A GIS-based Constraint Multi-objective decision-making approach to Determining Facility Location Using Spatial Data (Case study: Bank branch)

Abolfazl Ranjbar 1* and Farshad Hakimpour 2

1 School of Surveying and Geospatial Engineering, College of Engineering, University of Tehran, Tehran, Iran.; abranjbar57@ut.ac.ir and ranjbar57@yahoo.com
2 School of Surveying and Geospatial Engineering, College of Engineering, University of Tehran, Tehran, Iran.; fhakimpour@ut.ac.ir
* Correspondence: abranjbar57@ut.ac.ir; Tel.: +98 914 414 0763

Abstract: This paper presents a Geographic Information System (GIS)-based multi-objective decision-making approach for finding Location of bank branches under competitive conditions planning in an urban region. In a market setting, banks need to find quickly best locating until to attract the maximum customers in the competitive environment. This paper determines the location of bank branches under competitive conditions with different attractive conditions with a new approach. Finding an optimum location of branches depends on many factors and constraints, that these problems are known as NP-hard problems. From a marketing point of view, the total network-based distance needed for customers to travel to the newly established and existing branches should be minimized; also the network-based distance between the newly established branch and other existing branches of the same bank should be Maximizes. In addition to the two objectives mentioned, we consider two constraints is this paper, first the total number of customers for the new branch should not be less than a specified number, second the new branch should not attract customers of old branches of the same bank more than a threshold. For these purposes is an example of multi-objective optimization problems, involving two objectives and two constraints. In this paper, we compare NSGA-II, MOPSO and SPEA2 algorithms approaches optimization algorithm is applied for solving this model that has conflicting purposes and constraints. To fulfill this proposes a part of the Tabriz city was selected for implementation. The evaluation reveals that NSGA-II algorithm is able to achieve many of the solutions on the Pareto Front, furthermore, the distribution of the solutions on the Pareto Front and their convergence has been better than MOPSO and SPEA2 approaches. The results of statistical and test indicate that the accuracy and convergence speed of the NSGA-II algorithm has the capability to find the optimum location of new branches under competitive conditions with opposite purpose and multiple constraints. Application results show the usefulness of the developed model in supporting Location of bank branches under competitive conditions planning in an urban region.

Keywords: Multi-Objective Particle Swarm Optimization (MOPSO); Non-dominated Sorting Genetic Algorithm-II (NSGA-II); Bank, Competitive Facility Location, Constraints Multi-objective Optimization; Strength Pareto Evolutionary Algorithm 2 (SPEA2), Meta-Heuristics, Pareto Front.
1-Introduction

The distributing of facilities in each system is a strategic decision for every company. The facility location and customer allocation problem constitutes the core of distribution systems [1]. Competitive facility location problems arise in contexts where firms operating a chain of facilities such as offices, retail stores, bank branches, schools, hospitals or other kinds of chain outlets must decide, in the presence of competitors, where to open the next (set of) facilities or to move existing facilities to new locations [1,2]. Many factors influence such decisions, including the available budget, the location of the existing same facilities, location of competitors facilities and their performance, demographics (including population, age, level of attractive facilities and etc.) of the demand-generating population, target customer segments, availability and logistical suitability of candidate locations [2]. Many applications of location models in cities can be found in a book by Drezner and Hamacher [3] or Heragu [4].

Although the volume of practices and customer’s traffic to their branches has been reduced because of spreading the electronic banking right now, still the quality of branches’ performance plays a major role in the efficiency of these financial-service enterprises. Therefore, it should be considered that the locating of branches is performed in such a way that leads to customer’s satisfaction due to enhancing the quality of branches performance.

Nowadays, to perform particular financial tasks bank customers often need to be present at their bank. For the sake of its customers, a bank should increase its branches in the city to attract more customers in the race with competing banks. However, establishing new branches is too expensive and banks prefer to carry out an optimal location finding the nearest open facility. After them, where new facilities must be located on a network space to compete with a number of existing (competitor) facilities, with the assumption that a customer always visits the nearest open facility. After them, using the metaheuristic methods is also another approach to solve the locating problems. Genetic Algorithms have received considerable attention in the facility location literature. Several studies have attempted to solve variants of p-median, p-center, and other related problems [24-26].

