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Hybrid Logarithm Similarity Measure based MAGDM Strategy under SVNS environment

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Abstract: The objective of the paper is to introduce new similarity measure for single valued neutrosophic sets based on logarithm function. We define logarithm similarity measure and their weighted similarity measure for single valued neutrosophic sets. Then we define hybrid logarithm similarity measure and weighted hybrid logarithm similarity measure for single valued neutrosophic sets. We prove the basic properties of the proposed measures. We then define an entropy function using logarithm function to determine unknown attribute weights. We develop a novel multi attribute group decision making strategy for single valued neutrosophic sets based on the weighted hybrid logarithm similarity measure. We address an illustrative example to demonstrate the effectiveness and aptness of the proposed strategies. We conduct a sensitivity analysis of the developed strategy. We also make a comparison between the obtained results from proposed strategies and different existing strategies in the literature.

Keywords: single valued neutrosophic set; logarithm similarity measure; logarithm entropy function; ideal solution; multi attribute group decision making

1. Introduction

Smarandache [1] introduced neutrosophic sets (NSs) to pave the way to deal with problems involving uncertainty, indeterminacy and inconsistency. Wang et al. [2] grounded the concept of single valued neutrosophic sets (SVNSs), a subclass of NSs to tackle engineering and scientific problems. SVNSs have been applied to solve various problems in different fields such as medical problems [3–5] decision making problems [6–9], conflict resolution [10], social problems [11] engineering problems [12,13] image processing problems [14–16] and so on.

The concept of similarity measure is very significant in studying almost every practical field. In the literature, few studies have addressed similarity measures for SNVSs [17–20]. Peng et al. [21] developed SVNSs based multi attribute decision making (MADM) strategy employing MABAC

(Multi-Attributive Border Approximation area Comparison and similarity measure), TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) and a new similarity measure.

Ye [22] proposed cosine similarity measure based neutrosophic multiple criteria decision making (MADM) strategy. In order to overcome some disadvantages in the definition of cosine similarity measure, Ye [23] proposed 'improved cosine similarity measures' based on cosine function. Biswas et al. [24] studied cosine similarity measure based MCDM with trapezoidal fuzzy neutrosophic numbers. Mondal and Pramanik [25] developed tangent similarity measure of NSs and applied it to MADM. Ye and Fu [26] studied medical diagnosis problem using a SVNSs similarity measure based on tangent function. Can and Ozguven [27] studied a MADM problem for adjusting the proportional-integral-derivative (PID) coefficients based on neutrosophic Hamming, Euclidean, set-theoretic, Dice, and Jaccard similarity measures.

Several studies [28–30] have been reported in the literature for multi-attribute group decision making (MAGDM) in neutrosophic environment. Ye [31] studied the similarity measure based on distance function of SVNSs and applied it to MAGDM.

Lu and Ye [32] proposed logarithmic similarity measure for interval valued fuzzy set [33] and applied it in fault diagnosis method. In the literature of neutrosophic decision making, logarithm function based similarity measure is yet to appear. To fill the gap, we propose hybrid logarithm similarity measures of SVNSs and establish their basic properties. We also propose a logarithm entropy function to determine unknown attribute weights. We also show an illustrative example of the proposed similarity measures for a MAGDM problem.

The structure of the paper is as follows. Section 2 presents basic concepts of NSs, operations on NSs, SVNSs and operations on SVNSs. Section 3 proposes logarithm similarity measures and weighted logarithm similarity measures, hybrid logarithm similarity measure (HLSM), weighted hybrid logarithm similarity measure (WHLSM) in SVNSs environment. Section 4 proposes an entropy measure based on logarithm function to calculate unknown attribute weights and proves basic properties of entropy function. Section 5 presents a MAGDM strategy based novel weighted hybrid logarithm similarity measure. Section 6 presents an illustrative example to demonstrate the applicability and feasibility of the proposed strategies. Section 7 presents a sensitivity analysis for the results of the numerical example. Section 8 conducts a comparative analysis with the other existing strategies. Section 9 summarizes the paper and discusses future scope of research.

2. Preliminaries

In this section, the concepts of NSs, SVNSs, operations on NSs and SVNSs are outlined.

2.1. *Neutrosophic Sets (NSs)*

Assume that X be a universal set of neutrosophic sets [1]. Then the neutrosophic set N can be defined as follows:

$$N = \{< x: T_N(x), I_N(x), F_N(x) > \mid x \in X\}.$$

Here the functions T, I and F define respectively the membership degree, the indeterminacy degree, and the non-membership degree of the element $x \in X$ to the set N. The three functions T, I and F satisfy the following the conditions:

$$T, I, F: X \rightarrow]^-0,1^+[\text{ and } ^-0 \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3^+$$

For two neutrosophic sets $M = \{< x: T_M(x), I_M(x), F_M(x) > \mid x \in X\}$ and $N = \{< x, T_N(x), I_N(x), F_N(x) > \mid x \in X\}$, the two relations are defined as follows:

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- $M \subseteq N$ if and only if $T_M(x) \le T_N(x)$, $I_M(x) \ge I_N(x)$, $F_M(x) \ge F_N(x)$
- $M = N$ if and only if $T_M(x) = T_N(x)$, $I_M(x) = I_N(x)$, $F_M(x) = F_N(x)$.

2.2. *Single valued neutrosophic sets (SVNSs)*

Assume that X be a universal set of NNs. A SVNS [2] P in X is formed by a TMF (truth-membership function) $T_P(x)$, IMF (an indeterminacy membership function) $I_P(x)$, and a FMF (falsity membership function) $F_P(x)$. For each point x in X, $T_P(x)$, $I_P(x)$, and $F_P(x) \in [0, 1]$.

