 doi:10.20944/preprints201804.0293.v1

Applications of Double Framed T-Soft Fuzzy Sets in BCK/BCI-Algebras

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Abstracts:

The aim of this article is introduced the concept of double framed T-soft fuzzy set (DFT-soft fuzzy set) which is the combination of soft set and fuzzy set. We also defined the notions and apply this concept in BCK/BCI-algebras. By using example, we also discussed the concept of double framed T-soft fuzzy algebra (DFT-soft fuzzy algebra) and double framed B-soft fuzzy algebra (DFB-soft fuzzy algebra) and also investigated their properties. Each double framed T-soft fuzzy algebra is double framed B-soft fuzzy algebra but by using example, we proved that converse may or may not be possible.

Keywords: double framed T-soft fuzzy set; double framed T-soft fuzzy algebra; double framed B-soft fuzzy algebra

1. Introduction

Traditionally mathematics uses crisp set theory [1] to explain the properties of any substance and this set theory consists of two possibilities that are either false or true. But with the passage of time, the vague concepts became a virus in different fields of our life like pharmacology, engineering, medical application and economics but classical mathematical tools are failed to solve these problems. Then in 1965, Zadeh [2] introduced most successful theory which is known as fuzzy set theory which dealt with the vague concepts. This theory is used openly in different fields like pharmacology, engineering, medical application and economics, among others. We know that fuzzy set theory is strongly based on membership function. We can determine the membership grade of an element of a set with the help of membership function. The fuzzy set theory has become very popular. But there was a difficulty to set a membership grade of a membership function. To solve this problem Moldstov [3] presented the theory which is known as soft set theory. This theory is used successfully in different areas such as game theory, Riemann-integration, smoothness of function, Perron-integration, etc. In 2002 Maji et al. [4] introduced the application of soft set theory in decision making problems. Also Maji [5] in 2003 studied the theoretical work on soft set theory to polish this concept so that readers could easily understand and contributed their role to extend the scope of this theory in different fields of life. After theoretical discussion, now we discussed about the contributions of those researchers whose applied this concept in different fields of algebra like Cagman et al. [6] studied the soft set and soft groups and also introduced the notation of soft groups. They also defined the relation between fuzzy set, rough set and soft set and discuss the properties. Ali et.al [7] introduced some new operations on soft sets.

For the study of BCK/BCI algebras we refer the readers to books [8, 9]. Jun et al. [10] worked on intersectional soft sets and defined its applications in BCK/BCI algebras. They also defined the notations of intersectional soft sets. Also Jun [11], studied the union soft sets and defined its application in BCK/BCI algebras. Further, Jun and Park [12] elaborated applications of soft ideal theory in BCK/BCI algebras. Jun and Ahn [13] studied the applications of double framed soft sets. They also introduced the notations of double framed soft algebras and discussed their properties by giving several examples. Naz [14] introduced some operations on double framed double soft sets. Hadipour [15] applied the concept of double framed soft set in BF-algebras and introduced the notations of double framed BF-algebras. He also investigated its properties. Cho et al. [16] studied the concept of double framed soft near ring. Shabir and Samreena [17], worked on double framed soft topological spaces and defined its notation. For further information, we refer to reader papers [18-29] regarding soft algebraic structures.

From inspiring above literature, In section 2,we discussed basic definitions related to soft set, double framed soft set and BCK/BCI algebras. In section 3. our basic purpose to combine the concept of soft set and fuzzy set in pair form and then this concept is used in BCK/BCI algebras and introduced its notations. We investigated the properties of double framed T-soft fuzzy algebra and double framed B-soft fuzzy algebras. We elaborated that each double framed T-soft fuzzy algebra is also double framed B-soft fuzzy algebra but converse is may or may not possible.

2. Preliminaries

Let X be a non-empty set. Then $A = \{< x, \mu_A(x) > | x \in X\}$ is called fuzzy set, where μ_A is a membership function which map each element of X in $[0, 1]$. Here we say that A is fuzzy subset of X. The set of all fuzzy subsets of a set X is denoted by $FP(X)$. Two fuzzy sets A and B are equal if and only if $\mu_A(x) = \mu_B(x)$ for all $x \in X$. Where μ_A and μ_B are membership functions which map each element of X in $[0, 1]$. The complement of a fuzzy set A is denoted by A^c and is defined by,

$$\mu_{A^c} = 1 - \mu_A,$$

Where μ_A is a membership function which map each element of X in $[0, 1]$. Let X be a non-empty set. The union of two fuzzy sets A and B with membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively is denoted and defined by,

$$\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(x)\}\ for\ all\ x \in X.$$

Here μ_A and μ_B are membership functions which map each element of X in $[0,1]$. In abbreviated form $\mu_C = \mu_A \vee \mu_B$, where $C = A \cup B$. The intersection of two fuzzy sets A and B with membership functions $\mu_A\,(x)$ and $\mu_B(x)$ respectively is denoted and defined by,

$$\mu_{A\cap B}(x) = min\{\mu_A\,(x), \mu_B(x)\}\, for\ all\ x \in X.$$

Here μ_A and μ_B are membership functions which map each element of X in $[0,1]$. In abbreviated form $\mu_C = \mu_A \wedge \mu_B$, where $C = A \cap B$.

Cagman [6] introduced the new definition of soft set in this way,

A pair (Γ, E) is indicated to be soft set over if and only if Γ is a mapping from E to the all subset of U e.g

$$(\Gamma, E) = \{(e, \Gamma(e))|\ e \in E\ and\ \Gamma(e) \in P(U)\}$$

Where $P(U)$ is power set of U and $\Gamma: E \to P(U)$. The function Γ is an approximation function of the soft set (Γ, E). It is easy to see that soft set is parameterized family of subsets of U.

The set of all soft sets over U is denoted by $S(U)$.