Recently, many of meta-heuristics optimization algorithms have been used for tackling multi-objective facility location problems [27-29]. For instance, Min [30] combined the fuzzy goal programming and decision support system in order to locate the bank branches. In another study, Chen [31] has also used a hybrid heuristic method in locating of the hub. Furthermore, Caballero, Gonzalez and Guerrero [32] have used a metaheuristic method based on tabu search for locating. After them, Chen and Ting [33] combined a Lagrangian heuristic method and Ant Colony System in order to locate...

the single source capacitated location problem. For example, Beheshtifar and Alimohammadi [12] applied Multi-Objective Evolutionary Algorithm (MOEA) for modeling site suitability for health-care facilities and Neema and Ohgai [34] presented a Genetic Algorithm (GA) based MO model to obtain optimum locations for urban parks and open spaces and they proved that this algorithm is effective. Li and Yeh [35] Integrated Geospatial Information System (GIS) and GA for searching the optimal locations and compared Simulate Annealing (SA) with GA and then conclude that GA shows better performance than SA. In another study, Rahmati, et al. [36] presented multi-objective in a multi-server facility allocation problem with Multi-Objective Harmony Search (MOHS) and compared it with two popular algorithms NSGA-II and Non-dominated Ranking Genetic Algorithm (NRGA) and according to their result MOHS has better performance than two mentioned algorithms in terms of computational time. In another study, Xiao, et al. [37] applied MOEAs to optimize the shape and location of sites by considering the cost surface. Li, et al. [38] used the ant colony optimization technique to solve the site selection problem by minimization of the total costs. Masoomi, et al. [39] presented MOPSO algorithm to find the optimum arrangement of urban land uses in parcel level and their result shows that MOPSO is effective. Duh and Brown [40] develop a knowledge-informed Pareto SA approach to tackle specifically multi-objective allocation problems that consider spatial patterns as objectives, and then proved that is effective. Yeh and Chow [41] presented the integration of GIS and a location-allocation model for public facilities planning. Beheshtifar and Alimoahmmadi [42] applied NSGA-II optimization that combined geographical information system (GIS) analysis with a multi-objective genetic algorithm. Optimum sites for new clinics were determined by considering four objectives: minimizing total travel cost, minimizing inequity in access to clinics, minimizing the land-use incompatibility in the study area, and minimizing the costs of land acquisition and facility establishment. They used a posteriori preference method, Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS), was applied to assess and compare the Pareto-optimal solutions and to select the best solution according to different weight vectors. Luis, Salhi and Nagy [43] provided constructive and adaptive heuristics to generate initial solutions. Mohammadi, Malek and Alesheikh [44] used a new method that uses two genetic algorithms for capacitated multi-source Weber problem. The first, solve the location problem while the second, solve the allocation problem. A GRASP-based heuristic was also proposed by Luis, Salhi and Nagy [45] where adaptive learning is used to construct the restricted candidate list. Akyüzet, Altinel and Oncan [46] proposed two types of branch and bound algorithms for the capacitated MSWP. The first is an allocation space based-branch and bound algorithm whereas the second is based on the partition of the location space. Brimberg, Drezner and Mladenovic [47] proposed a new local search approach is embedded within Variable Neighborhood Search for solving the multi-source Weber problem. The algorithm switches between the continuous model and its discrete counterpart until no further improvement can be found in either.

Bashiri and Bakhtiarifar [8] used simulated annealing for a problem with one objective, but the present study has three objectives, which complicates the problem space. In the former study, the capacity constraint and ordered demands were also omitted. Yu and Solvang [48] used multi-objective location-allocation for the management of municipal solid waste, but the capacity constraint was omitted, and their results showed that the CPU running time increased significantly with an increase in the size of the problem. For future development, they proposed using an efficient algorithm such as a genetic algorithm. Ma and Zhao [49] used a multi-objective artificial immune optimization algorithm in land use allocation. However, they did not integrate their model into a GIS environment and the capacities of facilities and the order of demands were not considered. In another research, Bolouri at el. [50] the ordered capacitated multi-objective location-allocation problem for fire stations is presented with some incompatible objectives, such as minimizing the network-based distance between the demand and the station, minimizing the arrival time from the station, and maximizing the station’s coverage.
As mentioned above, location selection problem in banking is an important issue for the commercial success in a competitive environment [51-54]. Like many practical problems, finding Location of Bank Branches under Competitive Conditions (LBBCC) can have more than one objective and constraint. From a marketing point of view, the optimum location of a new branch should be far enough from other branches; also the sum of network-based distances between all customers and the new branch should be minimized (customer focus). In reality planning, these objectives conflict each other. Also, we consider two conditions in this paper, first the total number of customers for the new branch should not be less than a specified number, second the new branch should not attract customers of old branches of the same bank more than a threshold. Therefore, finding LBBCC is categorized as a constraint multi-objective problem. This paper presents a constraint multi-objective spatial optimization model to solve this problem. To fulfill this aim, we applied and compared the NSGA-II, MOPSO and SPEA2 multi-objective optimization algorithms with a constraint-handling method proposed by Oyama[55], for determining optimum LBBCC using spatial data of Tabriz city.