For continuous case, a SVNS P can be expressed as follows:

$$P = \int_x \frac{< T_P(x), I_P(x), F_P(x) >}{x} : x \in X \ ,$$

For discrete case, a SVNS P can be expressed as follows:

$$P = \sum_{i=1}^{n} \frac{< T_P(x_i), I_P(x_i), F_P(x_i) >}{x_i} : x_i \in X$$

For two SVNSs $P = \{< x: T_P(x), I_P(x), F_P(x) > \mid x \in X\}$ and $Q = \{< x: T_Q(x), I_Q(x), F_Q(x) > \mid x \in X\}$, some definitions are stated below:

- $P \subseteq Q$ if and only if $T_P(x) \le T_Q(x)$, $I_P(x) \ge I_Q(x)$, and $F_P(x) \ge F_Q(x)$.
- $P \supseteq Q$ if and only if $T_P(x) \ge T_Q(x)$, $I_P(x) \le I_Q(x)$, and $F_P(x) \le F_Q(x)$.
- $P = Q$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, and $F_P(x) = F_Q(x)$ for any $x \in X$.
- Complement of P i.e. $P^c = \{< x: F_P(x), 1 - I_P(x), T_P(x) > \mid x \in X\}$.

3. Similarity measures based on logarithm function of SVNSs

In this section, we define two types of logarithm similarity measures and their hybrid and weighted hybrid similarity measures.

3.1. Logarithm similarity measures of SVNSs

Definition 1. Let $A = <x(T_A(x_i), I_P(x_i), F_P(x_i))>$ and $B = <x(T_B(x_i), I_B(x_i), F_B(x_i))>$ be any two SVNSs. The logarithm similarity measures between SVNSs A, B are defined as follows:

$$L_1(A,B) = \frac{1}{n}\sum_{i=1}^{n} \log_4\left(4 - \left(\left|T_A(x_i) - T_B(x_i)\right| + \left|I_A(x_i) - I_B(x_i)\right| + \left|F_A(x_i) - F_B(x_i)\right|\right)\right) \quad (1)$$

$$L_2(A,B) = \frac{1}{n}\sum_{i=1}^{n} \log_2\left(2 - \max\left(\left|T_A(x_i) - T_B(x_i)\right|, \left|I_A(x_i) - I_B(x_i)\right|, \left|F_A(x_i) - F_B(x_i)\right|\right)\right) \quad (2)$$

Theorem 1. The logarithm similarity measure $L_t(A,B)$, (t = 1, 2) between any two SVNSs A and B satisfy the following properties:

P1. $0 \le L_t(A,B) \le 1$

P2. $L_t(A,B) = 1$, if and only if $A = B$

P3. $L_t(A,B) = L_t(B,A)$

P4. If C is a SVNS in X and $A \subseteq B \subseteq C$ then $L_t(A,C) \le L_t(A,B)$ and $L_t(A,C) \le L_t(B,C)$; $t = 1, 2$.

Proof 1. From the definition of SVNS, we write,

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ and $0 \le T_B(x) + I_B(x) + F_B(x) \le 3$

$$\Rightarrow 0 \le \left|T_A(x_i) - T_B(x_i)\right| + \left|I_A(x_i) - I_B(x_i)\right| + \left|F_A(x_i) - F_B(x_i)\right| \le 3;$$

$$0 \le \max\left(\left|T_A(x_i)-T_B(x_i)\right|, \left|I_A(x_i)-I_B(x_i)\right|, \left|F_A(x_i)-F_B(x_i)\right|\right) \le 1$$

$\Rightarrow 0 \le L_t(A, B) \le 1$ for t = 1, 2.

Proof 2. For any two SVNSs A and B,

$A = B$

$\Rightarrow T_A(x) = T_B(x),\ I_A(x) = I_B(x),\ F_A(x) = F_B(x)$

$\Rightarrow \left|T_A(x)-T_B(x)\right| = 0, \left|I_A(x)-I_B(x)\right| = 0, \left|F_A(x)-F_B(x)\right| = 0$

$\Rightarrow L_t(A, B) = 1$ for t = 1, 2.

Conversely,

for $L_t(A, B) = 1 (t = 1, 2)$, we have,

$\Rightarrow \left|T_A(x)-T_B(x)\right| = 0, \ \left|I_A(x)-I_B(x)\right| = 0, \ \left|F_A(x)-F_B(x)\right| = 0$

$\Rightarrow T_A(x) = T_B(x), \ I_A(x) = I_B(x), \ F_A(x) = F_B(x)$

$\Rightarrow \ A = B.$

Proof 3. We have,

$$\left|T_A(x)-T_B(x)\right| = \left|T_B(x)-T_A(x)\right|, \left|I_A(x)-I_B(x)\right| = \left|I_B(x)-I_A(x)\right|, \left|F_A(x)-F_B(x)\right| = \left|F_B(x)-F_A(x)\right|$$

$\Rightarrow L_t(A, B) = L_t(B, A)$ for t = 1, 2.

Proof 4. For $A \subseteq B \subseteq C$, we have,

$T_A(x) \le T_B(x) \le T_C(x),\ I_A(x) \ge I_B(x) \ge I_C(x),\ F_A(x) \ge F_B(x) \ge F_C(x)$ for $x \in X$.

$$\Rightarrow \left|T_A(x)-T_B(x)\right| \le \left|T_A(x)-T_C(x)\right|, \left|T_B(x)-T_C(x)\right| \le \left|T_A(x)-T_C(x)\right|;$$

$$\left|I_A(x)-I_B(x)\right| \le \left|I_A(x)-I_C(x)\right|, \left|I_B(x)-I_C(x)\right| \le \left|I_A(x)-I_C(x)\right|;$$

$$\left|F_A(x)-F_B(x)\right| \le \left|F_A(x)-F_C(x)\right|, \left|F_B(x)-F_C(x)\right| \le \left|F_A(x)-F_C(x)\right|.$$

$\Rightarrow L_t(A, C) \le L_t(A, B)$ and $L_t(A, C) \le L_t(B, C)$; t = 1, 2.

3.2. Weighted logarithm similarity measures of SVNSs

Definition 2. Let $A = <x(T_A(x_i), I_P(x_i), F_P(x_i))>$ and $B = <x(T_B(x_i), I_B(x_i), F_B(x_i))>$ be any two SVNSs. Then the weighted logarithm similarity measures between SVNSs A, B are defined as follows:

$$L_1^w(A, B) = \sum_{i=1}^{n} w_i \log_4\left(4-\left(\left|T_A(x_i)-T_B(x_i)\right| + \left|I_A(x_i)-I_B(x_i)\right| + \left|F_A(x_i)-F_B(x_i)\right|\right)\right) \quad (3)$$

$$L_2^w(A, B) = \sum_{i=1}^{n} w_i \log_2\left(2-\max\left(\left|T_A(x_i)-T_B(x_i)\right|, \left|I_A(x_i)-I_B(x_i)\right|, \left|F_A(x_i)-F_B(x_i)\right|\right)\right) \quad (4)$$

Here, $0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i = 1$.

