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Article

Resonance dipole-dipole interaction between two accelerated atoms in the presence of a reflecting plane boundary

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Abstract: We study the resonant dipole-dipole interaction energy between two uniformly accelerated identical atoms, one excited and the other in the ground state, prepared in a correlated *Bell-type* state, and interacting with the scalar field or the electromagnetic field nearby a perfectly reflecting plate. We suppose the two atoms moving with the same uniform acceleration, parallel to the plane boundary, and that their separation is constant during the motion. We separate the contributions of vacuum fluctuations and radiation reaction field to the resonance energy shift of the two-atom system, and show that Unruh thermal fluctuations do not affect the resonance interaction, which is exclusively related to the radiation reaction field. However, nonthermal effects of acceleration in the radiation-reaction contribution, beyond the Unruh acceleration-temperature equivalence, affect the resonance interaction energy. By considering specific geometric configurations of the two-atom system relative to the plate, we show that the presence of the mirror significantly modifies the resonance interaction energy between the two accelerated atoms. In particular, we find that new and different features appear with respect to the case of atoms in the free space, related to the presence of the boundary and to the peculiar structure of the quantum electromagnetic field vacuum in the locally inertial frame. Our results suggest the possibility to exploit the resonance interaction between accelerated atoms, as a probe for detecting the elusive effects of atomic acceleration on radiative processes.

Keywords: Dipole-dipole interaction; Unruh effect; Quantum field theory in curved space

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1. Introduction

Quantum field theory in accelerated backgrounds has led to deep insights into the fundamental notions of *vacuum* and *particle*, forcing to reconsider these basic concepts as observer-dependent notions. One of the most prominent manifestation of this feature is given by the Unruh effect [1], according to which a uniformly accelerating observer in the Minkowsky vacuum perceives vacuum field fluctuations as a *thermal bath* at a Unruh temperature proportional to its proper acceleration, a,

$$T_U = \frac{\hbar}{2\pi k_B c} a \tag{1}$$

(k_B is the Boltzmann's constant, c the speed of light, $\hbar$ the Planck constant).

An analogous effect, in a curved space-time, is the so-called Hawking radiation from a black-hole: an observer outside a black hole should experience a bath of thermal radiation at the temperature $T_H = \hbar g/(2\pi k_B c)$, g being the local acceleration due to gravity [2].

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As far as paradoxical the concept of thermal radiation from vacuum may appear, the Unruh effect is a clear manifestation of the *non-unicity* of the notion of quantum vacuum (and of particle), as extensively discussed in the seminal paper by Fulling [3] and in following papers on the subject [4,5]. This conceptually subtle effect, merging classical general relativity and quantum field theory, has been object of intense investigations in the literature, with different and sometimes conflicting conclusions on its physical interpretation [6–12]. Also, from Eq. (1) we have (cgs units)

$$T_U \sim \left(10^{-23} a\right) \text{ K}, \tag{2}$$

and therefore extremely high accelerations, of the order of 10^{23}cm/s^2, are necessary to get a Unruh thermal bath of a few Kelvin, thus making dramatically difficult the detection of this effect in the laboratory [6,8].

Many experimental proposals for detecting the Unruh effect have been discussed recently [13–18], but the effect has not been observed yet. Whilst the absence of any direct detection of the Unruh effect has induced to question the very existence of the effect [12], it has been argued that the Unruh effect is a fundamental requirement to ensure the consistency of quantum field theory [19]. Anyway, a direct verification of the effect could allow to solve some fundamental controversies about its physical interpretation.

Recently, the effects of an accelerated motion on the radiative properties of atoms/molecules in vacuum have been discussed in the literature [20–26]. Changes in the spontaneous emission rate [20,27–29] or in the Lamb-shift of single uniformly accelerating atoms [21,22], as well as the dispersion Casimir-Polder interaction between a uniformly accelerated atom and a reflecting plate [30–34] or between two uniformly accelerated atoms [35,36], have been investigated, and their relation with the Unruh effect was discussed. These studies also aimed to investigate possibility of detection of the Unruh effect through changes in the radiative properties of atoms in non-inertial motion in the vacuum space. Along this direction, it has been recently proposed that van der Waals/Casimir-Polder interactions between accelerated atoms, could be used as a probe for detecting thermal effects of the atomic acceleration [35,36]. The effect of non-equilibrium boundaries on radiative properties of atoms has been also considered [37,38].

A related question, recently addressed in the literature, is whether the effect of a uniform acceleration is truly equivalent to a thermal field and what is the physical meaning of this equivalence. For example, it has been recently shown that non-thermal features (related to a uniform acceleration) appear in the radiative properties of single accelerated atoms as well as in the resonance and van der Waals/Casimir-Polder interaction between uniformly accelerated atoms in the free space [25,26,36,39].

Motivated by these issues, in this paper we discuss the effect of a non-inertial motion on the resonance interaction energy between two atoms, moving with the same uniform acceleration in the presence of a reflecting plane boundary. As it is known, the presence of the boundary modifies vacuum field fluctuations and the density of states of the quantized radiation field, and, thus, it can significantly influence radiative properties of atoms placed nearby [40–45]. Our aim is to explore in detail physical manifestations of atomic acceleration in the radiation-mediated resonance interaction between the two atoms, located in proximity of a reflecting plate.

Resonance and dispersion Casimir-Polder interactions are long-range interactions between neutral objects such as atoms or molecules [46,47], originating from the zero-point fluctuations of the quantum electromagnetic field [47–49]. When one or more atoms are in their excited state, a resonance interaction between the atoms can occur, due to the exchange of real photons between them. If the two atoms are prepared in a factorized state, the resonance interaction is a fourth-order effect in the coupling, and scales as R^{-2} for large interatomic separations, $R \gg \lambda$ (λ is the wavelength associated to the main atomic transition and R is the interatomic distance) [50]. These interactions have been recently investigated in the literature, also in connection with some controversial results concerning the presence or not of space oscillating terms [51–54]. A different physical phenomenon occurs when two identical

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atoms are prepared in a correlated *Bell*-type state, with one atom excited and the other in the ground state, so that the excitation is delocalized between the two atoms. In this case, the resonance interaction is a second-order effect in the coupling. Such interaction is usually much more intense than dispersion interactions and can be of very long range, scaling asymptotically as R^{-1}. Resonance interactions, and the related Förster energy transfer [55], have been extensively investigated in the literature [56]. The possibility to manipulate (enhance or inhibit) the dispersion and resonance interactions through a structured environment has been also recently investigated [57–61].

We consider two atoms moving with the same uniform acceleration, in a direction parallel to a reflecting boundary, and interacting with the scalar and the electromagnetic field in the vacuum state. Following a procedure originally due to Dalibard, Dupont-Roc and Cohen-Tannoudji [62,63], we separate, at the second-order in the coupling, the contributions of vacuum fluctuations and radiation reaction field to the resonance interaction energy between the two accelerated atoms [25,39,44,64]. This approach has been recently used to investigate radiative process of atoms at rest in the presence of a boundary [44,65] or in a cosmic string spacetime [66], and it has been recently generalized to fourth order to evaluate the dispersion Casimir-Polder interaction between two atoms accelerating in the vacuum space [36]. We show that only the radiation reaction field (source field) contributes to the interatomic resonance interaction energy, while vacuum field fluctuations do not. As a consequence, the resonance interaction does not display any signature of *thermal* Unruh effect (which is exclusively related to the vacuum field fluctuations). However, we show that non-thermal effects of acceleration present in the radiation-reaction contribution, significantly influence the resonance interaction energy between the two accelerated atoms. To explore these effects, we consider two specific geometric configurations of the two-atom system: atoms aligned perpendicular or parallel to the plane boundary. We show that the presence of the mirror significantly modifies the character of the resonance interaction energy between the two accelerated atoms. By an appropriate choice of the orientation of the two dipole moments, we show that new effects of atomic acceleration (not present for atoms at rest) appear, yielding a nonvanishing resonance interaction energy even for specific configurations where the interaction for stationary atoms is zero. This result also suggests new possibilities of observing the effects of a uniform acceleration through a modification of the resonance interatomic interaction between two identical entangled atoms. Thus, our findings could have a relevance for a possible detection of effects of acceleration in radiation-mediated interaction between non-inertial atoms.

