


Article

Respiratory Rate Estimation Based on Spectrum Decomposition

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1 **Abstract:** We propose an electrocardiogram (ECG) signal-based algorithm to estimate the respiratory
2 rate is a significant informative indicator of physiological state of a patient. The consecutive
3 ECG signals reflect the information about the respiration because inhalation and exhalation make
4 transthoracic impedance vary. The proposed algorithm extracts the respiration-related signal by
5 finding out the commonality between the frequency and amplitude features in the ECG pulse train.
6 The respiration rate can be calculated from the principle components after the procedure of the
7 singular spectrum analysis. We achieved 1.7569 breaths per min of root-mean-squared error and
8 1.7517 of standard deviation with a 32-seconds signal window of the Capnobase dataset, which
9 gives notable improvement compared with the conventional Autoregressive model based estimation
10 methods.

11 **Keywords:** respiratory sinus arrhythmia (RSA), R-peak amplitude (RPA), QRS amplitude

12 1. Introduction

13 Respiratory rate is one of the significant vital signs to indicate clinical deterioration[1,2]. Even
14 though continuous monitoring of respiration rate is obviously helpful to track the health condition
15 of patients in hospital, home and community settings[3], it is still often not recorded. In [4], this
16 fact is due to the lack of time and clinical knowledge from nurses (usually, the respiratory rate is
17 estimated through stethoscope or by counting the chest wall movements.) as well as the lack of a
18 reliable respiratory rate monitoring devices.

19 Recent studies have tried to estimate the respiratory rate derived from physical or physiological
20 signal changes due to the respiration activity. There are several ways to estimate the respiratory rate by
21 using physical signal changes. The spirometry[5] can help calculating the respiratory rate as it can
22 measure the flow of air in and out of the mouth during breathing. However, this method need extra
23 equipment to measure air flow in and out. Another sensor for catching the physical signal changes
24 resulted from the respiration, the accelerometer, which can measure acceleration and estimate the
25 quantity of movements, is tried to apply the respiratory rate estimation in non-invasive way[6,7]. The
26 accelerometer is too sensitive to movement noise. For more robust estimation, it is required to apply
27 extra sensors or information.

28 Another type of solution for detecting the respiration activity is catching the physiological signal
29 changes. The impedance pneumography technique[8] can measure the thoracic volume changes
30 during the respiratory cycle by passing the low-amplitude, high-frequency current between two
31 ECG electrodes. Even though it is currently known as the most reliable technique for measuring
32 the respiratory rate, it has worse signal-to-noise ratio. The ECG signal also has the reflection of
33 the respiration activity. Due to the respiration activity, there are three types of modulation can be
34 observed[9] : baseline wander, amplitude modulation and frequency modulation. By using the
35 phenomena, several studies have shown the ways to extract the respiratory signal from the modulated
36 signals through autoregressive modeling methods[4,10–12]. In [13], it introduces that variations of the
37 angle of mean electrical axis (AMEA) of the heart reflect the respiratory patterns.

38 In this paper, we focus on the respiratory rate estimation algorithm based on single-lead ECG
39 signals. There are several benefits to utilize single-lead ECG. First, a single-lead ECG system can be
40 easily incorporated in a wearable type device. A single-lead ECG system gives higher comfortability
41 compared with other complicated sensors include multi-lead ECG system. Second, the device has an
42 ability to monitor cardiac and respiratory activities simultaneously.

43 The remaining part of paper is organized as follows. Section 2 describes physiological relationship
44 between ECG signal and respiration. We also introduce some kinds of approaches about extracting the
45 respiration signal from the given ECG signal. Section 3 depicts our proposed algorithm for estimating
46 respiratory rate in a fusion way. Section 4 shows the results of the performance analysis of our proposed
47 algorithm. Section 5 gives a discussion and a conclusion of this paper.

48 2. Preliminaries

49 Respiration affects ECG signals in several ways : baseline wander, amplitude modulation and
50 frequency modulation[9]. Generally, the frequency range of respiration is located between of 0.1
51 and 0.5 Hz[14]. The frequency of the respiration activity on the baseline of the ECG signals is easily
52 contaminated by various types of artifacts such as movement artifacts, which is not predicted. Due to
53 the characteristics, the baseline wandering effect is not suitable for utilizing to estimate respiratory
54 rate, hence, many studies used to concentrate on the modulated signals to obtain respiratory rate from
55 ECG signals.