BCK/BCI-algebras defined by K. Iseki, in such a way,

A non-empty set X under binary operation $" * "$ with θ is called a BCI-algebra of type $(2, \theta)$ is denoted by $(X, *, \theta)$ and defined as

$$(1)\big((u * v) * (u * w)\big) * (w * v) = \theta\ for\ all\ u, v, w \in X,$$

$$(2)\ \big(u * (u * v)\big) * v = \theta\ for\ all\ u, v \in X,$$

$$(3)\ u * u = \theta\ for\ all\ u \in X,$$

$$(4)\ u * v = \theta, v * u = \theta\ impies\ u = v\ for\ all\ u, v \in X.$$

If a BCI-algebra X satisfies, $\theta * u = \theta\ for\ all\ u \in X$ then X is called BCK-algebra. For any BCI/BCK-algebra the following hold, $u * \theta = u\ for\ all\ u \in X$.

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Jun at al. [13] introduced the double framed soft set in such a way,

A double framed pair $((F,\mathfrak{G}),A)$ over U is said to double framed soft set, in pair form$(F,\mathfrak{G})$ both are soft sets over U. A double framed pair $((F,\mathfrak{G}),A)$ over U is said to be double framed soft algebra if it satisfies

$F(u*v) \geq F(u) \cap F(v)$ and $\mathfrak{G}(u*v) \subseteq \mathfrak{G}(u) \cup \mathfrak{G}(v)$ for all $u,v \in A$.

3. Double framed T-soft fuzzy algebra and double framed B-soft fuzzy algebra

In this section, We elaborate the double framed T-soft fuzzy set, double framed T-soft fuzzy algebra, double framed B-soft fuzzy algebra and investigate their properties. In all examples, all defined BCI/BCK algebras are taken from [13] except 3.12. Example.

Note that we take $\delta = X$, which is BCI/BCK algebras, where δ is a set of parameters, and for sub algebras, we use A, B, C of δ, otherwise mentioned.

3.1. Definition

Let (F,A) is a soft set over U and $f: A \rightarrow I(\,where\ I = [0,1])$ is a fuzzy set then double framed pair $((F,f),A)$ over $(U,[0,1])$ is called double framed T-soft fuzzy set (DFT-soft fuzzy set).

3.2. Definition

Let $((F,f),A)$ and $((\mathfrak{G},g),B)$ be two "DFT-soft fuzzy set" over $(U,[0,1])$ then $((F,f),A)$ is said to be double framed T-soft fuzzy subset of $((\mathfrak{G},g),B)$ if

(1) $A \subseteq B$,

(2) $F(a) \subseteq \mathfrak{G}(a)$ and $f(a) \geq g(a)$ for all $a \in A$.

We can write $((F,f),A) \mathrel{\tilde{\tilde{\subset}}} ((\mathfrak{G},g),B)$. In case $((F,f),A)$ is super set of $((\mathfrak{G},g),B)$.

3.3. Definition

Let $((F,f),A)$ and $((\mathfrak{G},g),B)$ be two "DFT-soft fuzzy set" over $(U,[0,1])$ then $((F,f),A)$ is said to be double framed T-soft fuzzy twisted subset of $((\mathfrak{G},g),B)$ if

(1) $A \subseteq B$,

(2) $F(a) \supseteq \mathfrak{G}(a)$ and $f(a) \leq g(a)$ for all $a \in A$.

We can write $((F,f),A) \mathrel{\hat{\tilde{\subset}}} ((\mathfrak{G},g),B)$. In case $((F,f),A)$ is super set of $((\mathfrak{G},g),B)$.

3.4. Definition

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A "DFT-soft fuzzy set" $((F, f), A)$ over $(U, [0,1])$ is said to be a "DFT-soft fuzzy algebra" over $(U, [0,1])$ if it satisfies

$F(x * y) \subseteq F(x) \cup F(y)$ *and* $f(x * y) \geq f(x) \wedge f(y)$ for all $x, y \in A$.

3.5. Example

Suppose $\delta = \{p_\theta,\ p_1,\ p_2,\ p_3\}$ be the set of parameter, (where "$p_\theta = beautiful$", "$p_1 = cheap$", "$p_2 = in\ good\ location$", $p_3 = in\ green\ surroundings$) with the following binary operation

*	p_θ	p_1	p_2	p_3
p_θ	p_θ	p_θ	p_θ	p_θ
p_1	p_1	p_θ	p_1	p_1
p_2	p_2	p_2	p_θ	p_2
p_3	p_3	p_3	p_3	p_θ

$(\delta, *, p_\theta)$ is BCK-algebra And $U = \{h_1, h_2, h_3, h_4, h_5\}$ is the initial universe set that containing five houses.

Now we define a "DFT-soft fuzzy set" as follows

$$F: \delta \to P(U),\ y \mapsto \begin{cases} U & \text{if } y = p_\theta \\ \{h_1, h_2, h_5\} & \text{if } y = p_1 \\ \{h_1, h_2, h_3, h_5\} & \text{if } y = p_2 \\ \{h_1, h_2\} & \text{if } y = p_3 \end{cases}$$

Define $f: \delta \to [0\ 1]$ such that $f(p_\theta) = 0.7$, $f(y) = .5$. Then $((F, f), \delta)$ is "DFT-soft fuzzy set" $(U, [0,1])$.

3.6. Example

Consider the initial universe set $U = \{h_1, h_2, h_3, h_4, h_5\}$ and a set of parameters $\delta = \{p_\theta,\ p_1,\ p_2,\ p_3\}$ which is defined in above equation under the binary operation "*" such that

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$*$	p_θ	p_1	p_2	p_3
p_θ	p_θ	p_θ	p_θ	p_θ
p_1	p_1	p_θ	p_1	p_1
p_2	p_2	p_2	p_θ	p_2
p_3	p_3	p_3	p_3	p_θ

And $A = \{p_\theta,\ p_2,\ p_3\}$ is a subalgebra of δ. Now consider $((F, f), \delta)$ "DFT-soft fuzzy set" as follows

$$F: \delta \to P(U),\ y \mapsto \begin{cases} \{\ h_2\} & \text{if } y = p_\theta \\ \{h_1, h_2, h_3\} & \text{if } y = p_2 \\ \{h_1, h_2\} & \text{if } y = p_3 \end{cases}$$

Define $f: \delta \to [0\ 1]$ such that $f(p_\theta) = 0.8$, $f(p_2) = 0.6$ and $f(p_3) = 0.5$. Then $((F, f), A)$ is "DFT-soft fuzzy set" $(U, [0, 1])$.

