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Article

Duality Based Risk Mitigation Method for Construction of Joint Hydro-Wind Coordination Short-Run Marginal Cost Curves

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Abstract: This study analyses the short-run hydro generation scheduling for the wind power differences from the contracted schedule. The approach for construction of the joint short-run marginal cost curve for the hydro-wind coordinated generation is proposed and applied on the real example. This joint short-run marginal cost (SRMC) curve is important for its participation in the energy markets and for economic feasibility assessment of such coordination. The approach credibly describes the short-run marginal costs which this coordination bears in "real life". The approach is based on the duality framework of a convex programming and as a novelty combines the shadow price of risk mitigation capability and the water shadow price. The proposed approach is formulated as a stochastic linear program and tested on the case of the Vinodol hydropower system and the wind farm Vrataruša in Croatia. The result of the case study is a family of 24 joint short-run marginal cost curves.

Keywords: Convex programming; Wind power, Hydropower; Risk mitigation; CVaR; Short-run marginal cost curve

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Nomenclature

Symbols

c'	Short-run marginal cost function (€/MW·h).
d	Difference between forecasted Y and actual wind generation y_w (MW).
d^+	Positive wind difference (MW).
d^-	Negative wind difference (MW).
e	Natural water inflow (MW).
F_α	Special function used for risk shaping of CVaR (€).
g	Net outflow of hydro generation (MW)
I	Hourly revenue (€).
k_{St}	Maximal capacity of reservoir (MW·h).
k_{Rm}	Parameter used for risk exposure reduction in risk shaping procedure (€).
k_{Tu}	Hydro turbine maximal capacity (MW).
${k_{Tu}}^w$	Wind turbine maximal capacity (MW).
n_{St}	Minimal capacity of reservoir (MW·h).
n_{Tu}	Hydro turbine minimal capacity (MW).
${n_{Tu}}^w$	Wind turbine minimal capacity (MW).
s	Energy stock, amount of water in reservoir in t, (MW·h).
s_0	Energy stock at the beginning of planning interval (MW·h)

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s_T	Energy stock surplus or deficit at the end of planning interval (MWh)
y	Hydro generation (MW).
Y	Contracted wind generation (MW).
y_w	Actual wind generation (MW).
Greek	
α	Percentile used for the CVaR where 1-α defines the worst events (%).
δ	Shadow prices associated with constraint of CVaR's helping variable η (€).
ε	Shadow prices associated with CVaR's hourly revenue constraint.
ζ	The decision variable which defines the Value at Risk (€).
η	Variable used for obtainment of the CVaR (€).
κ^{Tu}	Shadow price of hydro generation maximum capacity (€/MW·h).
κ^{St}	Shadow price of reservoir maximum capacity (€/MW·h).
λ	Shadow price of energy stock surplus/deficit at the end of operation (€/MW·h).
ν^{Tu}	Shadow price of hydro generation minimum capacity (€/MW·h).
ν^{St}	Shadow price of reservoir minimum capacity (€/MW·h).
ξ	Shadow price of risk mitigation capability (dimensionless).
π	Price of electricity (€/MW·h).
ψ	Shadow price of water (€/MW·h).
Spaces	
$[0, T]$	Planning interval, subspace of the real line $t \in [0, T] \subset \mathbb{R}$.

1. Introduction

In this paper the hydro-wind coordination is formulated as a hydro-economic river basin model (HERBM). The coordination assumes that hydro generation is scheduled for firming wind generation. In other words, the short-run hydro generation is scheduled for the wind power differences from the contracted schedule. This wind power difference from contracted schedule impacts the short-run marginal cost curve of the hydro generation curve in the coordination [1]. Generally, a short-run marginal cost curve of a hydro generation is obtained from the water shadow price and as such is important for economic feasibility assessment of the coordination. In this paper, the joint short-run marginal cost curve for electricity generation of hydro-wind coordinated generation is obtained. To obtain these joint short-run marginal curves, the primal problem and its dual are formulated as convex optimization problems. The primal problem is defined as a short-run revenue maximization problem, while the dual problem minimizes the usage costs of the limited resources (hydro and wind turbine capacity, reservoir capacity, water inflow resource, wind difference resource, risk mitigation capability).