The remainder of the paper is organized as follows: Section 2, first defined Pareto front and performance metrics concepts, the fundamental concept of utilized constrain handling method for the multi-objective algorithm and later in this section, the approach of NSGA-II, MOPSO and SPEA2 multi-objective optimization are described in brief. In sections 3, we illustrate the problem of locating bank branches under competitive conditions and then provide a model formulation, which is tightly connected to the GIS platform we have used. In section 4, we illustrate AHP method for determining to weigh existing banks in competitive and then we explain Gravity model for select the best bank from the customer view. In section 5, we have introduced area study briefly. We describe implementation and results of our case study that derives from a real dataset in the city of Tabriz, IRAN and some relevant issues are discussed remarks are presented in Section 6. Finally, conclusion and suggestions for later work are discussed.

2- Multi-Objective Backgrounds

In this section, first, we defined Pareto front and performance metrics (Spacing and Coverage of Two Set) for evaluated multi-objective optimization. Moreover, all there NSGA-II, MOPSO and SPEA2 algorithms are described in brief. Also, Oyama constrains handling method is defined at end of this section.

2-1- Dominance and Pareto front

Consider the MO problems with k objectives,

Find \( \mathbf{x} \) that minimize

\[
F(\mathbf{x}) = f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})
\]

subject to

\[
g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \ldots, h
\]

(1)

(2)

Where \( \mathbf{X} = (x_1, x_2, \ldots, x_{nVar}) \) is the vector of solution that minimizes objective function(s), \( F(\mathbf{x}) \) while satisfying the constraint(s), \( g_i(\mathbf{x}) \leq 0 \). The numbers of design parameter(s), objective function(s) and constraint(s) are \( nVar, k \) and \( h \) respectively[56].

The concept of dominance in MO is generally used to compare two solutions \( x_i \) and \( x_j \). The solution \( x_i \) is said to dominate solution \( x_j \) if \( f_i(x_i) \) is no worse than \( f_j(x_j) \) for all the objectives \( (l = 1, \ldots, k) \) and is better for at least one of them, i.e.,[57-60];

\[
f_i(x_i) \leq f_i(x_j), \quad \forall l = 1, 2, \ldots, k \quad \text{and} \quad f_m(x_i) < f_m(x_j), \quad \exists m \in \{1, 2, \ldots, k\}
\]

(3)
So, a set of solutions is said to be a Pareto Front if no solution can dominate any solution in this set (Neema and Ohgai, 2010). For more details about Pareto optimal solutions, one can refer to Coello and Lechuga [61] and Deb [62].

2-2- Performance Metrics

2-2-1- Spacing (S)

The S metric numerically describes the spread of the vectors in PFknown [58,63]. This metric does not require the researcher to know PFtrue. Note that this becomes important in the deception problems where all Pareto Front solutions are equally spaced. Equations 4 and 5 define this metric.

\[
S = \left[ \frac{1}{n-1} \sum_{i=1}^{n} (d_i - \bar{d})^2 \right]^{1/2} \quad \text{where} \quad \bar{d} = \frac{1}{n-1} \sum_{i=1}^{n} d_i
\]  

\[d_i = \min_{j \neq i} \left[ \sum_{i=1}^{k} |f_i(x_i) - f_j(x_j)| \right], i \neq j\]

Where \( n \) is the number of solutions in PFknown. When \( S = 0 \), all members are spaced evenly apart.

2-2-2- Coverage of Two Set (CS)

In order to compare the dominance relationship between two populations resulting from two different MOEAs, Zitzler, et al. [64] propose the CS [65], that is measured to show how the final population of one algorithm dominates the final population of another algorithm. The CS value can be calculated as follows:

\[
CS(X', X^*) = \left| \left\{ a^* \in X^*; \exists a' \in X': a' \leq a^* \right\} \right| / |X^*|
\]

Where \( X' \) and \( X^* \) are two sets of solutions resulting from different algorithms. Where \( a' \leq a^* \) means that \( a' \) dominate \( a^* \) if and only if \( a' < a^* \) or \( a' = a^* \). Function CS is defined as the mapping of the ordered pair \((X', X^*)\) to the interval \([0,1]\). In general, if all solutions in \( X' \) dominate all solutions in \( X^* \), then \( CS(X', X^*) = 1 \). Also, \( CS(X', X^*) = 0 \) implies that none of the solutions in \( X^* \) are dominated by \( X' \). Note that both \( CS(X', X^*) \) and \( CS(X^*, X') \) need to be considered independently since they have the distinct meanings since \( CS(X', X^*) \) is not necessarily equal to \( 1 - CS(X^*, X') \). The advantage of this Pareto compliant metric is that it is easy to calculate and provides a relative comparison based upon dominance numbers between two MOEAs [66].