The paper is organized as follows. In Section 2, after a brief introduction of the method used, we discuss the resonance interaction energy between two accelerated atoms interacting with the relativistic scalar field, in the presence of a reflecting mirror. In Section 3 we extend our investigation to the case of atoms interacting with the electromagnetic field. Final remarks and conclusions are given in Section 4.

Throughout the paper, we adopt units such that $\hbar = c = k_B = 1$.

2. Resonance interaction between two uniform accelerated atoms: the scalar field case

Let us consider two identical atoms, A and B, interacting with the relativistic scalar field in the vacuum state and in the presence of a perfectly reflecting plate satisfying Dirichlet boundary conditions. The two atoms are modeled as point-like systems with two internal energy levels, $\mp\omega_0/2$, associated with the eigenstates $| \, g\rangle$ and $| \, e\rangle$, respectively. We suppose that the mirror is located at $z = 0$, and that the two atoms move in a direction parallel to the mirror, with the same uniform proper acceleration, perpendicular to their (constant) separation. The atom-field Hamiltonian in the multipolar coupling scheme and in the dipole approximation, in the instantaneous inertial frame of the two atoms is [25,36,48,68]

$$H = \omega_0\sigma_3^A(\tau) + \omega_0\sigma_3^B(\tau) + \sum_{\mathbf{k}} \omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \frac{dt}{d\tau} - \lambda \left(\sigma_2^A(\tau)\phi(x_A(\tau)) + \sigma_2^B(\tau)\phi(x_B(\tau))\right), \quad (3)$$

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where $\sigma_3 = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$ and $\sigma_2 = \frac{i}{2}(|g\rangle\langle e| - |e\rangle\langle g|)$ are the pseudospin atomic operators, $a^\dagger_{\mathbf{k}}$ and $a_{\mathbf{k}}$ the creation and annihilation operators of the scalar field, λ the coupling constant, and $x_\xi(\tau)(\xi = A, B)$ the trajectory of atom ξ (τ is the proper time of the atoms). Finally, $\phi(x(\tau))$ is the scalar field operator, satisfying Dirichlet boundary conditions on the surface of the plane. Eq. (3) is expressed in the comoving (i.e. locally inertial) frame of the two atoms, and we work in the Heisenberg representation.

We assume the two identical atoms prepared in one of the two correlated, symmetrical or antisymmetrical, states ($|\psi_+\rangle$ or $|\psi_-\rangle$, respectively)

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|g_A, e_B\rangle \pm |e_A, g_B\rangle). \tag{4}$$

In the Dicke model, these states are the superradiant (symmetric) and subradiant (antisymmetric) state. To investigate the resonance dipole-dipole interaction energy between the two atoms, we exploit a procedure originally due to Dalibard, Dupont-Roc and Cohen-Tannoudji [62,63], consisting in separating the contributions of vacuum fluctuations and radiation reaction field to the interaction energy between the two atoms. As discussed in [25,36,62,63], this leads to introduce an effective Hamiltonian that governs the time evolution of the atomic observables, pertaining to atom A (B), consisting of a sum of two terms (similar expressions can be obtained for atom B, by exchanging A with B),

$$(H_A^{eff})_{vf} = -\frac{i}{2}\lambda^2 \int_{\tau_0}^{\tau} d\tau' C^F(x_A(\tau), x_A(\tau'))[\sigma_2^{Af}(\tau), \sigma_2^{Af}(\tau')], \tag{5}$$

$$(H_A^{eff})_{sr} = -\frac{i}{2}\lambda^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x_A(\tau), x_A(\tau'))\{\sigma_2^{Af}(\tau), \sigma_2^{Af}(\tau')\} - \frac{i}{2}\lambda^2 \int_{\tau_0}^{\tau} d\tau' \left[\chi^F(x_A(\tau), x_B(\tau')) \times \{\sigma_2^{Af}(\tau), \sigma_2^{Bf}(\tau')\}\right], \tag{6}$$

where the functions $C^F(x_A(\tau), x_A(\tau'))$ and $\chi^F(x_A(\tau), x_A(\tau'))$ are the symmetric correlation function and the linear susceptibility of the field, respectively,

$$C^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|\{\phi(x(\tau)), \phi(x(\tau'))\}|0\rangle, \tag{7}$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|[\phi(x(\tau)), \phi(x(\tau'))]|0\rangle. \tag{8}$$

The contributions of vacuum fluctuations and radiation reaction field to the total energy shift of the two-atom system, are obtained by taking the average value of the effective Hamiltonians $(H_{A(B)}^{eff})_{vf}$ and $(H_{A(B)}^{eff})_{sr}$, on the correlated state (4),

$$(\delta E_A)_{vf} = -i\lambda^2 \int_{\tau_0}^{\tau} d\tau' C^F(x_A(\tau), x_A(\tau'))\chi^A(\tau, \tau'), \tag{9}$$

and

$$(\delta E_A)_{sr} = -i\lambda^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x_A(\tau), x_A(\tau'))C^A(\tau, \tau') - i\lambda^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x_A(\tau), x_B(\tau'))C^{AB}(\tau, \tau') \tag{10}$$

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where $\tau_0 \to -\infty$ and $\tau \to \infty$ are the initial and final times (similar expressions are obtained for atom B). $\chi^{AB}(\tau, \tau')$ and $C^{AB}(\tau, \tau')$ are respectively the antisymmetric and symmetric statistical functions of the two atoms

$$\chi^{AB}(\tau, \tau') = \frac{1}{2}\langle\psi_\pm|[\sigma_2^{Af}(\tau), \sigma_2^{Bf}(\tau')]|\psi_\pm\rangle, \quad (11)$$

$$C^{AB}(\tau, \tau') = \frac{1}{2}\langle\psi_\pm|\{\sigma_2^{Af}(\tau), \sigma_2^{Bf}(\tau')\}|\psi_\pm\rangle. \quad (12)$$

As discussed in [25], Eq. (9) does not depend on the interatomic distance, and it is the vacuum fluctuations contribution to the Lamb shift of each atom (A or B), as if the other were absent. Hence, this term does not contribute to the resonance force between the atoms. Similar considerations apply to the first term in the right-hand side of (10). On the contrary, the second term in the right-hand side of (10) depends on the distance between the two atoms, and thus it is the only contribution relevant to the interatomic interaction energy at the order considered. Therefore, the interatomic resonant energy shift is obtained as

$$\delta E = -i\int_{\tau_0}^{\tau} d\tau' \chi^F(x_A(\tau), x_B(\tau'))C^{AB}(\tau, \tau') + (A \rightleftharpoons B). \quad (13)$$

From the considerations above, it is clear that the resonance interaction is entirely related to the radiation reaction contribution. This is indeed expected from a physical point of view, since the resonance interaction is due to the exchange of a (real and virtual) photon between the two correlated atoms, and therefore it is related to the field radiated by the two atoms (source field). This property has important consequences when we consider the resonance interaction between accelerated atoms. In fact, as discussed in [25,26], this interaction energy does not display any signature of the Unruh thermal effect (which is exclusively related to the correlations of vacuum field fluctuations in the locally inertial frame). However, the atomic acceleration can determine a qualitative change of the interaction between the two atoms, not equivalent to a thermal effect.