Theorem 2. The weighted logarithm similarity measures $L_t^w(A, B)$ (t = 1, 2) between SVNSs A and B satisfy the following properties:

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P1. $0 \le L_t^w(A,B) \le 1$

P2. $L_t^w(A,B) = 1$, if and only if $A = B$

P3. $L_t^w(A,B) = L_t^w(B,A)$

P4. If C is a SVNS in X and $A \subseteq B \subseteq C$, then $L_t^w(A,C) \le L_t^w(A,B)$ and $L_t^w(A,C) \le L_t^w(B,C)$; (t = 1, 2);

$\sum_{i=1}^{n} w_i = 1$.

Proof 1. From the definition of SVNSs A and B, we write,

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ and $0 \le T_B(x) + I_B(x) + F_B(x) \le 3$

$$\Rightarrow 0 \le \max\left(\left|T_A(x_i) - T_B(x_i)\right|, \left|I_A(x_i) - I_B(x_i)\right|, \left|F_A(x_i) - F_B(x_i)\right|\right) \le 1$$

$$\Rightarrow 0 \le \left|T_A(x_i) - T_B(x_i)\right| + \left|I_A(x_i) - I_B(x_i)\right| + \left|F_A(x_i) - F_B(x_i)\right| \le 3,$$

$$\Rightarrow 0 \le L_t^w(A,B) \le 1 \text{ for t = 1, 2 since } \sum_{i=1}^{n} w_i = 1.$$

Proof 2. For any two SVNSs A and B if $A = B$, then we have,

$T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

$$\Rightarrow \left|T_A(x) - T_B(x)\right| = 0, \ \left|I_A(x) - I_B(x)\right| = 0, \ \left|F_A(x) - F_B(x)\right| = 0$$

$$\Rightarrow L_t^w(A,B) = 1, \text{ (t = 1, 2), since } \sum_{i=1}^{n} w_i = 1.$$

Conversely,

For $L_t^w(A,B) = 1$ (t = 1, 2), then we have,

$$\Rightarrow \left|T_A(x) - T_B(x)\right| = 0, \ \left|I_A(x) - I_B(x)\right| = 0, \ \left|F_A(x) - F_B(x)\right| = 0$$

$$\Rightarrow T_A(x) = T_B(x), \ I_A(x) = I_B(x), \ F_A(x) = F_B(x)$$

$$\Rightarrow A = B, \text{ since } \sum_{i=1}^{n} w_i = 1.$$

Proof 3. For any two SVNSs A and B, we have,

$$\left|T_A(x) - T_B(x)\right| = \left|T_B(x) - T_A(x)\right|, \left|I_A(x) - I_B(x)\right| = \left|I_B(x) - I_A(x)\right|, \left|F_A(x) - F_B(x)\right| = \left|F_B(x) - F_A(x)\right|$$

$$\Rightarrow L_t^w(A,B) = L_t^w(B,A) \text{ for } t = 1, 2.$$

Proof 4. For $A \subseteq B \subseteq C$, we have,

$T_A(x) \le T_B(x) \le T_C(x)$, $I_A(x) \ge I_B(x) \ge I_C(x)$, $F_A(x) \ge F_B(x) \ge F_C(x)$ for $x \in X$.

$$\Rightarrow \left|T_A(x) - T_B(x)\right| \le \left|T_A(x) - T_C(x)\right|, \left|T_B(x) - T_C(x)\right| \le \left|T_A(x) - T_C(x)\right|;$$

$$\left|I_A(x) - I_B(x)\right| \le \left|I_A(x) - I_C(x)\right|, \left|I_B(x) - I_C(x)\right| \le \left|I_A(x) - I_C(x)\right|;$$

$$\left|F_A(x) - F_B(x)\right| \le \left|F_A(x) - F_C(x)\right|, \left|F_B(x) - F_C(x)\right| \le \left|F_A(x) - F_C(x)\right|.$$

$$\Rightarrow L_t^w(A,C) \le L_t^w(A,B) \text{ and } L_t^w(A,C) \le L_t^w(B,C) \text{ since } \textstyle\sum_{i=1}^{n} w_i = 1, \ t = 1, 2.$$

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3.3. Hybrid logarithm similarity measures of SVNSs

Definition 3. Let $A = \langle x(T_A(x_i), I_P(x_i), F_P(x_i))\rangle$ and $B = \langle x(T_B(x_i), I_B(x_i), F_B(x_i))\rangle$ be any two SVNSs. The hybrid logarithm similarity measure between SVNSs A, B is defined as follows:

$$L_{Hyb}(A,B)=\frac{1}{n}\left[\begin{array}{l}\lambda\left\{\sum_{i=1}^{n}\log_4\left(4-\left(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|\right)\right)\right\}\\ +(1-\lambda)\left\{\sum_{i=1}^{n}\log_2\left(2-\max\left(|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|\right)\right)\right\}\end{array}\right] \quad (5)$$

Here, $0\le\lambda\le1$.

Theorem 3. The hybrid logarithm similarity measure $L_{Hyb}(A,B)$ between any two SVNSs A and B satisfy the following properties:

P1. $0\le L_{Hyb}(A,B)\le1$

P2. $L_{Hyb}(A,B)=1$, if and only if $A = B$

P3. $L_{Hyb}(A,B) = L_{Hyb}(B,A)$

P4. If C is a SVNS in X and $A\subseteq B\subseteq C$ then

$L_{Hyb}(A,C)\le L_{Hyb}(A,B)$ and $L_{Hyb}(A,C)\le L_{Hyb}(B,C)$.

