 doi:10.20944/preprints201804.0119.v1

Vertex Domination in t-Norm Fuzzy Graphs

Mohammadesmail Nikfar

Department of Mathematics, Payame Noor University, P. O. Box: 19395-3697, Tehran, Iran

* m.s.nikfar@member.ams.org; m.s.nikfar@gmail.com

Abstract

For the first time, We do fuzzification the concept of domination in crisp graph on a generalization of fuzzy graph by using membership values of vertices, α-strong edges and edges. In this paper, we introduce the first variation on the domination theme which we call vertex domination. We determine the vertex domination number γ_v for several classes of t-norm fuzzy graphs which include complete t-norm fuzzy graph, complete bipartite t-norm fuzzy graph, star t-norm fuzzy graph and empty t-norm fuzzy graph. The relationship between effective edges and α-strong edges is obtained. Finally, we discuss about vertex dominating set of a fuzzy tree with respect to a t-norm $\otimes$ by using the bridges and α-strong edges equivalence.

Keywords: t-norm Fuzzy graph, t-norm, fuzzy tree, bridge, α-strong edges, vertex domination

AMS Subject Classification: 05C72, 05C69, 03E72, 94D05

1 Introduction

In 1965, Zadeh published his seminal paper "fuzzy sets" (Ref. [24]) as a way for representing uncertainty. In 1975, fuzzy graphs were introduced by Rosenfeld (Ref. [23]) and Yeh and Bang (Ref. [24]) independently as fuzzy models which can be used in problems handling uncertainty. In 1998, the concept of domination in fuzzy graphs was introduced by A. Somasundaram and S. Somasundaram (Ref. [20]) as the classical problems of covering chess board with minimum number of chess pieces.. They defined domination in fuzzy graph by using effective edges (Refs. [20] and [21]). The works on domination in fuzzy graphs was also done such as domination (Refs. [5] and [12]), strong domination (Refs. [4] and [7]), $(1,2)$-vertex domination (Ref. [19]), 2-domination (Ref. [13]), connected domination (Ref. [6]), total domination (Ref. [8]), Independent domination (Ref. [15]), Complementary nil domination (Ref. [2]), Efficient domination (Ref. [22]), strong (weak) domination (Ref. [14]), Vertex domination (Ref. [16]) and etc. In 2018, the concept of t-norm fuzzy graphs which is a generalization of fuzzy graphs was introduced by J.N. Mordeson (Ref. [10]) as real-world applications to human trafficking for creating better models of certain real-world situations. Motivated by some applications are better modeled with a t-norm other than minimum, we define one of types of domination on t-norm fuzzy graphs.

The rest of this paper is organized as follows. In Section 2, we lay down the preliminary results which recall some basic concept. In Section 3, the vertex domination number of a t-norm fuzzy graph is defined in a classic way, Definition (3.2), (3.4), (3.5).

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 10 April 2018 doi:10.20944/preprints201804.0119.v1

We introduce t-norm bipartite fuzzy graph and t-norm star fuzzy graph, Definition (3.13). We determine vertex domination number for several classes of t-norm fuzzy graphs consists of complete t-norm fuzzy graph, Proposition (3.10), empty t-notm fuzzy graph, Proposition (3.11), star t-norm fuzzy graph, Proposition (3.16), complete bipartite t-norm fuzzy graph, Proposition (3.17). We determine the results about α-strong edges of several classes of t-norm fuzzy graphs, Corollary (3.9), Proposition (3.14), Corollary (3.15). We give the relationship between effective edges and α-strong edges, Corollary (3.14). The bridges and α-strong edges are equivalence, Corollary (3.20). Finally, some results about fuzzy forest, fuzzy trees and the vertex dominating sets of them is studied, Corollary (3.28), Proposition (3.29).

2 Preliminary

We provide some basic background for the paper in this section.

Definition 2.1 (**Ref.** [17], Definition 5.1.1, pp. 82, 83)**.** A binary operation $\otimes : [0,1] \times [0,1] \to [0,1]$ is a ***t*-norm** if it satisfies the following for $x, y, z, w \in [0,1]$:

1. $1 \otimes x = x$
2. $x \otimes y = y \otimes x$
3. $x \otimes (y \otimes z) = (x \otimes y) \otimes z$
4. If $w \leq x$ and $y \leq z$ then $w \otimes y \leq x \otimes z$

We concern with a t-norm fuzzy graph which is defined on a crisp graph. So we recall the basic concepts of crisp graph.