3.7. Example

Let $(\delta, *, p_\theta)$ is BCI-algebra, where $\delta = \{\ p_\theta, p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$, defined by

$*$	p_θ	p_1	p_2	p_3	p_4	p_5	p_6	p_6
p_θ	p_θ	p_θ	p_θ	p_θ	p_4	p_4	p_4	p_4
p_1	p_1	p_θ	p_θ	p_θ	p_5	p_4	p_4	p_4
p_2	p_2	p_2	p_θ	p_θ	p_6	p_6	p_4	p_4
p_3	p_3	p_2	p_1	p_θ	p_7	p_6	p_5	p_4
p_4	p_4	p_4	p_4	p_4	p_θ	p_θ	p_θ	p_θ
p_5	p_5	p_4	p_4	p_4	p_1	p_θ	p_θ	p_θ
p_6	p_6	p_6	p_4	p_4	p_2	p_2	p_θ	p_θ
p_7	p_7	p_6	p_5	p_4	p_3	p_2	p_1	p_θ

Now consider a "DFT-soft fuzzy set" $((F, f), \delta)$, as follows

$$F: \delta \to P(U), y \mapsto \begin{cases} \{h_1, h_3, h_5, h_7\} & \text{if } y \in \{ p_\theta, p_1, p_2, p_3\} \\ \{h_1, h_2, h_4\} & \text{if } y \in \{ p_4, p_5, p_6, p_7\} \end{cases}$$

where $U = \{h_1, h_2, h_3, h_4, h_5, h_6\ h_7\}$.

Define $f: \delta \to [0\ 1]$ such that $f(p_\theta) = 0.8$, $f(p_1) = 0.7$, $f(p_2) = 0.6$ and $f(p_3) = 0.5$, $f(p_4) = 1 = f(p_5)$, $f(p_6) = 0.9 = f(p_7)$.

Then we have

$F(p_6 * p_5) = F(p_2) = \{h_1, h_3, h_5, h_7\} \nsubseteq \{h_1, h_2, h_4\} = F(p_6) \cup F(p_5)$

And

$f(p_6 * p_5) = f(p_2) = .6 \ngeq .9 = .9 \wedge .9 = f(p_6) \wedge f(p_5)$

Implies $((F, f), \delta)$ is not a "DFT-soft fuzzy algebra" over $(U, [0, 1])$.

3.8. Lemma

Every "DFT-soft fuzzy algebra" $((F, f), \delta)$ over $(U, [0, 1])$ satisfies the following condition

$F(\theta) \subseteq F(x)\ and\ f(\theta) \geq f(x)$ for all $x \in \delta$.

Proof

Since $F(x * x) \subseteq F(x) \cup F(x)$ for all $x \in \delta$ implies $F(x * x) = F(\theta) \subseteq F(x)$ because $x * x = \theta$.

Now $f(x * x) \geq f(x) \wedge f(x)$ for all $x \in \delta$. Since $x * x = \theta$ so $f(x * x) = f(\theta) \geq f(x)$.

Hence, proof is complete.

3.9. Theorem

For a "DFT-soft fuzzy algebra" $((F, f), \delta)$ over $(U, [0, 1])$, the following are equivalent;

(1) $F(\theta) = F(x)\ and\ f(\theta) = f(x)$ for all $x \in \delta$,

(2) $F(x * y) \subseteq F(y)\ and\ f(x * y) \geq f(y)$ for all $x, y \in \delta$.

Proof

(1) $\Rightarrow$ (2) , We suppose that (1) is valid. Now by definition of $((F, f), \delta)$, we have

(for all $x \in \delta$) $F(x * y) \subseteq F(x) \cup F(y) = F(\theta) \cup F(y) = F(y)$ because $F(\theta) = F(x)$ for all $x \in \delta$,

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Now (for all $x \in \delta$)$f(x * y) \geq f(x) \wedge f(y) = f(\theta) \wedge f(y) = f(y)$ because $f(\theta) = f(x)$ for all $x \in \delta$.

(2) ⇒ (1), Assume that (2) is valid. Then for $y = \theta$ such that $F(x * \theta) \subseteq F(\theta)$ and $f(x * \theta) \geq f(\theta)$ implies $F(x) \subseteq F(\theta)$ and $f(x) \geq f(\theta)$ because $x * \theta = x$, then by 3.8. Lemma we have, $F(\theta) \subseteq F(x)$ *and* $f(\theta) \geq f(x)$ for all $x \in \delta$, so $F(x) = F(\theta)$ and $f(x) = f(\theta)$.

3.10. Proposition

In a BCI-algebra X, every "DFT-soft fuzzy algebra" $((F, f), A)$ over $(U, [0,1])$ satisfies the following condition

For all $u, v \in A$ $\begin{cases} F\big(u * (\theta * v)\big) \subseteq F(u) \cup F(v) \\ f\big(u * (\theta * v)\big) \geq f(u) \wedge f(v) \end{cases}$

Proof: Let $u, v \in A$, we have

$F\big(u * (\theta * v)\big) \subseteq F(u) \cup F(\theta * v) \subseteq F(u) \cup \big(F(\theta) \cup F(v)\big) \because ((F, f), A)$ is double framed soft fuzzy algebra implies $F\big(u * (\theta * v)\big) \subseteq F(u) \cup F(v)$ by 3.8. Lemma.