2-3- NSGA-II Algorithm

Srinivas and Deb [67] introduced NSGA to deal with MO problems. In this algorithm, Goldberg’s non-domination criterion is used to determine solution ranks, where fitness sharing is used to control the diversity of solutions in the search space. Deb, Pratap, Agarwal and Meyarivan [59] have proposed an improved version of the NSGA algorithm, called NSGA-II that includes a second-order sorting criterion called crowding distance, which is faster and more reliable than NSGA. It uses this crowding distance in its selection operator to keep a diverse front by making sure each member stays a crowding distance apart. This keeps the population diverse and helps the algorithm to explore the fitness landscape.
In an evolution cycle of the NSGA-II, a mating pool is first created and filled using binary tournament selection. Then, crossover and mutation operators are applied to the members of the mating pool. Next, the old set of solutions and newly created solutions are merged to create a larger population. This new population is sorted based on two criterions: (i) rank and (ii) Crowding Distance (CD). Finally, a certain amount of individuals in the sorted population is selected and others are deleted. These steps are repeated until a stopping condition is met. After NSGA-II terminates, non-dominated solutions of the final population are the approximate Pareto Front of MO problem.

2-4- MOPSO Algorithm

The MOPSO is an extension of PSO proposed by Coello, Pulido and Lechuga [58] to deal with MO problems. MOPSO stores the non-dominated solutions found so far in an external archive of solutions, namely “repository.” The members of the repository are not dominating each other and they provide an approximation of real Pareto Front of the optimization problem. In MOPSO, each particle randomly selects a solution in the repository as its leader, instead of a unique global best for all particles.

2-5- SPEA2 algorithm

SPEA2 is an improved version of the Strength Pareto Evolutionary Algorithm (SPEA), was proposed by Zitzler and Thiele (1999) [65]. Compared with SPEA, a fine-grained fitness assignment strategy which incorporates density information is employed in SPEA2. The fixed archive size is adopted, that is, whenever the number of non-dominated individuals is less than the predefined archive size, the archive is filled up by dominated individuals. Moreover, an alternative truncation method is used to replace the clustering technique in original SPEA but does not lose boundary points, which can guarantee the preservation of boundary solution. Finally, SPEA2 only makes members of the archive participate in the mating selection process [68].

2-6- Constrain handling method for the Multi-objective optimization approach

Consider the general form of a constrained multi-objective optimization problem, as

\[
F(x) = (f_1(x),...,f_k(x))
\]

subject to

\[
G(x) = (g_1(x),...,g_h(x)) \leq 0
\]

Where \(x=(x_1,...,x_{nVar})\) is the vector of solution that minimizes objective function(s) \(F(x)\) while satisfying the constraint(s) \(G(x)\leq 0\). The numbers of design parameter(s), objective function(s) and constraint(s) are \(nVar, k\) and \(h\), respectively.

Multi-Objective Evolutionary Algorithms (MOEAs) are robust and efficient multi-objective optimization algorithms, however, EAs do not have any explicit mechanism to handle constraints while most real-world design multi-objective optimization problems have multiple constraints[55]. The penalty function method is a traditional approach for handling the constraints of single-objective optimization problems. However, this method requires careful tuning of the penalty function coefficients to obtain a satisfactory design. Moreover, application of this method to a multi-objective optimization problem raises another problem; how to combine multiple constraints with multiple objectives[55,69].

Many previous constraint-handling methods need to tune some parameters to balance between the objective(s) and constraint(s). In this research, we employ a constraint-handling method proposed by Oyama [55], which does not need any parameters to be tuned for constraint handling and it can always be used even when all individuals in the initial population are infeasible or the amount of violation of each constraint is significantly different. The method is described as follows.

\[
\text{Preprints (www.preprints.org)  |  NOT PEER-REVIEWED  |  Posted: 17 April 2018  
doi:10.20944/preprints201804.0222.v1}
\]
Definition 1 (Constrained Pareto dominance): Solution $i$ is said to constrained-dominate solution $j$ if any of the following conditions are true,

1. Solutions $i$ and $j$ are both feasible and solution $i$ dominates solution $j$ in the objective function space. It should be noted that the solution $x_i$ is said to dominate solution $x_j$ if $f_k(x_i) \leq f_k(x_j)$ for all objectives and it is better for at least one of them, \([57,59]\):

$$f_k(x_i) \leq f_k(x_j), \quad \forall i = 1, 2, \ldots, k \text{ and } f_k(x_i) < f_k(x_j), \quad \exists i \in \{1, 2, \ldots, k\}$$

So, a set of solutions is said to be a Pareto front or Pareto solution if no element of this set dominates any other solutions\([34]\). For acquisition more detail about on Pareto optimal solutions, one can be referred to\([62,70]\).