This paper is based on the pioneering works [2-3] which systematically address the problem of the short run profit maximization of the hydro and the pumped storage units using continuous functions and [4] where method for the contraction of short-run marginal cost curves for hydro generation is given. These works are based on the ideas of conjugate duality and optimization [5]. The issue of hydro-wind coordination is addressed extensively and among the most significant are the following papers. The sizing method for the pumped hydro storage which uses the wind power surplus is presented in [6]. A methodology for increasing profits of a generation company that owns wind and pumped-storage plants while accounting the wind power uncertainty is presented in [7]. In [8] the problem of introduction of the new pumped-storage station to the existing hydropower system owned by a new subject is addressed. In [9] a combined strategy for bidding and operating in a power exchange is presented. It considers the combination of a wind generation company and a hydro-generation company. The [10] discusses the impact of joint bidding of wind power plant and

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a hydro generating unit on the wind farm revenue in a pool-based electricity market, considering the uncertainty of wind power prediction. The [11] proposes methodology that can reflect different risk profiles of decision makers in wind–hydro-thermal coordination and in [12] a joint operation between a wind farm and a hydro-pump plant is proposed to decrease costs of wind farm imbalances.

This paper will complement these studies by tackling the issue of contraction of joint short-run marginal cost curves for the hydro-wind coordination. Obtaining a joint short-run marginal cost curves for a hydro-wind coordination is hard issue due to facts that electricity generation from the hydro-wind coordination is: a) storable and; b) has a negligible direct generation cost. Therefore, usual approach (first derivative by output of the total cost function) for obtainment of short-run marginal cost curves is not sufficient in this case. This issue was not tackled until the fundamental works [4] where issue of constructing short-run marginal cost curve for a hydro producer is analyzed and [1] where the impact of a wind power difference from contracted schedule on the water shadow price is analyzed. This paper, as an extension of the research done in [1] and [4], contributes with the systematic approach for construction of the joint short-run marginal cost curves for a hydro-wind coordination.

Therefore, this research contributes with the approach based on duality method of convex programming for construction of the joint short-run marginal cost curves for electricity generation of a hydro-wind coordination which as a novelty combines: i) a shadow price of risk mitigation capability, ξ; ii) and a water shadow price, ψ. The approach enables quantification of hourly cost of risk mitigation through the shadow price of risk mitigation capability, ξ. The proposed approach is formulated as a stochastic linear program and is easy to implement in various optimization problems. The approach is tested on the case study of Vinodol hydropower system and the wind farm Vrataruša in Croatia resulting in a family of 24 joint short-run marginal cost curves. The cost of risk mitigation is also provided. To implement risk mitigation, the CVaR risk measure is used, which enables measurement and limitation of extreme financial losses. It is a coherent risk measure [14-18] which means that it satisfies properties of monotonicity, sub-additivity, positive homogeneity and translational invariance. For the risk measure which satisfies these properties it is said that it describes well a real life occurring risks. The properties of sub-additivity and positive homogeneity can be replaced by property of convexity which is important when implementing in convex optimization problems in order not to undermine its convexity[1].

Besides the CVaR, the second important term is the *water shadow price*. The idea behind the water shadow price is that generation of 1 MW·h in particular hour means not being able to produce that 1 MW·h in other future hours (it is important to notice that the water shadow price can also be regarded as an opportunity cost of water generation in particular hour). Generally, the shadow price is equal to the marginal utility of relaxing particular constraint or marginal cost when constraint is strengthened. In this case, generation of 1 MW·h means strengthening water balance constraint by 1 MW·h, in particular hour. Therefore, by definition, a water shadow price can be regarded as a marginal cost which makes it appropriate for construction of short-run marginal cost curves[2]. In fundamental works [20,21] the water shadow price is assumed to be constant over the planning period [0, T]. For more realistic approach the water shadow price should change over the planning period, such as in [1-4], every time when reservoir limits are reached, which is also assumed here.

In Chapter II, the short-run dual for valuing limited hydro-wind generation resources is set as the double infinite linear programming problem. The dual is reformulated for shadow pricing of water in the coordinated hydro-wind generation. The wind power difference from contracted schedule and CVaR are implemented in the dual problem through the duality framework of convex

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[1] See Corollary 11 in [18] for more about convexity of CVaR.

[2] For more about issue of shadow pricing of water, see [19] where hydro generation with constant head is analyzed.

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programming. In the Chapter III cascaded hydropower system Vinodol is carefully modeled as the HERBM [22,23] and the wind farm Vrataruša power difference is implemented within it. The results are presented and discussed in the same Chapter. In the Chapter IV conclusions are given.