Now $f\big(u * (\theta * v)\big) \geq f(u) \wedge f(\theta * v) \geq f(u) \wedge \big(f(\theta) \wedge f(v)\big) \because ((F, f), A)$ is double framed soft fuzzy algebra implies $f\big(u * (\theta * v)\big) \geq f(u) \wedge f(v)$ by 3.8. Lemma.

Hence proved.

3.11. Definition

A "DFT-soft fuzzy set" $((F, f), \delta)$ over $(U, [0,1])$ is said to be a "DFB-soft fuzzy algebra" over $(U, [0,1])$ if it satisfies

$F(x * y) \subseteq F(x) \cup F(y)$ *and* $f(x * y) \geq f(x).f(y)$ for all $x, y \in \delta$.

3.12. Example

Let $\delta = \{\xi_\theta, \xi_1, \xi_2, \xi_3\}$ be a set with binary operation " $*$ " defined by

$*$	ξ_θ	ξ_1	ξ_2	ξ_3
ξ_θ	ξ_θ	ξ_1	ξ_2	ξ_3
ξ_1	ξ_1	ξ_θ	ξ_3	ξ_2
ξ_2	ξ_2	ξ_3	ξ_θ	ξ_1
ξ_3	ξ_3	ξ_2	ξ_1	ξ_θ

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Then $(\delta, *, \xi_\theta)$ is a BCI-algebra. Define

$$F: \delta \to P(U), y \mapsto \begin{cases} \{h_1, h_3, h_5\} & \text{if } y = \xi_\theta \\ U & \text{if } y \in \{\xi_1, \xi_2, \xi_3\} \end{cases} \quad \text{where } U = \{h_1, h_2, h_3, h_4, h_5, h_6\, h_7\}.$$

And

$f: \delta \to [0\ 1]$ such that $f(\xi_\theta) = 0.6$, $f(y) = 0.7$ for all $y \in \{\xi_1, \xi_2, \xi_3\}$

It is routine to verify $((F, f), \delta)$ is a "DFB-soft fuzzy algebra" over $(U, [0, 1])$.

3.13. Theorem

Let $((F, f), \delta)$ be a "DFT-soft fuzzy set" over $(U, [0, 1])$. Then prove that "DFT-soft fuzzy algebra" of δ over $(U, [0, 1])$ is also "DFB-soft fuzzy algebra" of δ over $(U, [0, 1])$.

Proof

Let $((F, f), \delta)$ be a "DFT-soft fuzzy algebra" over $(U, [0, 1])$ to prove $((F, f), \delta)$ be a "DFB-soft fuzzy algebra" over $(U, [0, 1])$.

As $((F, f), \delta)$ be a "DFT-soft fuzzy algebra" over $(U, [0, 1])$ then for any $u, v \in \delta$, we have

$F(u * v) \subseteq F(u) \cup F(v)$ is obvious,

$f(u * v) \geq f(u) \wedge f(v) \geq f(u).f(v)$ implies $f(u * v) \geq f(u).f(v)$.

Hence, $((F, f), \delta)$ is "DFB-soft fuzzy algebra" over $(U, [0, 1])$.

Note that converse of above theorem may or may not be true.

3.12. Example represented that $((F, f), \delta)$ is a "DFB-soft fuzzy algebra" of δ over $(U, [0, 1])$ but not a "DFT-soft fuzzy algebra" of δ over $(U, [0, 1])$ because

$f(\xi_1 * \xi_1) = f(\xi_\theta) = 0.6 \ngeq 0.7 = 0.7 \wedge 0.7 = f(\xi_1) \wedge f(\xi_1)$.

3.14. Lemma

Each "DFB-soft fuzzy algebra" $((F, f), \delta)$ over $(U, [0, 1])$ satisfies the following condition

$F(\theta) \subseteq F(x)$ *and* $f(\theta) \geq (f(x))^2$ for all $x \in \delta$.

Proof

Since $F(x * x) \subseteq F(x) \cup F(x)$ for all $x \in \delta$ implies $F(x * x) = F(\theta) \subseteq F(x)$ because $x * x = \theta$.

Now $f(x * x) \geq f(x).f(x)$ for all $x \in \delta$, since $x * x = \theta$ so $f(x * x) = f(\theta) \geq (f(x))^2$.

Hence, proof is complete.

3.15. Proposition

In a BCI-algebra X, every "DFB-soft fuzzy algebra" $((F,f),A)$ over $(U,[0,1])$ satisfies the following condition

For all $u,v \in A \begin{cases} F\big(u*(\theta*v)\big) \subseteq F(u) \cup F(v) \\ f\big(u*(\theta*v)\big) \geq f(u).f(v)^3 \end{cases}$

Proof: Let $u,v \in A$, we have

$F\big(u*(\theta*v)\big) \subseteq F(u) \cup F(\theta*v) \subseteq F(u) \cup \big(F.(\theta) \cup F(v)\big) \because ((F,f),A)$ is double framed soft fuzzy algebra over $(U,[0,1])$ implies $F\big(u*(\theta*v)\big) \subseteq F(u) \cup F(v)$, by 3.14. Lemma.

Now $f\big(u*(\theta*v)\big) \geq f(u).f(\theta*v) \geq f(u).\big(f(\theta).f(v)\big) \because ((F,f),A)$ is double framed soft fuzzy algebra over $(U,[0,1])$,
Implies $f\big(u*(\theta*v)\big) \geq f(u) \wedge f(v) \geq f(u).(f(v)^2.f(v))$, by 3.14. Lemma,
Implies $f\big(u*(\theta*v)\big) \geq f(u).f(v)^3$.

Hence, proof is complete.

