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NC-Cross Entropy Based MADM under Neutrosophic Cubic Set Environment

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Abstract: Neutrosophic cubic set (NCS) is one of the important family members of neutrosophic hybrid sets. Neutrosophic cubic set has more strength than other family members of neutrosophic hybrid sets to express incomplete information due to the presence of interval valued neutrosophic set (IVNS) and single valued neutrosophic set (SVNS) in its structure. Cross entropy measure is one of the best way to calculate the divergence of any variable from the priori one variable. In this paper we first define a new cross entropy measure under NCSs environment which we call NC- cross entropy measure. We investigate the basic properties of NC-cross entropy. We also propose weighted NC-cross entropy and investigate its basic properties. We develop a novel multi attribute decision making (MADM) strategy based on weighted NC-cross entropy. To show the feasibility and applicability, we solve a MADM problem using the proposed strategy.

Keywords: single valued neutrosophic set; interval neutrosophic set; neutrosophic cubic set; multi attribute decision making; NC-cross entropy

1. Introduction

In 1998, Smarandache [1] introduced neutrosophic set (NS) by considering membership (truth), indeterminacy, non-membership (falsity) functions as independent component to uncertain, inconsistent and incomplete information. In 2010, Wang *et al*. [2] defined single valued neutrosophic set (SVNS), a subclass of neutrosophic set to deal with real and scientific and engineering applications. Some theoretical and applications of NS and SVNS are found in [3-21]. In 2005 Wang *et al*. [22] introduced the interval neutrosophic set (INS) taking membership function, non-membership function and indeterminacy function as independent functions which assume the values in the unit interval [0, 1]. Various important applications of INS are reported in the literature [23-28].
Ali *et al.* [29] proposed the concept of neutrosophic cubic set (NCS) by hybridizing NS and INS and defined external and internal neutrosophic cubic sets, and established some of their properties. In the same study, Ali *et al.* [29] proposed an adjustable strategy to NCS-based decision making. Jun *et al.* [30] also defined NCS by combining NS and INS.
Cross entropy measure is an important measure to calculate the divergence of any variable from prior one variable. In 1968, Zadeh [31] first proposed fuzzy entropy to calculate the divergence of two fuzzy

variables. Thereafter, Deluca and Termini [32] introduced some axiom of fuzzy entropy based on Shannon's function [33] to describe the fuzziness degree of fuzzy sets. Szmidt and Kacprzyk [34] proposed an entropy measure for IFSs by employing a geometric interpretation of IFS. Majumder and Samanta [35] introduced an entropy measure and similarity measure for SVNS to solve multi criteria decision making (MCDM) problems under SNVS environment. Further, Aydogdu [36] proposed an entropy measure for INS. Ye and Du [37] proposed some entropy measures for INS based on the distances as the extension of the entropy measures of interval valued IFS. Shang and Jiang [38] defined cross entropy in fuzzy environment between two fuzzy variables. Vlachos and Sergiadis [39] defined intuitionistic fuzzy cross-entropy and showed a mathematical connection between the notions of entropy for fuzzy sets and IFSs in terms of fuzziness and intuitionism. Ye [40] applied intuitionistic fuzzy cross entropy to MCDM. Maheshwari and Srivastava [41] proposed new cross entropy in intuitionistic fuzzy environment and applied it to decision making for medical diagnosis. Zhang et al. [42] defined entropy and cross entropy measure for interval-intuitionistic sets and discuss their properties. Ye [43] proposed an interval intuitionistic fuzzy cross entropy measure and applied to MCDM problem.

Ye [44] defined cross entropy for SVNSs by extending intuitionistic fuzzy cross entropy and applied it to MCDM problems. Due to some drawbacks of cross entropy proposed by Ye [44], Ye [45] proposed an improved single valued neutrosophic cross entropy and extended it to INS environment and applied it to MCDM problems. Thereafter, Tian et al. [46] proposed a cross entropy for INS environment and applied it to MCDM problem. Sahin [47] proposed an interval neutrosophic cross entropy measure based on fuzzy cross entropy and single valued neutrosophic cross entropy measures and applied it to MCDM problem. Recently, Pramanik et al. [48] proposed a new cross entropy namely, NS-cross entropy in SVNS environment and proved its basic properties. In the same study, Pramanik et al. [48] also proposed weighted NS-cross entropy and employed it to multi attribute group decision making problem (MAGDM). Further, Dalapati et al. [49] extended NS-cross entropy in INS environment and employed it for solving MADM problem. Pramanik et al. [50] developed two new MADM strategies based on cross entropy measures under bipolar and interval bipolar neutrosophic set environment.

In NCS environment, very few studies on the operations and applications of NCS [51-58] have been reported in the literature.

Research gap:

This study answers the following research questions:

i. Is it possible to introduce a cross entropy measure under NCS environment?
ii. Is it possible to introduce a weighted cross entropy measure under NCS environment?
iii. Is it possible to develop a novel MADM strategy based on the proposed cross entropy measure under NCS environment?
iv. Is it possible to develop a novel MAGDM strategy based on the proposed weighted cross entropy measure under NCS environment?

Motivation:

The above-mentioned studies [51-58] reveal that cross entropy measure is yet to appear under NCS environment. The studies in [44-49] motivate us to propose a comprehensive NC-cross entropy-based strategy for tackling MADM under the NCS environment. This study develops a novel NC-cross entropy-based MADM strategy.

The objectives of the paper are:

i. To introduce a cross entropy measure and prove its basic properties under NCS environment.
ii. To introduce a weighted cross measure and prove its basic properties under NCS environment.
iii. To develop a novel MADM strategy based on weighted NC-cross entropy measure under NCS environment.

To fill the research gap, we propose NC-cross entropy-based MADM.

The main contributions of this paper are summarized below:

i. We introduce a NC-cross entropy measure and prove its basic properties under NCS environment.
ii. We introduce a weighted NC-cross entropy measure and prove its basic properties under NCS environment.
iii. We develop a novel MADM strategy based on weighted NC- cross entropy to solve MADM problems.
iv. We solved a MADM problem using proposed strategy under NCS environment.

Rest of the paper is organized as follows. In section 2, we describe the basic definitions and operation of SVNS, IVNS, NCS. In section 3, we propose a NC-cross entropy measure and a weighted NC-cross entropy measure and establish their basic properties. Section 4 devotes to develop MADM strategy using NC-cross entropy. In section 5 an illustrative numerical example is provided to demonstrate the applicability and validity of proposed strategy under NCS environment. Section 6 offers conclusions and future direction of this research.

2. Preliminaries

In this section, some basic concepts and definitions of SVNS, INS and NCS are presented which will be utilized to develop the paper.

Definition 1. SVNS

Assume that U be a space of points (objects) with generic elements u ∊ U. A SVNS [2] H in U is characterized by a truth-membership function $T_H(u)$, an indeterminacy-membership function $I_H(u)$, and a falsity-membership function $F_H(u)$, where $T_H(u)$, $I_H(u)$, $F_H(u)$ ∈ [0, 1] for each point u in U. Therefore, a SVNS A can be expressed as H = {u, T_H (u), I_H (u), F_H (u) ǀ u∈ U}, whereas, the sum of $T_H(u)$, $I_H(u)$ and $F_H(u)$ satisfies the condition $0 \leq T_H(u) + I_H(u) + F_H(u) \leq 3$.

The order triplet < T, I, F > is called single valued neutrosophic number (SVNN).

Example 1.

Let H be any SVNS in U, then H can be expressed as: H = {u, (.7, .3, .5) ǀ u∈ U} and SVNN presented H = < 0.7, 0.3, 0.5>.

Definition 2. Inclusion of SVNS

The inclusion of any two SVNSs [2] H_1 and H_2 in U is denoted by $H_1 \subseteq H_2$ and defined as follows:

$H_1 \subseteq H_2$, iff $T_{H_1}(u) \leq T_{H_2}(u), I_{H_1}(u) \geq I_{H_2}(u), F_{H_1}(u) \geq F_{H_2}(u)$ for all u ∈ U.

Example 2.

Let H_1 and H_2 be any two SVNNs in U presented as follows: H_1 = < (0.7, 0.3, 0.5)> and H_2 = < (0.8, 0.2, 0.4)> for all u ∈ U. Using the property of inclusion of two SVNNs, we conclude that $H_1 \subseteq H_2$.

Definition 3. Equality of two SVNS

The equality of any two SVNSs [2] H_1 and H_2 in U denoted by $H_1 = H_2$ and is defined as follows:

$T_{H_1}(u) = T_{H_2}(u), I_{H_1}(u) = I_{H_2}(u)$ and $F_{H_1}(u) = F_{H_2}(u)$ for all u ∈ U.

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Definition 4. Complement of any SVNS

The complement of any SVNS [2] H in U denoted by H^c and defined as follows:

$H^c = \{u, 1-T_H, 1-I_H, 1-F_H \mid u \in U\}$.

Example 3.

Let H be any SVNN in U presented as follows:

H = < (0.7, 0.3, 0.5)>. Then compliment of H is obtained as H^c = < (0.3, 0.7, 0.5) >.

Definition 5. Union of two SVNSs

The union of two SVNSs [2] H_1 and H_2 is a neutrosophic set H_3 (say) written as $H_3 = H_1 \cup H_2$.

$$T_{H_3}(u) = \max\{T_{H_1}(u),\ T_{H_2}(u)\},\ I_{H_3}(u) = \min\{I_{H_1}(u),\ I_{H_2}(u)\},\ F_{H_3}(u) = \min\{F_{H_1}(u),\ F_{H_2}(u)\},\ \forall u \in U.$$

Example 4. Let H_1 and H_2 be two SVNNs in U presented as follows:

H_1 = <(0.6, 0.3, 0.4)> and H_2 = <(0.7, 0.3, 0.6)>. Then union of them is present as follows:

$H_1 \cup H_2$ = <(0.7, 0.3, 0.4)>.