A *graph* (**Ref.** [1], p. 1) G is a finite nonempty set of objects called *vertices* (the singular is *vertex*) together with a (possibly empty) set of unordered pairs of distinct vertices of G called *edges*. The *vertex set* of G is denoted by $V(G)$, while the *edge set* is denoted by $E(G)$.

We recall that a *fuzzy subset* in **Ref.** ([17], Definition 1.2.1, p. 3) of a set S is a function of S into the closed interval $[0,1]$.

we lay down the preliminary results which recall some basic concepts of fuzzy graph from **Ref.** [11].

A *fuzzy graph* in **Ref.** ([11], p. 19) is denoted by $G = (V, \sigma, \mu)$ such that $\mu(\{x, y\}) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$ where V is a vertex set, σ is a fuzzy subset of V, μ is a fuzzy relation on V and $\wedge$ denote the minimum. We call σ the fuzzy vertex set of G and μ the fuzzy edge set of G, respectively. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e., $\mu(\{x, x\}) = \sigma(x)$, for all x) and symmetric (i.e., $\mu(\{x, y\}) = \mu(\{y, x\})$, for all $x, y \in V$). In all the examples σ and μ is chosen suitably. In any fuzzy graph, the underlying crisp graph is denoted by $G^* = (V, E)$ where V and E are domain of σ and μ, respectively. The fuzzy graph $H = (\tau, \nu)$ is called a *partial fuzzy subgraph* of $G = (\sigma, \mu)$ if $\nu \subseteq \mu$ and $\tau \subseteq \sigma$. Similarly, the fuzzy graph $H = (\tau, \nu)$ is called a fuzzy subgraph of $G = (V, \sigma, \mu)$ induced by P in if $P \subseteq V, \tau(x) = \sigma(x)$ for all $x \in P$ and $\nu(\{x, y\}) = \mu(\{x, y\})$ for all $x, y \in P$. For the sake of simplicity, we sometimes call H a fuzzy subgraph of G. We say that the partial fuzzy subgraph (τ, ν) spans the fuzzy graph (σ, μ) if $\sigma = \tau$. In this case, we call (τ, ν) a spanning fuzzy subgraph of (σ, μ).

For the sake of simplicity, we sometimes write xy instead of $\{x, y\}$

A *path* P of length n in is a sequence of distinct vertices $u_0, u_1, \cdots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, \cdots, n$ and the degree of membership of a weakest edge is defined as its strength. If $u_0 = u_n$ and $n \geq 3$ then P is called a cycle and P is called a

 doi:10.20944/preprints201804.0119.v1

fuzzy cycle, if it contains more than one weakest edge. The strength of a cycle is the strength of the weakest edge in it. The *strength of connectedness* between two vertices x and y in is defined as the maximum of the strengths of all paths between x and y and is denoted by $\mu_G^\infty(x,y)$.

A fuzzy graph $G = (V, \sigma, \mu)$ is *connected* in if for every x, y in V, $\mu_G^\infty(x,y) > 0$.

Definition 2.2 (**Ref.** [10], Definition 3.1, p. 131)**.** Let $G = (V, E)$ be a graph. Let σ be a fuzzy subset of V and μ be a fuzzy subset of E. Then (σ, μ) is called a **fuzzy subgraph** of G with respect to a t-norm $\otimes$ if for all $uv \in E, \mu(uv) \leq \sigma(u) \otimes \sigma(v)$.

Let k be a positive integer. Define $\mu^k(u,v) = \vee\{\mu(uu_1) \otimes \cdots \otimes \mu(u_{n-1}v)|P$ $: u = u_0, u_1, \cdots, u_{n-1}, u_n = v$ is a path of length k from u to v } Let $\mu^\infty(u,v) = \vee\{\mu^k(u,v)|k \in \mathbb{N}\}$ where $\mathbb{N}$ denotes the positive integers in **Ref.** ([10], p. 131).

