11435,$$
$$B = \log\beta = \log\left(\frac{m_{\mathrm{e}}}{m_{\mathrm{pr}}}\right) \approx -3.26391. \tag{2}$$

We also define the fundamental dimensionless time and length ratios as follows:

$$T = \log\left(\frac{t}{t_{\mathrm{Pl}}}\right),\ L = \log\left(\frac{l}{l_{\mathrm{Pl}}}\right), \tag{3}$$

where t and l are the time and length scales of objects, respectively, and t_{Pl} and l_{Pl} are the Planck time and length, respectively. Using these dimensionless values, we define the similarity dimension D_A as

$$D_A = \left(\frac{T}{L}\right)^3 = \frac{A}{A+B} \approx 1.20592. \tag{4}$$

A new similarity dimension, D_B, is then introduced as follows:

$$D_B = \frac{A-B}{A+B} \approx 1.41184. \tag{5}$$

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The values of D_A and D_B correspond to the fractal dimensions for large-scale structures in the universe, reported by several authors ($D = 1.2$ [43, 44, 45, 46] and $D = 1.4$ [47]).

The hierarchical structures of the D_B SSS model are constructed according to the following sequences:

$$L_0 = 2(A + B) \approx 31.70089, \tag{6}$$

$$L_n = D_B^n L_0 \quad \text{for} \quad L > L_0, \tag{7}$$

$$L_m = (2 - D_B^m) L_0 \quad \text{for} \quad L < L_0, \tag{8}$$

where n and m are natural numbers that represent the hierarchical levels. The time scales of each hierarchy are also calculated using Eq. (4). Eqs. (7) and (8) indicate that L_n and L_m are self-similar to L_0, which corresponds to the CMB temperature [40].

3 Verification of the D_B SSS model

To verify the proposed D_B SSS model, we compared the model values with reference values. Tables 1 and 2 summarize the length and time scales of the Planck, weak, solar, and universe hierarchies. The values obtained using the D_B SSS model closely agree with the reference values. Figure 1 shows the hierarchy time scale as a function of the length scale. The coincidences observed in the figure confirm the validity of the SSS model.

Table 1: Length scales of the hierarchies of the universe.

Hierarchy	l (m)	L	D_B SSS model	Error (%)
Planck scale[a]	1.6×10^{-35}	0	0.21 ($m = 2$)	-
Weak scale[b]	10^{-16}	18.79	18.65 ($m = 1$)	-0.8
Solar scale[c]	1.4×10^{9}	43.93	44.76 ($n = 1$)	1.8
Universe scale[d]	4.1×10^{28}	63.40	63.19 ($n = 2$)	-0.3

[a] Planck length $l_{\text{Pl}} = \sqrt{\hbar G/c^3}$, where c is the speed of light in vacuum.
[b] Experimental results show that the range of the weak interaction is $r_w \leq 10^{-16}$m [48].
[c] Diameter of the sun, based on the nominal solar radius defined by the International Astronomical Union [49].
[d] Upper bound of the universe derived from the D_A SSS model [40].

4 Discussion

Using the gravitational coupling constant $\alpha_{\text{G}} = Gm_{\text{pr}}^2/\hbar c$ and Eq. (2), it is obtained that $2A = -\log \alpha_{\text{G}}$. The following relations are satisfied:

$$L_{n=1} + L_0 = 3L_0 - L_{m=1} = 4A, \tag{9}$$

$$L_{m=1} - L_0 = L_0 - L_{n=1} = 4B. \tag{10}$$

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Table 2: Time scales of the hierarchies of the universe.

Hierarchy	t (s)	T	D_B SSS model	Error (%)
Planck scale[a]	5.4×10^{-44}	0	0.23 ($m = 2$)	-
Weak scale[b]	6.6×10^{-27}	17.09	19.85 ($m = 1$)	13.9
Solar scale[c]	2.3×10^{5}	48.63	47.64 ($n = 1$)	-2.1
Universe scale[d]	1.7×10^{24}	67.49	67.26 ($n = 2$)	-0.3

[a] Planck time $t_{\rm Pl} = \sqrt{\hbar G/c^5}$.
[b] The electromagnetic and weak forces unify at 100 GeV; [50] $t = \hbar/10^{11}$ s.
[c] Sun's rotational period; [51] $t = 2.32 \times 10^5$ s.
[d] Time scale of the universe derived from the D_A SSS model [40].

Therefore, $\alpha_{\rm G}$ and β are important in forming the hierarchical structure of the universe. Regarding the similarity dimension, $D_A = (r_a - r_b)/(1 - r_b)$ (where $r_a = (D_A^3 + D_A^2 - 2)/D_A$ and $r_b = (2 - D_A^3)/(D_A^2 - D_A)$ are the ratios of the length scales of the hierarchies [40]) can be used to obtain a simple relation between r_a and r_b:

$$(\alpha\beta)^{r_a} = \alpha\beta^{r_b}. \tag{11}$$

Equation (11) can be interpreted as the basic formula for the similarity dimension, and indicates the correlation between the cosmic structure and fundamental dimensionless mass ratios [1]. Using Eq. (11), we obtain

$$D_B = \frac{2r_a - r_b - 1}{1 - r_b}. \tag{12}$$

However, the numerical relation between D_B and r_a is

$$\frac{r_a D_B}{\sqrt{2}} \approx 1.000009. \tag{13}$$

Equation (13) indicates the validity of D_B: if the D_B value is substituted into Eq. (8), then $L_{m=2}$ is consistent with the Planck length.