2. Solution $i$ is feasible and solution $j$ is not.

3. Solutions $i$ and $j$ are both infeasible, but solution $i$ dominates solution $j$ in the constraint space.

Definition 2 (Constraint space dominance): Solution $i$ is said to dominate solution $j$ in the constraint space if both of the following conditions are true,

1. Solutions $i$ is no worse than solution $j$ in all constraints, i.e.,

$$\forall G_n(x_i) \leq G_n(x_j)$$

and

2. Solution $i$ is strictly better than solution $j$ for at least one constraint, i.e.,

$$\forall G_n(x_i) < G_n(x_j)$$

where

$$G_n(x) = \max(0, g_n(x)), \quad n = 1, 2, \ldots, k$$

With Oyama’s constraint-handling approach, we applied niching based on the amount of constraint violations to infeasible solutions. Here, a standard fitness sharing (Goldberg and Richardson 1987)\([71]\) is applied to the infeasible designs based on their constraint violations as:

$$\text{rank}'(x_i) = \text{rank}(x_i) \times \text{Penalty}(x_i)$$

$$\text{Penalty}(x_i) = 1 + \sum_{j=1, j \neq i}^{\text{npop}} s_{hj}$$

$$s_{hj} = \begin{cases} 1 - \left( \frac{d_j}{\sigma_{\text{share}}} \right)^{\alpha} & d_j < \sigma_{\text{share}} \\ 0 & d_j \geq \sigma_{\text{share}} \end{cases}$$

$$\sigma_{\text{share}} = \frac{\sum_{n=1}^{k} (g_{\text{max}} - g_{\text{min}})}{\text{npop}}$$

$$d_j = \sqrt{\sum_{n=1}^{k} \left( g_n(x_i) - g_n(x_j) \right)^2}$$

$$g_{\text{max}} = \max(g_n(x_1), \ldots, g_n(x_{\text{npop}}))$$

$$g_{\text{min}} = \min(g_n(x_1), \ldots, g_n(x_{\text{npop}}))$$

Where $\text{npop}$ is population size and $\alpha$ is set to 0.4.

3- Model formulation

Given the diversity and complexity of issues in location-allocation problems, a number of classifications from various perspectives for this type of problems exist. But one of the most common classifications is based on the number of service centers: (1) single source and (2) multi-source problems. The latter is also known as Multi-Source Weber Problem (MSWP)\([72,73]\). Yet, MSWP itself is
classified into different types. One of them is called Capacitated Multi-Source Weber Problem (CMSWP) where every service center has a certain service capacity\cite{74}. Another form of this kind of problems is Multi-Facility Weber Problem (MFWP); in this case, each service center provides certain types of services. The MFWP is defined as locating simultaneously $m$ facilities with particular service types satisfying the demand of $n$ users, in order to minimize the total transportation cost for, each consumer from its closest facility\cite{74}.

In this paper, a modified version of MFWP is proposed for the location-allocation problem. Banks location-allocation problems under competitive condition assuming that each bank can have a different type of services and there is no limit to the number of their customers is classified under the UMFWP (Uncapacitated Multi-Facility Weber Problem). This is a non-convex optimization problem. In a competitive environment, selection of the best location for a new bank branch is an important decision that has significant effects on the efficiency of bank services. Because of the competitive environment, the goal is to attract more customers, so we must look for locations to establish new branches as far as possible from the branches of the same bank. Therefore, for solving this type of problem we need constraint multi-objective. The assumptions for the defined problem can be expressed in the following statements:

a) We consider four different banks (Melli, Mellat, Sepah and Meher) in our study area.
b) Each bank presents different levels of services (In the fourth part of the article).
c) Population densities in building blocks do not change during the study.
d) Population density (of people over 15 years of age) is available at the building block level.
e) Customers are interested in nearest bank branch with a higher level of attraction, to perform their financial tasks.
f) Banks have an infinite capacity for accepting customers.
g) New bank branches should have maximum network-based distance from branches of the same bank, so that;
   a. It attracts a minimum number of customers from branches of the same bank.
   b. It can attract maximum customers of competitor banks.
   c. And, also from all customers familiar banks not less than the threshold
h) Each customer refers to only one bank.
i) For customer convenience, sum of the network-based distance between customers and banks be minimized.

According to the above-mentioned assumptions, a mathematical model of the function for MO constrains is as follows:

**First objective**: Minimizes the total network-based distance needed for customers to travel to the newly established and existing branches. (Equation 14)

**Second objective**: Maximizes the network-based distance between the newly established branch and other existing branches of the same bank. (Equation 15)

In this facility location problem the following conditions must also be satisfied:

**First constraint**: Number of customers in the new established branch should not be less than a defined minimum. (Equation 16)

**Second constraint**: All the customers of other branches of the same bank should not be less than a threshold. (Equation 17)

Regarding the aforementioned procedure, the objective functions can be defined as follows:

**Parameters**:

$p_i$ : Number of customers in the building block $i$

$new$ : Total number of bank branches to be established

$b$ : Total number of existing bank branches in the study area

$C_{new}$ : Minimum capacity service of new established bank branches
%C_p: All changes in the type of bank customers to create new bank should not exceed this threshold.

p_{new}: The number of the customer, that are serviced by new establish branch.
d: The number of banks on own

Decision variables:

\( d_{i,j} \): Network-based distance between a customer at block \( i \) and a bank branch \( j \) (existing or to be established)

\( d_{k,j} \): Network-based distance between a new branch \( k \) \((k=1,..., \text{new})\) and an existing branch of the same bank \( j \).