Definition 6. Intersection of any two SVNSs

The intersection of two SVNSs [2] H_1 and H_2 denoted by H_4 and defined as

$H_4 = H_1 \cap H_2$ defined by $T_{H_4}(u) = \min\{T_{H_1}(u),\ T_{H_2}(u)\},\ I_{H_4}(u) = \max\{I_{H_1}(u),\ I_{H_2}(u)\}$

$F_{H_4}(u) = \max\{F_{H_1}(u),\ F_{H_2}(u)\},\ \forall u \in U.$

Example 5. Let H_1 and H_2 be any two SVNNs in U presented as follows:

H_1 = <(0.6, 0.3, 0.4)> and H_2 = <(0.7, 0.3, 0.6)>. Then intersection of H_1 and H_2 is presented as follows:

$H_1 \cap H_2$ = <(0.6, 0.3, 0.6)>

Definition 7. Some operation of single valued neutrosophic set

Let H_1 and H_2 be any two SVNSs [2]. Then, addition and multiplication are defined as:

1. $H_1 \oplus H_2 = < T_{H_1}(u) + T_{H_2}(u) - T_{H_1}(u) \cdot T_{H_2}(u),\ I_{H_1}(u) \cdot I_{H_2}(u),\ F_{H_1}(u) \cdot F_{H_2}(u) > \forall u \in U.$
2. $H_1 \otimes H_2 = < T_{H_1}(u) \cdot T_{H_2}(u),\ I_{H_1}(u) + I_{H_2}(u) - I_{H_1}(u) \cdot I_{H_2}(u),\ F_{H_1}(u) + F_{H_2}(u) - F_{H_1}(u) \cdot F_{H_2}(u) > \forall$ $u \in U.$

Example 6. Let H_1 and H_2 be any two SVNSs in U presented as follows:

H_1 = <0.6, 0.3, 0.4> and H_2 = <0.7, 0. 3, 0. 6>.

Then,

1. $H_1 \oplus H_2$ = <0.88, 0.09, 0.24>
2. $H_1 \otimes H_2$ = <0.42, 0.51, 0.76>.

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Definition 8. INSs

Assume that U be a space of points (objects) with generic element u ∈ U. An INS [22] J in U is characterized by a truth-membership function $T_J(u)$, an indeterminacy-membership function $I_J(u)$, and a falsity-membership function $F_J(u)$, where $T_J(u)=[T_J^-(u),T_J^+(u)]$, $I_J(u)=[I_J^-(u),I_J^+(u)]$, $F_J(u)=[F_J^-(u),F_J^+(u)]$ for each point u in U. Therefore, a INSs J can be expressed as J = {u, $[T_J^-(u),T_J^+(u)]$, $[I_J^-(u),I_J^+(u)]$, $[F_J^-(u),F_J^+(u)]$ | u∈ U}, where, $T_J^-(u),T_J^+(u)$, $I_J^-(u),I_J^+(u)$, $F_J^-(u),F_J^+(u) \subseteq [0,1]$.

Definition 9. Inclusion of two INSs

Let $J_1=\{u, [T_{J_1}^-(u),T_{J_1}^+(u)], [I_{J_1}^-(u),I_{J_1}^+(u)], [F_{J_1}^-(u),F_{J_1}^+(u)] \mid u\in U\}$ and $J_2=\{u, [T_{J_2}^-(u),T_{J_2}^+(u)], [I_{J_2}^-(u),I_{J_2}^+(u)], [F_{J_2}^-(u),F_{J_2}^+(u)] \mid u\in U\}$ be any two INSs [22] in U, then $J_1 \subseteq J_2$ iff $T_{J_1}^-(u) \le T_{J_2}^-(u)$, $T_{J_1}^+(u) \le T_{J_2}^+(u)$, $I_{J_1}^-(u) \ge I_{J_2}^-(u)$, $I_{J_1}^+(u) \ge I_{J_2}^+(u)$, $F_{J_1}^-(u) \ge F_{J_2}^-(u)$, $F_{J_1}^+(u) \ge F_{J_2}^+(u)$ for all u∈ U.

Definition 10. Complement of an INS

The complement J^c of an INS [22] J = {u, $[T_J^-(u),T_J^+(u)]$, $[I_J^-(u),I_J^+(u)]$, $[F_J^-(u),F_J^+(u)]$ | u∈ U} is defined as follows:

$J^c = \{u, [1-T_J^+(u), 1-T_J^-(u)], [1-I_J^+(u), 1-I_J^-(u)], [1-F_J^+(u), 1-F_J^-(u)] \mid u\in U\}$.

Definition 11. Equality of two INSs

Let $J_1=\{u, [T_{J_1}^-(u),T_{J_1}^+(u)], [I_{J_1}^-(u),I_{J_1}^+(u)], [F_{J_1}^-(u),F_{J_1}^+(u)] \mid u\in U\}$ and $J_2=\{u, [T_{J_2}^-(u),T_{J_2}^+(u)], [I_{J_2}^-(u),I_{J_2}^+(u)], [F_{J_2}^-(u),F_{J_2}^+(u)] \mid u\in U\}$ be any two INSs [22] in U, then $J_1 = J_2$ iff $T_{J_1}^-(u) = T_{J_2}^-(u)$, $T_{J_1}^+(u) = T_{J_2}^+(u)$, $I_{J_1}^-(u) = I_{J_2}^-(u)$, $I_{J_1}^+(u) = I_{J_2}^+(u)$, $F_{J_1}^-(u) = F_{J_2}^-(u)$, $F_{J_1}^+(u) = F_{J_2}^+(u)$ for all u∈ U.

Definition 12. Neutrosophic cubic set (NCS)

Assume that U be a space of points (objects) with generic elements u_i ∈ U. A NCS [29, 30] Q in U is a hybrid structure of INS and SVNS that can be expressed as follows:

Q = {u_i, < ($[T_Q^-(u_i),T_Q^+(u_i)]$, $[I_Q^-(u_i),I_Q^+(u_i)]$, $[F_Q^-(u_i),F_Q^+(u_i)]$), ($T_Q(u_i)$, $I_Q(u_i)$, $F_Q(u_i)$) > | u_i ∈ U}. Here, ($[T_Q^-(u_i),T_Q^+(u_i)]$, $[I_Q^-(u_i),I_Q^+(u_i)]$, $[F_Q^-(u_i),F_Q^+(u_i)]$) and ($T_Q(u_i)$, $I_Q(u_i)$, $F_Q(u_i)$) are INSs and SVNSs respectively in U. NCS can be simply presented as < ($[T_Q^-(u),T_Q^+(u)]$, $[I_Q^-(u),I_Q^+(u)]$, $[F_Q^-(u),F_Q^+(u)]$), ($T_Q(u)$, $I_Q(u)$, $F_Q(u)$) >, we call it as a neutrosophic cubic number (NCN).

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Definition 13. Inclusion of two NCSs

Let $Q_1 = \{u_i, ([T^-_{Q_1}(u_i), T^+_{Q_1}(u_i)], [I^-_{Q_1}(u_i), I^+_{Q_1}(u_i)], [F^-_{Q_1}(u_i), F^+_{Q_1}(u_i)]), (T_{Q_1}(u_i), I_{Q_1}(u_i), F_{Q_1}(u_i)) \mid u_i \in U\}$ and $Q_2 = \{u_i, [T^-_{Q_2}(u_i), T^+_{Q_2}(u_i)], [I^-_{Q_2}(u_i), I^+_{Q_2}(u_i)], [F^-_{Q_2}(u_i), F^+_{Q_2}(u_i)]), (T_{Q_2}(u_i), I_{Q_2}(u_i), F_{Q_2}(u_i)) \mid u_i \in U\}$ be any two NCSs [29, 30] in U. Then $Q_1 \subseteq Q_2$ iff $T^-_{Q_1}(u_i) \le T^-_{Q_2}(u_i)$, $T^+_{Q_1}(u_i) \le T^+_{Q_2}(u_i)$, $I^-_{Q_1}(u_i) \ge I^-_{Q_2}(u_i)$, $I^+_{Q_1}(u_i) \ge I^+_{Q_2}(u_i)$, $F^-_{Q_1}(u_i) \ge F^-_{Q_2}(u_i)$, $F^+_{Q_1}(u_i) \ge F^+_{Q_2}(u_i)$ and $T_{Q_1}(u_i) \le T_{Q_2}(u_i), I_{Q_1}(u_i) \ge I_{Q_2}(u_i), F_{Q_1}(u_i) \ge F_{Q_2}(u_i)$ for all $u_i \in U$.

Definition 14. Equality of two NCSs

Let $Q_1 = \{u_i, ([T^-_{Q_1}(u_i), T^+_{Q_1}(u_i)], [I^-_{Q_1}(u_i), I^+_{Q_1}(u_i)], [F^-_{Q_1}(u_i), F^+_{Q_1}(u_i)]), (T_{Q_1}(u_i), I_{Q_1}(u_i), F_{Q_1}(u_i)) \mid u_i \in U\}$ and $Q_2 = \{u_i, ([T^-_{Q_2}(u_i), T^+_{Q_2}(u_i)], [I^-_{Q_2}(u_i), I^+_{Q_2}(u_i)], [F^-_{Q_2}(u_i), F^+_{Q_2}(u_i)]), (T_{Q_2}(u_i), I_{Q_2}(u_i), F_{Q_2}(u_i)) \mid u_i \in U\}$ be any two NCSs [29, 30] in U. Then $Q_1 = Q_2$ iff $T^-_{Q_1}(u_i) = T^-_{Q_2}(u_i)$, $T^+_{Q_1}(u_i) = T^+_{Q_2}(u_i)$, $I^-_{Q_1}(u_i) = I^-_{Q_2}(u_i)$, $I^+_{Q_1}(u_i) = I^+_{Q_2}(u_i)$, $F^-_{Q_1}(u_i) = F^-_{Q_2}(u_i)$, $F^+_{Q_1}(u_i) = F^+_{Q_2}(u_i)$ and $T_{Q_1}(u_i) = T_{Q_2}(u_i), I_{Q_1}(u_i) = I_{Q_2}(u_i), F_{Q_1}(u_i) = F_{Q_2}(u_i)$ for all $u_i \in U$.