Proof 1. From the definition of SVNS, we write,

$0 \le T_A(x)+ I_A(x)+ F_A(x) \le 3$ and $0 \le T_B(x) + I_B(x) + F_B(x) \le 3$

$\Rightarrow 0\le \max\left(|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|\right)\le1$

$\Rightarrow 0\le|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|\le3;$

$\Rightarrow 0\le L_{Hyb}(A,B)\le1$.

Proof 2. For any two SVNSs A and B,

for $A = B$, we have,

$\Rightarrow T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$

$\Rightarrow |T_A(x)-T_B(x)|=0, |I_A(x)-I_B(x)|=0, |F_A(x)-F_B(x)|=0$

$\Rightarrow L_{Hyb}(A,B)=1$.

Conversely,

for $L_{Hyb}(A,B)=1$, we have,

$|T_A(x)-T_B(x)|=0,\ |I_A(x)-I_B(x)|=0,\ |F_A(x)-F_B(x)|=0$

$\Rightarrow T_A(x)=T_B(x),\ I_A(x)=I_B(x),\ F_A(x)=F_B(x)$

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$\Rightarrow A = B.$

Proof 3. For any two SVNSs A and B, we have,

$$|T_A(x)-T_B(x)|=|T_B(x)-T_A(x)|,\ |I_A(x)-I_B(x)|=|I_B(x)-I_A(x)|,\ |F_A(x)-F_B(x)|=|F_B(x)-F_A(x)|$$

$$\Rightarrow L_{Hyb}(A,B)=L_{Hyb}(B,A).$$

Proof 4. For $A\subseteq B\subseteq C$, we have,
$T_A(x)\le T_B(x)\le T_C(x)$, $I_A(x)\ge I_B(x)\ge I_C(x)$, $F_A(x)\ge F_B(x)\ge F_C(x)$ for $x\in X$.

$$\Rightarrow |T_A(x)-T_B(x)|\le|T_A(x)-T_C(x)|,\ |T_B(x)-T_C(x)|\le|T_A(x)-T_C(x)|;$$

$$|I_A(x)-I_B(x)|\le|I_A(x)-I_C(x)|,\ |I_B(x)-I_C(x)|\le|I_A(x)-I_C(x)|;$$

$$|F_A(x)-F_B(x)|\le|F_A(x)-F_C(x)|,\ |F_B(x)-F_C(x)|\le|F_A(x)-F_C(x)|.$$

$$\Rightarrow L_{Hyb}(A,C)\le L_{Hyb}(A,B) \text{ and } L_{Hyb}(A,C)\le L_{Hyb}(B,C).$$

3.4. Hybrid weighted logarithm similarity measures of SVNSs

Definition 4. Let $A = \langle x(T_A(x_i), I_P(x_i), F_P(x_i))\rangle$ and $B = \langle x(T_B(x_i), I_B(x_i), F_B(x_i))\rangle$ be any two SVNSs. The weighted hybrid logarithm similarity measure between SVNSs A and B is defined as follows:

$$L_{wHyb}(A,B)=\left[\begin{array}{l}\lambda\left\{\sum_{i=1}^{n}w_i\log_4\left(4-\left(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|\right)\right)\right\}\\+(1-\lambda)\left\{\sum_{i=1}^{n}w_i\log_2\left(2-\max\left(|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|\right)\right)\right\}\end{array}\right] \quad (6)$$

Here, $0\le\lambda\le1$.

Theorem 4. The weighted hybrid logarithm similarity measure $L_{wHyb}(A,B)$ between any two SVNSs A and B satisfy the following properties:

P1. $0\le L_{wHyb}(A,B)\le1$

P2. $L_{wHyb}(A,B)=1$, if and only if $A = B$

P3. $L_{wHyb}(A,B) = L_{wHyb}(B,A)$

P4. If C is a SVNS in X and $A\subseteq B\subseteq C$, then $L_{wHyb}(A,C)\le L_{wHyb}(A,B)$ and $L_{wHyb}(A,C)\le L_{wHyb}(B,C)$.

Proof 1. From the definition of SVNS, we write,
$0\le T_A(x)+I_A(x)+F_A(x)\le3$ and $0\le T_B(x)+I_B(x)+F_B(x)\le3$

$$\Rightarrow 0\le\max\left(|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|\right)\le1$$

$$\Rightarrow 0\le|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|\le3;$$

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$\Rightarrow 0 \le L_{wHyb}(A,B) \le 1$.

Proof 2. For any two SVNSs A and B, for $A = B$, we have,

$\Rightarrow T_A(x) = T_B(x),\ I_A(x) = I_B(x),\ F_A(x) = F_B(x)$

$\Rightarrow |T_A(x) - T_B(x)| = 0,\ |I_A(x) - I_B(x)| = 0,\ |F_A(x) - F_B(x)| = 0$

$\Rightarrow L_{wHyb}(A,B) = 1$.

Conversely,

for $L_{wHyb}(A,B) = 1$, we have,

$|T_A(x) - T_B(x)| = 0,\ |I_A(x) - I_B(x)| = 0,\ |F_A(x) - F_B(x)| = 0$

$\Rightarrow T_A(x) = T_B(x),\ I_A(x) = I_B(x),\ F_A(x) = F_B(x)$

$\Rightarrow A = B$.

Proof 3. For any two SVNSs A and B, we have,

$|T_A(x) - T_B(x)| = |T_B(x) - T_A(x)|,\ |I_A(x) - I_B(x)| = |I_B(x) - I_A(x)|,\ |F_A(x) - F_B(x)| = |F_B(x) - F_A(x)|$

$\Rightarrow L_{wHyb}(A,B) = L_{wHyb}(B,A)$.

Proof 4. For $A \subseteq B \subseteq C$, we have,

$T_A(x) \le T_B(x) \le T_C(x),\ I_A(x) \ge I_B(x) \ge I_C(x),\ F_A(x) \ge F_B(x) \ge F_C(x)$ for $x \in X$.

$\Rightarrow |T_A(x) - T_B(x)| \le |T_A(x) - T_C(x)|,\ |T_B(x) - T_C(x)| \le |T_A(x) - T_C(x)|$;

$|I_A(x) - I_B(x)| \le |I_A(x) - I_C(x)|,\ |I_B(x) - I_C(x)| \le |I_A(x) - I_C(x)|$;

$|F_A(x) - F_B(x)| \le |F_A(x) - F_C(x)|,\ |F_B(x) - F_C(x)| \le |F_A(x) - F_C(x)|$.