56 Respiration is known to modulate the heart rate such that it increases during inspiration and
57 decreases during expiration[14]. It is similar principle with frequency modulation, thus a waveform
58 can be extracted from the heart rate time series representing this modulation, which is referred to as
59 respiratory sinus arrhythmia (RSA). Many studies have focused on estimating a dominant frequency
60 in the modulated signals as an index of respiratory activity[11,15,16].

61 There is another signal, which is reflected on the respiration. During inspiration and expiration,
62 the filling and emptying of the lungs causes a rotation of the electrical axis of the heart and a
63 change in the impedance of the thorax. It yields changes in the electrocardiogram beat morphology.
64 Consequently, the R-peak amplitudes are modulated by the respiratory activity in an amplitude
65 modulation manner[14]. Respiratory rate can be derived from this modulation, which is referred as
66 the R-peak amplitude (RPA).

67 In order to estimate respiratory rate from the extracted RPA and RSA, spectral information related
68 to the respiration should be used. The main assumption is that the dominant frequency of the given
69 signals is regards as the respiration related signal. Common approach to obtain the respiration-related
70 frequency in RPA and RSA is spectral analysis or time-spectral analysis.

71 The traditional technique to find a dominant spectral peak in a given signal is a Fourier
72 Transform-based method[17]. It is simple, but it should be assumed that the signal is stationary or
73 locally stationary. Further, it can't extract the cluttered spectral information. In contrast, Autoregressive
74 (AR) spectral modeling[18–20] gives clean and uncluttered spectral information. However, the result
75 of AR modeling reflects not only the input, but also assumptions about its complexity.

76 Another approach is a time-frequency analysis such as continuous wavelet transform[21] or
77 empirical mode detection[17,22]. It tries to decompose the signal first, then, find a dominant spectral
78 peak in a decomposed signal.

79 Both of the traditional spectral analysis and the AR modeling method requires uniform sampling.
80 However, the RSA and the RPA signals are non-uniform sampled signals. Hence, interpolation
81 is required to analyze the spectral information of the signals. Linear interpolation or cubic spline
82 interpolation technique is traditional way of interpolation. The study by Berger[23] provides a good
83 performance in the ECG analysis.

84 Since the respiration-related activity is generally located at the range of 0.1 and 0.5 Hz, the RPA and
85 RSA signals are downsampled at 2 or 4 Hz. Especially, this downsampling gives an improvement of the
86 resolution of the phase angles and identifying the one corresponding to the respiratory frequency[4].

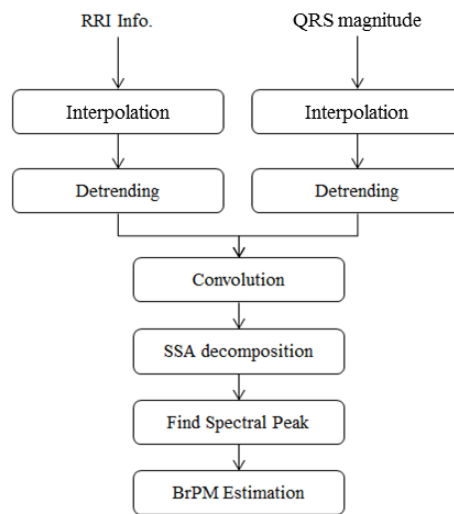


Figure 1. The procedure of the proposed algorithm

87 3. Spectrum Decomposition Algorithm for Respiratory Rate

88 In this paper, our proposed algorithm named as spectrum decomposition algorithm for respiratory
 89 rate (SDAR), focuses on the respiratory rate estimation based on spectrum decomposition with a
 90 single-lead ECG signal. As described in Section 2, respiratory rate can be derived from the relationship
 91 between respiration and the modulated signals. In the SDAR algorithm, a FM modulated signal, RSA
 92 and an AM modulated signal, QRS amplitude is utilized to estimate respiratory rate. Many studies
 93 used to utilize RPA, however, it is sensitive to noise and motion artifacts. To reduce the effects of
 94 the artifacts, the SDAR algorithm uses the difference of the maximum and the minimum of QRS
 95 complexes, which is defined as QRS amplitude, instead of using the RPA directly. For obtaining an
 96 accurate estimation result, the SDAR algorithm is based on fusion techniques. The SDAR algorithm
 97 employs the convolutional method to extract the common frequency component between the RSA
 98 signal and the QRS amplitude based signal.