Definition 15. Complement of a NCS

Let $Q = \{u_i, < ([T^-_Q(u_i), T^+_Q(u_i)], [I^-_Q(u_i), I^+_Q(u_i)], [F^-_Q(u_i), F^+_Q(u_i)]), (T_Q(u_i), I_Q(u_i), F_Q(u_i)) > \mid u_i \in U\}$ be any NCS [29, 30] in U. Then complement Q^c of Q is defined as follows:

$$Q^c = \{u_i, ([1 - T^+_Q(u_i), 1 - T^-_Q(u_i)], [1 - I^+_Q(u_i), 1 - I^-_Q(u_i)], [1 - F^+_Q(u_i), 1 - F^-_Q(u_i)]), (1 - T_Q(u_i), 1 - I_Q(u_i), 1 - F_Q(u_i)) > \mid u_i \in U\}.$$

3. NC-Cross-entropy measure under NCS environment

Definition 16. NC- cross entropy measure

Let Q_1 and Q_2 be any two NCSs in $U = \{u_1, u_2, u_3, \ldots, u_n\}$. Then, neutrosophic cubic cross-entropy measure of Q_1 and Q_2 is denoted by $CE_{NC}(Q_1, Q_2)$ and defined as follows:

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$$
\begin{aligned}
CE_{NC}(Q_1,Q_2)=\frac{1}{8}\Bigg\{\sum_{i=1}^{n}\Bigg(\Bigg[&\frac{2\left|T^-_{Q1}(u_i)-T^-_{Q2}(u_i)\right|}{\sqrt{1+\left|T^-_{Q1}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-T^-_{Q_1}(u_i))-(1-T^-_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-T^-_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^-_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|T^+_{Q1}(u_i)-T^+_{Q2}(u_i)\right|}{\sqrt{1+\left|T^+_{Q1}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-T^+_{Q_1}(u_i))-(1-T^+_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-T^+_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^+_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|I^-_{Q1}(u_i)-I^-_{Q2}(u_i)\right|}{\sqrt{1+\left|I^-_{Q1}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-I^-_{Q_1}(u_i))-(1-I^-_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-I^-_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^-_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|I^+_{Q1}(u_i)-I^+_{Q2}(u_i)\right|}{\sqrt{1+\left|I^+_{Q1}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-I^+_{Q_1}(u_i))-(1-I^+_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-I^+_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^+_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|F^-_{Q1}(u_i)-F^-_{Q2}(u_i)\right|}{\sqrt{1+\left|F^-_{Q1}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-F^-_{Q_1}(u_i))-(1-F^-_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-F^-_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^-_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|F^+_{Q1}(u_i)-F^+_{Q2}(u_i)\right|}{\sqrt{1+\left|F^+_{Q1}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-F^+_{Q_1}(u_i))-(1-F^+_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-F^+_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^+_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|T_{Q1}(u_i)-T_{Q2}(u_i)\right|}{\sqrt{1+\left|T_{Q1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}\Bigg]+\\
\Bigg[&\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}\Bigg]\Bigg)\Bigg\}
\end{aligned}
\qquad (1)
$$

Theorem 1.

Let Q_1, Q_2 be any two NCSs in U. The NC-cross entropy measure $CE_{NC}(Q_1,Q_2)$ satisfies the following properties:

i) $CE_{NC}(Q_1,Q_2)\geq 0$.

ii) $CE_{NC}(Q_1,Q_2)=0$ iff $T^-_{Q1}(u_i)=T^-_{Q2}(u_i)$, $T^+_{Q1}(u_i)=T^+_{Q2}(u_i)$, $I^-_{Q1}(u_i)=I^-_{Q2}(u_i)$, $I^+_{Q1}(u_i)=I^+_{Q2}(u_i)$, $F^-_{Q1}(u_i)=F^-_{Q2}(u_i)$,

$F^+_{Q1}(u_i)=F^+_{Q2}(u_i)$ and $T_{Q1}(u_i)=T_{Q2}(u_i), I_{Q1}(u_i)=I_{Q2}(u_i), F_{Q1}(u_i)=F_{Q2}(u_i)$ for all $\forall u_i \in U$.

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iii) $CE_{NC}(Q_1, Q_2) = CE_{NC}(Q_1^c, Q_2^c)$

iv) $CE_{NC}(Q_1, Q_2) = CE_{NC}(Q_2, Q_1)$

Proof: i)

For all values of $u_i \in U$, $\left|T_{Q1}(u_i)\right| \geq 0$, $\left|T_{Q2}(u_i)\right| \geq 0$, $\left|T_{Q1}(u_i) - T_{Q2}(u_i)\right| \geq 0$, $\sqrt{1+\left|T_{Q1}(u_i)\right|^2} \geq 0$, $\sqrt{1+\left|T_{Q2}(u_i)\right|^2} \geq 0$,

$\left|(1-T_{Q_1}(u_i))\right| \geq 0$, $\left|(1-T_{Q2}(u_i))\right| \geq 0$, $\left|(1-T_{Q_1}(u_i)) - (1-T_{Q2}(u_i))\right| \geq 0$, $\sqrt{1+\left|(1-T_{Q1}(u_i))\right|^2} \geq 0$, $\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2} \geq 0$,

Then, $$\left[\frac{2\left|T_{Q1}(u_i)-T_{Q2}(u_i)\right|}{\sqrt{1+\left|T_{Q1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}\right] \geq 0 \quad (2)$$

Similarly, $$\left[\frac{2\left|I_{Q1}(u_i)-I_{Q2}(u_i)\right|}{\sqrt{1+\left|I_{Q1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q2}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}\right] \geq 0 \quad (3)$$

and

$$\left[\frac{2\left|F_{Q1}(u_i)-F_{Q2}(u_i)\right|}{\sqrt{1+\left|F_{Q1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}\right] \geq 0 \quad (4)$$

Again,

For all values of $u_i \in U$, $\left|T^-_{Q1}(u_i)\right| \geq 0$, $\left|T^-_{Q2}(u_i)\right| \geq 0$, $\left|T^-_{Q1}(u_i) - T^-_{Q2}(u_i)\right| \geq 0$, $\sqrt{1+\left|T^-_{Q1}(u_i)\right|^2} \geq 0$, $\sqrt{1+\left|T^-_{Q2}(u_i)\right|^2} \geq 0$,

$\left|(1-T^-_{Q_1}(u_i))\right| \geq 0$, $\left|(1-T^-_{Q2}(u_i))\right| \geq 0$, $\left|(1-T^-_{Q_1}(u_i)) - (1-T^-_{Q2}(u_i))\right| \geq 0$, $\sqrt{1+\left|(1-T^-_{Q_1}(u_i))\right|^2} \geq 0$, $\sqrt{1+\left|(1-T^-_{Q2}(u_i))\right|^2} \geq 0$

$$\Rightarrow \left[\frac{2\left|T^-_{Q1}(u_i)-T^-_{Q2}(u_i)\right|}{\sqrt{1+\left|T^-_{Q1}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-T^-_{Q_1}(u_i))-(1-T^-_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-T^-_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^-_{Q_2}(u_i))\right|^2}}\right] \geq 0 \quad (5)$$

and $\left|T^+_{Q1}(u_i)\right| \geq 0$, $\left|T^+_{Q2}(u_i)\right| \geq 0$, $\left|T^+_{Q1}(u_i) - T^+_{Q2}(u_i)\right| \geq 0$, $\sqrt{1+\left|T^+_{Q1}(u_i)\right|^2} \geq 0$, $\sqrt{1+\left|T^+_{Q2}(u_i)\right|^2} \geq 0$,

$\left|(1-T^+_{Q_1}(u_i))\right| \geq 0$, $\left|(1-T^+_{Q2}(u_i))\right| \geq 0$, $\left|(1-T^+_{Q_1}(u_i)) - (1-T^+_{Q2}(u_i))\right| \geq 0$, $\sqrt{1+\left|(1-T^+_{Q_1}(u_i))\right|^2} \geq 0$, $\sqrt{1+\left|(1-T^+_{Q2}(u_i))\right|^2} \geq 0$

$$\Rightarrow \left[\frac{2\left|T^+_{Q1}(u_i)-T^+_{Q2}(u_i)\right|}{\sqrt{1+\left|T^+_{Q1}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-T^+_{Q_1}(u_i))-(1-T^+_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-T^+_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^+_{Q_2}(u_i))\right|^2}}\right] \geq 0 \quad (6)$$

Similarly, we can show that

$$\left[\frac{2\left|I^-_{Q1}(u_i)-I^-_{Q2}(u_i)\right|}{\sqrt{1+\left|I^-_{Q1}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q2}(u_i)\right|^2}}+\frac{2\left|(1-I^-_{Q_1}(u_i))-(1-I^-_{Q2}(u_i))\right|}{\sqrt{1+\left|(1-I^-_{Q1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^-_{Q_2}(u_i))\right|^2}}\right] \geq 0, \quad (7)$$

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$$\left[\frac{2\left|I^{+}_{Q_1}(u_i)-I^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^{+}_{Q_1}(u_i))-(1-I^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{+}_{Q_2}(u_i))\right|^2}}\right]\geq 0, \quad (8)$$

$$\left[\frac{2\left|F^{-}_{Q_1}(u_i)-F^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{-}_{Q_1}(u_i))-(1-F^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{-}_{Q_2}(u_i))\right|^2}}\right]\geq 0 \quad (9)$$

and

$$\left[\frac{2\left|F^{+}_{Q_1}(u_i)-F^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{+}_{Q_1}(u_i))-(1-F^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{+}_{Q_2}(u_i))\right|^2}}\right]\geq 0 \quad (10)$$

Adding Equation (2) to Equation (10), we obtain $CE_{NC}(Q_1, Q_2)\geq 0$.

ii).

$$\left[\frac{2\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}\right]=0,$$

$$\Leftrightarrow T_{Q_1}(u_i)=T_{Q_2}(u_i) \quad (11)$$

$$\left[\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow I_{Q_1}(u_i)=I_{Q_2}(u_i) \quad (12)$$

$$\left[\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}\right]=0,$$

$\Leftrightarrow F_{Q_1}(u_i)=F_{Q_2}(u_i)$, For all values of $u_i \in U$. (13)

Again,

$$\left[\frac{2\left|T^{-}_{Q_1}(u_i)-T^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T^{-}_{Q_1}(u_i))-(1-T^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{-}_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow T^{-}_{Q_1}(u_i)=T^{-}_{Q_2}(u_i) \quad (14)$$

$$\left[\frac{2\left|T^{+}_{Q_1}(u_i)-T^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T^{+}_{Q_1}(u_i))-(1-T^{+}_{JQ_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{+}_{Q_2}(u_i))\right|^2}}\right]=0$$