3 Main Results

In this section, we provide the main results.

Definition 3.1. Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $uv \in E$. We call that uv is α-strong edge if $\mu(uv) > \mu_{G-uv}^\infty(u,v)$.

Definition 3.2. Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $x, y \in V$. We say that x **dominates** y in G as α-strong if the edge $\{x, y\}$ is α-strong.

Example 3.3. Let (σ, μ) be a t-norm fuzzy graph with respect to $\wedge$. By attention to it In Figure (1), the edges v_2v_5, v_2v_4, v_3v_4 and v_1v_3 are α-strong and the edges v_1v_4, v_1v_2 and v_4v_5 are not α-strong.

Definition 3.4. Let $\otimes$ be a t-norm. Let (σ, μ) be a t-norm fuzzy graph with respect to $\otimes$. A subset S of V is called a α**-strong dominating set** in G if for every $v \notin S$, there exists $u \in S$ such that u dominates v as α-strong.

Definition 3.5. Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Let S be the set of all α-strong dominating sets in G. The **vertex domination number** of G is defined as $\min_{D \in S}[\Sigma_{u \in D}(\sigma(u) + \frac{d_s(u)}{d(u)})]$ and it is denoted by $\gamma_{\mathbf{v}}(\mathbf{G})$. If $d(u) = 0$, for some $u \in V$, then we consider $\frac{d_s(u)}{d(u)}$ equal with 0. The α-strong dominating set that is correspond to $\gamma_v(G)$ is called by **vertex dominating set**. We also say $\Sigma_{u \in D}(\sigma(u) + \frac{d_s(u)}{d(u)})$, **vertex weight** of D, for every $D \in S$ and it is denoted by $\mathbf{w_v}(\mathbf{D})$.

Example 3.6. Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to $\wedge$. In Figure (1), the set $\{v_2, v_3\}$ is the α-strong dominating set. This set is also vertex dominating set in t-norm fuzzy graph G with respect to $\wedge$. Hence $\gamma_v(G) = 1.75 + 0.9 + 0.7 = 3.35$. So $\gamma_v(G) = 3.35$.

Definition 3.7. Ref. [10], Definition 3.1, p. 131] Let (σ, μ) be a fuzzy graph with respect to $\otimes$. Then (σ, μ) is said to be complete with respect to $\otimes$, if for all $u, v \in V, \mu(uv) = \sigma(u) \otimes \sigma(v)$.

Proposition 3.8 (**Ref.** [10], Proposition 3.24. , pp. 135, 136)**.** *Let $G = (\sigma, \mu)$ be a complete fuzzy graph with respect to $\otimes$. Then*

1. $\mu_\otimes^\infty(u,v) = \mu(uv), \forall u, v \in V$
2. *G has no cutvertices.*

Corollary 3.9. *A complete t-norm fuzzy graph with respect to $\otimes$ is α-strong edgeless.*

 doi:10.20944/preprints201804.0119.v1

Proof. Let (σ, μ) be a complete t-norm fuzzy graph with respect to $\otimes$. For all $u, v \in V$, $\mu_{\otimes}^{\infty}(u,v) = \mu(u,v)$ by Proposition (3.8). So for all $u, v \in V$, $\mu_{\otimes}^{'\infty}(u,v) \geq \mu(u,v)$. Hence uv is not α-strong edge. The result follows. □

It is well known and generally accepted that the problem of determining the domination number of an arbitrary graph is a difficult one. Because of this, researchers have turned their attention to the study of classes of graphs for which the domination problem can be solved in polynomial time.

Proposition 3.10 (Complete t-norm fuzzy graph)**.** *Let $G = (\sigma, \mu)$ be a complete t-norm fuzzy graph with respect to $\otimes$. Then $G = K_n$, $\gamma_v(K_n) = p$.*

Proof. Since $G = (\sigma, \mu)$ be a complete t-norm fuzzy graph with respect to $\otimes$, none of edges are α-strong by Corollary (3.9). so we have

$$\gamma_v(G) = \min_{D \in S}[\Sigma_{u \in D}\sigma(u)] = \Sigma_{u \in v}\sigma(u) = p$$

by Definition (3.5). Hence we can write $\gamma_v(K_n) = p$ by our notations. □

Proposition 3.11 (Empty t-norm fuzzy graph)**.** *Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Then $\gamma_v(G) = p$, if G be edgeless, i.e $G = \bar{K}_n$.*