Regarding Λ, Eq. (1) can be written in an equivalent dimensionless form using $G = \hbar c/m_{\rm Pl}^2$:

$$l_{\rm Pl}^2 \Lambda = \alpha_f^{-6} \left(\frac{m_{\rm e}}{m_{\rm Pl}}\right)^6. \tag{14}$$

We employed the following formulas derived from the D_A SSS model [40] in Eq. (14):

$$\alpha_{\rm G} \simeq \tau_{\rm CMB}^{D_A}, \tag{15}$$
$$\beta^2 \simeq \tau_{\rm CMB}^{D_A - 1}, \tag{16}$$

where $\tau_{\rm CMB} = T_{\rm CMB}/T_{\rm Pl}$, with $T_{\rm CMB}$ being the CMB temperature and $T_{\rm Pl}$ the Planck temperature. Then, we obtain

$$\lambda(\xi) \simeq \xi^3 \tag{17}$$

[1] Using Eq. (11), we can derive another similarity dimension $D_C = -B/(A+B) \approx 0.20592$.

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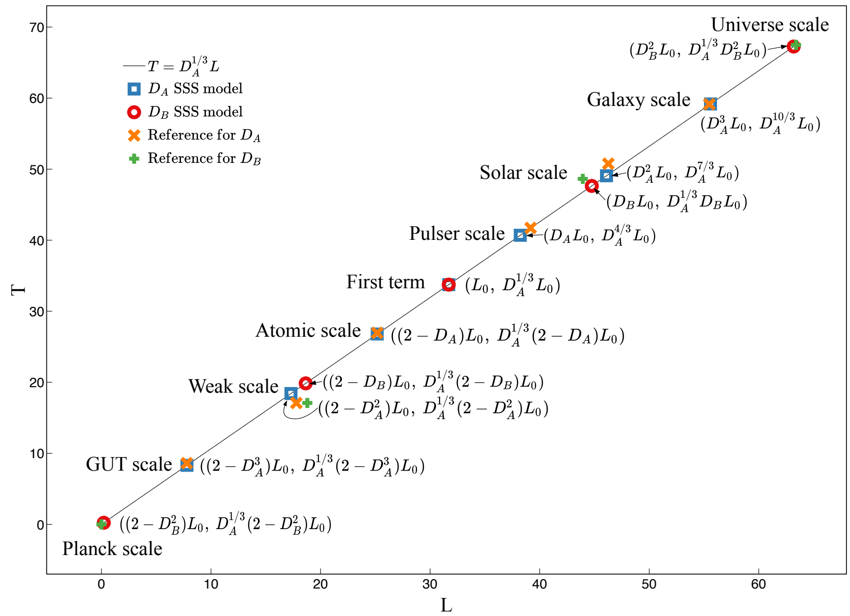

Figure 1: The time scale as a function of the length scale for the SSS model and reference values. The reference values for the D_A SSS model are taken from Ref. [40]. The lower and upper bounds of the universe are interpolated in the D_B SSS model. Note the symmetry of the first term L_0, which corresponds to the CMB temperature. This symmetry indicates that each hierarchy is self-similar to the CMB temperature.

where λ is the cosmological constant in reduced Planck units, $\lambda = l_{\text{Pl}}^2 \Lambda$, and we define $\xi \equiv \alpha_f^{-2} \alpha_{\text{G}}^{D_B/D_A} \simeq \alpha_f^{-2} \tau_{\text{CMB}}^{D_B}$. Equation (17) is based on the LNH.

If we employ $T_{\text{CMB}} = 2.725\text{K}$ as the current parameter in Eq. (17), then we obtain $\lambda_0 \approx 3.07 \times 10^{-122}$, which is consistent with the latest observational data [52].

If we adopt $T_{\text{CMB}} = T_{\text{Pl}}$ as the initial condition of the universe, and substitute this into Eqs. (15), (16), and (17), then we obtain $\alpha = \beta = 1$ and $\lambda_{T_{\text{CMB}}=T_{\text{Pl}}} = \alpha_f^{-6}$, which implies that the entire hierarchy was contained in a single point, and that a large λ can trigger cosmic inflation. The value of λ decreases with the decrease of $T_{\text{CMB}} \ll T_{\text{Pl}}$, while the size of the universe L expands according to $L \propto \log(T_{\text{Pl}}/T_{\text{CMB}})$. Assuming that $T_{\text{CMB}} \to 0$ is the ultimate fate of the universe, $L \to \infty$ and $\lambda \to 0$. This indicates that the universe falls into an inactive state as it expands to infinity.

5 Conclusions

We showed that the similarity dimension D_B based on the SSS model is valid for describing the hierarchy structure of the universe. Using D_B, Λ is expressed as a function of the CMB temperature. Linde [17] proposed that Λ is a function of temperature, and related it to the process of broken symmetries. Gasperini

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[20, 21] also argued that Λ can be interpreted as a measure of the temperature of the vacuum. Our result supports their arguments in this respect. The cosmological scenario of a dynamically decaying Λ gives a natural interpretation for sufficiently small Λ in the present epoch. Taking Eq. (17) together with Eq. (15), we can unify the two parameters Λ and G in Einstein's field equations as a single parameter of the CMB temperature. We should apply this result to Einstein's field equations to analyze further details in the future.

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