2. Construction of the joint short-run marginal cost curve

The primal problem is a short-run profit maximization problem shown in Eqs. (1) - (14). To achieve strong duality, the primal problem must be convex and needs to satisfy *Slater's condition*. The primal problem used here is a linear program, i.e. it is convex and it satisfies *Linearity constraint qualification* which is sufficient condition for strong duality. Strong duality is important here since it implies that the duality gap, i.e. difference between the primal and dual solutions is zero. The dual to the primal problem is shown in Eqs. (15) - (21) and is used for the construction of the joint short-run marginal cost curves for the hydro-wind coordination.

For hydro generation, it is assumed that minimal and maximal reservoir capacities are n_{St} and k_{St} (Eq. (6)) and are measured in terms of energy (MW·h) same as water stock measured which is stored at no cost. The natural water inflow e into the reservoir is measured over $[0,T]$ in MW. Wind power difference d is measured over $[0,T]$ in MW and can obtain positive d^+ and negative d^-values (Eq. (12)). The units are MW due to continuous-time dating (see Eq. (2) and Eq. (16)). It is defined as the difference between generated wind power and power indicated in contracted schedule. The hydro turbine operating range in Eq. (4) is limited by: its minimal and maximal capacity n_{Tu} and k_{Tu} measured in MW; the positive d^+ and the negative d^- wind difference. The performance curve of the hydro generation is considered linear and the head is fixed over the planning interval $[0,T] \in \mathbb{R}$. The goal function (Eq. (2)) maximizes the expected revenue obtained from day-ahead energy market over the planning interval $[0,T]$ which is multiplied with the probability $P(A)$ of event A. The y is the hydro generation measured in MW and π is the electricity price measured in €/MW·h since of continuous-time dating. The decision variables in Eq. (3), over which the goal function (Eq. (2)) is optimized, consist of the electricity generation variable y and (ζ,η) which are decision variables associated with CVaR risk measure.

According to the Theorem 14 in [18] the CVaR can be implemented in the optimization problem as a goal function, a constraint or both. Since the goal function is predefined here, then CVaR is implemented with Eq. (7) - (9). The risk mitigation is done through risk shaping of the special function F_α with the parameter k_{Rm}, according to the Theorem 16 in [18]. The k_{Rm} is measured in monetary units (€) and represents the minimum desired level of revenue the owner expect in worst $1-\alpha$ outcomes (usually 5%, where $\alpha = 95\%$) and is part of input data (Eq. (1)). The decision variable ζ in Eq. (3) defines the Value at Risk (VaR) measure and η is a variable used for determining hourly revenue in worst case events, both these are used for obtainment of the CVaR[3]. The electricity price π is expressed in €/MW·h. The s_0 defines the energy stock at the beginning of planning interval in MW·h and s_T energy stock surplus or deficit at the end of planning interval in MW·h. Both parameters are the result of the middle-term HERBM optimization and part of the input data (Eq. (1)). The input data is same for primal (Eq. (1)), and dual problem (Eq. (15)). The g in Eq. (10) defines the net outflow from the reservoir measured over $[0,T]$ in MW. When net outflow g is integrated over time (Eq. (5)) it equals amount of water left at the end of planning horizon, T, which should be above or equal to the predefined level, s_T. To preserve clarity, in depth analysis of the primal and dual problems is provided in the comments section. Therefore, a primal problem is (the variables denoted after the ";" in equations are the dual variables/shadow prices associated with the constraints):

Given

[3] For more about CVaR in optimization problems see [13] and Theorems 14 and 16 in [18]

$$(\pi;\ n_{Tu};\ k_{St}, n_{St}, k_{Tu}, s_0, s_T;\ d, e, k_{Rm}) \tag{1}$$

Maximize

$$\int_{t=0}^{T} \sum_{A\in\mathcal{F}_t} P(A)\cdot\pi(t,A)\cdot y(t,A)\,\mathrm{dt} \tag{2}$$

Over

$$(\zeta,\eta,y) \tag{3}$$

Subject to

$$n_{Tu} + d^{+}(t) \le y(t,A) \le k_{Tu} + d^{-}(t)\ ;\ \kappa^{Tu}(t,A), \nu^{\mathrm{Tu}}(t,A) \tag{4}$$

$$\int_{t=1}^{T} g(t,A)\,\mathrm{dt} \le s_T\ ;\ \lambda(A) \tag{5}$$

$$n_{St} \le s_0 - \int_{\tau=1}^{t} g(\tau,A)\,\mathrm{d\tau} \le k_{St}\ ;\ \nu^{\mathrm{St}}(t,A), \kappa^{\mathrm{St}}(t,A) \tag{6}$$