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$$\Leftrightarrow T^{+}_{Q_1}(u_i) = T^{+}_{Q_2}(u_i) \quad (15)$$

$$\left[\frac{2\left|I^{-}_{Q_1}(u_i) - I^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{-}_{Q_1}(u_i)\right|^2} + \sqrt{1+\left|I^{-}_{Q_2}(u_i)\right|^2}} + \frac{2\left\|(1-I^{-}_{Q_1}(u_i)) - (1-I^{-}_{Q_2}(u_i))\right\|}{\sqrt{1+\left|(1-I^{-}_{Q_1}(u_i))\right|^2} + \sqrt{1+\left|(1-I^{-}_{Q_2}(u_i))\right|^2}}\right] = 0$$

$$\Leftrightarrow I^{-}_{Q_1}(u_i) = I^{-}_{Q_2}(u_i) \quad (16)$$

$$\left[\frac{2\left|I^{+}_{Q_1}(u_i) - I^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{+}_{Q_1}(u_i)\right|^2} + \sqrt{1+\left|I^{+}_{Q_2}(u_i)\right|^2}} + \frac{2\left|(1-I^{+}_{Q_1}(u_i)) - (1-I^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{+}_{Q_1}(u_i))\right|^2} + \sqrt{1+\left|(1-I^{+}_{Q_2}(u_i))\right|^2}}\right] = 0$$

$$\Leftrightarrow I^{+}_{Q_1}(u_i) = I^{+}_{Q_2}(u_i) \quad (17)$$

$$\left[\frac{2\left|F^{-}_{Q_1}(u_i) - F^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{-}_{Q_1}(u_i)\right|^2} + \sqrt{1+\left|F^{-}_{Q_2}(u_i)\right|^2}} + \frac{2\left|(1-F^{-}_{Q_1}(u_i)) - (1-F^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{-}_{Q_1}(u_i))\right|^2} + \sqrt{1+\left|(1-F^{-}_{Q_2}(u_i))\right|^2}}\right] = 0$$

$$\Leftrightarrow F^{-}_{Q_1}(u_i) = F^{-}_{Q_2}(u_i) \quad (18)$$

$$\left[\frac{2\left|F^{+}_{Q_1}(u_i) - F^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{+}_{Q_1}(u_i)\right|^2} + \sqrt{1+\left|F^{+}_{Q_2}(u_i)\right|^2}} + \frac{2\left|(1-F^{+}_{Q_1}(u_i)) - (1-F^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i))\right|^2} + \sqrt{1+\left|(1-F^{+}_{Q_2}(u_i))\right|^2}}\right] = 0$$

$\Leftrightarrow F^{+}_{Q_1}(u_i) = F^{+}_{Q_2}(u_i)$, for all values of $u_i \in U$. (19)

From, Equation (11) to Equation (19), we obtain $CE_{NC}(Q_1, Q_2) = 0$ iff $T^{-}_{Q_1}(u_i) = T^{-}_{Q_2}(u_i)$, $T^{+}_{Q_1}(u_i) = T^{+}_{Q_2}(u_i)$, $I^{-}_{Q_1}(u_i) = I^{-}_{Q_2}(u_i)$, $I^{+}_{Q_1}(u_i) = I^{+}_{Q_2}(u_i)$, $F^{-}_{Q_1}(u_i) = F^{-}_{Q_2}(u_i)$, $F^{+}_{Q_1}(u_i) = F^{+}_{Q_2}(u_i)$ and $T_{Q_1}(u_i) = T_{Q_2}(u_i), I_{Q_1}(u_i) = I_{Q_2}(u_i), F_{Q_1}(u_i) = F_{Q_2}(u_i)$ for all $\forall u_i \in U$.

iii).

Using Definition (1), Definition (4) and Definition (10), we obtain the following expression:

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$$
CE_{NC}(Q_1^c, Q_2^c) = \frac{1}{8}\left\{\sum_{i=1}^{n}\left(\left[\frac{2\left|T^-_{Q_1^c}(u_i)-T^-_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|T^-_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-T^-_{Q_1^c}(u_i))-(1-T^-_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-T^-_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-T^-_{Q_2^c}(u_i))\right|^2}}\right]+\right.\right.
$$

$$
\left[\frac{2\left|T^+_{Q_1^c}(u_i)-T^+_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|T^+_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-T^+_{Q_1^c}(u_i))-(1-T^+_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-T^+_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-T^+_{Q_2^c}(u_i))\right|^2}}\right]+
$$

$$
\left[\frac{2\left|I^-_{Q_1^c}(u_i)-I^-_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|I^-_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-I^-_{Q_1^c}(u_i))-(1-I^-_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-I^-_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-I^-_{Q_2^c}(u_i))\right|^2}}\right]+
$$

$$
\left[\frac{2\left|I^+_{Q_1^c}(u_i)-I^+_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|I^+_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-I^+_{Q_1^c}(u_i))-(1-I^+_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-I^+_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-I^+_{Q_2^c}(u_i))\right|^2}}\right]+
$$

$$
\left[\frac{2\left|F^-_{Q_1^c}(u_i)-F^-_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|F^-_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-F^-_{Q_1^c}(u_i))-(1-F^-_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-F^-_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-F^-_{Q_2^c}(u_i))\right|^2}}\right]+
$$

$$
\left[\frac{2\left|F^+_{Q_1^c}(u_i)-F^+_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|F^+_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-F^+_{Q_1^c}(u_i))-(1-F^+_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-F^+_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-F^+_{Q_2^c}(u_i))\right|^2}}\right]+
$$

$$
\left[\frac{2\left|T_{Q_1^c}(u_i)-T_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|T_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1^c}(u_i))-(1-T_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2^c}(u_i))\right|^2}}\right]+
$$

$$
\left[\frac{2\left|I_{Q_1^c}(u_i)-I_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|I_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2^c}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1^c}(u_i))-(1-I_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2^c}(u_i))\right|^2}}\right]+
$$

$$
\left.\left.\left[\frac{2\left|F_{Q_1^c}(u_i)-F_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|F_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1^c}(u_i))-(1-F_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2^c}(u_i))\right|^2}}\right]\right)\right\}
$$

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$$
=\frac{1}{8}\left\{\sum_{i=1}^{n}\left(\left[\frac{2\left|(1-T^{-}_{Q_1}(u_i))-(1-T^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{-}_{Q_2}(u_i))\right|^2}}+\frac{2\left|T^{-}_{Q_1}(u_i)-T^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{-}_{Q_2}(u_i)\right|^2}}\right]+\right.\right.
$$

$$
\left[\frac{2\left|(1-T^{+}_{Q_1}(u_i))-(1-T^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{+}_{Q_2}(u_i))\right|^2}}+\frac{2\left|T^{+}_{Q_1}(u_i)-T^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{+}_{Q_2}(u_i)\right|^2}}\right]+
$$

$$
\left[\frac{2\left|(1-I^{-}_{Q_1}(u_i))-(1-I^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{-}_{Q_2}(u_i))\right|^2}}+\frac{2\left|I^{-}_{Q_1}(u_i)-I^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{-}_{Q_2}(u_i)\right|^2}}\right]+
$$

$$
\left[\frac{2\left|(1-I^{+}_{Q_1}(u_i))-(1-I^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{+}_{Q_2}(u_i))\right|^2}}+\frac{2\left|I^{+}_{Q_1}(u_i)-I^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{+}_{Q_2}(u_i)\right|^2}}\right]+
$$

$$
\left[\frac{2\left|(1-F^{-}_{Q_1}(u_i))-(1-F^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{-}_{Q_2}(u_i))\right|^2}}+\frac{2\left|F^{-}_{Q_1}(u_i)-F^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{-}_{Q_2}(u_i)\right|^2}}\right]+
$$

$$
\left[\frac{2\left|(1-F^{+}_{Q_1}(u_i))-(1-F^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{+}_{Q_2}(u_i))\right|^2}}+\frac{2\left|F^{+}_{Q_1}(u_i)-F^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{+}_{Q_2}(u_i)\right|^2}}\right]+
$$

$$
\left[\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}+\frac{2\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}}\right]+
$$

$$
\left[\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}+\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}\right]+
\left.\left.\left[\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}+\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}\right]\right)\right\}
$$

$$
\begin{aligned}
&=\frac{1}{8}\left\{\sum_{i=1}^{n}\left(\left[\frac{2\left|T_{Q_1}^{-}(u_i)-T_{Q_2}^{-}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}^{-}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}^{-}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}^{-}(u_i))-(1-T_{Q_2}^{-}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}^{-}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}^{-}(u_i))\right|^2}}\right]+\right.\right.\\
&\left[\frac{2\left|T_{Q_1}^{+}(u_i)-T_{Q_2}^{+}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}^{+}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}^{+}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}^{+}(u_i))-(1-T_{Q_2}^{+}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}^{+}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}^{+}(u_i))\right|^2}}\right]+\\
&\left[\frac{2\left|I_{Q_1}^{-}(u_i)-I_{Q_2}^{-}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}^{-}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}^{-}(u_i)\right|^2}}+\frac{2\left|(1-I_{Q_1}^{-}(u_i))-(1-I_{Q_2}^{-}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}^{-}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}^{-}(u_i))\right|^2}}\right]+\\
&\left[\frac{2\left|I_{Q_1}^{+}(u_i)-I_{Q_2}^{+}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}^{+}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}^{+}(u_i)\right|^2}}+\frac{2\left|(1-I_{Q_1}^{+}(u_i))-(1-I_{Q_2}^{+}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}^{+}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}^{+}(u_i))\right|^2}}\right]+\\
&\left[\frac{2\left|F_{Q_1}^{-}(u_i)-F_{Q_2}^{-}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}^{-}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}^{-}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}^{-}(u_i))-(1-F_{Q_2}^{-}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}^{-}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}^{-}(u_i))\right|^2}}\right]+\\
&\left[\frac{2\left|F_{Q_1}^{+}(u_i)-F_{Q_2}^{+}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}^{+}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}^{+}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}^{+}(u_i))-(1-F_{Q_2}^{+}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}^{+}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}^{+}(u_i))\right|^2}}\right]+\\
&\left[\frac{2\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}\right]+\\
&\left[\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}\right]+\\
&\left.\left.\left[\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}\right]\right)\right\}=CE_{NC}(Q_1,Q_2)
\end{aligned}
$$

iv).