$\Rightarrow L_{wHyb}(A,C) \le L_{wHyb}(A,B)$ and $L_{wHyb}(A,C) \le L_{wHyb}(B,C)$.

4. Logarithm entropy function

Entropy strategy [34] is an important contribution for determining indeterminate information. Zhang et al. [35] introduced the fuzzy entropy. Vlachos and Sergiadis [36] proposed entropy function for intuitionistic fuzzy sets. Majumder and Samanta [37] developed some entropy measures for SVNSs. When attribute weights are completely unknown to decision makers, the entropy measure is used to calculate attribute weights. In this paper, we develop an entropy strategy based on logarithm function for determining unknown attribute weights.

Definition 5. The logarithm entropy function of a SVNS $P = \langle T_{ij}^{P}, I_{ij}^{P}, F_{ij}^{P} \rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) is defined as follows:

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$$E_j(P) = 1 - \frac{1}{n}\sum_{i=1}^{m}\left[\left(T_{ij}^{P} + F_{ij}^{P}\right)\log_2\left(2 - 4\,I_{ij}^{P}.(I_{ij}^{P})^{c}\right)\right] \tag{7}$$

$$w_j = \frac{1 - E_j(P)}{n - \sum_{j=1}^{n} E_j(P)} \tag{8}$$

Here, $\sum_{j=1}^{n} w_j = 1$

Theorem 5. The logarithm entropy function $E_j(P)$ satisfies the following properties:

P1. $E_j(P) = 0$, if $T_{ij}=1, F_{ij}=I_{ij}=0$.

P2. $E_j(P) = 1$, if $\langle T_{ij}, I_{ij}, F_{ij}\rangle = \langle 0.5, 0.5, 0.5\rangle$.

P3. $E_j(P) \geq E_j(Q)$, if $T_{ij}^{P} + F_{ij}^{P} \leq T_{ij}^{Q} + F_{ij}^{Q};\ I_{ij}^{P}.(I_{ij}^{P})^{c} \geq I_{ij}^{Q}.(I_{ij}^{Q})^{c}$.

P4. $En_j(P) = En_j(P^{c})$.

Proof 1. $T_{ij}=1, F_{ij}=I_{ij}=0$

$$\Rightarrow E_j(P) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left[(1+0)\log_2(2)\right] = 1 - \frac{1}{n}.n = 0$$

Proof 2. $\langle T_{ij}, I_{ij}, F_{ij}\rangle = \langle 0.5, 0.5, 0.5\rangle$.

$$\Rightarrow E_j(P) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left[(0.5+0.5)\log_2(2-1)\right] = 1 - 0 = 1$$

Proof 3. $T_{ij}^{P} + F_{ij}^{P} \leq T_{ij}^{Q} + F_{ij}^{Q};\ I_{ij}^{P}.(I_{ij}^{P})^{c} \geq I_{ij}^{Q}.(I_{ij}^{Q})^{c}$

$$\Rightarrow \sum_{i=1}^{n}\left[\left(T_{ij}^{P} + F_{ij}^{P}\right)\log_2\left(2 - 4\,I_{ij}^{P}.(I_{ij}^{P})^{c}\right)\right] \leq \sum_{i=1}^{n}\left[\left(T_{ij}^{Q} + F_{ij}^{Q}\right)\log_2\left(2 - 4\,I_{ij}^{Q}.(I_{ij}^{Q})^{c}\right)\right]$$

$$\Rightarrow E_j(P) \geq E_j(Q)$$

Proof 4. Since $\langle T_{ij}, I_{ij}, F_{ij}\rangle^{c} = \langle F_{ij}, 1 - I_{ij}, T_{ij}\rangle$, we have $E_j(P) = E_j(P^{c})$.

Note 1: We propose logarithm entropy function to calculate unknown weights of each attribute. When uncertainty increases, criterion weight decreases.

5. MAGDM strategy based on weighted logarithm similarity measure for SVNSs

Assume that (P_1, P_2, ..., P_m) be the alternatives, (C_1, C_2, ..., C_n) be the criteria of each alternative, and {D_1, D_2, ..., D_r} be the decision makers. Decision makers provide the rating of alternatives based on the predefined attribute. Each decision maker constructs a neutrosophic decision matrix associated with the alternatives based on each attribute shown in Equation (9). Using the following steps, we present the MAGDM strategy based on weighted hybrid logarithm similarity measure (see Figure 1).

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Step 1: Determine the relation between alternatives and attribute

At first, each decision maker prepares decision matrix. The relation between alternatives P_i (i = 1, 2, ..., m) and the attribute C_j (j = 1, 2, ..., n) corresponding to each decision maker is presented in the Equation (9).

$$D_r[P|C] = \begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{array} \left(\begin{array}{cccc} C_1 & C_2 & \cdots & C_n \\ \langle T_{11}^{D_r}, I_{11}^{D_r}, F_{11}^{D_r} \rangle & \langle T_{12}^{D_r}, I_{12}^{D_r}, F_{12}^{D_r} \rangle & \cdots & \langle T_{1n}^{D_r}, I_{1n}^{D_r}, F_{1n}^{D_r} \rangle \\ \langle T_{21}^{D_r}, I_{21}^{D_r}, F_{21}^{D_r} \rangle & \langle T_{22}^{D_r}, I_{22}^{D_r}, F_{22}^{D_r} \rangle & \cdots & \langle T_{2n}^{D_r}, I_{2n}^{D_r}, F_{2n}^{D_r} \rangle \\ \cdots & \cdots & \ddots & \cdots \\ \langle T_{m1}^{D_r}, I_{m1}^{D_r}, F_{m1}^{D_r} \rangle & \langle T_{m2}^{D_r}, I_{m2}^{D_r}, F_{m2}^{D_r} \rangle & \cdots & \langle T_{mn}^{D_r}, I_{mn}^{D_r}, F_{mn}^{D_r} \rangle \end{array} \right) \quad (9)$$

Here, $\langle T_{ij}^{D_r}, I_{ij}^{D_r}, F_{ij}^{D_r} \rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) is the single valued neutrosophic rating value of the alternative P_i with respect to the criterion C_j corresponding to the decision maker D_r.