$$\zeta(t) - I(t,A) - \eta(t,A) \le 0\ ;\ \varepsilon(t,A) \tag{7}$$

$$\eta(t,A) \ge 0\ ;\ \delta(t,A) \tag{8}$$

$$F_\alpha\big(\eta(t,A),\zeta(t)\big) \ge k_{Rm}\ ;\ \xi(t) \tag{9}$$

Where

$$g(t,A) := y(t,A) - d(t,A) - e(t,A) \tag{10}$$

$$I(t,A) \coloneqq \pi(t,A)\cdot y(t,A) \tag{11}$$

$$d(t) := d^{-}(t) + d^{+}(t) \tag{12}$$

$$|d^{-}(t)| \wedge d^{+}(t) = 0 \tag{13}$$

$$F_\alpha\big(\eta(t,A),\zeta(t)\big) := \zeta(t) - \frac{1}{1-\alpha}\sum_{A\in\mathcal{F}} P(A)\cdot\eta(t,A) \tag{14}$$

And its dual:

Given

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$$(\pi;\ n_{Tu};k_{St},n_{St},k_{Tu},s_0,s_T;d,e,k_{Rm}) \tag{15}$$

Minimize

$$\int_0^T \sum_{A\in\mathcal{F}_t} \big(k_{Tu}+d^-(t,A)\big)\cdot\kappa^{Tu}(t,A)-\big(n_{Tu}+d^+(t,A)\big)\cdot\nu^{Tu}(t,A)+\ (k_{St}-s_0)\cdot\kappa^{St}(t,A) \\ +(s_0-n_{st})\cdot\nu^{St}(t,A)+\big(e(t)+d(t)\big)\cdot\psi(t,A)\,dt+\sum_{A\in\mathcal{F}_t} s_T\cdot\lambda(A) \\ +\int_0^T k_{Rm}(t)\cdot\xi(t)\,dt \tag{16}$$

Over

$$(\kappa^{Tu},\nu^{\mathrm{Tu}},\kappa^{St},\nu^{St},\lambda,\varepsilon,\delta,\xi) \tag{17}$$

Subject to

$$\psi(t,A)+\kappa^{Tu}(t,A)-\nu^{\mathrm{Tu}}(t,A)-\pi(t,A)\cdot\varepsilon(t,A)\geq\pi(t,A)\cdot P(A) \tag{18}$$

$$\frac{P(A)}{1-\alpha}\cdot\xi(t)-\varepsilon(t,A)-\delta(t,A)\geq 0 \tag{19}$$

$$\sum_{A\in\mathcal{F}_t}\big(\varepsilon(t,A)-\delta(t,A)\big)\geq 0 \tag{20}$$

Where

$$\psi(t,A):=\lambda(A)+\int_{\tau=t}^{T}\nu^{St}(\tau,A)-\kappa^{St}(\tau,A)\,d\tau \tag{21}$$

For hour t and for each event A, the pairs (y,ψ) are obtained, where $\psi(t,A)$ is the water shadow price and $y(t,A)$ the hydro generation. Each event A represents "one dot" in the short-run marginal cost curve for the t^{th} hour as shown in Fig. 1(d). The Fig. 1(d) is the final short-run marginal cost curve of the hydro-wind coordination. The idea behind the primal and dual problem presented in Eqs. (1) - (21) is the process of construction which starts with the short-run marginal cost curve of the hydro generation [4] without the impact of wind power difference on the water shadow price [1] as seen on Fig. 1(a). To include the impact of wind power difference on the water shadow price, the wind power difference d, d^+ and d^- are implemented as discussed earlier. The resulting curve is one in the Fig. 1(b) where the impact of wind power difference on the water shadow price, $\Delta\psi$, is added to short-run marginal cost of hydro generation. The sign of $\Delta\psi$ depends on the sign of the wind power difference i.e. the surplus of wind d^+ means $\Delta\psi<0$ which reduces the water shadow price while the shortage of wind d^- means $\Delta\psi>0$ which increases the water shadow price in the coordinated generation [1]. After accounting for the wind differences, the whole short-run marginal cost curve should be horizontally shifted for forecasted wind generation, Y, as shown in the Fig. 1(c). This can be done since the effects of the wind generation on the generation range of hydro generation and the availability of water in the reservoir and resulting opportunity costs are already accounted

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in the previous curve (Fig. 1(b)). Finally, the short-run marginal cost $c'(y_w)$ of wind generation y_w should be added in a merit order way (based on ascending order of price). In the Fig. 1(c) the $c'(y_w)$ is lower than the water shadow price ψ and is ranked before hydro generation, otherwise it would be ranked last, or somewhere in the middle while shifting right part of the of the curve.