Since, $\forall u_i \in U$, for single valued part we obtain:

$$\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|=\left|T_{Q_2}(u_i)-T_{Q_1}(u_i)\right|,\ \left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|=\left|I_{Q_2}(u_i)-I_{Q_1}(u_i)\right|,\ \left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|=\left|F_{Q_2}(u_i)-F_{Q_1}(u_i)\right|,$$

$$\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|=\left|(1-T_{Q_2}(u_i))-(1-T_{Q_1}(u_i))\right|,\ \left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|=\left|(1-I_{Q_2}(u_i))-(1-I_{Q_1}(u_i))\right|,$$

$$\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|=\left|(1-F_{Q_2}(u_i))-(1-F_{Q_1}(u_i))\right|.$$

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Then

$$\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_1}(u_i)\right|^2},\ \sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|I_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}$$

$$,\ \sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}=\sqrt{1+\left|(-T_{Q_2}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2},$$

$$\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}=\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2},$$

$$\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}=\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2},\ \forall u_i\in U.$$

For interval neutrosophic part, we obtain,

$$\left|T^-_{Q_1}(u_i)-T^-_{Q_2}(u_i)\right|=\left|T^-_{Q_2}(u_i)-T^-_{Q_1}(u_i)\right|,\ \left|I^-_{Q_1}(u_i)-I^-_{Q_2}(u_i)\right|=\left|I^-_{Q_2}(u_i)-I^-_{Q_1}(u_i)\right|,\ \left|F^-_{Q_1}(u_i)-F^-_{Q_2}(u_i)\right|=\left|F^-_{Q_2}(u_i)-F^-_{Q_1}(u_i)\right|,$$

$$\left|(1-T^-_{Q_1}(u_i))-(1-T^-_{Q_2}(u_i))\right|=\left|(1-T^-_{Q_2}(u_i))-(1-T^-_{Q_1}(u_i))\right|,\quad \left|(1-I^-_{Q_1}(u_i))-(1-I^-_{Q_2}(u_i))\right|=\left|(1-I^-_{Q_2}(u_i))-(1-I^-_{Q_1}(u_i))\right|,$$

$$\left|(1-F^-_{Q_1}(u_i))-(1-F^-_{Q_2}(u_i))\right|=\left|(1-F^-_{Q_2}(u_i))-(1-F^-_{Q_1}(u_i))\right|.$$

Then, we obtain,

$$\sqrt{1+\left|T^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|T^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|I^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|I^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q_1}(u_i)\right|^2},\ \sqrt{1+\left|F^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|F^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-T^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-T^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-T^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-T^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-I^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-I^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-I^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-I^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-F^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-F^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-F^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-F^-_{Q_1}(u_i)\right|^2},\ \forall u_i\in U.$$

Similarly, $\left|T^+_{Q_1}(u_i)-T^+_{Q_2}(u_i)\right|=\left|T^+_{Q_2}(u_i)-T^+_{Q_1}(u_i)\right|$, $\left|I^+_{Q_1}(u_i)-I^+_{Q_2}(u_i)\right|=\left|I^+_{Q_2}(u_i)-I^+_{Q_1}(u_i)\right|$,

$$\left|F^+_{Q_1}(u_i)-F^+_{Q_2}(u_i)\right|=\left|F^+_{Q_2}(u_i)-F^+_{Q_1}(u_i)\right|,\ \left|(1-T^+_{Q_1}(u_i))-(1-T^+_{Q_2}(u_i))\right|=\left|(1-T^+_{Q_2}(u_i))-(1-T^+_{Q_1}(u_i))\right|,$$

$\left|(1-I^+_{Q_1}(u_i))-(1-I^+_{Q_2}(u_i))\right|=\left|(1-I^+_{Q_2}(u_i))-(1-I^+_{Q_1}(u_i))\right|$, $\left|(1-F^+_{Q_1}(u_i))-(1-F^+_{Q_2}(u_i))\right|=\left|(1-F^+_{Q_2}(u_i))-(1-F^+_{Q_1}(u_i))\right|$, then

$$\sqrt{1+\left|T^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|T^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|I^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|I^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q_1}(u_i)\right|^2},\ \sqrt{1+\left|F^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|F^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-T^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-T^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-T^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-T^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-I^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-I^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-I^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-I^+_{Q_1}(u_i)\right|^2},$$

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$$\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-F^{+}_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-F^{+}_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i)\right|^2},\ \forall u_i\in U.$$

So, $CE_{NC}(Q_1,Q_2)=CE_{NC}(Q_2,Q_1)$.

Definition 17. Weighted NC-cross-entropy measure

We consider the weight w_i (i = 1, 2, 3,…, n) of u_i (i = 1, 2, 3, …, n) with $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$.

Then neutrosophic cubic weighted cross entropy measure between Q_1 and Q_2 can be defined as

$$
\begin{aligned}
CE^{w}_{NC}(Q_1,Q_2)=\frac{1}{8}\Bigg\langle \sum_{i=1}^{n} w_i \Bigg\{ & \left[\frac{2\left|T^{-}_{Q_1}(u_i)-T^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T^{-}_{Q_1}(u_i))-(1-T^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{-}_{Q_2}(u_i))\right|^2}}\right]+ \\
& \left[\frac{2\left|T^{+}_{Q_1}(u_i)-T^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T^{+}_{Q_1}(u_i))-(1-T^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{+}_{Q_2}(u_i))\right|^2}}\right]+ \\
& \left[\frac{2\left|I^{-}_{Q_1}(u_i)-I^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^{-}_{Q_1}(u_i))-(1-I^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{-}_{Q_2}(u_i))\right|^2}}\right]+ \\
& \left[\frac{2\left|I^{+}_{Q_1}(u_i)-I^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^{+}_{Q_1}(u_i))-(1-I^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{+}_{Q_2}(u_i))\right|^2}}\right]+ \\
& \left[\frac{2\left|F^{-}_{Q_1}(u_i)-F^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{-}_{Q_1}(u_i))-(1-F^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{-}_{Q_2}(u_i))\right|^2}}\right] \\
& +\left[\frac{2\left|F^{+}_{Q_1}(u_i)-F^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{+}_{Q_1}(u_i))-(1-F^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{+}_{Q_2}(u_i))\right|^2}}\right]+ \\
& \left[\frac{2\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}\right]+ \\
& \left[\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}\right]+ \\
& \left[\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}\right]\Bigg\}\Bigg\rangle
\end{aligned}
\tag{20}
$$

Theorem 2.

Let Q_1, Q_2 be any two NCSs in U. Then weighted NC-cross entropy measure $CE^{w}_{NC}(Q_1,Q_2)$ satisfies the following properties:

i) $CE^{w}_{NC}(Q_1,Q_2)\geq 0$.

ii) $CE^{w}_{NC}(Q_1,Q_2)=0$ iff $T^{-}_{Q_1}(u_i)=T^{-}_{Q_2}(u_i)$, $T^{+}_{Q_1}(u_i)=T^{+}_{Q_2}(u_i)$, $I^{-}_{Q_1}(u_i)=I^{-}_{Q_2}(u_i)$, $I^{+}_{Q_1}(u_i)=I^{+}_{Q_2}(u_i)$, $F^{-}_{Q_1}(u_i)=F^{-}_{Q_2}(u_i)$, $F^{+}_{Q_1}(u_i)=F^{+}_{Q_2}(u_i)$ and $T_{Q_1}(u_i)=T_{Q_2}(u_i), I_{Q_1}(u_i)=I_{Q_2}(u_i), F_{Q_1}(u_i)=F_{Q_2}(u_i)$ for all $\forall u_i \in U$.

iii) $CE^{w}_{NC}(Q_1,Q_2)=CE^{w}_{NC}(Q_1^c,Q_2^c)$

iv) $CE^{w}_{NC}(Q_1,Q_2)=CE^{w}_{NC}(Q_2,Q_1)$

Proof:

i).

For all values of $u_i \in U$, $|T_{Q_1}(u_i)|\geq 0$, $|T_{Q_2}(u_i)|\geq 0$, $|T_{Q_1}(u_i)-T_{Q_2}(u_i)|\geq 0$, $\sqrt{1+|T_{Q_1}(u_i)|^2}\geq 0$, $\sqrt{1+|T_{Q_2}(u_i)|^2}\geq 0$, $|(1-T_{Q_1}(u_i))|\geq 0$, $|(1-T_{Q_2}(u_i))|\geq 0$, $|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))|\geq 0$, $\sqrt{1+|(1-T_{Q_1}(u_i))|^2}\geq 0$, $\sqrt{1+|(1-T_{Q_2}(u_i))|^2}\geq 0$,

Then $$\left[\frac{2|T_{Q_1}(u_i)-T_{Q_2}(u_i)|}{\sqrt{1+|T_{Q_1}(u_i)|^2}+\sqrt{1+|T_{Q_2}(u_i)|^2}}+\frac{2|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))|}{\sqrt{1+|(1-T_{Q_1}(u_i))|^2}+\sqrt{1+|(1-T_{Q_2}(u_i))|^2}}\right]\geq 0 \quad (21)$$

Similarly, $$\left[\frac{2|I_{Q_1}(u_i)-I_{Q_2}(u_i)|}{\sqrt{1+|I_{Q_1}(u_i)|^2}+\sqrt{1+|I_{Q_2}(u)|^2}}+\frac{2|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))|}{\sqrt{1+|(1-I_{Q}(u_i))|^2}+\sqrt{1+|(1-I_{Q_2}(u_i))|^2}}\right]\geq 0 \quad (22)$$

and

$$\left[\frac{2|F_{Q_1}(u_i)-F_{Q_2}(u_i)|}{\sqrt{1+|F_{Q_1}(u_i)|^2}+\sqrt{1+|F_{Q_2}(u_i)|^2}}+\frac{2|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))|}{\sqrt{1+|(1-F_{Q_1}(u_i))|^2}+\sqrt{1+|(1-F_{Q_2}(u_i))|^2}}\right]\geq 0 \quad (23)$$