We now apply the procedure discussed above, to evaluate the resonance interaction energy between two uniformly accelerated atoms, interacting with the vacuum scalar field in the presence of the reflecting plane boundary. In order to do that, we first evaluate the linear susceptibility of the field. In the presence of a reflecting boundary, it can be expressed as a sum of two terms, a free term (χ_0^F) that coincides with the usual one obtained in free-space, and a boundary-dependent term (χ_b^F), related to the presence of the reflecting plate [67]

$$\chi^F(x_A(\tau), x_B(\tau')) = \chi_0^F(x_A(\tau), x_B(\tau')) + \chi_b^F(x_A(\tau), x_B(\tau')), \quad (14)$$

with

$$\chi_0^F(x_A(\tau), x_B(\tau')) = \frac{i}{8\pi|\Delta\mathbf{x}_-|}[\delta(\Delta t + |\Delta\mathbf{x}_-|) - \delta(\Delta t - |\Delta\mathbf{x}_-|)], \quad (15)$$

$$\chi_b^F(x_A(\tau), x_B(\tau')) = \frac{i}{8\pi|\Delta\mathbf{x}_+|}[\delta(\Delta t + |\Delta\mathbf{x}_+|)\delta(\Delta t - |\Delta\mathbf{x}_+|)], \quad (16)$$

where we have defined $x_A(\tau) = (t, x, y, z)$, $x_B(\tau') = (t', x', y', z')$, $\Delta t = t - t'$, and $|\Delta\mathbf{x}_\mp| = [(x - x')^2 + (y - y')^2 + (z \mp z')^2]^{1/2}$.

The atomic statistical function $C^{AB}(\tau, \tau')$ can be also easily obtained [25],

$$C^{AB}(\tau, \tau') = \pm\frac{1}{8}(e^{i\omega_0(\tau-\tau')} + e^{-i\omega_0(\tau-\tau')}) \,, \quad (17)$$

where $\pm$ signs refer to the symmetric or antisymmetric states (4), respectively.

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Expression (14) is general and can be applied to different situations, for example two atoms at rest in the presence of a mirror or uniformly accelerating near a plane boundary, provided the appropriate atomic trajectories, $x_A(\tau)$ and $x_B(\tau)$, are given.

We now specialize our considerations to two specific cases. We suppose the mirror located at $z = 0$ and assume that the two atoms accelerate in the half-space $z > 0$, with the same proper uniform acceleration, parallel to the reflecting plate. In this case the distance between the atoms is constant. We will consider two different geometric configurations of the two-atom system with respect to the plate: two atoms aligned along the z-axis, perpendicular to the boundary, and two atoms aligned in a direction parallel to the plate. This will permit to simplify our calculation and to discuss some relevant effects of the presence of the plate on the resonant interaction energy between the two accelerated atoms.

2.1. *Atoms aligned perpendicularly to the plate*

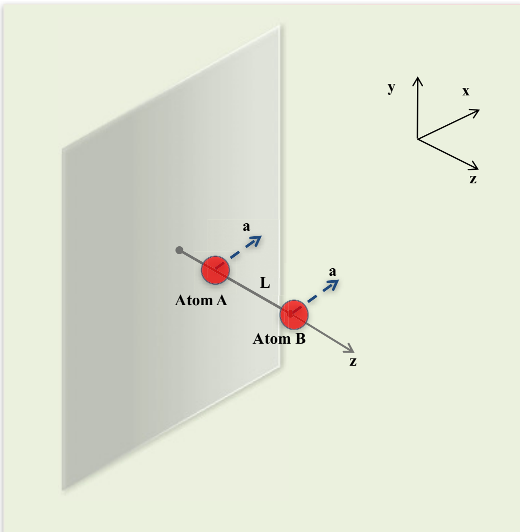

Figure 1. Pictorial description of the first geometrical configuration considered for the physical system: two atoms aligned along the z axis, perpendicular to the plate, and uniformly accelerating along the x direction.

Let first consider both atoms located along the z direction, perpendicular to the surface, as shown in Figure 1, and uniformly accelerating along the x direction, perpendicular to their (constant) separation. In the instantaneous locally inertial frame of the two-atom system, the atomic trajectories, as a function of the proper time τ of both atoms, are

$$t_A(\tau) = t_B(\tau) = \frac{1}{a}\sinh(a\tau), \quad x_A(\tau) = x_B(\tau) = \frac{1}{a}\cosh(a\tau),$$
$$y_A = y_B = 0, \quad z_A = z, \, z_B = z + L\,. \tag{18}$$

In order to obtain the distance-dependent energy shift of the two-atom system, we first evaluate the linear susceptibility of the scalar field for the atomic trajectories (18). Substituting Eq. (18) in the expressions of the linear susceptibility of the scalar field (Eqs. (15) and (16)), we obtain

$$\chi^F_\perp(x_A(\tau), x_B(\tau')) = -\frac{1}{8\pi^2}\int_0^\infty d\omega(e^{i\omega\Delta\tau} - e^{-i\omega\Delta\tau})\left(\frac{\sin(\frac{2\omega}{a}\sinh^{-1}(\frac{aL}{2}))}{L\sqrt{1+\frac{1}{4}a^2L^2}} - \frac{\sin(\frac{2\omega}{a}\sinh^{-1}(\frac{a\mathcal{R}}{2}))}{\mathcal{R}\sqrt{1+\frac{1}{4}a^2\mathcal{R}^2}}\right), \tag{19}$$

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where $\Delta\tau = \tau - \tau'$, L is the interatomic distance, and $\mathcal{R} = z_A + z_B = L + 2z$ is the distance between one atom and the image of the second atom with respect to the mirror.

The resonance dipole-dipole interaction between the two accelerated atoms is then obtained by substituting (17) and (19) into (13). We get

$$\delta E_\perp(z, L, a) = \mp \frac{\lambda^2}{16\pi} \left[\frac{\cos(\frac{2\omega_0}{a} \sinh^{-1}(\frac{aL}{2}))}{L\sqrt{1+\frac{1}{4}a^2L^2}} - \frac{\cos(\frac{2\omega_0}{a} \sinh^{-1}(\frac{a\mathcal{R}}{2}))}{\mathcal{R}\sqrt{1+\frac{1}{4}a^2\mathcal{R}^2}} \right], \tag{20}$$

where the $\mp$ sign refers to the symmetric or antisymmetric superposition of the atomic states, respectively.

The expression above describes the resonance dipole-dipole interaction energy, as a function of the proper acceleration of the two atoms and the atom-plate distances. It reduces to that for atoms at rest in the limit $a \to 0$. It consists of two terms: a term coinciding with the resonance interaction between two accelerating atoms in the free space, discussed in [25], and a new term, depending from $\mathcal{R}$, which is related to the presence of the mirror. The latter term, that describes the effect of the boundary on the resonance interaction energy shift, originates from the interaction of one atom (for example atom A) with the image of the other atom (B), with respect to the mirror. When both atoms are very distant from the reflecting boundary, the boundary-dependent term in (20) goes to zero, and we recover the resonance interaction between two atoms accelerating in free-space [25]. On the other hand, when the atoms are very close to the mirror, and we can approximate $\mathcal{R} \sim L$, the resonance interaction is strongly suppressed. Thus, in this limit the interaction between the two entangled atoms can be strongly inhibited by means of the nearby plate, analogously to the case of atoms at rest discussed in [44].