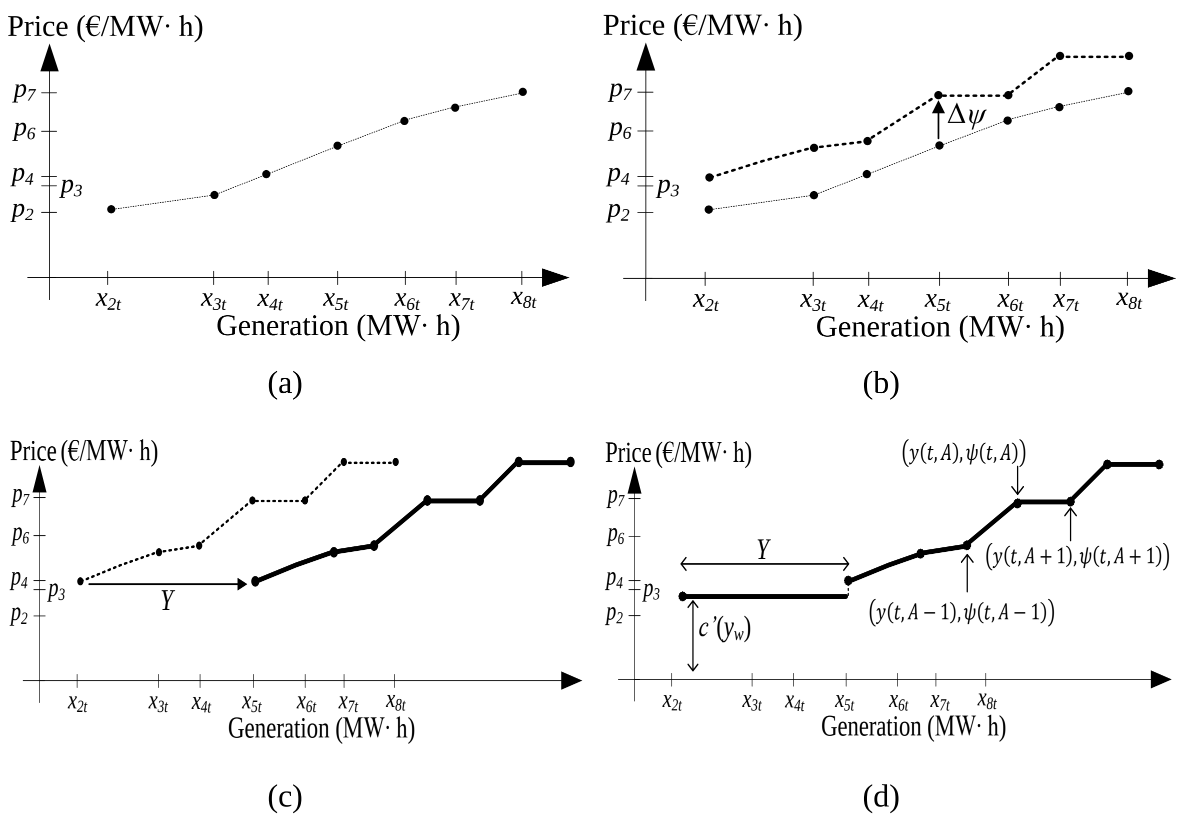

Fig. 1 The example of construction of the joint short-run marginal cost curves

For the low market price events A (and high opportunity cost) this approach would "force" marginal cost curve to adjust itself (horizontally and vertically) until the water shadow price ψ is less or equal than market price $\pi(t,A)$ for some hour t, i.e. $\psi(t,A) \leq \pi(t,A)$: $y(t,A) > 0$. Generally, there is no generation when water shadow price is greater than market price[4].

The implementation of the presented approach is simple, especially when having in mind that the only needed equation for constructing marginal cost curves is the water shadow price ψ, equation, Eq. (21). This equation can be used in the primal problems since the tools for modelling and optimization usually enable readout of the optimal values of the dual variables, i.e. readout of the shadow prices of each constraint. For calculating the water shadow price, the readout of tripe consisting of shadow prices $(\lambda, \nu^{St}, \kappa^{St})$ is needed.