3.16. Theorem

Let $\big((F,f),A\big)$ over $(U,[0,1])$ be a double framed T-soft fuzzy subset of a double framed soft fuzzy set $\big((\mathfrak{G},g),B\big)$ over $(U,[0,1])$ where $F(p)$and $\mathfrak{G}(p)$ both are identically approximations and $f(p)$ and $g(p)$ both are identical approximations for all $p \in A$. If $\big((\mathfrak{G},g),B\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$ then $\big((F,f),A\big)$ is both "DFT-soft fuzzy algebra" over $(U,[0,1])$ and "DFB-soft fuzzy algebra" over $(U,[0,1])$.

Proof

Let $u,v \in A$ then $,v \in B \because A \subseteq B$.

As $\big((\mathfrak{G},g),B\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$ so

$\mathfrak{G}(u) \cup \mathfrak{G}(v) = F(u) \cup F(v) \supseteq F(u*v) = \mathfrak{G}(u*v)$ implies $F(u) \cup F(v) \supseteq F(u*v)$.

$\because F(p)$and $\mathfrak{G}(p)$ are identical approximations.

$g(u) \wedge g(v) = f(u) \wedge f(v) \leq f(u*v) = g(u*v)$ implies $f(u) \wedge f(v) \leq f(u*v)$.

$\because f(p)$ and $g(p)$ are identical approximations.

Hence $\big((F,f),A\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$.

Now, $f(u).f(v) = g(u).g(v) \leq f(u) \wedge f(v) = g(u) \wedge g(v) = f(u) \wedge f(v) \leq f(u*v) = g(u*v)$ implies $f(u).f(v) \leq f(u*v)$. $\because f(p)$ and $g(p)$ are identical approximations.

Hence $\big((F,f),A\big)$ is a double frame B-soft fuzzy algebra over $(U,[0,1])$.

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3.17. Theorem

Let $\big((F,f),A\big)$ over $(U,[0,1])$ be a double framed B-soft fuzzy subset of a double framed soft fuzzy set $\big((\mathfrak{G},g),B\big)$ over $(U,[0,1])$ where $F(p)$and $\mathfrak{G}(p)$ both are identically approximations and $f(p)$ and $g(p)$ both are identical approximations for all $p \in A$. If $\big((\mathfrak{G},g),B\big)$ is a "DFB-soft fuzzy algebra" over $(U,[0,1])$ then $\big((F,f),A\big)$ is a "DFB-soft fuzzy algebra" over $(U,[0,1])$.

Proof

Let $u,v \in A$ then $u,v \in B \because A \subseteq B$.

As $\big((\mathfrak{G},g),B\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$ so

$\mathfrak{G}(u) \cup \mathfrak{G}(v) = F(u) \cup F(v) \supseteq F(u * v) = \mathfrak{G}(u * v)$ implies $F(u) \cup F(v) \supseteq F(u * v)$.

$\because F(p)$ and $\mathfrak{G}(p)$ are identical approximations.

$g(u).g(v) = f(u).f(v) \leq f(u * v) = g(u * v)$ implies $f(u).f(v) \leq f(u * v)$. $\because f(p)$ and $g(p)$ are identical approximations.

Hence $\big((F,f),A\big)$ is a double frame B-soft algebra over $(U,[0,1])$.

Remark

(1) Converse of above theorem is may or may not be possible.

(2) $\big((F,f),A\big)$ is may or may not be "DFT-soft fuzzy algebra" over $(U,[0,1])$ because 3.12. example represented that each "DFB-soft fuzzy algebra" is not "DFT-soft fuzzy algebra".

3.18. Definition

Let $V_{(F,f)}$ and $B_{(\mathfrak{G},g)}$ are two "DFT-soft fuzzy set" over $(U,[0,1])$. Then extended uni-int "DFT-soft fuzzy set" of $V_{(F,f)}$ and $B_{(\mathfrak{G},g)}$ is defined as a "DFT-SS" $(V \cup B\,)_{(F \tilde{\cup} \mathfrak{G}, f \bar{\cap} g)}$,

Where $F \tilde{\cup} \mathfrak{G} : (V \cup B\,) \longrightarrow P(U)$ defined by

$$p \longrightarrow \begin{cases} F(p)\ if\ p \in V - B \\ \mathfrak{G}(p)\ if\ p \in B - V \\ F(p) \cup \mathfrak{G}(p)\ if\ p \in V \cap B \end{cases}$$

and $f \bar{\cap} g : (V \cup B\,) \longrightarrow [0,1]$ defined by

$$p \longrightarrow \begin{cases} f(p)\ if\ p \in V - B \\ g(p)\ if\ p \in B - V \\ f(p) \wedge g(p)\ if\ p \in V \cap B. \end{cases}$$

It is denoted by $V_{(F,f)} \sqcup_{\mathcal{E}} B_{(\mathfrak{G},g)} = (V \cup B\,)_{(F \tilde{\cup} \mathfrak{G}, f \bar{\cap} g)}$. We shall call this uni-int T-"DFT-soft fuzzy set" over $(U,[0,1])$ as union of "DFT-soft fuzzy set" over $(U,[0,1])$.

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3.19. Theorem

The extended uni-int "DFT-soft fuzzy set" over $(U,[0,1])$ of two double-framed T-soft fuzzy algebras $\big((F,f),A\big)$ and $\big((\mathfrak{G},g),A\big)$ over $(U,[0,1])$ is "DFT-soft fuzzy algebra" over $(U,[0,1])$.

Proof

Let $u,v\in A$ then we have

$$(F\,\tilde{\cup}\,\mathfrak{G})(u*v)=F(u*v)\cup\mathfrak{G}(u*v)\subseteq\big(F(u)\cup F(v)\big)\cup\big(\mathfrak{G}(u)\cup\mathfrak{G}(v)\big)$$

$$=\big(F(u)\cup\mathfrak{G}(u)\big)\cup\big(F(v)\cup\mathfrak{G}(v)\big)=(F\,\tilde{\cup}\,\mathfrak{G})(u)\cup(F\,\tilde{\cup}\,\mathfrak{G})(v),$$

$$(f\,\bar{\cap}\,g)(u*v)=f(u*v)\wedge g(u*v)\geq\big(f(u)\wedge f(v)\big)\wedge\big(g(u)\wedge g(v)\big)$$

$$=\big(f(u)\wedge g(u)\big)\wedge\big(f(v)\wedge g(v)\big)=(f\,\bar{\cap}\,g)(u)\wedge(f\,\bar{\cap}\,g)(v).$$

Hence, $V_{(F,f)}\,\overline{\sqcup_{\varepsilon}}\,B_{(\mathfrak{G},g)}$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$.