The formulation of the constraint problem is as follows:

\[
\begin{align*}
\min & \sum_i \sum_j p_i d_{i,j} \\
\max & \sum_k \sum_j d_{k,j}, \quad \forall \ j,
\end{align*}
\]

subject to:

\[
\begin{align*}
\sum_k p_{new} & \geq C_{new} \quad k = 1,...,\text{new} \\
\sum_i p_i - \sum_j d_{new} & < \%C_p
\end{align*}
\]

4- AHP (Analytic Hierarchy Process)

The AHP which is a powerful tool in applying Multi Criteria Decision Making (MCDA) was introduced and developed by Saaty in 1980. In comparison with other MCDM methods, the AHP method has widely been used in MCDM and has been applied successfully in many practical decision-making problems [75]. In AHP the decision maker starts by laying out the overall hierarchy of the decision. This hierarchy reveals the factors to be considered as well as the various alternatives in the decision. Here both qualitative and quantitative criteria can be compared using a number of pairwise comparisons, which result in the determination of factor weights. Finally, the alternative with the highest total weighted score is selected as the best alternative [75].

- **Gravity model**

According to gravity model, the probability that a customer at \( i \) refer at a bank \( j \) is given by [5]

\[
p_{ij} = \frac{a_j}{d_{ij}^2}
\]

where \( a_j \) represents the quality of service \( j \), \( d_{ij} \) is the network-based distance from customer point \( i \) to bank \( j \).

To obtain the attractive National, Mellat, Sepah and Meher banks, according to criteria such as quality of service, financial, human, physical; AHP is used.

- **Selecting the best bank**

The major steps for selecting best bank are organized as follows:

1. Define the goal (selecting the best bank)
2. Selecting the bank parameters for human selecting for referring.
Based on a review by experts on attracting customers to the Banks (Melli, Mellat, Sepah and Meher) the findings of the field investigation, literature review and collected 4 parameters (including Services, Financial, Human and Physical parameters) were found relevant to select best position bank.

3. Structure the hierarchy from the top through the intermediate levels to the lowest level, which usually contains the list of alternatives. The goal is to attract customers to the bank. The criteria include four standard level of service, financial, human and physical requirements and at the option of the four banks (Melli, Mellat, Sepah and Meher) have been considered. Figure 1 describes the hierarchy of a decision making problem.

4. Design the format of questionnaire items as to process according to the hierarchy in step 2. And then collect the input by a pairwise comparison of decision elements (according to tables 1).

5. Use the eigenvalue method to estimate the consistency index. Then, determine whether the input data satisfies a “consistency check”. If it does not, go back to step 4 and redo the pairwise comparisons (according to tables 1).

After the necessary calculations are obtained attractiveness of banks (according to table 2).

![Figure 1. Hierarchy model of selecting the bank](image-url)

### Table 1.a. The pairwise comparisons matrix A–B1–4. (CR=0.09<0.1)

<table>
<thead>
<tr>
<th>Selecting the best bank</th>
<th>Services</th>
<th>Financial</th>
<th>Human</th>
<th>Physical</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Services</td>
<td>1</td>
<td>1/7</td>
<td>3</td>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>Financial</td>
<td></td>
<td>1</td>
<td>7</td>
<td>6</td>
<td>0.68</td>
</tr>
<tr>
<td>Human</td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>0.09</td>
</tr>
<tr>
<td>Physical</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0.07</td>
</tr>
</tbody>
</table>

### Table 1.b. The pairwise comparisons matrix B1–C1–4. (CR=0.03<0.1)

<table>
<thead>
<tr>
<th>Services</th>
<th>Melli</th>
<th>Mellat</th>
<th>Sepah</th>
<th>Meher</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melli</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>0.39</td>
</tr>
<tr>
<td>Mellat</td>
<td></td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>0.38</td>
</tr>
<tr>
<td>Sepah</td>
<td></td>
<td></td>
<td>1/2</td>
<td>1</td>
<td>0.10</td>
</tr>
<tr>
<td>Meher</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0.13</td>
</tr>
</tbody>
</table>

### Table 1.c. The pairwise comparisons matrix B2–C1–4. (CR=0.05<0.1)

<table>
<thead>
<tr>
<th>Financial</th>
<th>Melli</th>
<th>Mellat</th>
<th>Sepah</th>
<th>Meher</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melli</td>
<td>1</td>
<td>2</td>
<td>9</td>
<td>2</td>
<td>0.47</td>
</tr>
<tr>
<td>Mellat</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Table 1.d. The pairwise comparisons matrix B3–C1–4. (CR=0.06<0.1)