3. Case study and Results

The primal and the dual problems are implemented with the GAMS Rev 239 modelling system under the CPLEX solver. The study is done for the 5th December in 2016 on the case of wind farm Vrataruša and hydropower system Vinodol, both located in Croatia. That specific week is considered as the worst-case scenario from the standpoint of water scarcity and wind power differences. It is assumed that wind farm Vrataruša and Vinodol hydropower system operate in hydro-wind coordination and that they completely avoid balancing cost. The modelled Vinodol hydropower system consists of: 3 reservoirs, 2 pumped-storage plants (PSP) and 1 hydropower plant (HP), with

[4] See Theorem 4.9 in [2].

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details given in Table 1 and Fig. 2. The energy stock at the beginning/end of a day and the natural water inflows are shown in Table 2. The difference between contracted and actual generated wind power is given in Fig. 4. These input parameters are obtained from the real life middle-term operation planning.

Table 1: HPS Vinodol facilities parameters.

Reservoir	k_{St} (GWh)	Power Plant	k_{Tu} / n_{Tu} (m^3/s)	k_{Tu} / n_{Tu} (MW)
Lokve	52	PSP Fužine	10/9	4.6/4.8
Bajer	1.9	HPP Vinodol	18.6	94.5
Lepenica	5.9	PSP Lepenica	6.2/5.3	1.14/1.25

Table 2: Energy stock in percent of maximum reservoir capacity k_{St} at the beginning of and the end of planning interval

Reservoir	s_0	s_T	e (m^3/s)
Lokve	66	46	0,68
Bajer	64	64	0,89
Lepenica	58	58	0,21

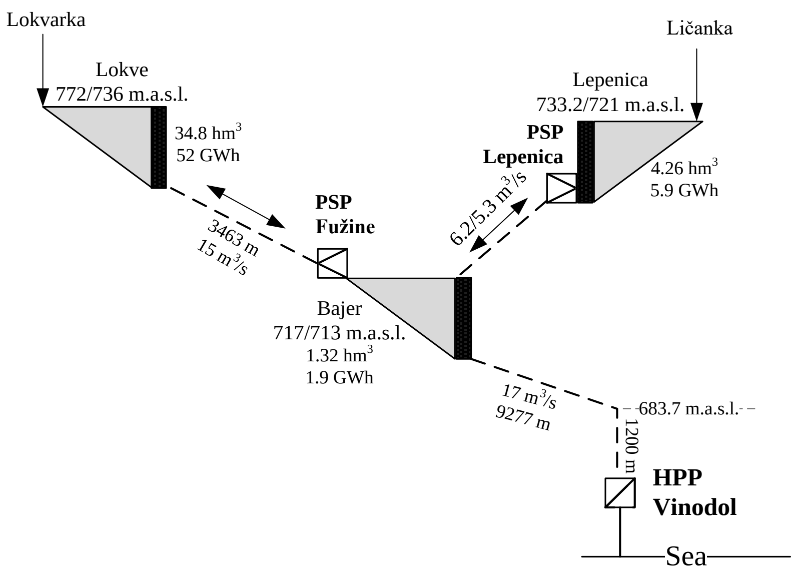

Fig. 2 Model of the Vinodol hydropower system

The wind farm Vrataruša consists of 14 units, each with 3 MW of installed power. The total installed power is 42 MW with minimum power output 1.1 MW. For the computational effectiveness, only the hydro power plant Vinodol profit is maximized. There is no significant loss in accuracy since the revenue of hydro power plant Vinodol participates with 95% in total system's revenue. The ramp rate limits of the hydro power plant Vinodol are: MSR (up) 255.15 (MW/min); MSR (down) 340.2 (MW/min). Therefore, there are no constraints on ramping capabilities in the model. The real hydropower system Vinodol has conversion coefficient ρ equal to 5.08 (MW/m^3) and is used for frequency regulation ancillary services [32]. A total of 10 electricity price events is generated with the random number generator based on the statistical analyses of the Croatian nodal electricity prices of the first week of December 2016 with the same probability of each event equal to $P(A) = \frac{1}{10}, \forall t \in T\ \forall A \in \mathcal{F}_t$. The CVaR percentile α is 85%.