Step 2: Determine the central decision matrix

We form a new decision matrix, called central decision matrix to combine all the decision maker's opinions into a group opinion. Central decision matrix minimizes the biasness which is imposed by different decision makers and hence credibility to the final decision increases. The central decision matrix is presented in Equation (10).

$$D[P|C] =$$

$$\left(\begin{array}{ccccc} & C_1 & C_2 & \cdots & C_n \\ P_1 & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{11}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{11}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{11}^{D_t} \right\rangle & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{12}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{12}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{12}^{D_t} \right\rangle & \cdots & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{1n}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{1n}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{1n}^{D_t} \right\rangle \\ P_2 & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{21}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{21}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{21}^{D_t} \right\rangle & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{22}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{22}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{22}^{D_t} \right\rangle & \cdots & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{2n}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{2n}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{2n}^{D_t} \right\rangle \\ \vdots & \cdots & \cdots & \ddots & \cdots \\ P_m & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{m1}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{m1}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{m1}^{D_t} \right\rangle & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{m2}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{m2}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{m2}^{D_t} \right\rangle & \cdots & \left\langle \frac{1}{r}\Sigma_{t=1}^{r} T_{mn}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} I_{mn}^{D_t}, \frac{1}{r}\Sigma_{t=1}^{r} F_{mn}^{D_t} \right\rangle \end{array} \right) \quad (10)$$

Step 3: Determine the ideal solution

The evaluation of attributes can be categorized into benefit criterion and cost attributes. An ideal alternative can be determined by using a maximum operator for the benefit attributes and a minimum operator for the cost attributes for determining the best value of each attribute among all the alternatives. An ideal alternative [38] is presented as follows:

$$P^* = \{C_1^*, C_2^*, \ldots, C_m^*\}.$$

where the benefit attribute is

$$C_j^* = \left\langle \max_i T_{C_j}{}^{(P_i)}, \min_i I_{C_j}{}^{(P_i)}, \min_i F_{C_j}{}^{(P_i)} \right\rangle \quad (11)$$

and the cost attribute is

$$C_j^* = \left\langle \min_i T_{C_j}{}^{(P_i)}, \max_i I_{C_j}{}^{(P_i)}, \max_i F_{C_j}{}^{(P_i)} \right\rangle \quad (12)$$

Step 4: Determine the attribute weights

Using Equation (8), determine the weights of the attribute.

Step 5: Determine the weighted similarity measures

Using Equation (6), calculate the weighted similarity measures for each alternative.

Step 6: Ranking the priority

All the alternatives are preference ranked based on the decreasing order of calculated measure values. The highest value reflects the best alternative.

Step 7: End.

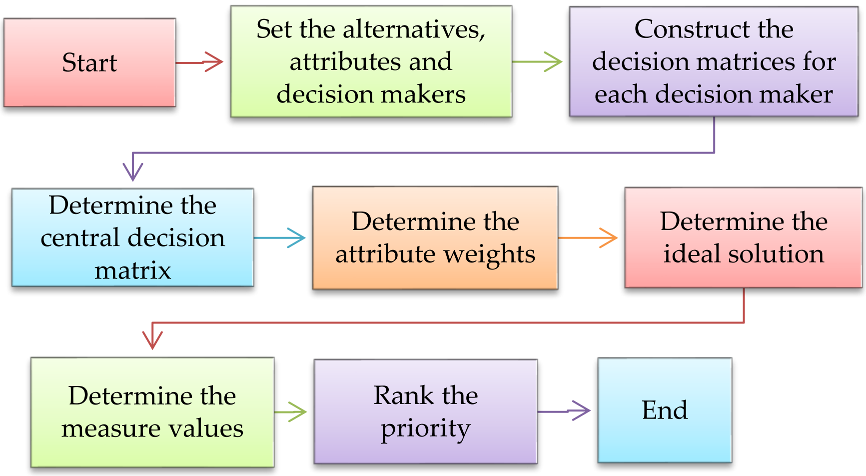

Figure 1 A flow chart of the proposed MAGDM strategy

Note 2: In Figure 1, we represent the steps of hybrid logarithm similarity measure strategy for SVNSs.

6. An illustrative example

Suppose that a state government wants to construct an eco-tourism park for the development of state tourism and especially for mental refreshment of children. After initial screening, five potential spots namely, spot-1 (P_1), spot-2 (P_2), spot-3 (P_3), spot-4 (P_4), spot-5 (P_5) remain for further selection. A team of four decision makers, namely, D_1, D_2, D_3, and D_4 has been constructed for selecting the most suitable spot with respect to the following attributes.

- Ecology (C_1),
- Costs (C_2),
- Technical facility (C_3),
- Transport (C_4),
- Risk factors (C_5)

The steps of decision making strategy to select the best potential spot to construct an eco-tourism park based on the proposed strategy are stated below:

6.1. Steps of MAGDM strategy

Step 1: Determine the relation between alternatives and attributes

The relation between alternatives P_1, P_2 and P_3 and the attribute set $\{C_1, C_2, C_3, C_4, C_5\}$ corresponding to the set of decision makers $\{D_1, D_2, D_3\}$ are presented in Equations (13), (14), (15) and (16).