Proof. Since G is edgeless, Hence V is only α-strong dominating set in G and none of arcs are α-strong. so we have $\gamma_v(G) = p$ by Definition (3.5). In other words, $\gamma_v(\bar{K}_n) = p$ by our notations. □

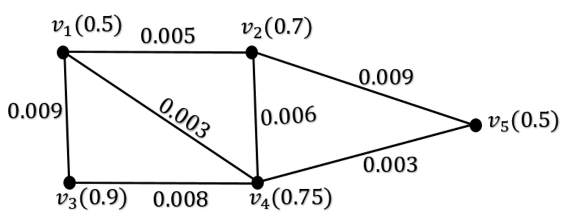

Figure 1. Vertex domination

It is interesting to note the converse of Proposition (3.11) that does not hold.

Example 3.12. We show the converse of Proposition (3.11) does not hold. For this purpose, Let $V = \{v_1, v_2, v_3, v_4, v_5\}$. We define σ on V by $\sigma : V \to [0,1]$ such that

$$\sigma(v_1) = 0.5, \sigma(v_2) = 0.7, \sigma(v_3) = 0.9, \sigma(v_4) = 0.75, \sigma(v_5) = 0.5$$

Now, The function $\mu : V \times V \to [0,1]$ is defined by

$$\mu(v_1v_2) = 0.005, \mu(v_1v_4) = 0.003, \mu(v_1v_3) = 0.009, \mu(v_2v_4) = 0.006, \mu(v_2v_5) = 0.009,$$

$\mu(v_3v_4) = 0.008, \mu(v_4v_5) = 0.003$ such that $\forall u, v \in V, \mu(u,v) \leq \sigma(u) \otimes \sigma(v)$ and $\otimes$ is defined as a t-norm $\wedge$. Finally, Let V, σ, and μ be the vertices, value of vertices and value of edges respectively. In other words, By attention to fuzzy graph with respect to $\wedge$ In Figure (1), the edges v_2v_5, v_2v_4, v_3v_4 and v_1v_3 are α-strong and the edges v_1v_4, v_1v_2 and v_4v_5 are not α-strong. So the set $\{v_2, v_3\}$ is the α-strong dominating set. This set is also vertex dominating set in t-norm fuzzy graph G. Hence $\gamma_v(G) = 1.75 + 0.9 + 0.7 = 3.35 = \Sigma_{u \in v}\sigma(u) = p$. So $G \neq \bar{K}_5$ but $\gamma_v(G) = p$.

Definition 3.13. A t-norm fuzzy graph G with respect to a t-norm $\otimes$ is said **bipartite**, if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Moreover, if $\mu(uv) = \sigma(u) \otimes \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a **complete bipartite t-norm fuzzy graph** and is denoted by K_{σ_1, σ_2}, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2. In this case, If $|V_1| = 1$ or $|V_2| = 1$ then a complete bipartite t-norm fuzzy graph is said a **star t-norm fuzzy graph** which is denoted by $K_{1,\sigma}$.

Proposition 3.14. *A complete bipartite t-norm fuzzy graph is α-strong edgeless.*

Proof. Let $G = (\sigma, \mu)$ be a complete bipartite t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $u \in V_1, v \in V_2$. the strength of path P from u to v is of the form $\sigma(u) \otimes \cdots \otimes \sigma(v) \leq \sigma(u) \otimes \sigma(v) = \mu(uv)$. So $\mu_\otimes^\infty(u,v) \leq \mu(uv)$. uv is a path from u to v such that $\mu(u,v) = \sigma(u) \otimes \sigma(v)$. So $\mu_\otimes^\infty(u,v) \geq \mu(uv)$. Hence $\mu_\otimes^\infty(u,v) = \mu(uv)$. So $\mu_\otimes^{'\infty}(u,v) \geq \mu(uv)$ that induce uv is not α-strong edge. The result follows. □