99 3.1. Extracting Modulated Signals

100 The SDAR algorithm utilizes QRS amplitude and RSA from a given ECG signal. At the begin of
 101 the algorithm, QRS complexes should be detected. The SDAR algorithm employs Pan and Tompkins
 102 QRS complex detector[24], which is one of the widely used as the peak detector. After obtaining the R
 103 peak indices, RSA signal can be easily organized by R to R inter-beat intervals with the corresponding
 104 time indices. QRS amplitude is calculated by Eq.1.

$$QRS_{amp}^i = \max_{t \in S} ECG_S - \min_{t \in S} ECG_S \quad (1)$$

105 where i is the index of the given peak data, S is a set of data in the searching window, which has width
 106 w centered at the i th peak, t means the timing index within the searching window S and ECG_S depicts
 107 ECG signals within searching window S .

108 The SDAR algorithm is based on a fusion technique with both of modulated signals. To obtain
 109 the spectral information from the modulated signals, it should make the signals uniform sampled. The
 110 algorithm applies interpolation to each of RSA and QRS amplitude. Berger's method[23] is applied for
 111 interpolating RSA signal and spline interpolation is applied for obtaining uniformly sampled QRS
 112 amplitude signal. The results of the interpolation from a given ECG signal are shown in Fig.2.

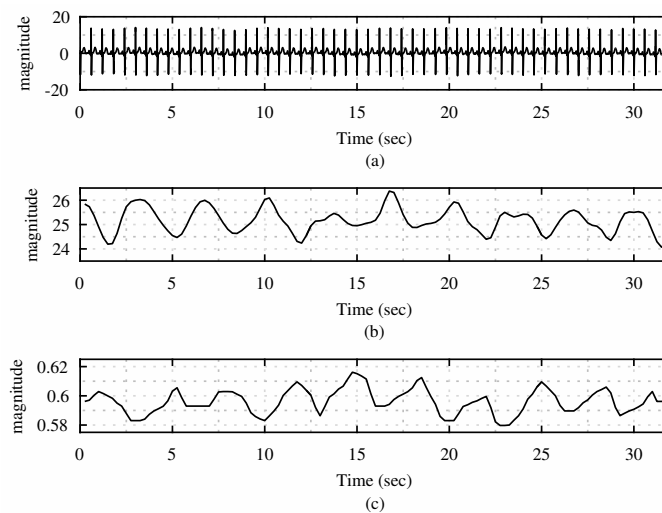


Figure 2. QRS amplitude and RSA signals extracted from a given ECG signal (a) ECG signal (b) interpolated QRS amplitude (c) RSA

113 3.2. Detrending

114 Spectral analysis inherently assumes that the signal is weakly stationary at least[25]. However,
 115 heart rate variability (HRV) signal such as RSA and QRS amplitude is not stationary[26]. Detrending is
 116 the process to remove the non-stationarities in the signal. In this paper, The SDAR algorithm employs
 117 a detrending method using smoothness prior approach (SPA) [25]. According to the SPA, a series of
 118 data can be written into stationary part and non-stationary part as

$$z = z_{stat} + z_{trend} \quad (2)$$

119 where z_{stat} implies the stationary part and z_{trend} is the non-stationary part of the given series of data.
 120 The non-stationary component z_{trend} can be modeled with a linear observation model as

$$z_{trend} = H\theta + v \quad (3)$$

121 where $H \in \mathbb{R}$ is the observation matrix, $\theta \in \mathbb{R}$ are the regression parameters and v is the observation
 122 error. The estimation of the regression parameter $\hat{\theta}$ can be obtained by the regularized least squares
 123 method.

$$\hat{\theta} = \arg \min_{\theta} \{ \|H\theta - z\|^2 + \lambda^2 \|D_d(H\theta)\|^2 \} \quad (4)$$

124 As shown in Fig.3, the extracted signals have trend components. The dashed line of Fig.3 (a) and
 125 (c) is the trending line by the SPA method. After subtracting the fitting line from the extracted signals,
 126 the detrended signals are shown in Fig.3 (b) and (d).

127 3.3. Normalization and Convolution

128 The SDAR algorithm is based on the fusion technique using both of RSA and QRS amplitude
 129 information. Since the energy level of the extracted signals is different each other, normalization is
 130 required to avoid a biased result. In order to make the signals same energy levels, each signal is
 131 divided by the root mean square (RMS) of the signal.