$$D_1[P\,|\,C] = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\left(\begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \\ \langle 0.7,0.4,0.4\rangle & \langle 0.7,0.4,0.3\rangle & \langle 0.8,0.1,0.1\rangle & \langle 0.7,0.2,0,1\rangle & \langle 0.6,0.5,0.5\rangle \\ \langle 0.4,0.3,0.6\rangle & \langle 0.5,0.2,0.5\rangle & \langle 0.6,0.2,0.2\rangle & \langle 0.7,0.3,0.3\rangle & \langle 0.4,0.3,0.4\rangle \\ \langle 0.4,0.2,0.3\rangle & \langle 0.8,0.1,0.3\rangle & \langle 0.5,0.4,0.4\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.7,0.3,0.2\rangle \\ \langle 0.5,0.3,0.2\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.7,0.3,0.1\rangle & \langle 0.6,0.3,0.2\rangle & \langle 0.5,0.1,0.2\rangle \\ \langle 0.6,0.2,0.5\rangle & \langle 0.6,0.4,0.4\rangle & \langle 0.6,0.2,0.2\rangle & \langle 0.5,0.3,0.5\rangle & \langle 0.6,0.5,0.5\rangle \end{array} \right) \quad (13)$$

$$D_2[P\,|\,C] = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\left(\begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \\ \langle 0.5,0.2,0.3\rangle & \langle 0.7,0.4,0.4\rangle & \langle 0.8,0.2,0.2\rangle & \langle 0.5,0.2,0,2\rangle & \langle 0.5,0.5,0.4\rangle \\ \langle 0.5,0.4,0.4\rangle & \langle 0.5,0.2,0.4\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.8,0.3,0.3\rangle & \langle 0.4,0.1,0.4\rangle \\ \langle 0.4,0.2,0.5\rangle & \langle 0.8,0.2,0.2\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.7,0.2,0.2\rangle & \langle 0.7,0.4,0.2\rangle \\ \langle 0.6,0.6,0.2\rangle & \langle 0.5,0.3,0.1\rangle & \langle 0.3,0.4,0.2\rangle & \langle 0.5,0.3,0.4\rangle & \langle 0.5,0.5,0.2\rangle \\ \langle 0.4,0.6,0.5\rangle & \langle 0.8,0.3,0.3\rangle & \langle 0.4,0.4,0.4\rangle & \langle 0.6,0.3,01\rangle & \langle 0.6,0.1,0.1\rangle \end{array} \right) \quad (14)$$

$$D_3[P\,|\,C] = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\left(\begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \\ \langle 0.7,0.4,0.3\rangle & \langle 0.8,0.2,0.1\rangle & \langle 0.6,0.3,0.3\rangle & \langle 0.7,0.2,0,5\rangle & \langle 0.5,0.6,0.5\rangle \\ \langle 0.6,0.2,0.3\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.7,0.4,0.4\rangle & \langle 0.5,0.3,0.4\rangle & \langle 0.3,0.4,0.4\rangle \\ \langle 0.6,0.2,0.3\rangle & \langle 0.6,0.4,0.2\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.7,0.4,0.2\rangle & \langle 0.5,0.6,0.4\rangle \\ \langle 0.7,0.3,0.4\rangle & \langle 0.7,0.1,0.3\rangle & \langle 0.6,0.3,0.3\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.6,0.2,0.2\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.5,0.4,0.5\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.6,0.3,0.4\rangle \end{array} \right) \quad (15)$$

$$D_4[P\,|\,C] = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\left(\begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \\ \langle 0.5,0.2,0.2\rangle & \langle 06,0.2,0.4\rangle & \langle 0.6,0.2,0.2\rangle & \langle 0.5,0.2,0,4\rangle & \langle 0.4,0.4,0.2\rangle \\ \langle 0.5,0.3,0.3\rangle & \langle 0.5,0.3,0.4\rangle & \langle 0.6,0.3,0.3\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.5,0.4,0.4\rangle \\ \langle 0.6,0.2,0.1\rangle & \langle 0.6,0.1,0.1\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.5,0.4,0.2\rangle & \langle 0.5,0.3,0.4\rangle \\ \langle 0.6,0.4,0.4\rangle & \langle 0.5,0.3,0.2\rangle & \langle 0.4,0.2,0.2\rangle & \langle 0.4,0.1,0.4\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.4,0.2,0.4\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.5,0.4,0.3\rangle & \langle 0.6,0.3,0.2\rangle \end{array} \right) \quad (16)$$

Step 2: Determine the central decision matrix

Using Equation (10), we construct the central decision matrix for all decision makers shown in Equation (17).

$$D[P\,|\,C] = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\left(\begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \\ \langle 0.6,0.3,0.3\rangle & \langle 0.7,0.3,0.3\rangle & \langle 0.7,0.2,0.2\rangle & \langle 0.6,0.2,0.3\rangle & \langle 0.5,0.5,0.4\rangle \\ \langle 0.5,0.3,0.4\rangle & \langle 0.5,0.2,0.4\rangle & \langle 0.6,0.3,0.3\rangle & \langle 0.6,0.3,0.3\rangle & \langle 0.4,0.3,0.4\rangle \\ \langle 0.5,0.2,0.3\rangle & \langle 0.7,0.2,0.2\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.6,0.3,0.2\rangle & \langle 0.6,0.4,0.3\rangle \\ \langle 0.6,0.4,0.3\rangle & \langle 0.6,0.2,0.2\rangle & \langle 0.5,0.3,0.2\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.6,0.3,0.4\rangle & \langle 0.6,0.3,0.3\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.6,0.3,0.3\rangle \end{array} \right) \quad (17)$$

Step 3: Determine the ideal solution

Here, C_3 and C_4 denote benefit attributes and C_1, C_2 and C_5 denote cost attributes. Using Equations (11) and (12), the ideal solutions are calculated as follows:

$$P^{*} = \{\langle 0.5,0.3,0.4\rangle, \langle 0.5,0.3,0.4\rangle, \langle 0.7,0.2,0.2\rangle, \langle 0.6,0.2,0.2\rangle, \langle 0.4,0.5,0.5\rangle\}.$$

Step 4: Determine the attribute weights

Using Equation (8), we calculate the criteria weights as follows:
$[w_1, w_2, w_3, w_4, w_5]$ = [0.2092, 0.1592, 0.2665, 0.2378, 0.1273]

Step 5: Determine the weighted hybrid logarithm similarity measures

Using Equation (6), we calculate similarity values for alternatives shown in Table 1.

Step 6: Ranking the alternatives

Ranking order of alternatives is prepared as the descending order of similarity values. Highest value indicates the best alternative. Ranking results are shown in Table 1 for different values of λ.

Step 7. End.

7. Sensitivity analysis

In this section, we discuss the variation of ranking results for different values of λ. From the results shown in Tables 1, we observe that the proposed strategy provides the same ranking order for different values of λ. However, the ranking order for different values of λ changes.