Again,

For all values of $u_i \in U$, $|T^{-}_{Q_1}(u_i)|\geq 0$, $|T^{-}_{Q_2}(u_i)|\geq 0$, $|T^{-}_{Q_1}(u_i)-T^{-}_{Q_2}(u_i)|\geq 0$, $\sqrt{1+|T^{-}_{Q_1}(u_i)|^2}\geq 0$, $\sqrt{1+|T^{-}_{Q_2}(u_i)|^2}\geq 0$, $|(1-T^{-}_{Q_1}(u_i))|\geq 0$, $|(1-T^{-}_{Q_2}(u_i))|\geq 0$, $|(1-T^{-}_{Q_1}(u_i))-(1-T^{-}_{Q_2}(u_i))|\geq 0$, $\sqrt{1+|(1-T^{-}_{Q_1}(u_i))|^2}\geq 0$, $\sqrt{1+|(1-T^{-}_{Q_2}(u_i))|^2}\geq 0$

$$\Rightarrow\left[\frac{2|T^{-}_{Q_1}(u_i)-T^{-}_{Q_2}(u_i)|}{\sqrt{1+|T^{-}_{Q_1}(u_i)|^2}+\sqrt{1+|T^{-}_{Q_2}(u_i)|^2}}+\frac{2|(1-T^{-}_{Q_1}(u_i))-(1-T^{-}_{Q_2}(u_i))|}{\sqrt{1+|(1-T^{-}_{Q_1}(u_i))|^2}+\sqrt{1+|(1-T^{-}_{Q_2}(u_i))|^2}}\right]\geq 0 \quad (24)$$

and $|T^{+}_{Q_1}(u_i)|\geq 0$, $|T^{+}_{Q_2}(u_i)|\geq 0$, $|T^{+}_{Q_1}(u_i)-T^{+}_{Q_2}(u_i)|\geq 0$, $\sqrt{1+|T^{+}_{Q_1}(u_i)|^2}\geq 0$, $\sqrt{1+|T^{+}_{Q_2}(u_i)|^2}\geq 0$, $|(1-T^{+}_{Q_1}(u_i))|\geq 0$, $|(1-T^{+}_{Q_2}(u_i))|\geq 0$, $|(1-T^{+}_{Q_1}(u_i))-(1-T^{+}_{Q_2}(u_i))|\geq 0$, $\sqrt{1+|(1-T^{+}_{Q_1}(u_i))|^2}\geq 0$, $\sqrt{1+|(1-T^{+}_{Q_2}(u_i))|^2}\geq 0$

$$\Rightarrow\left[\frac{2|T^{+}_{Q_1}(u_i)-T^{+}_{Q_2}(u_i)|}{\sqrt{1+|T^{+}_{Q_1}(u_i)|^2}+\sqrt{1+|T^{+}_{Q_2}(u_i)|^2}}+\frac{2|(1-T^{+}_{Q_1}(u_i))-(1-T^{+}_{Q_2}(u_i))|}{\sqrt{1+|(1-T^{+}_{Q_1}(u_i))|^2}+\sqrt{1+|(1-T^{+}_{Q_2}(u_i))|^2}}\right]\geq 0 \quad (25)$$

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Similarly, we can show that

$$\left[\frac{2\left|I^-_{Q_1}(u_i)-I^-_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^-_{Q_1}(u_i))-(1-I^-_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^-_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^-_{Q_2}(u_i))\right|^2}}\right]\geq 0, \tag{26}$$

$$\left[\frac{2\left|I^+_{Q_1}(u_i)-I^+_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^+_{Q_1}(u_i))-(1-I^+_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^+_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^+_{Q_2}(u_i))\right|^2}}\right]\geq 0, \tag{27}$$

$$\left[\frac{2\left|F^-_{Q_1}(u_i)-F^-_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^-_{Q_1}(u_i))-(1-F^-_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^-_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^-_{Q_2}(u_i))\right|^2}}\right]\geq 0 \tag{28}$$

and

$$\left[\frac{2\left|F^+_{Q_1}(u_i)-F^+_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^+_{Q_1}(u_i))-(1-F^+_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^+_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^+_{Q_2}(u_i))\right|^2}}\right]\geq 0 \tag{29}$$

Adding Equation (21) to Equation (29), and using $w_i\in[0,1], \sum_{i=1}^{n} w_i=1$, we have, $CE^{w}_{NC}(Q_1,Q_2)\geq 0$. Hence complete the proof.

ii).

$$\left[\frac{2\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}\right]=0,$$

$$\Leftrightarrow T_{Q_1}(u_i)=T_{Q_2}(u_i) \tag{30}$$

$$\left[\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow I_{Q_1}(u_i)=I_{Q_2}(u_i) \tag{31}$$

$$\left[\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}\right]=0,$$

$\Leftrightarrow F_{Q_1}(u_i)=F_{Q_2}(u_i)$, For all values of $u_i\in U$. (32)

Again,

$$\left[\frac{2\left|T^{-}_{Q_1}(u_i)-T^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T^{-}_{Q_1}(u_i))-(1-T^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{-}_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow T^{-}_{Q_1}(u_i)=T^{-}_{Q_2}(u_i) \qquad (33)$$

$$\left[\frac{2\left|T^{+}_{Q_1}(u_i)-T^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T^{+}_{Q_1}(u_i))-(1-T^{+}_{JQ_2}(u_i))\right|}{\sqrt{1+\left|(1-T^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^{+}_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow T^{+}_{Q_1}(u_i)=T^{+}_{Q_2}(u_i) \qquad (34)$$

$$\left[\frac{2\left|I^{-}_{Q_1}(u_i)-I^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^{-}_{Q_1}(u_i))-(1-I^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{-}_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow I^{-}_{Q_1}(u_i)=I^{-}_{Q_2}(u_i) \qquad (35)$$

$$\left[\frac{2\left|I^{+}_{Q_1}(u_i)-I^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^{+}_{Q_1}(u_i))-(1-I^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{+}_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow I^{+}_{Q_1}(u_i)=I^{+}_{Q_2}(u_i) \qquad (36)$$

$$\left[\frac{2\left|F^{-}_{Q_1}(u_i)-F^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{-}_{Q_1}(u_i))-(1-F^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{-}_{Q_2}(u_i))\right|^2}}\right]=0$$

$$\Leftrightarrow F^{-}_{Q_1}(u_i)=F^{-}_{Q_2}(u_i) \qquad (37)$$

$$\left[\frac{2\left|F^{+}_{Q_1}(u_i)-F^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{+}_{Q_1}(u_i))-(1-F^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{+}_{Q_2}(u_i))\right|^2}}\right]=0$$

$\Leftrightarrow F^{+}_{Q_1}(u_i)=F^{+}_{Q_2}(u_i)$, For all values of $u_i \in U$. (38)

Using Equation (30) to Equation (38) and $w_i \in [0,1], \sum_{i=1}^{n} w_i = 1, w_i \geq 0$, we can show that $CE^{w}_{NC}(Q_1, Q_2)=0$ iff $T^{-}_{Q_1}(u_i)=T^{-}_{Q_2}(u_i)$, $T^{+}_{Q_1}(u_i)=T^{+}_{Q_2}(u_i)$, $I^{-}_{Q_1}(u_i)=I^{-}_{Q_2}(u_i)$, $I^{+}_{Q_1}(u_i)=I^{+}_{Q_2}(u_i)$, $F^{-}_{Q_1}(u_i)=F^{-}_{Q_2}(u_i)$, $F^{+}_{Q_1}(u_i)=F^{+}_{Q_2}(u_i)$ and $T_{Q_1}(u_i)=T_{Q_2}(u_i), I_{Q_1}(u_i)=I_{Q_2}(u_i), F_{Q_1}(u_i)=F_{Q_2}(u_i)$ for all $u_i \in U$.

iii).

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Using Definition (20), Definition (4) and Definition (10), we obtain the following expression:

$$CE^{w}_{NC}(Q_1^c,Q_2^c)=\frac{1}{8}\left\langle \sum_{i=1}^{n} w_i\left\{\left[\left[\frac{2\left|T^-_{Q_1^c}(u_i)-T^-_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|T^-_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-T^-_{Q_1^c}(u_i))-(1-T^-_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-T^-_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-T^-_{Q_2^c}(u_i))\right|^2}}\right]+\right.\right.\right.$$

$$\left[\frac{2\left|T^+_{Q_1^c}(u_i)-T^+_{J_2^c}(u_i)\right|}{\sqrt{1+\left|T^+_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-T^+_{Q_1^c}(u_i))-(1-T^+_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-T^+_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-T^+_{Q_2^c}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|I^-_{Q_1^c}(u_i)-I^-_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|I^-_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-I^-_{Q_1^c}(u_i))-(1-I^-_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-I^-_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-I^-_{Q_2^c}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|I^+_{Q_1^c}(u_i)-I^+_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|I^+_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-I^+_{Q_1^c}(u_i))-(1-I^+_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-I^+_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-I^+_{Q_2^c}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|F^-_{Q_1^c}(u_i)-F^-_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|F^-_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-F^-_{Q_1^c}(u_i))-(1-F^-_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-F^-_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-F^-_{Q_2^c}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|F^+_{Q_1^c}(u_i)-F^+_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|F^+_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-F^+_{Q_1^c}(u_i))-(1-F^+_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-F^+_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-F^+_{Q_2^c}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|T_{Q_1^c}(u_i)-T_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|T_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1^c}(u_i))-(1-T_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2^c}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|I_{Q_1^c}(u_i)-I_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|I_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2^c}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1^c}(u_i))-(1-I_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2^c}(u_i))\right|^2}}\right]+$$

$$\left.\left.\left.\left[\frac{2\left|F_{Q_1^c}(u_i)-F_{Q_2^c}(u_i)\right|}{\sqrt{1+\left|F_{Q_1^c}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2^c}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1^c}(u_i))-(1-F_{Q_2^c}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1^c}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2^c}(u_i))\right|^2}}\right]\right]\right\}\right\rangle$$

$$=\frac{1}{8}\left\langle \sum_{i=1}^{n} w_i\left\{\left[\left[\frac{2\left|(1-T^-_{Q_1}(u_i))-(1-T^-_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T^-_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T^-_{Q_2}(u_i))\right|^2}}+\frac{2\left|T^-_{Q_1}(u_i)-T^-_{Q_2}(u_i)\right|}{\sqrt{1+\left|T^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q_2}(u_i)\right|^2}}\right]+\right.\right.\right.$$