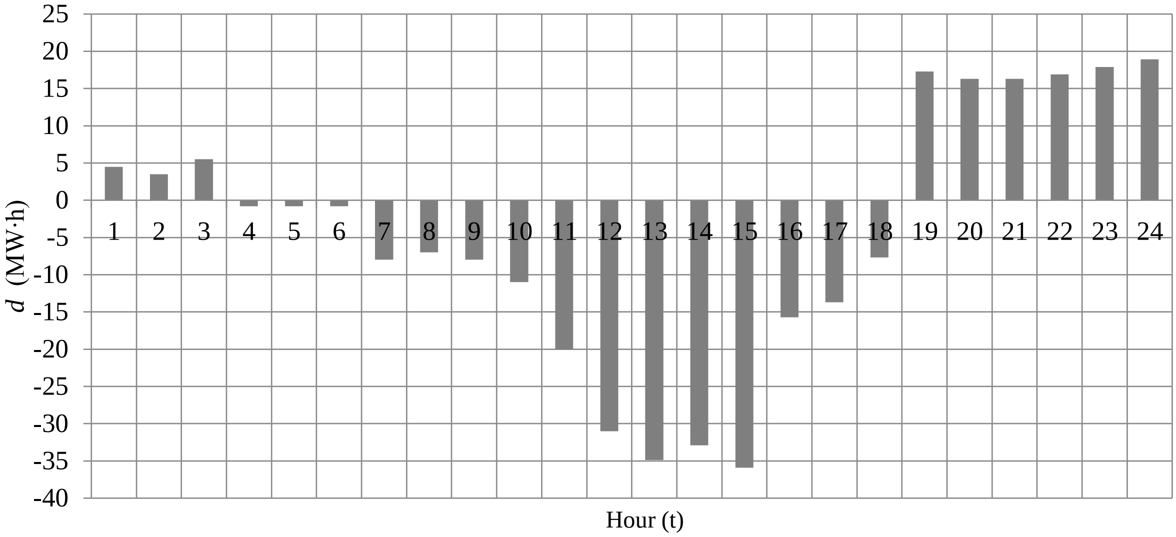

Fig. 4 The wind power difference (d) for 5th December of 2016.

3.1 Results

The results are shown in Table 3 and Fig. 5. The Table 3 shows the dependence of expected daily revenue on the risk mitigation parameter k_{Rm} used for hourly risk exposure reduction (it defines the minimum expected return in the 1-α worst outcomes for each hour). The curves (Fig. 5) are for the hourly risk exposure of 1380 € and 5 €/ MW·h of short-run marginal costs of Vrataruša wind farm generation.

Table 3 The expected daily revenue and associated risk exposure reduction $k_{Rm}, \forall t$

Revenue (€)	k_{Rm} **(€)**
98885	0
97995	400
97100	800
96332	1380

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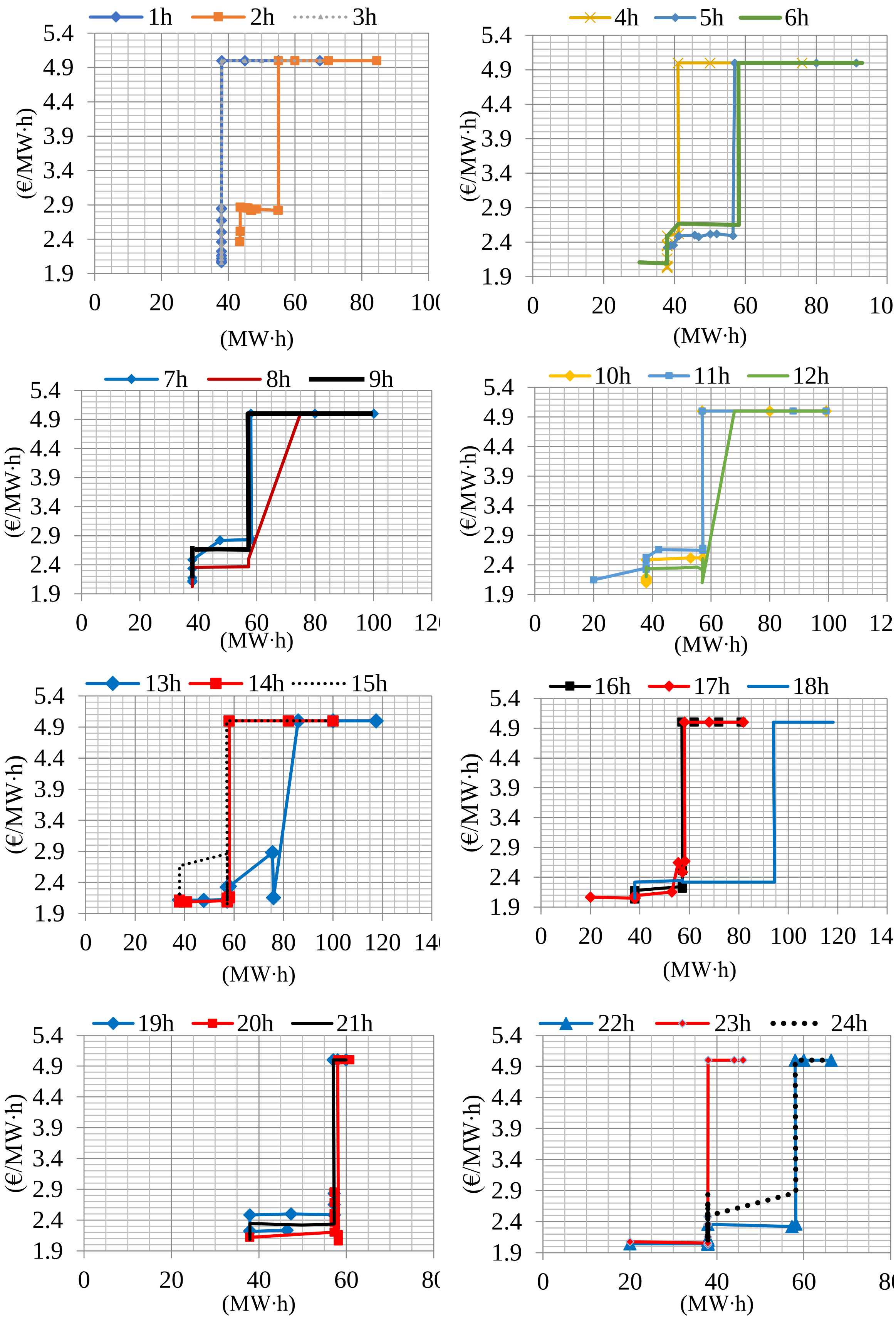