Corollary 3.15. *A star t-norm fuzzy graph has no α-strong edges.*

Proof. Obviously, the result is hold by using Proposition (3.14). □

Proposition 3.16 (Star t-norm fuzzy graph)**.** *Let $G = (\sigma, \mu)$ be a star t-norm fuzzy graph with respect to a t-norm $\otimes$. Then $G = K_{1,\sigma}$ and $\gamma_v(K_{1,\sigma}) = \sigma(u)$ where u is center of G.*

Proof. Let $G = (\sigma, \mu)$ be a star t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $V = \{u, v_1, v_2, \cdots, v_n\}$ such that u and v_i are center and leaves of G, for $1 \leq i \leq n$, respectively. The edge $uv_i, 1 \leq i \leq n$ is omly path between u and v_i. So $\{u\}$ is vertex dominating set in G. G is α-strong edgeless by Corollary (3.16). So $\gamma_v(K_{1,\sigma}) = \sigma(u)$. □

Proposition 3.17 (Complete bipartite t-norm fuzzy graph)**.** *Let $G = (\sigma, \mu)$ be a star t-norm fuzzy graph with respect to a t-norm $\otimes$ which is not star t-norm fuzzy graph. Then $G = K_{\sigma_1,\sigma_2}$ and $\gamma_v(K_{\sigma_1,\sigma_2}) = \min_{u \in V_1, v \in V_2}(\sigma(u) + \sigma(v))$.*

Proof. Let $G \neq K_{1,\sigma}$ be complete bipartite t-norm fuzzy graph with respect to $\otimes$. Then both of V_1 and V_2 include more than one vertex. In K_{σ_1,σ_2}, none of edges are α-strong by Proposition (3.14). Also, each vertex in V_1 is adjacent with all vertices in V_2 and conversely. Hence in K_{σ_1,σ_2}, the α-strong dominating sets are V_1 and V_2 and any sets containing 2 vertices, one in V_1 and other in V_2. Hence $\gamma_v(K_{\sigma_1,\sigma_2}) = \min_{u \in V_1, v \in V_2}(\sigma(u) + \sigma(v))$. So the proposition is proved. □

Definition 3.18 (**Ref.** [10], Definition 3.2., p.131)**.** Let (σ, μ) be a fuzzy graph with respect to $\otimes$. Let $xy \in E$. Then xy is called a **bridge** if $\mu_\times^{'\infty}(u,v) < \mu_\otimes^\infty(u,v)$ for some $u, v \in V$, where $\mu^{'}(xy) = 0$ and $\mu^{'} = \mu$ otherwise.

Theorem 3.19 (**Ref.** [10], Theorem 3.3., p.132)**.** *Let (σ, μ) be a fuzzy graph with respect to $\otimes$. Let $xy \in E$. Let $\mu^{'}$ be the fuzzy subset of E such that $\mu^{'}(xy) = 0$ and $\mu^{'} = \mu$ otherwise. Then* $(3) \Rightarrow (2) \Leftrightarrow (1)$:

(1) *xy is a bridge with respect to $\otimes$;*

(2) $\mu_\otimes^{'\infty}(x,y) < \mu(xy)$;

(3) *xy is not a weakest edge of any cycle.*

Corollary 3.20. *Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to $\otimes$. Let $xy \in E$. xy is a α-strong edge if and only if xy is a bridge.*

Proof. Obviously, The result is hold by Theorem (3.19). □

Definition 3.21 (**Ref.** [10], Definition 3.2., p.133)**.** Let (σ, μ) be a fuzzy graph with respect to $\otimes$. Then an edge uv is said to be **effective**, if $\mu(uv) = \sigma(u) \otimes \sigma(v)$.

Proposition 3.22 (**Ref.** [10], proposition 3.10., p.133)**.** *Let (σ, μ) be a fuzzy graph with respect to $\otimes$. If the edge uv is effective, then $\mu(uv) = \mu_\otimes^\infty(u,v)$.*

 doi:10.20944/preprints201804.0119.v1

Corollary 3.23. *Let (σ, μ) be a fuzzy graph with respect to $\otimes$. If the edge uv is effective, then uv is not α-strong.*

Proof. Let uv be a edge of (σ, μ). So $\mu(uv) = \mu^{\infty}_{\otimes}(u, v)$ by Proposition (3.22). Hence $\mu(uv) \leq \mu^{\infty}_{\otimes}(u, v)$. It means the edge uv is not α-strong. □

The following example illustrates this concept.