Most importantly, Eq. (20) shows that the effects of the atomic acceleration are not in the form of *thermal* terms. Nevertheless, the relativistic acceleration significantly affect the interaction energy, giving a different scaling of the interaction with the interatomic distance. In fact, in analogy to the results in [25,36] for atoms accelerating in the unbounded space, we can identify a characteristic length scale related to the acceleration, $z_a = 1/a$, at which curvature effects must be considered. For distances larger than z_a the effects of relativistic acceleration can significantly change the interaction between the two non-inertial atoms; in fact, when $\mathcal{R} > L \gg z_a$, we get

$$\delta E_\perp(z, L, a) \sim \mp \frac{\lambda^2}{8\pi a} \left[\frac{1}{L^2} \cos(\frac{2\omega_0}{a} \ln(\frac{aL}{2})) - \frac{1}{\mathcal{R}^2} \cos(\frac{2\omega_0}{a} \ln(\frac{a\mathcal{R}}{2})) \right], \tag{21}$$

while in the *near*-zone limit, $\mathcal{R}, L \ll z_a$, we recover the well-known result for inertial (static) atoms

$$\delta E_\perp(z, L, a) \sim \mp \frac{\lambda^2}{16\pi} \left[\frac{1}{L} \cos(\omega_0 L) - \frac{1}{\mathcal{R}} \cos(\omega_0 \mathcal{R}) \right]. \tag{22}$$

In the intermediate zone, $\mathcal{R} \gg z_a \gg L$, when the separation between the two atoms is smaller than the characteristic length z_a, but their distance from the mirror is such that $\mathcal{R} \gg z_a$, we obtain

$$\delta E_\perp(z, L, a) \sim \mp \frac{\lambda^2}{8\pi} \left[\frac{1}{2L} \cos(\omega_0 L) - \frac{1}{a\mathcal{R}^2} \cos(\frac{2\omega_0}{a} \ln(\frac{a\mathcal{R}}{2})) \right], \tag{23}$$

Thus the relativistic acceleration and the presence of the boundary can significantly affect the qualitative features of the resonance interaction, in particular its power-law distance dependence, decreasing at large distances more rapidly than in the usual inertial case. Also, in the presence of a boundary, the noninertial character of acceleration modifies the interatomic interaction energy, even when the separation between the two atoms is much smaller then z_a. Actually, such a result can be expected on a physical ground: the boundary-dependent term, as mentioned, can be interpreted as the interaction of one atom with the image of the other atom with respect to the plate. When the atoms are

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accelerating, the distance traveled by the photon emitted by one atom to reach the other atom, after reflection from the mirror, increases with time; if $R \gg z_a$, this effect becomes relevant and causes an overall decrease of the interaction strength between the two atoms.

2.2. Atoms aligned parallel to the plate

We now investigate if effects similar to those found in the previous subsection, manifest also for different geometric configurations of the atoms-plate system. To this purpose, we consider two atoms aligned in the y direction, parallel to the mirror, as shown in Figure 2, and uniformly accelerating in the x direction, perpendicular to their (constant) separation. In this case, the atomic trajectories are

$$\begin{aligned} &t_A(\tau) = t_B(\tau) = \frac{1}{a}\sinh(a\tau), \;\; x_A(\tau) = x_B(\tau) = \frac{1}{a}\cosh(a\tau), \\ &y_A = 0, \;\; y_B = D, \;\; z_A = z_B = z, \end{aligned} \tag{24}$$

with $D > 0$.

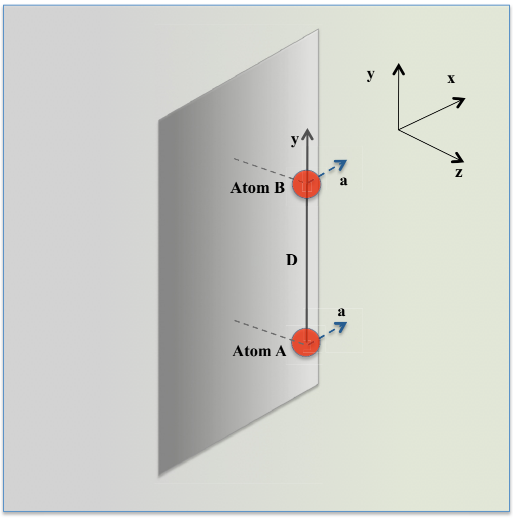

Figure 2. Pictorial description of the second geometrical configuration considered for the physical system: two atoms aligned along the y axis, parallel to the plate, and uniformly accelerating along the x direction.

Following the same procedure of the previous subsection, we first obtain the linear susceptibility of the scalar field

$$\chi_{\parallel}^{F}(x_A(\tau), x_B(\tau')) = -\frac{1}{8\pi^2}\int_0^\infty d\omega (e^{i\omega\Delta\tau} - e^{-i\omega\Delta\tau})\left(\frac{\sin(\frac{2\omega}{a}\sinh^{-1}(\frac{aD}{2}))}{D\sqrt{1+\frac{1}{4}a^2D^2}} - \frac{\sin(\frac{2\omega}{a}\sinh^{-1}(\frac{aR}{2}))}{R\sqrt{1+\frac{1}{4}a^2R^2}}\right), \tag{25}$$

where D is the interatomic distance, $\Delta\tau = \tau - \tau'$, and we have defined $R = R(z, D) = \sqrt{D^2 + 4z^2}$.

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Substitution of (25) and (17) into (13) yields, after algebraic calculations, the resonance dipole-dipole interaction between the two accelerated atoms

$$\delta E_{\parallel}(z,D,a) = \mp\frac{\lambda^2}{16\pi}\left[\frac{\cos(\frac{2\omega_0}{a}\sinh^{-1}(\frac{aD}{2}))}{D\sqrt{1+\frac{1}{4}a^2D^2}} - \frac{\cos(\frac{2\omega_0}{a}\sinh^{-1}(\frac{aR}{2}))}{R\sqrt{1+\frac{1}{4}a^2R^2}}\right]. \tag{26}$$

As before, we find that the resonance interaction energy consists of two terms. The first term in the right-hand side of (26) is that for atoms uniformly accelerating in free-space [25], while the second new term is related to the presence of the boundary. In the static (inertial) limit, we recover the expression of the resonance interaction between two atoms at rest near the mirror in the configuration considered [44],

$$\delta E_{\parallel}(z,D) = \mp\frac{\lambda^2}{16\pi}\left[\frac{\cos(\omega_0 D)}{D} - \frac{\cos(\omega_0\sqrt{D^2+4z^2})}{\sqrt{D^2+4z^2}}\right]. \tag{27}$$

It is worth to note that the expression of $\delta E_{\parallel}(z,D,a)$ given by Eq. (26) is formally equal to that obtained for $\delta E_{\perp}(z,L,a)$ in (20)), provided $\mathcal{R}$ is replaced by R. This is indeed expected, since the distance $R=\sqrt{D^2+4z^2}$ is the distance between one atom and the image of the other one with respect to the mirror. In order to compare the results obtained in the two geometric configurations, in Fig. 3 we have plotted the expressions (20) and (26) of the resonance interaction, as a function of the atomic acceleration. The plots show that the resonance interaction significantly depends from the geometric configuration of the two atoms with respect to the plate (perpendicular or parallel alignment), and that it can be enhanced or suppressed depending on the atomic acceleration.