Fig. 5 The short-run marginal costs curves for electricity generation of a Vinodol and Vrataruša hydro-wind coordination.

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3.2 Discussion

Risk mitigation and daily revenue. As parameter used for hourly risk exposure reduction k_{Rm} increases from 0 € to 1380 €, the expected daily revenue decreased from 98 885 € to 96 332 € (Table 3).

Risk mitigation and the vertical shift of marginal cost curves. As parameter for hourly risk exposure reduction k_{Rm} increases from 0 € to 1380 €, the short-run marginal cost curves (Fig. 5) are vertically shifted upwards from 1.9 €/MW·h to approximately 2.5 €/MW·h. This means that the hourly risk exposure reduction has a certain cost that can be calculated from the marginal cost curves in Fig. 5. In average this cost is 2.5 €/MW·h – 1.9 €/MW·h = 0.6 €/MW·h. Therefore 0.6 €/MW·h is the price of electricity which ensures owner with at least 1380 € in the worst-case scenario. Of course, this insurance is at the expense of the daily revenue, which decreases.

Range of generation in marginal cost curves. The range of generation of Vrataruša-Vinodol coordination is usually from 38 MW·h to 58 MW·h (Fig. 5). The one general conclusion can be drawn regarding range of generation in joint marginal cost curves, that for low electricity prices (1h-3h and 22h- 24h) there is no desire to produce less than 38 MW·h. It is obvious that in hours with higher prices (13h-18h) coordination want to produce even more than 58 MW·h and up to 80-90 MW·h.

Prices in marginal cost curves. The short-run marginal cost curves are characterised by relatively low (compared to market prices) cost which is a result of a low water shadow price. Perfectly inelastic parts of the short-run marginal cost curves are obtained for the values 38 MW·h and 58 MW·h (Fig. 5).

4. Conclusion

The approach presented in this paper analyses the case of short-run hydro-wind coordination which participates on the electricity market. This research contributes with the approach based on duality method of convex programming for determining the joint short-run marginal cost curves for an electricity generation of the hydro-wind coordination, which as a novelty combines risk mitigation and the water shadow price. The proposed approach is easy to implement in various optimization problems. Proposed method is suitable for investors with various risk preferences, from risk-averse to risk-neutral or risk-seeking, since it enables risk mitigation. It has been shown that the perfectly inelastic short-run marginal cost curves are appropriate for Vrataruša-Vinodol hydro wind coordination which follows from the fact that the water shadow price was constant over the various price events in particular hour. Therefore, it is justified to use simplified method for calculating water shadow price which is constant over the planning period. It has been shown that the risk mitigation has a cost and that this cost is easily quantified which is exemplified and discussed in the paper.

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Conflicts of Interest: The authors declare no conflict of interest.

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