3.20. Definition

Let $V_{(F,f)}$ and $B_{(\mathfrak{G},g)}$ are two "DFSS" over U. Then extended int-uni "DFT-SS" of $V_{(F,f)}$ and $B_{B_{(\mathfrak{G},g)}}$ is defined as a "DFSS" $(V\cup B\,)_{(\,F\tilde{\cap}\mathfrak{G}\,,f\bar{\cup}g)}$ over $(U,[0,1])$,

Where $F\,\tilde{\cap}\,\mathfrak{G}:(V\cup B\,)\longrightarrow P(U)$ defined by

$$p\longrightarrow\begin{cases}F(p)\;if\;p\in V-B\\ \mathfrak{G}(p)\;if\;p\in B-V\\ F(p)\cap\mathfrak{G}(p)\;if\;p\in V\cap B\end{cases}$$

and $f\,\bar{\cup}\,g:(V\cup B\,)\longrightarrow[0,1]$ defined by

$$p\longrightarrow\begin{cases}f(p)\;if\;p\in V-B\\ g(p)\;if\;p\in B-V\\ f(p)\vee g(p)\;if\;p\in V\cap B\end{cases}$$

It is denoted by $V_{(F,f)}\,\overline{\sqcap_{\varepsilon}}\,B_{(\mathfrak{G},g)}=(V\cup B\,)_{(\mathfrak{G}\tilde{\cap}g,f\bar{\cup}g)}$. We shall call this int-uni "DFT-soft fuzzy set" over $(U,[0,1])$ as intersection of double framed T- soft fuzzy set over $(U,[0,1])$.

3.21. Theorem

The extended int-uni "DFT-soft fuzzy set" of two double-framed T-soft fuzzy algebras $\big((F,f),A\big)$ and $\big((\mathfrak{G},g),A\big)$ over $(U,[0,1])$ is "DFT-soft fuzzy algebra" over $(U,[0,1])$ if $((F,f),A)\ \tilde{\tilde{\subseteq}}\ ((\mathfrak{G},g),A)$.

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Proof

Let $u, v \in A$ then we have

$$(F \,\tilde{\cap}\, \mathfrak{G})(u * v) = F(u * v) \cap \mathfrak{G}(u * v) = F(u * v) \subseteq F(u) \cup F(v)$$

$$= \big(F(u) \cup \mathfrak{G}(u)\big) \cap \big(F(v) \cup \mathfrak{G}(v)\big) = (F \,\tilde{\cap}\, \mathfrak{G})(u) \cup (F \,\tilde{\cap}\, \mathfrak{G})(v),$$

$$\because \text{ if } A \subseteq C \text{ and } B \subseteq D \text{ then } A \cup B = (A \cap C) \cup (B \cap D).$$

$$(f \,\bar{\cup}\, g)(u * v) = f(u * v) \vee g(u * v) = f(u * v) \geq f(u) \wedge f(v)$$

$$= \big(f(u) \vee g(u)\big) \wedge \big(f(v) \vee g(v)\big) = (f \,\bar{\cup}\, g)(u) \wedge (f \,\bar{\cup}\, g)(v).$$

Hence, $V_{(F,f)} \,\overline{\sqcap}_{\mathcal{E}}\, B_{(\mathfrak{G},g)}$ is a “DFT-soft fuzzy algebra” over $(U, [0,1])$.

3.22. Theorem

The extended uni-int “DFT-soft fuzzy set” of two double-framed B-soft fuzzy algebras $\big((F,f),A\big)$ and $\big((\mathfrak{G},g),A\big)$ over $(U,[0,1])$ is “DFB-soft fuzzy algebra” over $(U,[0,1])$.

Proof

Let $u, v \in A$ then we have

$$(F \,\tilde{\cup}\, \mathfrak{G})(u * v) = F(u * v) \cup \mathfrak{G}(u * v) \subseteq \big(F(u) \cup F(v)\big) \cup \big(\mathfrak{G}(u) \cup \mathfrak{G}(v)\big)$$

$$= \big(F(u) \cup \mathfrak{G}(u)\big) \cup \big(F(v) \cup \mathfrak{G}(v)\big) = (F \,\tilde{\cup}\, \mathfrak{G})(u) \cup (F \,\tilde{\cup}\, \mathfrak{G})(v),$$

$$(f \,\bar{\cap}\, g)(u * v) = f(u * v) \wedge g(u * v) \geq \big(f(u).f(v)\big) \wedge \big(g(u).g(v)\big) \;\ldots\ldots\text{(a)}$$

Case (1)

If $\big(f(u).f(v)\big) \wedge \big(g(u).g(v)\big) = \big(f(u).f(v)\big)$ then (a) becomes

$$(f \,\bar{\cap}\, g)(u * v) \geq \big(f(u).f(v)\big) \geq \big(f(u) \wedge g(u)\big).\big(f(v) \wedge g(v)\big) = (f \,\bar{\cap}\, g)(u).(f \,\bar{\cap}\, g)(v)$$

Implies $(f \,\bar{\cap}\, g)(u * v) \geq (f \,\bar{\cap}\, g)(u).(f \,\bar{\cap}\, g)(v)$.