132 After the normalization, our algorithm takes the convolution of detrended QRS amplitude and
 133 detrended RSA signals. The convolution makes the common frequency components salient and
 134 suppresses the frequency components which are not observed in common in the RSA and QRS
 135 amplitude signals. To reduce the edge effect of the signal, the algorithm takes a hanning window

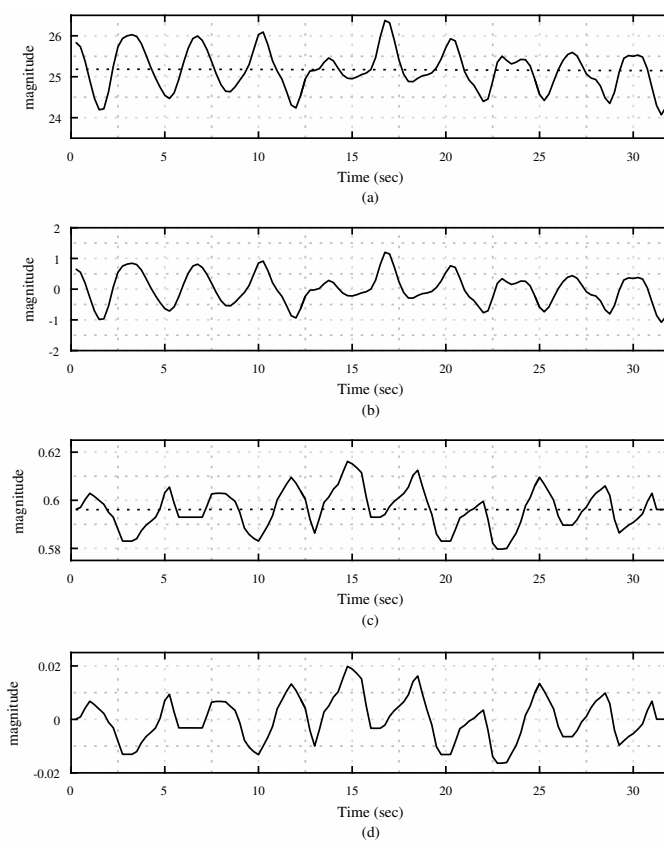


Figure 3. Detrending (a) QRS amplitude without detrending and fitting line (dashed) (b) detrended QRS amplitude (c) RSA without detrending and fitting line(dashed) (d) detrended RSA

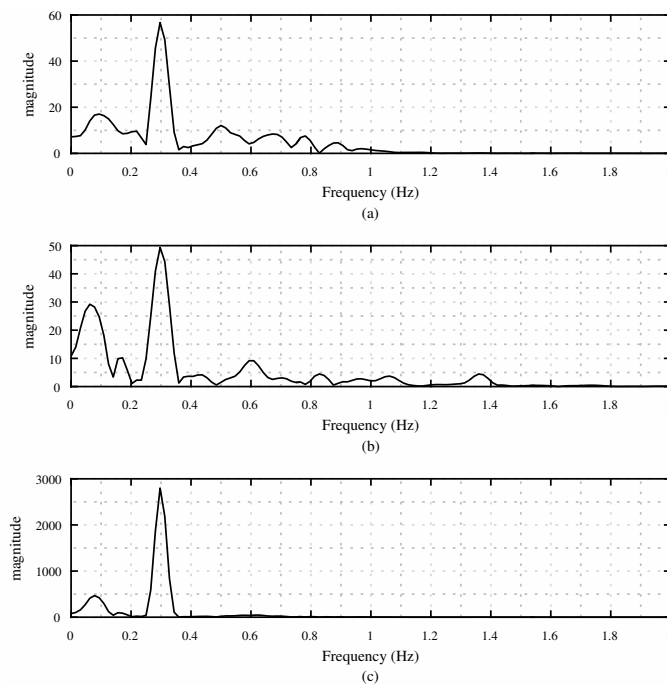


Figure 4. Fusion (a) spectrum of normalized QRS amplitude (b) spectrum of normalized RSA (c) spectrum of convolution

136 before convolution. Fig.4 depicts the normalized signals and convolution of QRS amplitude and RSA
137 signals.

138 3.4. Singular Spectrum Analysis

139 The singular spectrum analysis (SSA) technique is a powerful technique of time series analysis
140 incorporating the elements of classical time series analysis, multivariate statistics, multivariate
141 geometry, dynamical systems and signal processing[27,28]. The aim of SSA is to make a decomposition
142 of the original series into the sum of a small number of independent and interpretable components.