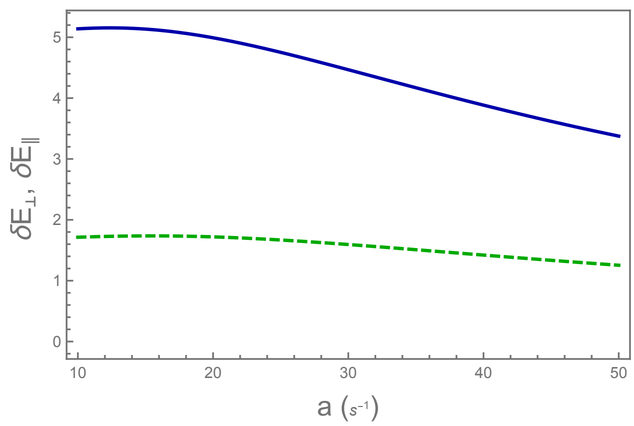

Figure 3. The resonance interaction energy between the two atoms (arbitrary units), as a function of the atomic acceleration, for two different geometric configurations. Blue continuous line: atoms aligned along the z axis, that is perpendicular to the plate. Green dashed line: atoms in the y axis, that is parallel to the plate. The plots show that the resonance interaction between accelerated atoms depends significantly on the geometric configuration of the two-atom system with respect to the mirrors. Parameters, in the units used, are chosen such that: $L = D = 7.5\times10^{-2}\,\mathrm{eV}^{-1}$, $z = 2.0\times10^{-2}\,\mathrm{eV}^{-1}$, $\omega_0 = 4.17\,\mathrm{eV}^{-1}$.

3. Resonance interaction between two accelerated atoms: the electromagnetic field case

We now extend our investigations to the case of two uniformly accelerated identical atoms interacting with the electromagnetic field in the vacuum state, in the presence of a perfectly reflecting boundary. As before, the atoms have a uniform acceleration a in a direction parallel to the plane boundary, located at $z=0$, and are separated by a constant distance. Our aim is to discuss if new and further effects of acceleration may manifest in the resonance interaction, as a consequence of the vector nature of the electromagnetic field.

The Hamiltonian of the atom-field interacting system in the instantaneous inertial (comoving) frame of both atoms is

$$H = \omega_0 \sigma_3^A(\tau) + \omega_0 \sigma_3^B(\tau) + \sum_{\mathbf{k},\lambda} \omega_k a^\dagger_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda} \frac{dt}{d\tau} - \boldsymbol{\mu}_A(\tau) \cdot \mathbf{E}(x_A(\tau)) - \boldsymbol{\mu}_B(\tau) \cdot \mathbf{E}(x_B(\tau)), \tag{28}$$

where $\lambda = 1,2$ is the polarization index, $\boldsymbol{\mu} = e\mathbf{r}$ the atomic dipole moment operator (restricted to the subspace of the two atomic levels considered), and $\mathbf{E}(x(\tau))$ the electric field operator, satisfying the appropriate boundary conditions on the reflecting plate.

As shown in the previous Section, the resonant interaction energy is related only to the radiation-reaction contribution and can be obtained by evaluating the expectation value of the effective Hamiltonian $(H_A^{eff})_{sr} + (H_B^{eff})_{sr}$ (of atoms A and B, respectively) on the correlated state $|\psi_\pm\rangle$, taking only terms depending on the interatomic distance,

$$\delta E = -i \int_{\tau_0}^{\tau} d\tau' \chi^F_{ij}(x_A(\tau), x_B(\tau')) C^{AB}_{ij}(\tau,\tau') + (A \rightleftharpoons B) \tag{29}$$

$(i,j = x,y,z)$, where the atomic symmetric correlation function C^{AB}_{ij} and the electromagnetic field susceptibility χ^F_{ij} will be now obtained.

The susceptibility of the electromagnetic field in the accelerated frame can be obtained from the two-point field correlation function in the proper reference frame of the two accelerated atoms (Rindler noise) [67]. In the presence of a perfectly reflecting boundary, the two-point correlation function of the electric field operator

$$g_{ij}(x_A, x_B) = \langle 0|E_i(x_A)E_j(x_B)|0\rangle \tag{30}$$

can be expressed as the sum of a free part, $g^{(0)}_{ij}(x_A, x_B)$, and a boundary-dependent term, $g^{(b)}_{ij}(x_A, x_B)$,

$$g_{ij}(x_A, x_B) = g^{(0)}_{ij}(x_A, x_B) + g^{(b)}_{ij}(x_A, x_B), \tag{31}$$

where

$$g^{(0)}_{ij}(x_A, x_B) = -\frac{1}{4\pi^2}(\delta_{ij}\partial_0\partial_{0'} - \partial_i\partial_{j'})\frac{1}{(\Delta t - i\epsilon)^2 - |\Delta\mathbf{x}_-|^2}, \tag{32}$$

$$g^{(b)}_{ij}(x_A, x_B) = \frac{1}{4\pi^2}[(\delta_{ij} - 2n_i n_j)\partial_0\partial_{0'} - \partial_i\partial_{j'}]\frac{1}{(\Delta t - i\epsilon)^2 - |\Delta\mathbf{x}_+|^2}, \tag{33}$$

and $\mathbf{n}$ is the unit vector along the line joining the two atoms.

As in the previous section, we now specialize our considerations to the two specific configurations considered for the scalar field case and illustrated in Figures (1) and (2), that is atoms aligned in a direction perpendicular or parallel to the plate, respectively.

3.1. Atoms aligned perpendicularly to the plate

We first consider two atoms aligned along the z direction, perpendicular to the boundary, and uniformly accelerating along the x direction, as shown in Figure (1). Thus they moves on the trajectory given by Eq. (18). Due to the vector structure of the electromagnetic field, the calculation of the field susceptibility turn out to be more complicated than for the scalar field [67]. After lengthy algebraic calculations, involving Lorentz transformations of the electromagnetic field to the comoving reference system, we obtain (in the locally inertial frame of the two atoms),

$$g_{\perp_{ij}}(x_A, x_B) = g^{(0)}_{\perp_{ij}}(x_A, x_B) + g^{(b)}_{\perp_{ij}}(x_A, x_B), \tag{34}$$

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where

$$g^{(0)}_{\perp_{ij}}(x_A,x_B) = \frac{a^4}{16\pi^2}\frac{1}{(\sinh^2(\frac{a}{2}(\Delta\tau-i\epsilon))-\frac{1}{4}a^2L^2)^3}\times\Big\{\frac{1}{4}a^2L^2(\delta_{ij}-2n_in_j) + \Big[\delta_{ij}+\frac{1}{2}a^2L^2(\delta_{ij}-k_ik_j-2n_in_j)+aL(k_in_j-k_jn_i)\Big]\sinh^2\Big(\frac{a}{2}\Delta\tau\Big)\Big\} \quad (35)$$

is the two-point correlation function of two atoms uniformly accelerated in vacuum [25], and

$$g^{(b)}_{\perp_{ij}}(x_A,x_B) = -\frac{a^4}{16\pi^2}\frac{(1-2n_in_j)}{(\sinh^2(\frac{a}{2}(\Delta\tau-i\epsilon))-\frac{1}{4}a^2\mathcal{R}^2)^3}\times\Big\{\frac{1}{4}a^2\mathcal{R}^2(\delta_{ij}-2n_in_j) + \Big[\delta_{ij}+\frac{1}{2}a^2\mathcal{R}^2(\delta_{ij}-k_ik_j-2n_in_j)+a\mathcal{R}(k_in_j+k_jn_i)\Big]\sinh^2\Big(\frac{a}{2}\Delta\tau\Big)\Big\} \quad (36)$$

is the contribution due to the presence of the boundary. In the equations above, $\mathbf{k}=(1,0,0)$ is the unit vector along the direction of the acceleration. As discussed in Ref. [25], the function $g_{\perp_{ij}}(x_A,x_B)$ evaluated on the atomic trajectories of the two non inertial atoms is not isotropic and displays a nondiagonal component. Actually, in the present case, we have two characteristic directions in space, namely the direction of the acceleration and that perpendicular to the plate. Similar anisotropies were already found in the case of a single uniformly accelerated atom near a boundary [31], or two accelerated atoms in the free-space [25]. They can be ascribed to the spatially extended structure of the two-atom-plate system here considered, as well as to the vector nature of the electromagnetic field. This peculiarity, as we will now show, has deep consequences on the resonance interaction between the two atoms.