Case (2)

If $\big(f(u).f(v)\big) \wedge \big(g(u).g(v)\big) = \big(g(u).g(v)\big)$ then (a) becomes

$(f \,\overline{\cap}\, g)(u * v) \geq \big(g(u).\, g(v)\big) \geq \big(f(u) \wedge g(u)\big).\big(f(v) \wedge g(v)\big) = (f \,\overline{\cap}\, g)(u).(f \,\overline{\cap}\, g)(v)$

Implies $(f \,\overline{\cap}\, g)(u * v) \geq (f \,\overline{\cap}\, g)(u).(f \,\overline{\cap}\, g)(v)$.

Hence, proof is complete.

Hence, $V_{(F,f)} \,\overline{\sqcup_{\mathcal{E}}}\, B_{(\mathfrak{G},g)}$ is a "DFB-soft fuzzy algebra" over $(U, [0,1])$.

3.23. Theorem

The extended int-uni "DFT-soft fuzzy set" of two double-framed B-soft fuzzy algebras $\big((F, f), A\big)$ and $\big((\mathfrak{G}, g), A\big)$ over $(U, [0,1])$ is "DFB-soft fuzzy algebra" over $(U, [0,1])$ if $((F, f), A) \;\tilde{\tilde{\subseteq}}\; ((\mathfrak{G}, g), A)$.

Proof

Let $u, v \in A$ then we have

$$(F \,\tilde{\cap}\, \mathfrak{G})(u * v) = F(u * v) \cap \mathfrak{G}(u * v) = F(u * v) \subseteq F(u) \cup F(v)$$

$$= \big(F(u) \cup \mathfrak{G}(u)\big) \cap \big(F(v) \cup \mathfrak{G}(v)\big) = (F \,\tilde{\cap}\, \mathfrak{G})(u) \cup (F \,\tilde{\cap}\, \mathfrak{G})(v),$$

$$\because \text{if } A \subseteq C \text{ and } B \subseteq D \text{ then } A \cup B = (A \cap C) \cup (B \cap D).$$

$$(f \,\overline{\cup}\, g)(u * v) = f(u * v) \vee g(u * v) = g(u * v) \geq \big(g(u) \wedge g(v)\big)$$

$$= \big(f(u) \vee g(u)\big).\big(f(v) \vee g(v)\big) = (f \,\overline{\cap}\, g)(u) \wedge (f \,\overline{\cap}\, g)(v).$$

Hence, $V_{(F,f)} \,\overline{\sqcap_{\mathcal{E}}}\, B_{(\mathfrak{G},g)}$ is a "DFB-soft fuzzy algebra" over $(U, [0,1])$.

For a "DFT-soft fuzzy set" $\big((F, f), A\big)$, we defined $\psi - exclusive$ and $\alpha - cut$ respectively as follows, where $\psi \subseteq U$ and $\alpha \in [0,1]$

$$\psi - exclusive = \{u \in A \mid \psi \supseteq F(u)\},$$

$$\alpha - cut = \{u \in A \mid \alpha \leq f(u)\},$$

and denoted as $e_A(F; \psi)$ and f_α, respectively.

The set $\mathrm{DFT_A}(F, f)_{(\psi,\alpha)} = \{u \in A \mid \psi \supseteq F(u), \alpha \leq f(u)\}$ is called double framed including set.

3.24. Theorem

For a "DFT-soft fuzzy set" $\big((F, f), \delta\big)$ over $(U, [0,1])$, the following are equivalent

(1) $\big((F, f), \delta\big)$ is double framed soft fuzzy algebra over $(U, [0,1])$,

(2) for every $\psi \subseteq U$ and $\beta \in [0,1]$ with $\psi \in Im(F)$ and $\alpha \in Im(f)$ then $\psi - exclusive$ set and $\alpha - cut$ of $\big((F,f),\delta\big)$ are sub algebras of δ.

Proof

(1) ⇒ (2)

Consider $\big((F,f),\delta\big)$ is "DFT-soft fuzzy algebra" over $(U,[0,1])$. Let $u,v \in \delta$ be such that $u,v \in e_\delta(F;\psi)$ and $u,v \in f_\alpha$ for every $\psi \subseteq U$ and $\alpha \in [0,1]$ respectively, with $\psi \in Im(F)$ and $\alpha \in Im(f)$. Then by definition of $\big((F,f),\delta\big)$ such that

$$F(u*v) \subseteq F(u) \cup F(v) \subseteq \psi, \qquad g(u*v) \geq g(u) \wedge g(v) \geq \alpha.$$

Implies $u*v \in e_\delta(F;\psi)$ and $u*v \in f_\alpha$.

Hence, $e_\delta(F;\psi)$ and f_α are subalgebras of δ.

(2) ⇒ (1), Suppose that (2) is valid to prove $\big((F,f),\delta\big)$ is double framed soft fuzzy algebra over $(U,[0,1])$. Let $u,v \in \delta$ be such that $F(u) = \psi_u, F(v) = \psi_v$ and $f(u) = \alpha_u, f(v) = \alpha_v$. By taking $\psi = \psi_u \cup \psi_v$ and $\alpha = \alpha_u \wedge \alpha_v$. Then $u*v \in e_\delta(F;\psi)$ and $u*v \in f_\alpha$.

As $e_\delta(F;\psi)$ and f_α are double framed T-soft fuzzy subalgebra so

$$F(u*v) \subseteq \psi = \psi_u \cup \psi_v = F(u) \cup F(v), \qquad g(u*v) \geq \alpha = \alpha_u \wedge \alpha_v = g(u) \wedge g(v).$$

Hence, $\big((F,f),\delta\big)$ is "DFT-soft fuzzy algebra" over $(U,[0,1])$.

3.25. Corollary

If $\big((F,f),\delta\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$ then double framed including set of $\big((F,f),\delta\big)$ is subalgebra X.