143 The SSA comprises two steps, decomposition and reconstruction steps. At decomposition step,
144 one-dimensional time series $Y_T = (y_1, \dots, y_T)$ are transferred into the multi-dimensional series
145 X_1, \dots, X_T with vectors $X_i = (y_i, \dots, y_{i+L-1})'$. This process is named as embedding. The single
146 parameter of the embedding is the window length $L(2 \leq L \leq T)$. After embedding, the trajectory
147 matrix $X = [X_1, \dots, X_K]$ is obtained. Decomposition is based on singular value decomposition (SVD) of
148 the obtained trajectory matrix. After SVD, the trajectory matrix is decomposed into *rank* 1 matrices,
149 and the vectors of *principle components* (PCs) by the *eigentriples*, $(\sqrt{\lambda_i}, U_i, V_i)$.

150 Fig.5 shows the principle components, which have the largest eigenvalues in numerical order,
151 decomposed by SSA.

152 According to the applications, the reconstruction steps is required. However, our main goal to
153 use SSA in our application decomposes the given signal in independent time series.

154 3.5. Finding a spectral peak

155 The principle components obtained at the SSA decomposition step should be sorted in numerical
156 descending order. That means the first principle component has largest energy occupied in a given
157 signal. Our assumption is that the RSA and QRS amplitude have respiration-related frequency
158 components, which are dominant in the signal. Hence, the dominant frequency of the first principle
159 component could be the respiration-related frequency.

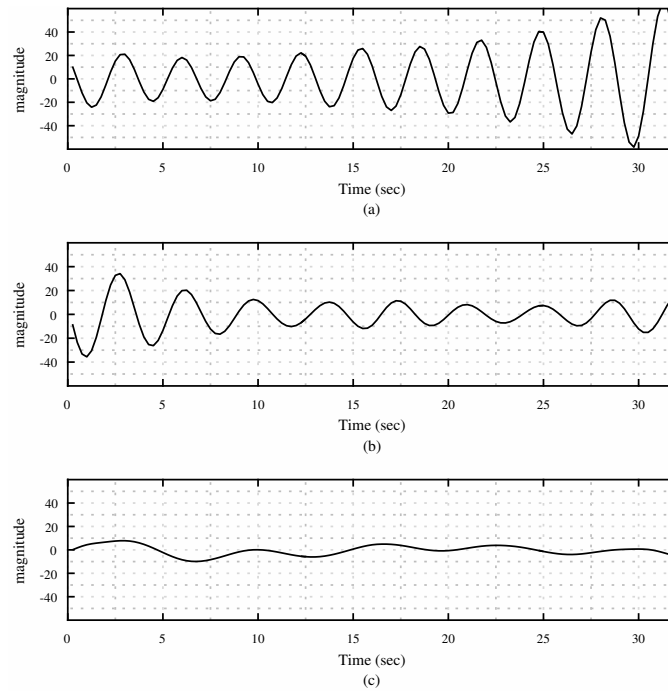


Figure 5. Decomposed time series signals by SSA

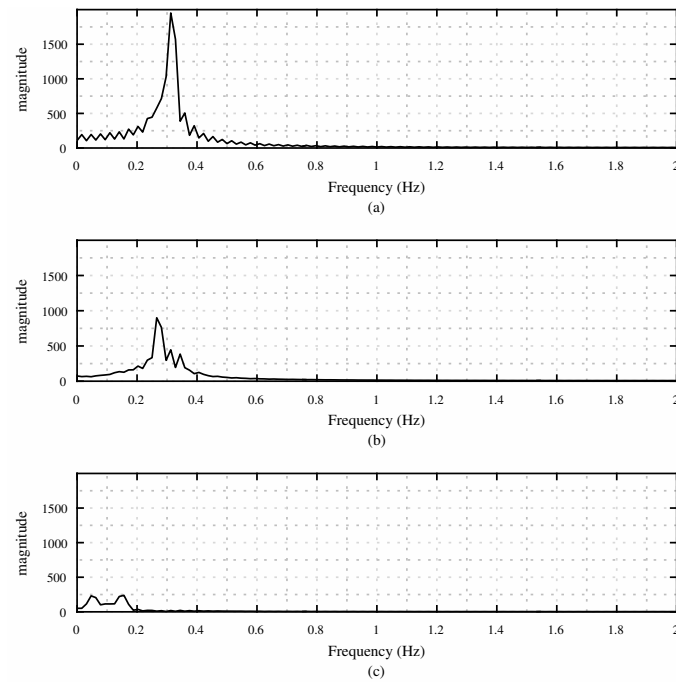


Figure 6. Power spectral density of the SSA decomposed signals

160 To search for the spectral peak of respiratory activities, the algorithm should transform the time
 161 series principle components into spectral domain. Fig.6 shows the transformed principle components.