Table 1 Ranking order for different values of λ.

Similarity measures	(λ)	Measure values	Ranking order
$L_{wHyb}(P^*, P_i)$	0.1	$L_{wHyb}(P^*, P_1) = 0.9599$; $L_{wHyb}(P^*, P_2) = 0.9210$; $L_{wHyb}(P^*, P_3) = 0.8889$; $L_{wHyb}(P^*, P_4) = 0.8899$; $L_{wHyb}(P^*, P_5) = 0.8998$	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$
$L_{wHyb}(P^*, P_i)$	0.25	$L_{wHyb}(P^*, P_1) = 0.9623$; $L_{wHyb}(P^*, P_2) = 0.9258$; $L_{wHyb}(P^*, P_3) = 0.8949$; $L_{wHyb}(P^*, P_4) = 0.8961$; $L_{wHyb}(P^*, P_5) = 0.9043$	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$
$L_{wHyb}(P^*, P_i)$	0.40	$L_{wHyb}(P^*, P_1) = 0.9671$; $L_{wHyb}(P^*, P_2) = 0.9355$; $L_{wHyb}(P^*, P_3) = 0.9069$; $L_{wHyb}(P^*, P_4) = 0.9073$; $L_{wHyb}(P^*, P_5) = 0.9133$	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$
$L_{wHyb}(P^*, P_i)$	0.55	$L_{wHyb}(P^*, P_1) = 0.9707$; $L_{wHyb}(P^*, P_2) = 0.9428$; $L_{wHyb}(P^*, P_3) = 0.9158$; $L_{wHyb}(P^*, P_4) = 0.9221$; $L_{wHyb}(P^*, P_5) = 0.9241$	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$
$L_{wHyb}(P^*, P_i)$	0.70	$L_{wHyb}(P^*, P_1) = 0.9743$; $L_{wHyb}(P^*, P_2) = 0.9501$; $L_{wHyb}(P^*, P_3) = 0.9248$; $L_{wHyb}(P^*, P_4) = 0.9257$; $L_{wHyb}(P^*, P_5) = 0.9268$	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$
$L_{wHyb}(P^*, P_i)$	0.90	$L_{wHyb}(P^*, P_1) = 0.9791$; $L_{wHyb}(P^*, P_2) = 0.9598$; $L_{wHyb}(P^*, P_3) = 0.9368$; $L_{wHyb}(P^*, P_4) = 0.9379$; $L_{wHyb}(P^*, P_5) = 0.9398$	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$

8. Comparative analysis

In this section, we solve the problem with different existing strategies [23, 25, 26, 38]. Outcomes are furnished in the Table 2 and Figure 2.

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Table 2 Ranking order for different existing strategies

Similarity measures	Measure values of P_1, P_2, P_3, P_4, and P_5	Ranking order
Mondal and Pramanik [25]	0.8922, 0.8549, 0.7921, 0.8137, 0.8001	$P_1 \succ P_2 \succ P_4 \succ P_5 \succ P_3$
Ye [23]	0.8401, 0.8239, 0.7436, 0.8007, 0.7792	$P_1 \succ P_2 \succ P_4 \succ P_5 \succ P_3$
Biswas et al. [38] (λ=0.55)	0.9659, 0.9511, 0.9089, 0.9111, 0.9201	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$
Ye and Fu [26]	0.9761, 0.9158, 0.7321, 0.7756, 0.8417	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$
Proposed strategy (λ=0.55)	0.9707, 0.9428, 0.9158, 0.9221, 0.9241	$P_1 \succ P_2 \succ P_5 \succ P_4 \succ P_3$

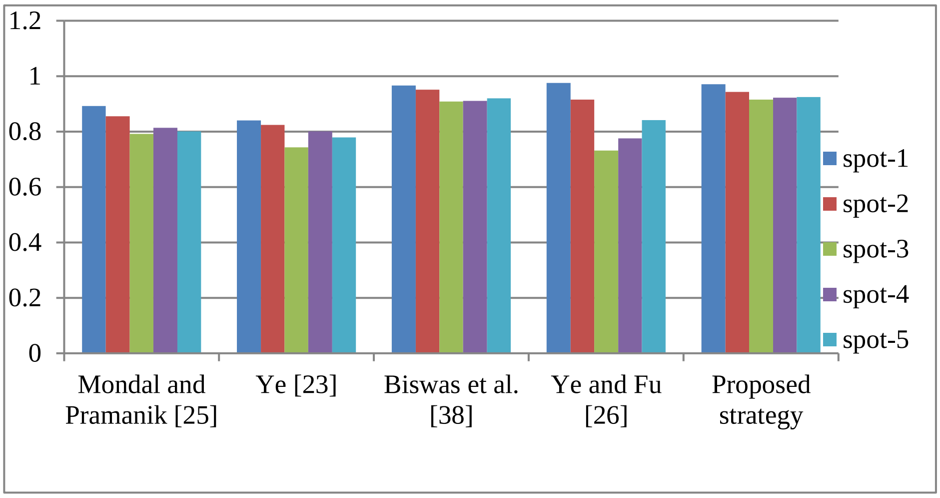

Figure 2 Ranking order of different strategies

9. Conclusions

The conclusions of the paper are concisely presented as follows:

- We have proposed hybrid logarithm similarity measure and weighted hybrid logarithm similarity measure for dealing uncertainty in decision making situation.
- We have defined the logarithm entropy function to determine unknown attribute weights.
- We have developed a new MAGDM strategy based on the proposed weighted hybrid logarithm similarity measure strategy.
- We have presented a numerical example to illustrate the proposed strategy.
- We have conducted a sensitivity analysis
- We have presented comparative analyses between the obtained results from the proposed strategies and different existing strategies in the literature.
- The proposed weighted hybrid logarithm similarity measure strategy can be applied to solve MADM and MAGDM problems in fault diagnosis [12], logistics center selection [39], Weaver selection [40], teacher selection [41], brick selection [42], renewable energy selection [43], etc.
- Future research can be continued to investigate the proposed similarity measures in neutrosophic hybrid environment for tackling uncertainty, inconsistency and indeterminacy in decision making situation.

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