In order to evaluate the resonance energy, we first focus our attention on the boundary-dependent term, and calculate the linear susceptibility of the electric field. Using Eq. (36), after lengthly algebraic calculations, involving a Fourier transform of the statistical function of the field, we finally get

$$\chi^{F(b)}_{\perp_{ij}}(x_A(\tau),x_B(\tau')) = \frac{1}{8\pi^2}\int_0^\infty d\omega(e^{i\omega\Delta\tau}-e^{-i\omega\Delta\tau})\Big(f^{\perp(b)}_{ij}(a,\mathcal{R},\omega)\cos\Big(\frac{2\omega}{a}\sinh^{-1}\Big(\frac{a\mathcal{R}}{2}\Big)\Big) + h^{\perp(b)}_{ij}(a,\mathcal{R},\omega)\sin\Big(\frac{2\omega}{a}\sinh^{-1}\Big(\frac{a\mathcal{R}}{2}\Big)\Big)\Big), \quad (37)$$

where we have introduced the functions $f^{\perp(b)}_{ij}(a,\mathcal{R},\omega)$ and $h^{\perp(b)}_{ij}(a,\mathcal{R},\omega)$ given in the Appendix A [see Eqs. (A1) and (A2)].

Substituting (37) and the atomic symmetric statistical function,

$$C^{AB}_{ij}(\tau,\tau') = \frac{1}{2}(\mu^A_{ge})_i(\mu^B_{ge})_j(e^{i\omega_0\Delta\tau}+e^{-i\omega_0\Delta\tau}), \quad (38)$$

into Eq. (29), we finally obtain the boundary-dependent contribution to the resonant energy shift of the two accelerated atoms

$$\delta E^{(b)}_{\perp} = \mp\frac{1}{4\pi}[\delta_{ij}(\mu^A_{ge})_i(\mu^B_{eg})_jP^{\perp(b)}_{ij}(a,\mathcal{R},\omega_0)\pm((\mu^A_{ge})_x(\mu^B_{eg})_z+(\mu^A_{ge})_z(\mu^B_{eg})_x)P^{\perp(b)}_{xz}(a,\mathcal{R},\omega_0)], \quad (39)$$

where we introduced the function $P^{\perp(b)}_{ij}(a,\mathcal{R},\omega_0)$

$$P^{\perp(b)}_{ij}(a,\mathcal{R},\omega_0) = f^{\perp(b)}_{ij}(a,\mathcal{R},\omega_0)\sin\Big(\frac{2\omega_0}{a}\sinh^{-1}\Big(\frac{a\mathcal{R}}{2}\Big)\Big) - h^{\perp(b)}_{ij}(a,\mathcal{R},\omega_0)\cos\Big(\frac{2\omega_0}{a}\sinh^{-1}\Big(\frac{a\mathcal{R}}{2}\Big)\Big), \quad (40)$$

which modulates the interaction, as a function of the distance $\mathcal{R}$ and the atomic acceleration.

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With a similar procedure, evaluation of the boundary-independent contribution, $\delta E_\perp^{(0)}$, to the resonance interaction energy yields [25]

$$\delta E_\perp^{(0)} = \pm \frac{1}{4\pi}[\delta_{ij}(\mu_{ge}^A)_i(\mu_{eg}^B)_j \mathrm{P}_{ij}^{\perp(0)}(a, L, \omega_0) \pm ((\mu_{ge}^A)_x(\mu_{eg}^B)_z - (\mu_{ge}^A)_z(\mu_{eg}^B)_x)\mathrm{P}_{xz}^{\perp(0)}(a, L, \omega_0)], \quad (41)$$

where

$$\mathrm{P}_{ij}^{\perp(0)}(a, L, \omega_0) = \mathrm{f}_{ij}^{\perp(0)}(a, L, \omega_0)\sin\left(\frac{2\omega_0}{a}\sinh^{-1}\left(\frac{aL}{2}\right)\right) - \mathrm{h}_{ij}^{\perp(0)}(a, L, \omega_0)\cos\left(\frac{2\omega_0}{a}\sinh^{-1}\left(\frac{aL}{2}\right)\right), \quad (42)$$

and the functions $f_{ij}^{\perp(0)}(a, L, \omega)$ and $h_{ij}^{\perp(0)}(a, l, \omega)$ are given by Eqs. (A3) and (A4 of Appendix A).

The complete resonance interaction energy of the accelerated two-atom system is then obtained by summing expressions (39) and (41),

$$\delta E_\perp = \delta E_\perp^{(0)} + \delta E_\perp^{(b)}. \quad (43)$$

The result (43) is valid for any value of the parameters a, L and $\mathcal{R}$. It is easy to show that in the *near-zone* limit, $L \ll a^{-1}$ and $\mathcal{R} \ll a^{-1}$, the linear susceptibility is well described by its stationary counterpart, and we recover the expression of the resonance interaction for two atoms at rest [25,44]. On the other hand, at higher orders in $a\mathcal{R}$ (and/or aL), corrections related to the atomic accelerated motion become relevant, yielding a change of the distance dependence of the resonance interaction, in analogy to the scalar-field case discussed in the previous Section. Interestingly, a comparison with the scalar-field case shows the emergence of new features in the resonance interaction, originating from the presence of the boundary, and related to the anisotropic structure of the susceptibility of the electromagnetic field. In fact, from Eq. (39) it follows that the effect of the atomic acceleration on the resonance interaction can be enhanced or inhibited through an appropriate choice of the orientation of the two dipoles, as well as of the distance of the two atoms from the plate. For example, when the dipole moments are orthogonal to each other, with one of them along x and the other along z, the diagonal term in (39) vanishes, and only the second (nondiagonal) term survives. The nondiagonal term is present only for $a \neq 0$, and thus its contribution is a peculiar characteristic of the accelerated motion, giving in this specific configuration a nonvanishing interaction energy, while the interaction for stationary atoms is zero. This term is thus a signature of the accelerated motion.

The results above suggest to investigate if similar effects of acceleration manifest also for other geometric configurations of the two-atom system, for example when both atoms are aligned parallel to the reflecting plane boundary. This configuration will be considered in the next subsection.

3.2. Atoms aligned parallel to the plate

We now consider the configuration of two atoms aligned along the y direction, parallel to the boundary, which move with uniform acceleration along the x-direction, so that their trajectories are those given in (24). As before, the distance between the two atoms remains constant during their motion. This configuration is illustrated in Fig. 2.