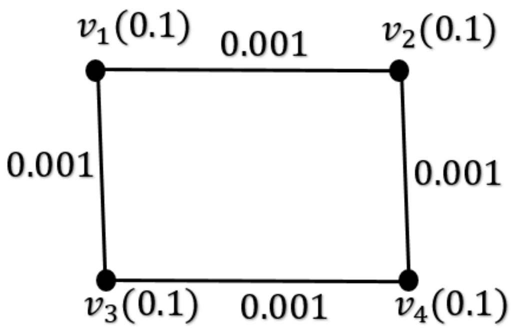

Figure 2. M**-strong arcs and** α**-strong arcs**

Example 3.24. In Figure (2) , all edges are M-strong but there is no α-strong edges in this t-norm fuzzy graph with respect to a t-norm $\wedge$.

Remark 3.25 (**Ref.** [10], p.133). A (crisp) graph that has no cycles is called **acyclic** or a **forest**. A connected forest is called a **tree**. A fuzzy graph is called a **forest** if the graph consisting of its nonzero edge is a forest and a **tree** if this graph is also connected. We call the fuzzy graph $G = (\sigma, \mu)$ a **fuzzy forest** if it has a partial fuzzy spanning subgraph which is a forest, where for all edges xy not in $F[\nu(xy) = 0]$, we have $\mu(xy) < \nu^{\infty}(x, y)$. In other words, if xy is in G, but not F, there is a path in F between x and y whose strength is greater than $\mu(xy)$. It is clear that a forest is a fuzzy forest.

Definition 3.26 (**Ref.** [10], Definition 3.12., p.133)**.** Let $\otimes$ be a t-norm. A fuzzy graph (σ, μ) is a **fuzzy tree** with respect to $\otimes$. If (σ, μ) has a partial fuzzy spanning subgraph $F = (\tau, \nu)$ which is a tree and $\forall xy$ not in F, $\mu(xy) < \nu^{\infty}_{\otimes}(x, y)$.

Theorem 3.27 (**Ref.** [10], Theorem 3.30, p.137)**.** *Let $G = (\sigma, \mu)$ be a fuzzy forest with respect to $\otimes$. Then rhe edges of $F = (\tau, \nu)$ are just the bridges of G.*

Corollary 3.28. *Let $G = (\sigma, \mu)$ be a fuzzy tree with respect to $\otimes$. Then the edges of $F = (\tau, \nu)$ are just the α-strong edges of G.*

Proof. Obviously, the results follows by Theorem (3.27) and Corollary (3.20). □

Proposition 3.29. *Let $T = (\sigma, \mu)$ be a fuzzy tree with respect to $\otimes$. Then $D(T) = D(F) \cup D(S)$, where $D(T), D(F)$ and $D(S)$ are vertex dominating sets of T, F and S, respectively. S is a set of vertices which has no edge with connection to F.*

Proof. By Corollary (3.28), the edges of $F = (\tau, \nu)$ are just the α-strong edges of G. So the result follows by using Definition (3.26). □

4 Applications

According to some applications of t-norm fuzzy graph in **Ref.** ([10], Section 4, p.137), increasing numbers of people from Asia and Africa are seeking to enter the US illegally over the Mexican border. The vast majority of immigrants detained were from the Americas. However, a significant number were from Asian and African countries. We can obtain vertex dominating set by α-strong connections between these countries and vertex domination. In other words, We can find the countries which dominate others as

α-strong from many countries which are increasing and they have a significant number. So We can study the main illegal immigration routes to the United States precisely, usefully and deeply.

5 Conclusion

At present, domination is considered to be one of the fundamental concepts in graph theory and its various applications to ad hoc networks, biological networks, distributed computing, social networks and web graphs partly explain the increased interest. t-norm fuzzy graphs are the vast subject which have the fresh topics and many applications from the real-world problems that make the future better. So we defined domination which is a strong tools for analyzing data, on t-norm fuzzy graphs, for the first time. We hope this concept is useful for studying theoretical topics and applications on t-norm fuzzy graphs.