<table>
<thead>
<tr>
<th>Human</th>
<th>Melli</th>
<th>Mellat</th>
<th>Sepah</th>
<th>Meher</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melli</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>0.48</td>
</tr>
<tr>
<td>Mellat</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>0.30</td>
</tr>
<tr>
<td>Sepah</td>
<td>1</td>
<td>1/3</td>
<td>1</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Meher</td>
<td></td>
<td></td>
<td>1</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.e. The pairwise comparisons matrix B4–C1–4. (CR=0.05<0.1)

<table>
<thead>
<tr>
<th>Physical</th>
<th>Melli</th>
<th>Mellat</th>
<th>Sepah</th>
<th>Meher</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melli</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>0.40</td>
</tr>
<tr>
<td>Mellat</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>Sepah</td>
<td>1</td>
<td>4</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meher</td>
<td></td>
<td>1</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Criteria weights of all factors.

<table>
<thead>
<tr>
<th>Name of bank</th>
<th>Melli</th>
<th>Mellat</th>
<th>Sepah</th>
<th>Meher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attractiveness</td>
<td>0.45</td>
<td>0.28</td>
<td>0.09</td>
<td>0.18</td>
</tr>
</tbody>
</table>

5- Study area

A major part of the district 3 of Tabriz (612850m²) was selected as the study area. A total population of 19,387 (over 15 years of age) with an average population density of 68 people per hectare. The number of existing bank branches in the study area are 13 (Melli 4 branches - Mellat 3 branches – Sepah 5 branches - Meher 1 branch according to figure 2). In this study, we locate a new branch for Mellat bank in the study area.

Figure 2. Banks spatial distribution and dispersion of the population in the study area

6- Determining Optimum Location Banks under Competitive Conditions Using Spatial Data (Implementation and Results)
Determining the optimum location for bank branches under competitive conditions using spatial data often concerns multiple objectives and constraint under complex, highly nonlinear constraints. In reality, different objectives often conflict each other, and sometimes, truly optimal solutions may not exist at all, and some compromise and approximations are often needed. In addition to these challenges and complexity, a location problem is subjected to various constraints, limited by a number of population attracted to the new bank. So, we have implemented the NSGA-II, MOPSO and SPEA2 multi-objective optimization methods in Matlab and for determining optimum location banks (branches) under competitive conditions using spatial data problems. Finally, to evaluate quality and accuracy of the above algorithms, several iterations are performed.

Constraint optimization methods with two objectives and two constraints introduced in section 3, implementation and results are displayed below.

Figure 3 show the graphical results (Pareto Front) produced by NSGA-II, MOPSO and SPEA2 for the defined problem in the article. In all these figures, the vertical axis is obtained from (14) and the horizontal axis is obtained from (15). The Pareto Front of the problem is shown as a circle point. It is also clear from figures that the distribution of answers by NSGA-II method is better than the other two methods.

Figure 4 show S metric by NSGA-II, MOPSO and SPEA2 in the defined problem. NSGA-II method shows better convergence than the other two methods and can be seen in the figures. We can see clearly that our NSGA-II algorithm indeed converges almost exponentially. Table 3 shows the comparison of results among the three algorithms considering the S metrics.
Figure 4. (a). Convergence with spacing obtained by NSGA-II, (b). Convergence with spacing obtained by MOPSO and (c). Convergence with spacing obtained by SPEA2

Table 3. Comparison of NSGA-II, MOPSO and SPEA2 in the study shows the mean and SD (Standard Division) values for S (Spacing) and a number of Pareto Front (n=50 and iteration=50 for three approaches).

<table>
<thead>
<tr>
<th>Approach</th>
<th>Spacing (Mean)</th>
<th>Spacing (SD)</th>
<th>Number of Pareto Front</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSGA-II</td>
<td>2074.0</td>
<td>204.4</td>
<td>50</td>
</tr>
<tr>
<td>SPEA2</td>
<td>2765.8</td>
<td>253.2</td>
<td>50</td>
</tr>
<tr>
<td>MOPSO</td>
<td>4031.2</td>
<td>369.9</td>
<td>33</td>
</tr>
</tbody>
</table>

Figure 5 show number of Pareto Front at each iteration algorithm by NSGA-II, the SPEA2, MOPSO and SPEA2 in the defined problem. As can be seen in Table 3, after 50 iterations of each of the algorithms, SPEA2 and NSGA-II methods obtained 50 solutions on the Pareto Front while MOPSO method obtained 33 solutions on the Pareto Front.

Figure 5. (a). Number of Pareto Front at each iteration algorithm obtained by NSGA-II (b). Number of Pareto Front at each iteration algorithm obtained by SPEA2
(c). Number of Pareto Front at each iteration algorithm obtained by MOPSO

Figure 6 presented optimum locations of branch bank based on two objectives and two constraints in three methods.