The two-point correlation function of the field in the locally inertial frame of both atoms is

$$g_{\|_{ij}}(x_A, x_B) = g_{\|_{ij}}^{(0)}(x_A, x_B) + g_{\|_{ij}}^{(b)}(x_A, x_B) , \quad (44)$$

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where $g^{(0)}_{\|ij}(x_A, x_B)$ is the two-point correlation function in free-space [25], and $g^{(b)}_{\|ij}(x_A, x_B)$ is the boundary-dependent contribution, that consists of a diagonal term

$$g^{(b)}_{\|ij}(x_A, x_B) = -\frac{a^4}{16\pi^2}\frac{(\delta_{ij}-2n_in_j)}{(\sinh^2(\frac{a}{2}(\Delta\tau - i\epsilon)) - \frac{1}{4}a^2R^2)^3}\left\{\frac{1}{4}a^2\tilde{R}^2(n_in_j - p_ip_j) + \frac{1}{4}a^2R^2k_ik_j + \left[1+\frac{1}{2}a^2\tilde{R}^2(1-k_ik_j-2p_ip_j)\right]\sinh^2\left(\frac{a}{2}\Delta\tau\right)\right\} \quad (i=j)\,, \tag{45}$$

which is non-vanishing only for $i = j$, and a non-diagonal term

$$g^{(b)}_{\|ij}(x_A, x_B) = -\frac{a^4}{16\pi^2}\frac{1}{(\sinh^2(\frac{a}{2}(\Delta\tau - i\epsilon)) - \frac{1}{4}a^2R^2)^3}\left\{-a^2zD(p_in_j - p_jn_i) + [aD(k_ip_j - k_jp_i) + 2az(k_in_j + k_jn_i) - 2a^2zD(p_in_j - p_jn_i)]\sinh^2\left(\frac{a}{2}\Delta\tau\right)\right\} \quad (i \neq j), \tag{46}$$

which is different from zero only for $i \neq j$. We have here introduced the unit vector $\mathbf{p} = (0,1,0)$ and the distances $R = \sqrt{D^2+4z^2}$, $\tilde{R} = \sqrt{D^2-4z^2}$). The boundary-dependent contribution to the linear susceptibility of the field, is then obtained as

$$\chi^{F(b)}_{\|ij}(x_A(\tau), x_B(\tau')) = \frac{1}{8\pi^2}\int_0^\infty d\omega(e^{i\omega\Delta\tau} - e^{-i\omega\Delta\tau})\left(f^{\|(b)}_{ij}(a,D,z,\omega)\cos\left(\frac{2\omega}{a}\sinh^{-1}\left(\frac{aR}{2}\right)\right) + h^{\|(b)}_{ij}(a,D,z,\omega)\sin\left(\frac{2\omega}{a}\sinh^{-1}\left(\frac{aR}{2}\right)\right)\right), \tag{47}$$

where the functions $f^{\|(b)}_{ij}(a,D,z,\omega)$ and $h^{\|(b)}_{ij}(a,D,z,\omega)$, given in Eqs. (A5) and (A6) of Appendix A, modulate the resonance interaction energy with the distance D and the atomic acceleration a.

Using Eqs. (47) and (38) in Eq. (29), we find the boundary-dependent contribution to the resonant energy shift

$$\delta E^{(b)}_{\|} = -\frac{1}{4\pi}\Big[\delta_{ij}(\mu^A_{ge})_i(\mu^B_{eg})_jP^{\|(b)}_{ij}(a,D,z,\omega_0) + \left((\mu^A_{ge})_x(\mu^B_{eg})_y - (\mu^A_{ge})_y(\mu^B_{eg})_x\right)P^{\|(b)}_{xy}(a,D,z,\omega_0) + \left((\mu^A_{ge})_x(\mu^B_{eg})_z + (\mu^A_{ge})_z(\mu^B_{eg})_x\right)P^{\|(b)}_{xz}(a,D,z,\omega_0) + \left((\mu^A_{ge})_y(\mu^B_{eg})_z - (\mu^A_{ge})_z(\mu^B_{eg})_y\right)P^{\|(b)}_{yz}(a,D,z,\omega_0)\Big]\,, \tag{48}$$

where

$$P^{\|(b)}_{ij}(a,D,z,\omega_0) = f^{\|(b)}_{ij}(a,D,z,\omega_0)\sin\left(\frac{2\omega_0}{a}\sinh^{-1}\left(\frac{aR}{2}\right)\right) - h^{\|(b)}_{ij}(a,D,z,\omega_0)\cos\left(\frac{2\omega_0}{a}\sinh^{-1}\left(\frac{aR}{2}\right)\right). \tag{49}$$

The resonance interaction energy between the two accelerating atoms is finally obtained by adding (48) to the known interaction energy in absence of the boundary $\delta E^{(0)}_{\|}$, which is given by [25]

$$\delta E^{(0)}_{\|} = \frac{1}{4\pi}\left[\delta_{ij}(\mu^A_{ge})_i(\mu^B_{eg})_j\mathrm{P}^{\|(0)}_{ij}(a,D,\omega_0) + \left((\mu^A_{ge})_x(\mu^B_{eg})_y - (\mu^A_{ge})_y(\mu^B_{eg})_x\right)\mathrm{P}^{\|(0)}_{xy}(a,D,\omega_0)\right]\,, \tag{50}$$

with

$$\mathrm{P}^{\|(0)}_{ij}(a,D,\omega_0) = \mathrm{f}^{\|(0)}_{ij}(a,D,\omega_0)\sin\left(\frac{2\omega_0}{a}\sinh^{-1}\left(\frac{aR}{2}\right)\right) - \mathrm{h}^{\|(0)}_{ij}(a,D,\omega_0)\cos\left(\frac{2\omega_0}{a}\sinh^{-1}\left(\frac{aR}{2}\right)\right)\,, \tag{51}$$

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(the functions $\mathrm{f}_{ij}^{\parallel(0)}(a,D,\omega)$ and $\mathrm{h}_{ij}^{\parallel(0)}(a,D,\omega)$ can be obtained from Eqs. (A3) and (A4) in Appendix A by exchanging subscripts z and y).

A comparison with the case of accelerated atoms aligned along the z axis, considered in the previous subsection, shows the emergence of a new effect, related to the specific geometric configuration of the two-atom system with respect to the plane boundary. In fact, from the equations above, it follows that when the dipole moments are orthogonal to each other, one of them along y and the other in the plane xz, a new nonvanishing contribution to the interaction energy (not present for atoms located perpendicular to the boundary) arises. This contribution is present only for $a \neq 0$, and thus it is peculiar of an accelerated motion in the specific configuration considered. This gives new additional possibilities to exploit the resonance interaction between accelerated atoms as a probe for detecting nonthermal effects of acceleration and, in general, physical effects of the accelerated motion on radiation-mediated interactions between atoms.

4. Summary

We have discussed the resonance interaction between two identical atoms, one excited and the other in the ground-state, prepared in a *Bell-type* state, and moving with uniform acceleration, in the presence of a perfectly reflecting plate. The atoms interact with the massless scalar field or the electromagnetic field in the vacuum state. We have considered the contributions of vacuum fluctuations and radiation-reaction field to the resonance interaction, and shown that the Unruh thermal fluctuations do not affect the resonance interatomic interaction, which is exclusively given by the radiation-reaction term (source field). We have shown that, in both cases of scalar and electromagnetic fields, the presence of the boundary significantly affects the resonance interaction between the accelerated atoms, and that nonthermal effects related to the atomic acceleration appear, yielding a change of the distance dependence of the resonance interaction. Finally, in the case of electromagnetic field we have shown, for specific geometric configurations of the two-atom-plate system, the emergence of new and different effects in the resonance interaction energy, for example a nonvanishing interaction energy in configurations/dipole orientations where the interaction is zero for inertial atoms. These effects, not present for atoms at rest, therefore provide a unique signature of the non inertial motion of the two-atom system. These findings could be exploited for a detection of the non-thermal effects of atomic acceleration in radiation-mediated interactions between non-inertial atoms.

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Appendix A

In this Appendix, we give the expressions of the functions $f_{ij}^{\perp(\parallel)}$ and $h_{ij}^{\perp(\parallel)}$ used in Section 3.