3.26. Definition

Let $\big((F,f),\delta\big)$ be a "DFT-soft fuzzy set" over $(U,[0,1])$. Then "DFT-soft fuzzy set" $\big((F^*,f^*),\delta\big)$ over $(U,[0,1])$ is defined by

$$F^* : \delta \longrightarrow P(U) \;\; p \longrightarrow \begin{cases} F(p) & p \in e_\delta(F;\psi) \\ M & otherwise \end{cases}$$

$$f^* : \delta \longrightarrow [0,1] \;\; p \longrightarrow \begin{cases} f(p) & if\ p \in f_\alpha \\ \gamma & therwise \end{cases}$$

Where $\psi, M \subseteq U$, $\alpha, \gamma \in [0,1]$ and $F(p) \subset M$, $f(p) > \gamma$.

3.27. Theorem

If $\big((F,f),\delta\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$. Then prove that $\big((F^*,f^*),\delta\big)$ over $(U,[0,1])$ is also a "DFT-soft fuzzy algebra".

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Proof

Let $\big((F,f),\delta\big)$ be a "DFT-soft fuzzy algebra" over $(U,[0,1])$, to prove $\big((F^*,f^*),\delta\big)$ is also a "DFT-soft fuzzy algebra" over $(U,[0,1])$.

As $\big((F,f),\delta\big)$ be a "DFT-soft fuzzy algebra" over $(U,[0,1])$ so for every $\psi \subseteq U$ and $\alpha \in [0,1]$, $e_\delta(F;\psi)$ and f_α are sub algebras with $\psi \in Im(F)$ and $\alpha \in Im(f)$.

Let $u,v \in \delta$ if $u,v \in e_\delta(F;\psi)$ then $u*v \in e_\delta(F;\psi)$.

Thus, $F^*(u*v) = F(u*v) \subseteq F(u) \cup F(v) = F^*(u) \cup F^*(v)$,

If $u \notin e_\delta(F;\psi)$ or $v \notin e_\delta(F;\psi$ then $F^*(u) = M$ or $F^*(v) = M$,

Thus, $F^*(u*v) \subseteq M = F^*(u) \cup F^*(v)$ $\because F(p) \subset M$ if $p \notin e_\delta(F;\psi)$.

Now, if $u,v \in i_\delta(f;\alpha) = f_\alpha$ then $u*v \in f_\alpha$.

Thus, $f^*(u*v) = f(u*v) \geq f(u) \wedge f(v) = f^*(u) \wedge f^*(v)$

Implies, $f^*(u*v) \geq f^*(u) \wedge f^*(v)$.

If $u \notin f_\alpha$ or $v \notin f_\alpha$ then $f^*(u) = \gamma$ or $f^*(v) = \gamma$,

Thus,

$f^*(u*v) = f(u*v) \geq \gamma = f(u) \wedge f(v) = f^*(u) \wedge f^*(v) \because f(p) > \gamma$ if $p \notin f_\alpha$.

3.28. Definition

Let $\big((F,f),A\big)$ and $\big((F,f),B\big)$ be "DFT-soft fuzzy set" over $(U,[0,1])$. Then their product is defined as

$$F_{A\vee B}: A \times B \longrightarrow P(U),$$

$$f_{A\wedge B}: A \times B \longrightarrow [0,1]$$

Such that $(x,y) \longrightarrow \begin{cases} F(x) \cup F(y) \\ f(x) \wedge f(y) \end{cases}$

And denoted by $\big((F_\vee,f_\wedge),A\times B\big)$.

3.29. Theorem

Let $\big((F,f),A\big)$ and $\big((F,f),B\big)$ be double framed T-soft fuzzy algebras over $(U,[0,1])$. Then prove that $\big((F_\vee,f_\wedge),A\times B\big)$ is also a "DFT-soft fuzzy algebra" over $(U,[0,1])$.

Proof

We know that $(A \times B, \circledast, (\theta,\theta)$ is also a BCK/BCI algebra. Then we only prove that $\big((F_\vee,f_\wedge),A\times B\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$.

Let $(x,y),(u,v)\in A\times B$, we have

$$F_{A\vee B}\big((x,y)\circledast(u,v)\big)=F_{A\vee B}(x*u,y*v)$$

$$=F(x*u)\cup F(y*v)\subseteq(F(x)\cup F(u))\cup(F(y)\cup F(v))$$

$$=(F(x)\cup F(y))\cup(F(u)\cup F(v))$$

$$=F_{A\vee B}(x,u)\cup F_{A\vee B}(y,v).$$

$$f_{A\wedge B}\big((x,y)\circledast(u,v)\big)=f_{A\wedge B}(x*u,y*v)=f(x*u)\wedge f(y*v)$$

$$\geq(f(x)\wedge f(u))\wedge(f(y)\wedge f(v))$$

$$=(f(x)\wedge f(y))\wedge(f(u)\wedge f(v))$$

$$=f(x*y)\wedge f(u*v)$$

$$=f_{A\wedge B}(x,y)\wedge f_{A\wedge B}(u,v).$$

Hence, $\big((F_\vee,f_\wedge),A\times B\big)$ is a "DFT-soft fuzzy algebra" over $(U,[0,1])$.

3.30. Theorem

Let $\big((F,f),A\big)$ and $\big((F,f),B\big)$ be double framed T-soft fuzzy algebras over $(U,[0,1])$. Then prove that $\big((F_\vee,f_\wedge),A\times B\big)$ is also a "DFB-soft fuzzy algebra" over $(U,[0,1])$.

Proof

We similar to above theorem by using definition of double framed T-soft fuzzy algebra and previous theorem.

Conclusion:

We defined the concept of "DFT-soft fuzzy set" which is combination of double framed T soft fuzzy set and introduced their notions. Further, proved that extended uni-int of "DFT-soft fuzzy set" is also double framed T-soft fuzzy algebra and "DFB-soft fuzzy algebra" but converse is not true because, we proved that each double framed T soft fuzzy algebra is also "DFB-soft fuzzy algebra" but converse is not true. And we also discussed the product of double framed T-soft fuzzy algebra and "DFB-soft fuzzy algebra".

Further, we will apply this concept in different algebraic structure and discussed its operations.

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