![Image of Figure 6](image-url)

Red triangles show existing competitor branches, red triangles with blue stars show branches of the same bank and yellow stars show proposed result set for the new branch.

**Figure 6.** Optimum locations of branch bank based on two objectives which are obtained by (a) SPEA2 method, (b) NSGA-II method and (c) MOPSO method.

Implement and compare it with respect to the NSGA-II, MOPSO and SPEA2 algorithms using the constraint problems in Coverage of two Set metrics. Table 4 shows the Coverage of two Set e metric values of the three approaches, averaged on 50 independent runs. A careful inspection of Tables 4 reveals that in terms of Coverage of two Set metrics, the final solutions obtained by NSGA-II is better than (high probability dominate the results of the other two methods) those obtained by SPEA2 and MOPSO for this problem.

**Table 4.** Comparison of NSGA-II, MOPSO and SPEA2 in the study shows the mean and Standard Division (SD) values for set coverage of two sets metrics (n=50 and iteration=50 for three approaches).

<table>
<thead>
<tr>
<th>Approach</th>
<th>SPEA2</th>
<th>MOPSO</th>
<th>NSGA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NSGA-II</td>
<td>MOPSO</td>
<td>SPEA2</td>
</tr>
<tr>
<td>Mean</td>
<td>0.08</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>SD</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

To check if the final results obtained with the best performing algorithm differ from the final results of rest of the competing algorithms in a statistically significant manner, the Wilcoxon’s Ranksum test for independent samples [76] is used at 5% significance level, as presented in Table 5. The numerical values -1, 0, 1 correspond to whether the other methods are inferior to, equal to, and superior to our proposed algorithm, as indicated in Tables 5. The results of S obtained by NSGA-II, MOPSO and SPEA2 and besides Wilcoxon’s Ranksum test are presented in Table 5. From Table 5, NSGA-II outperforms SPEA2 and MOPSO in the problem.

**Table 5.** Comparison between NSGA-II and other algorithms on Average S metrics and the basis of Wilcoxon’s Ranksum test. (-1→worse, 0→equal and +1→better) (Chen et al. 2014) (Independent runs=30 and pop.=50)

<table>
<thead>
<tr>
<th>Problem</th>
<th>NSGA-II</th>
<th>SPEA2</th>
<th>MOPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.72E-03</td>
<td>3.32E-02</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>1.38E-03</td>
<td></td>
<td>-1</td>
</tr>
</tbody>
</table>
The results of statistical and final tests indicate that the accuracy and convergence speed of NSGA-II optimization algorithm in finding optimal solutions for LBBCC with conflicting purposes in part of Tabriz. Research results show the usefulness of the developed model using major performance measures in locating bank branches under competitive conditions for an urban region.

7- Conclusions

This paper aims at solving the multi-objective problem of locating bank branches under competitive conditions by using NSGA-II, MOPSO and SPEA2 algorithms. On this issue, the following objectives were considered for the location of the branches under competitive conditions. In accordance with the defined problem, in the content of the article, two objective functions and two constraints were considered as follows; First objective: Minimizes the total network-based distance needed for customers to travel to the newly established and existing branches. Second objective: Maximizes the network-based distance between the newly established branch and other existing branches of the same bank. First constraint: Number of customers in the new established branch should not be less than a defined minimum. Second constraint: All the customers of other branches of the same bank should not be less than a threshold.

In this study, we compare the results of the NSGA-II, MOPSO and SPEA2 constraint algorithms. From the obtained results for an LBBCC, we find that NSGA-II outperforms two other contemporary MOEAs: MOPSO and SPEA2 in terms of finding a diverse set of solutions, Number of the solution on the Pareto Front, in the speed of convergence. Finally, the results of NSGA-II method, with high probability dominate the results the other two methods (MOPSO and SPEA2) that this indicates that high quality solutions are NSGA-II method.

From the simulation results on a number of difficult test problems, we find that NSGA-II outperforms two other contemporary MOEAs: Pareto-Archived Evolution Strategy (PAES) and multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) in terms of finding a diverse set of solutions and in converging near the true Pareto-optimal set.

Reference

8. Aghamohammadi, H.; Mesgari, M.S.; Molaei, D.; Aghamohammadi, H. Development a heuristic method to locate and allocate the medical centers to minimize the earthquake relief operation time. Iranian journal of public health 2013, 42, 63.
24. Dibble, C.; Densham, P.J. In Generating interesting alternatives in gis and sdss using genetic algorithms, 1993; AMER SOC PHOTOGRAMMETRY & REMOTE SENSING.


57. Van Veldhuizen, D.A. Multiobjective evolutionary algorithms: Classifications, analyses, and new innovations; Air Force Institute of Technology, Wright-Patterson AFB: Ohio, USA, 1999.