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$$\left[\frac{2\left|(1-T_{Q_1}^{+}(u_i))-(1-T_{Q_2}^{+}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}^{+}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}^{+}(u_i))\right|^2}}+\frac{2\left|T_{Q_1}^{+}(u_i)-T_{Q_2}^{+}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}^{+}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}^{+}(u_i)\right|^2}}\right]+$$

$$\left[\frac{2\left|(1-I_{Q_1}^{-}(u_i))-(1-I_{Q_2}^{-}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}^{-}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}^{-}(u_i))\right|^2}}+\frac{2\left|I_{Q_1}^{-}(u_i)-I_{Q_2}^{-}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}^{-}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}^{-}(u_i)\right|^2}}\right]+$$

$$\left[\frac{2\left|(1-I_{Q_1}^{+}(u_i))-(1-I_{Q_2}^{+}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}^{+}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}^{+}(u_i))\right|^2}}+\frac{2\left|I_{Q_1}^{+}(u_i)-I_{Q_2}^{+}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}^{+}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}^{+}(u_i)\right|^2}}\right]+$$

$$\left[\frac{2\left|(1-F_{Q_1}^{-}(u_i))-(1-F_{Q_2}^{-}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}^{-}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}^{-}(u_i))\right|^2}}+\frac{2\left|F_{Q_1}^{-}(u_i)-F_{Q_2}^{-}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}^{-}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}^{-}(u_i)\right|^2}}\right]+$$

$$\left[\frac{2\left|(1-F_{Q_1}^{+}(u_i))-(1-F_{Q_2}^{+}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}^{+}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}^{+}(u_i))\right|^2}}+\frac{2\left|F_{Q_1}^{+}(u_i)-F_{Q_2}^{+}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}^{+}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}^{+}(u_i)\right|^2}}\right]+$$

$$\left[\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}+\frac{2\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}}\right]+$$

$$\left[\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}+\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}\right]+$$

$$\left.\left.\left[\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}+\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}\right]\right\}\right\rangle$$

$$=\frac{1}{8}\left\langle\sum_{i=1}^{n} w_i\left\{\left[\frac{2\left|T_{Q_1}^{-}(u_i)-T_{Q_2}^{-}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}^{-}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}^{-}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}^{-}(u_i))-(1-T_{Q_2}^{-}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}^{-}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}^{-}(u_i))\right|^2}}\right]+\right.\right.$$

$$\left[\frac{2\left|T_{Q_1}^{+}(u_i)-T_{Q_2}^{+}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}^{+}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}^{+}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}^{+}(u_i))-(1-T_{Q_2}^{+}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}^{+}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}^{+}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|I_{Q_1}^{-}(u_i)-I_{Q_2}^{-}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}^{-}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}^{-}(u_i)\right|^2}}+\frac{2\left|(1-I_{Q_1}^{-}(u_i))-(1-I_{Q_2}^{-}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}^{-}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}^{-}(u_i))\right|^2}}\right]+$$

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$$\left[\frac{2\left|I^{+}_{Q_1}(u_i)-I^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|I^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-I^{+}_{Q_1}(u_i))-(1-I^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I^{+}_{Q_2}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|F^{-}_{Q_1}(u_i)-F^{-}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{-}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{-}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{-}_{Q_1}(u_i))-(1-F^{-}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{-}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{-}_{Q_2}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|F^{+}_{Q_1}(u_i)-F^{+}_{Q_2}(u_i)\right|}{\sqrt{1+\left|F^{+}_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^{+}_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F^{+}_{Q_1}(u_i))-(1-F^{+}_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F^{+}_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F^{+}_{Q_2}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|}{\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}}\right]+$$

$$\left[\frac{2\left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|}{\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u)\right|^2}}+\frac{2\left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}}\right]+$$

$$\left.\left.\left[\frac{2\left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|}{\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}}+\frac{2\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|}{\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}}\right]\right\}\right\rangle = CE^{w}_{NC}(Q_1,Q_2), \forall u_i \in U.$$

iv).

Since, $\forall u_i \in U$, for single valued part we obtain:

$$\left|T_{Q_1}(u_i)-T_{Q_2}(u_i)\right|=\left|T_{Q_2}(u_i)-T_{Q_1}(u_i)\right|,\ \left|I_{Q_1}(u_i)-I_{Q_2}(u_i)\right|=\left|I_{Q_2}(u_i)-I_{Q_1}(u_i)\right|,\ \left|F_{Q_1}(u_i)-F_{Q_2}(u_i)\right|=\left|F_{Q_2}(u_i)-F_{Q_1}(u_i)\right|,$$

$$\left|(1-T_{Q_1}(u_i))-(1-T_{Q_2}(u_i))\right|=\left|(1-T_{Q_2}(u_i))-(1-T_{Q_1}(u_i))\right|,\quad \left|(1-I_{Q_1}(u_i))-(1-I_{Q_2}(u_i))\right|=\left|(1-I_{Q_2}(u_i))-(1-I_{Q_1}(u_i))\right|,$$

$$\left|(1-F_{Q_1}(u_i))-(1-F_{Q_2}(u_i))\right|=\left|(1-F_{Q_2}(u_i))-(1-F_{Q_1}(u_i))\right|.$$

Then, we obtain

$$\sqrt{1+\left|T_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|T_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|T_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|I_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|I_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|I_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|F_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|F_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|F_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_2}(u_i))\right|^2}=\sqrt{1+\left|(-T_{Q_2}(u_i))\right|^2}+\sqrt{1+\left|(1-T_{Q_1}(u_i))\right|^2},$$

$$\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}=\sqrt{1+\left|(1-I_{Q_2}(u_i))\right|^2}+\sqrt{1+\left|(1-I_{Q_1}(u_i))\right|^2},$$

$$\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}=\sqrt{1+\left|(1-F_{Q_2}(u_i))\right|^2}+\sqrt{1+\left|(1-F_{Q_1}(u_i))\right|^2},\ \forall u_i \in U.$$

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For interval neutrosophic part, we have

$$\left|T^-_{Q_1}(u_i)-T^-_{Q_2}(u_i)\right|=\left|T^-_{Q_2}(u_i)-T^-_{Q_1}(u_i)\right|,\ \left|I^-_{Q_1}(u_i)-I^-_{Q_2}(u_i)\right|=\left|I^-_{Q_2}(u_i)-I^-_{Q_1}(u_i)\right|,\ \left|F^-_{Q_1}(u_i)-F^-_{Q_2}(u_i)\right|=\left|F^-_{Q_2}(u_i)-F^-_{Q_1}(u_i)\right|,$$

$$\left|(1-T^-_{Q_1}(u_i))-(1-T^-_{Q_2}(u_i))\right|=\left|(1-T^-_{Q_2}(u_i))-(1-T^-_{Q_1}(u_i))\right|,\quad \left|(1-I^-_{Q_1}(u_i))-(1-I^-_{Q_2}(u_i))\right|=\left|(1-I^-_{Q_2}(u_i))-(1-I^-_{Q_1}(u_i))\right|,$$

$$\left|(1-F^-_{Q_1}(u_i))-(1-F^-_{Q_2}(u_i))\right|=\left|(1-F^-_{Q_2}(u_i))-(1-F^-_{Q_1}(u_i))\right|.$$

Then, we obtain

$$\sqrt{1+\left|T^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|T^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|T^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|I^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|I^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|I^-_{Q_1}(u_i)\right|^2},\ \sqrt{1+\left|F^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|F^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|F^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-T^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-T^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-T^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-T^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-I^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-I^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-I^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-I^-_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-F^-_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-F^-_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-F^-_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-F^-_{Q_1}(u_i)\right|^2},\ \forall u_i\in U.$$

Similarly, $\left|T^+_{Q_1}(u_i)-T^+_{Q_2}(u_i)\right|=\left|T^+_{Q_2}(u_i)-T^+_{Q_1}(u_i)\right|$, $\left|I^+_{Q_1}(u_i)-I^+_{Q_2}(u_i)\right|=\left|I^+_{Q_2}(u_i)-I^+_{Q_1}(u_i)\right|$,

$\left|F^+_{Q_1}(u_i)-F^+_{Q_2}(u_i)\right|=\left|F^+_{Q_2}(u_i)-F^+_{Q_1}(u_i)\right|$, $\left|(1-T^+_{Q_1}(u_i))-(1-T^+_{Q_2}(u_i))\right|=\left|(1-T^+_{Q_2}(u_i))-(1-T^+_{Q_1}(u_i))\right|$,

$\left|(1-I^+_{Q_1}(u_i))-(1-I^+_{Q_2}(u_i))\right|=\left|(1-I^+_{Q_2}(u_i))-(1-I^+_{Q_1}(u_i))\right|$, $\left|(1-F^+_{Q_1}(u_i))-(1-F^+_{Q_2}(u_i))\right|=\left|(1-F^+_{Q_2}(u_i))-(1-F^+_{Q_1}(u_i))\right|$, then

$$\sqrt{1+\left|T^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|T^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|T^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|I^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|I^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|I^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|F^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|F^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|F^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-T^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-T^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-T^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-T^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-I^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-I^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-I^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-I^+_{Q_1}(u_i)\right|^2},$$

$$\sqrt{1+\left|(1-F^+_{Q_1}(u_i)\right|^2}+\sqrt{1+\left|(1-F^+_{Q_2}(u_i)\right|^2}=\sqrt{1+\left|(1-F^+_{Q_2}(u_i)\right|^2}+\sqrt{1+\left|(1-F^+_{Q_1}(u_i)\right|^2},\ \forall u_i\in U.$$

And $w_i\in[0,1], \sum_{i=1}^{n} w_i=1, w_i\geq 0$.

So, $CE^{w}_{NC}(Q_1,Q_2)=CE^{w}_{NC}(Q_2,Q_1)$.

Hence complete the proof.

4. MADM strategy using proposed NC-cross entropy measure in NCS environment

In MADM, decision maker evaluates the alternatives based on the attribute values. But it is not easy to rate the performance of alternatives with respect to predefined attributes in terms of crisp numbers due to the uncertain and inconsistent nature of decision making problem. NCS can express this type information. In this section, we develop a MADM strategy using the proposed NC-cross entropy measure.

Description of the MADM problem

The MADM problem can be consider as follows:

Let $A = \{A_1, A_2, A_3, ..., A_m\}$ and $G = \{G_1, G_2, G_3, ..., G_n\}$ be the discrete set of alternatives and attribute respectively. Let $W = \{w_1, w_2, w_3, ..., w_n\}$ be the weight vector of attributes G_j (j = 1, 2, 3, …, n), where $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$.