162 The SSA decomposition may make the given signal split into different principle components,
 163 even though they have similar range of frequency components. Further, if there are some dominant
 164 components out of range of the respiration (0.1 ~ 0.7 Hz), the first principle component may not be
 165 the component which has a respiratory signal. It can result in the error in estimation. In order to
 166 avoid estimation error, our algorithm selects five of the most influential principle components and
 167 reconstructs the signal to find the spectral peak.

168 4. Simulation Results and Analysis

169 4.1. Database used

170 In this paper, Capnabase 8-min database, which is available for download from the online
 171 database[29], is used to evaluate the algorithm. The database is composed of forty two 8-min segments
 172 from 29 pediatrics and 13 adults cases. Each recording contains raw PPG and ECG signals with
 173 annotations by an expert rater, reference CO₂ signals and derived instantaneous respiration rate for all
 174 cases. The capnometric waveform was used as the reference gold standard recording for estimating
 175 respiration rate. The evaluation is performed with 35 recordings out of 42 recordings, since they have
 176 signal distortion or out of respiration rate range (0.1 ~ 0.7 Hz).

177 4.2. Parameters

178 Generally, human respiratory frequency is located at the range of 0.1 and 0.5 Hz[14]. In this paper,
 179 the frequency region of interest is set to 0.1 and 0.7 Hz, since the capnometric waveform in Capnabase
 180 dataset, is observed at the range of 0.1 and 0.7 Hz. Further, 32-sec window is used as the time window
 181 for obtaining the modulated signals. The modulated signals are extracted and interpolated at 4 Hz,
 182 hence 128-point interpolated signals are obtained in time-frequency analysis.

183 4.3. Evaluation

184 The SDAR is compared with the conventional respiratory rate estimation algorithms, which are
 185 based on AR methods using RPA and RSA. The comparison is operated with AR with RPA only, AR
 186 with RSA only, AR with fusion technique[4] and SDAR (proposed). The respiratory rate estimation
 187 result of each method is shown in Fig.7. A solid gray line represents the reference respiratory rate
 188 based on the capnometry signal. Dashed lines demonstrate the estimation results by the conventional
 189 methods based on AR modeling. Single modulated signal based method such as AR with RPA only
 190 or AR with RSA only shows more unstable estimation results than fusion-based method. AR with
 191 fusion technique shows better performance than the single modulated signal based method, since it
 192 determines the respiratory rate by selecting the better estimation between RPA and RSA. A black solid
 193 line, which depicts the estimation results by the SDAR, demonstrate minimum tracking errors out of
 194 the given methods as shown in Fig.7.

195 Over the whole segments in the dataset, the accuracy of the estimation results is shown in Fig.8.
 196 Gray markers represent the respiratory rate estimation results based on the conventional methods,
 197 and black markers shows the results of SDAR algorithm. In a ideal case, the estimation results should
 198 be on the bold gray line in the center of the graph. In other words, the estimation accuracy is superior
 199 if the estimation results are concentrated on the gray line.

200 To quantify the performance, RMS error and standard deviation of the error are used as the metric
 201 of the performance. RMS is defined as