Acknowledgments

We thank just about everybody. The author is highly thankful to the Editor-in-Chief and the referees for their valuable comments and suggestions for improving the paper.

References

1. G. Chartrand and L. Lesniak, *IGraphs and Digraphs*, Third edition, CRC Press, 1996.
2. M. Ismayil and I. Mohideen, *Complementary nil domination in fuzzy graphs*, Annals of Fuzzy Mathematics and Informatics (2014) 1-8.
3. G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*,Prentice Hall PTR, Upper Saddle River, 1995.
4. O.T. Manjusha and M.S. Sunitha, *Strong Domination in Fuzzy Graphs*, Fuzzy Inf. Eng. 7 (2015) 369-377.
5. O.T. Manjusha and M.S. Sunitha, *Notes on domination in fuzzy graphs*, Journal of Intelligent and Fuzzy systems 27 (2014) 3205-3212.
6. O.T. Manjusha and M.S. Sunitha, *Connected domination in fuzzy graphs using strong arcs*, Annals of Fuzzy Mathematics and Informatics 10 (6) (2015) 979-994.
7. O.T. Manjusha and M.S. Sunitha, *The Strong Domination Alteration Sets in Fuzzy Graphs*, International Journal of Mathematics and its Applications 4 (2-D) (2016) 109-123.
8. O.T. Manjusha and M.S. Sunitha, *Total Domination in Fuzzy Graphs Using Strong Arcs*, Annals of Pure and Applied Mathematics 9 (1) (2015) 23-33.
9. S. Mathew and M.S. Sunitha, *Types of arcs in a fuzzy graph*, Information Sciences 179 (2009) 1760-1768.
10. J.N. Mordeson and S. Mathew, *t-Norm Fuzzy Graphs*, New Mathematics and Natural Computation 14 (1) (2018) 129-143.
11. J.N. Mordeson and P.S. Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-Verlag, 2000.

12. A. Nagoorgani and V.T. Chandrasekaran, *Domination in fuzzy graph*, Adv. in fuzzy sets and systems 1 (1) (2006) 17-26.
13. A. Nagoor Gani and K. Prasanna Devi, 2-*Domination in Fuzzy Graphs*, International Journal of Fuzzy Mathematical Archive 9 (1) (2015) 119-124.
14. C. Natarajan and S.K. Ayyaswamy, *on Strong (weak) domination in Fuzzy Graphs*, International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering 4 (7) (2010) 1035-1037.
15. A. Nagoorgani and P.Vadivel, *Relations between the parameters of Independent Domination and Irredundance in Fuzzy Graphs*, International Journal of Algorithms, Computing and Mathmatics 2 (1) (2009) 15-19.
16. M. Nikfar, *The Results on Vertex Domination in Fuzzy Graphs*. Preprints 2018, 2018040085 (doi: 10.20944/preprints201804.0085.v1).
17. H.T. Nguyen and E.A. Walker, *A First course in fuzzy logic*, CRC Press, 2006.
18. A. Rosenfeld, *Fuzzy Graphs*, In Fuzzy Sets and their Applications to Cognitive and Decision Processes, eds. L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, Academic Press, New York (1975) 77-95.
19. N. Sarala and T. Kavitha, $(1,2)$-*Vertex Domination in Fuzzy Graphs*, International Journal of Innovative Research in Science, Engineering and Technology 5 (7) (2016) 16501-16505.
20. A. Somasundaram and S. Somasundaram, *Domination in fuzzy graphs-I*, Pattern Recognition Letters 19 (1998) 787-791.
21. A. Somasundaram, *Domination in fuzzy graphs-II*, J. Fuzzy Math. 13 (2) (2005) 281-288.
22. Dr.S. Vimala and J.S. Sathya, *Efficient Domination number and Chromatic number of a Fuzzy Graph*, International Journal of Innovative Research in Science, Engineering and Technology 3 (3) (2014)9965-9970.
23. R.T. Yeh, S.Y. Bang, *Fuzzy relations, Fuzzy relations, fuzzy graphs and their applications to clustering analysis*, in: Fuzzy Sets and Their Applications to Cognitive and Decision Processes, eds. L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, Academic Press, (1975) 125-149.
24. L.A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353.