The explicit expressions of the functions $f_{ij}^{\perp(b)}(a,\mathcal{R},\omega)$ and $h_{ij}^{\perp(b)}(a,\mathcal{R},\omega)$ are

$$\begin{cases} f_{xx}^{\perp(b)} = \frac{\omega(1+a^2\mathcal{R}^2)}{\mathcal{N}^4\mathcal{R}^2}, \\ f_{yy}^{\perp(b)} = \frac{\omega(1+\frac{1}{2}a^2\mathcal{R}^2)}{\mathcal{N}^2\mathcal{R}^2}, \\ f_{zz}^{\perp(b)} = \frac{\omega(2+\frac{1}{4}a^2\mathcal{R}^2+\frac{1}{8}a^4\mathcal{R}^4)}{\mathcal{N}^4\mathcal{R}^2}, \\ f_{xz}^{\perp(b)} = f_{zx}^{\perp(b)} = -\frac{a\omega(1-\frac{1}{2}a^2\mathcal{R}^2)}{2\mathcal{N}^4\mathcal{R}}, \end{cases} \tag{A1}$$

$$\begin{cases} h^{\perp(b)}_{xx} &= -\frac{1+\frac{1}{2}a^2\mathcal{R}^2+\frac{1}{4}a^4\mathcal{R}^4}{\mathcal{N}^5\mathcal{R}^3} + \frac{\omega^2}{\mathcal{N}^3\mathcal{R}}, \\ h^{\perp(b)}_{yy} &= -\frac{1}{\mathcal{N}^3\mathcal{R}^3} + \frac{\omega^2}{\mathcal{N}\mathcal{R}}, \\ h^{\perp(b)}_{zz} &= -\frac{2(1+\frac{5}{8}a^2\mathcal{R}^2)}{\mathcal{N}^5\mathcal{R}^3} + \frac{a^2\mathcal{R}\omega^2}{4\mathcal{N}^3}, \\ h^{\perp(b)}_{xz} &= h^{\perp(b)}_{zx} = \frac{a(1+a^2\mathcal{R}^2)}{2\mathcal{N}^5\mathcal{R}^2} + \frac{a\omega^2}{2\mathcal{N}^3}, \end{cases} \tag{A2}$$

with $\mathcal{N} = \mathcal{N}(a,\mathcal{R}) = \sqrt{1+\frac{1}{4}a^2\mathcal{R}^2}$.

Explicit expressions of $\mathrm{f}^{\perp(0)}_{ij}(a,L,\omega)$ and $\mathrm{h}^{\perp(0)}_{ij}(a,L,\omega)$ are

$$\begin{cases} \mathrm{f}^{\perp(0)}_{xx} &= \frac{\omega(1+a^2L^2)}{\mathrm{N}^4L^2}, \\ \mathrm{f}^{\perp(0)}_{yy} &= \frac{\omega(1+\frac{1}{2}a^2L^2)}{\mathrm{N}^2L^2}, \\ \mathrm{f}^{\perp(0)}_{zz} &= -\frac{\omega(2+\frac{1}{4}a^2L^2+\frac{1}{8}a^4L^4)}{\mathrm{N}^4L^2}, \\ \mathrm{f}^{\perp(0)}_{xz} &= -\mathrm{f}^{\perp(0)}_{zx} = \frac{a\omega(1-\frac{1}{2}a^2L^2)}{2\mathrm{N}^4L}, \end{cases} \tag{A3}$$

$$\begin{cases} \mathrm{h}^{\perp(0)}_{xx} &= -\frac{1+\frac{1}{2}a^2L^2+\frac{1}{4}a^4L^4}{\mathrm{N}^5L^3} + \frac{\omega^2}{\mathrm{N}^3L}, \\ \mathrm{h}^{\perp(0)}_{yy} &= -\frac{1}{\mathrm{N}^3L^3} + \frac{\omega^2}{\mathrm{N}^{1/2}L}, \\ \mathrm{h}^{\perp(0)}_{zz} &= \frac{2(1+\frac{5}{8}a^2L^2)}{\mathrm{N}^5L^3} - \frac{a^2L\omega^2}{4\mathrm{N}^3}, \\ \mathrm{h}^{\perp(0)}_{xz} &= -\mathrm{h}^{\perp(0)}_{zx} = -\frac{a(1+a^2L^2)}{2\mathrm{N}^5L^2} - \frac{a\omega^2}{2\mathrm{N}^3}, \end{cases} \tag{A4}$$

with $\mathrm{N} = \mathrm{N}(a,L) = \sqrt{1+\frac{1}{4}a^2L^2}$.

Explicit expressions of $f^{\parallel(b)}_{ij}(a,D,z,\omega)$ and $h^{\parallel(b)}_{ij}(a,D,z,\omega)$ are

$$\begin{cases} f^{\parallel(b)}_{xx} &= \frac{\omega(1+a^2R^2)}{\tilde{N}^4R^2}, \\ f^{\parallel(b)}_{yy} &= \frac{\omega[4z^2-2D^2-\frac{1}{4}a^2R^2(D^2-12z^2)-\frac{1}{8}a^4R^4(D^2-4z^2)]}{\tilde{N}^4R^4}, \\ f^{\parallel(b)}_{zz} &= \frac{\omega[z^2(16+2a^2R^2+a^4R^4)-D^2(2+\frac{3}{2}a^2R^2+\frac{1}{4}a^4R^4)]}{2\tilde{N}^4R^4}, \\ f^{\parallel(b)}_{xy} &= -f^{\parallel(b)}_{yx} = -\frac{\omega aD(1-\frac{1}{2}a^2R^2)}{2\tilde{N}^4R^2}, \\ f^{\parallel(b)}_{xz} &= f^{\parallel(b)}_{zx} = -\frac{\omega az(1-\frac{1}{2}a^2R^2)}{\tilde{N}^4R^2}, \\ f^{\parallel(b)}_{yz} &= -f^{\parallel(b)}_{zy} = -\frac{2\omega zD(3+a^2R^2+\frac{1}{4}a^4R^4)}{\tilde{N}^4R^4}, \end{cases} \tag{A5}$$

$$\begin{cases} h^{\parallel(b)}_{xx} &= -\frac{1+\frac{1}{2}a^2R^2+\frac{1}{4}a^4R^4}{\tilde{N}^5R^3} + \frac{\omega^2}{\tilde{N}^3R}, \\ h^{\parallel(b)}_{yy} &= \frac{2D^2-4z^2+\frac{1}{4}a^2R^2(5D^2-4z^2)}{\tilde{N}^5R^5} + \frac{\omega^2[4z^2-\frac{1}{4}a^2R^2(D^2-4z^2)]}{\tilde{N}^3R^3}, \\ h^{\parallel(b)}_{zz} &= \frac{D^2(1+\frac{1}{4}a^2R^2)-8z^2(1+\frac{5}{8}a^2R^2)}{\tilde{N}^5R^5} + \frac{\omega^2[a^2z^2R^2-D^2(1+\frac{1}{4}a^2R^2)]}{\tilde{N}^3R^3}, \\ h^{\parallel(b)}_{xy} &= -h^{\parallel(b)}_{yx} = \frac{aD(1+a^2R^2)}{2\tilde{N}^5R^3} + \frac{\omega^2aD}{2\tilde{N}^3R}, \\ h^{\parallel(b)}_{xz} &= h^{\parallel(b)}_{zx} = \frac{az(1+a^2R^2)}{\tilde{N}^5R^3} + \frac{\omega^2az}{\tilde{N}^3R}, \\ h^{\parallel(b)}_{yz} &= -h^{\parallel(b)}_{zy} = \frac{6zD(1+\frac{1}{2}a^2R^2)}{\tilde{N}^5R^5} - \frac{2\omega^2zD(1+\frac{1}{2}a^2R^2)}{\tilde{N}^3R^3}, \end{cases} \tag{A6}$$

with $\tilde{N} = \tilde{N}(a,R) = \sqrt{1+\frac{1}{4}a^2R^2}$.

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