$$RMS = \sqrt{\frac{1}{n} \sum_{i=1}^n (Br_{ref}^i - Br_{est}^i)^2} \quad (5)$$

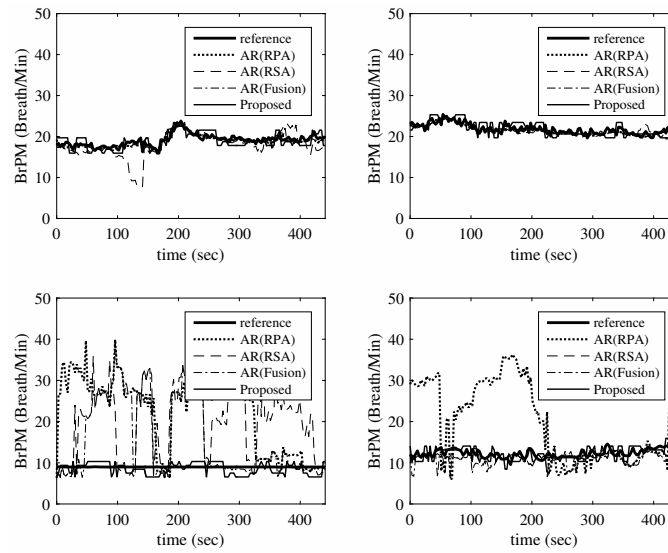


Figure 7. Comparison with conventional respiratory rate estimation algorithm

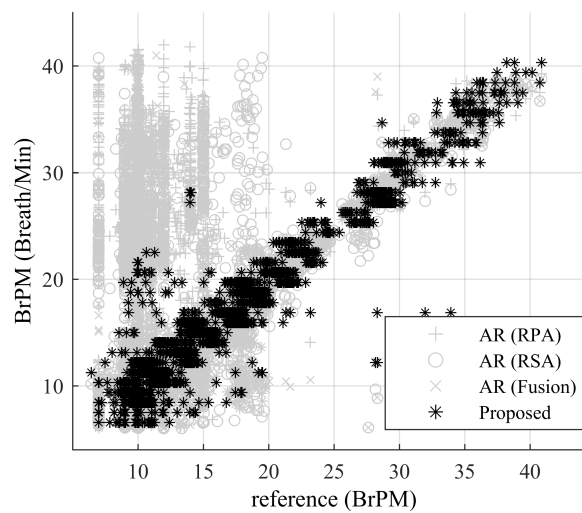


Figure 8. Comparison with conventional respiratory rate estimation algorithm

Table 1. Performance analysis compared with the conventional methods

	RMS Error (Breath/min)	std. (error)
AR with RPA only	11.4016	9.3401
AR with RSA only	7.1853	6.6895
AR with fusion	5.5391	5.4216
SDAR (proposed)	1.7568	1.7517

where n is the number of observations and Br_{ref} and Br_{est} are the reference and the estimation of the respiratory rate respectively. In Table 1, the estimation result of SDAR gives minimum RMS error over the given method. The standard deviation of the error is smallest, that means it gives stable in various cases.

AR modeling based methods estimates the respiratory rate utilizing the resonant frequency of the spectral peak by calculating the phase angle of the corresponding pole. For this reason, AR modeling approaches have a good frequency resolution. On the other hand, Fourier transform-based method gives less resolution than AR modeling approaches. In frequency domain, the resolution is determined by how many samples is used in spectral analysis. The given simulation uses 32-sec time window for single estimation, thus, the resolution is 1/32 Hz, which implies 1.875 Br/min. The SDAR uses a frequency interpolation method for improving the resolution in Br/min. In spite of the limitation, The SDAR gives better accuracy than AR modeling approaches, since it has a good performance to search for a common frequency component between given signals, which is highly related to the respiratory activities.

5. Conclusion

In this paper, we focus on a respiratory rate estimation with single-lead ECG signals in a non-invasive way. Our new approach, spectrum decomposition algorithm for respiratory rate (SDAR), fuses frequency information from the respiration modulated signal by extracting the common frequency components. Further, it decomposes several principles components with singular spectrum analysis (SSA). Finally, we validate SDAR's performance by comparing with the conventional AR modeling approaches utilizing Capnobase dataset. Even though the frequency resolution is inferior that implies SDAR uses short-length of time windows (32-sec), SDAR has less estimation error than AR modeling approaches. To improve the frequency resolution, longer-length of time windows is required, however, it results in deterioration in the time resolution. Since SDAR is a secondary analysis technique, which implies that the analysis is required to obtain the modulated signals, the first analysis is quite important. To improve the performance of the estimation, the preprocessing to remove misdetecting beats or compensate missing beats.

Abbreviations

The following abbreviations are used in this manuscript:

AMEA	Angle of Mean Electrical Axis
AR	Autoregressive
ECG	Electrocardiogram
HRV	Heart Rate Variability
RMS	Root Mean Square
RPA	R-Peak Amplitude
RSA	Respiratory Sinus Arrhythmia
SDAR	Spectrum Decomposition Algorithm for Respiratory Rate
SPA	Smoothness Prior Approach
SSA	Singular Spectrum Analysis
SVD	Singular Value Decomposition

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