Now, we describe the steps of MADM strategy using NC-cross entropy measure

MADM strategy using NC-cross entropy **measure**

Step: 1. Formulate the decision matrices

For MADM with neutrosophic cubic information, the rating values of the alternatives A_i $(i=1,2,3,...,m)$ on the basis of criterion $G_j (j=1,2,3,...,n)$ by the decision maker can be expressed in NCN as $a_{ij} = <[T^-_{ij}, T^+_{ij}],[I^-_{ij}, I^+_{ij}],[F^-_{ij}, F^+_{ij}], (T_{ij}, I_{ij}, F_{ij})>$ $(i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n)$. We present these rating values of alternatives provided by the decision maker in matrix form as follows:

$$M = \begin{pmatrix} & G_1 & G_2 & \ldots & G_n \\ A_1 & a_{11} & a_{12} & \cdots & a_{1n} \\ A_2 & a_{21} & a_{22} & & a_{2n} \\ . & . & & \ldots & . \\ A_m & a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad (39)$$

Step: 2. Formulate priori/ ideal decision matrix

In the MADM processes, the priori decision matrix is used to select the best alternatives among the set of collected feasible alternatives. In the decision making situation, we use the following decision matrix as priori decision matrix.

$$P = \begin{pmatrix} & G_1 & G_2 & \ldots & G_n \\ A_1 & a^*_{11} & a^*_{12} & \cdots & a^*_{1n} \\ A_2 & a^*_{21} & a^*_{22} & & a^*_{2n} \\ . & . & & \ldots & . \\ A_m & a^*_{m1} & a^*_{m2} & \cdots & a^*_{mn} \end{pmatrix} \quad (40)$$

Where, $a^*_{ij} = <[1,1],[0,0],[0,0]>$ for benefit type attributes and $a^*_{ij} = <[0,0],[1,1],[1,1]>$ for cost type attributes, (i = 1, 2, 3, …, m; j = 1, 2, 3, …, n).

Step: 3. Formulate the weighted NC- cross entropy matrix

Using Equation (20), we calculate weighted cross entropy values between decision matrix and priori matrix. The cross entropy value can be presented in matrix form as follows:

$$^{NC}M_{CE}^{w} = \begin{pmatrix} CE_{NC}^{w}(A_1) \\ CE_{NC}^{w}(A_2) \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ CE_{NC}^{w}(A_m) \end{pmatrix} \quad (41)$$

Step: 4. Rank the priority

Smaller value of the cross entropy reflects that an alternative is closer to the ideal alternative. Therefore, the preference ranking order of all the alternatives can be determined according to the increasing order of the cross entropy values $CE_{NC}^{w}(A_i)$ (i = 1, 2, 3, …, m). The smallest cross entropy value reflects the best alternative and the greatest cross entropy value reflects the worst alternative.

A conceptual model of the proposed strategy is shown in Figure 1.

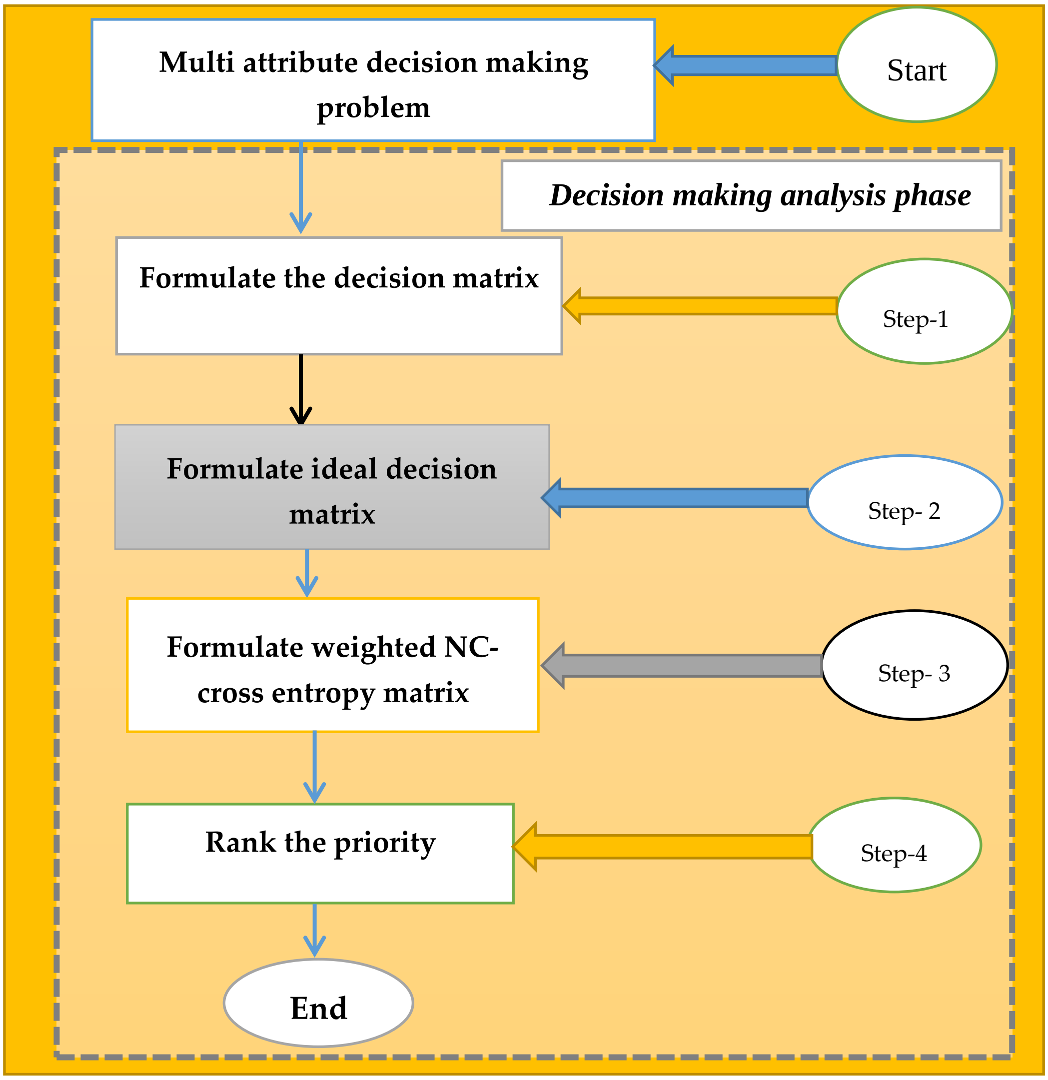

Fig.1. A flow chart of the NC- cross entropy based MADM strategy

5. Illustrative examples

In this section, we solve an illustrative example of MADM problem to reflect the feasibility and efficiency of our proposed strategy under NCSs environments.

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Now, we use an example [59] for cultivation and analysis. A venture capital firm intends to make evaluation and selection to five enterprises with the investment potential:

1) Automobile company (A_1)
2) Military manufacturing enterprise (A_2)
3) TV media company (A_3)
4) Food enterprises (A_4)
5) Computer software company (A_5)

On the basis of four attributes namely:

1) Social and political factor (G_1)
2) The environmental factor (G_2)
3) Investment risk factor (G_3)
4) The enterprise growth factor (G_4).

The investment firm makes a panel of three decision makers $E = \{E_1, E_2, E_3\}$ having their weight vector $\lambda = \{.42, .28, .30\}$ and weight vector of attributes is $W = \{.24, .25, .23, .28\}$.

The steps of decision making strategy to rank alternatives presented as follows:

Step: 1. Formulate the decision matrix

The decision maker represents the rating values of alternative Ai (i = 1, 2, 3, 4, 5) with respect to the attribute Gj (j = 1, 2, 3, 4) in terms of NCNs and constructs the decision matrix M as follows:

M =

$$\begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & <[.6,.8],[.2,.3],[.3,.4],(.8,.3,.4)> & <[.5,.6],[.2,.4],[.4,.4],(.6,.4,.4)> & <[.6,.8],[.2,.3],[.2,.4],(.8,.4,.4)> & <[.6,.7],[.3,.4],[.3,.4],(.7,.4,.5)> \\ A_2 & <[.5,.7],[.2,.3],[.3,.4],(.7,.3,.4)> & <[.7,.8],[.2,.3],[.2,.4],(.8,.3,.4)> & <[.6,.8],[.2,.4],[.3,.4],(.8,.4,.4)> & <[.6,.8],[.2,.3],[.2,.3],(.8,.2,.3)> \\ A_3 & <[.6,.8],[.2,.4],[.3,.4],(.8,.4,.4)> & <[.6,.8],[.2,.3],[.2,.3],(.8,.3,.3)> & <[.8,.9],[.3,.5],[.3,.5],(.9,.5,.5)> & <[.6,.7],[.2,.3],[.2,.4],(.7,.3,.4)> \\ A_4 & <[.5,.7],[.4,.5],[.3,.5],(.7,.5,.5)> & <[.4,.6],[.1,.3],[.3,.4],(.6,.3,.4)> & <[.5,.6],[.1,.2],[.3,.4],(.6,.2,.4)> & <[.5,.7],[.3,.4],[.4,.5],(.7,.4,.5)> \\ A_5 & <[.7,.8],[.2,.4],[.2,.3],(.8,.4,.4)> & <[.4,.6],[.2,.4],[.2,.4],(.6,.4,.4)> & <[.5,.7],[.2,.4],[.3,.4],(.7,.4,.4))> & <[.6,.8],[.4,.5],[.4,.5],(.8,.5,.5> \end{pmatrix} \quad (42)$$

Step: 3. Formulate priori/ ideal decision matrix

Priori/ ideal decision matrix

$$M^1 = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \\ A_2 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \\ A_3 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \\ A_4 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \\ A_5 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \end{pmatrix} \quad (43)$$

Step: 4. Calculate the weighted INS cross entropy matrix

Using equation (20), we calculate weighted NC-cross entropy values between ideal matrixes (43) and decision matrix (42).

$$^{NC}M^{w}_{CE} = \begin{pmatrix} 0.66 \\ 0.58 \\ 0.60 \\ 0.74 \\ 0.71 \end{pmatrix} \quad (44)$$

Step: 5. Rank the priority

The position of cross entropy values of alternatives arranging in increasing order is $0.58 < 0.60 < 0.66 < 0.71 < 0.74$. Since, smallest values of cross entropy indicate the alternative is closer to the ideal alternative, the ranking priority of alternatives is $A_2 > A_3 > A_1 > A_5 > A_4$. Hence, military manufacturing enterprise (A_2) is the best alternative for investment.

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Graphical representation of alternatives versus cross entropy is shown in Figure 2. From the Figure 2, we see that A_2 is the best preference alternative and A_4 is the least preference alternative. Figure 3 presents relation between cross entropy value and preference ranking of the alternative.

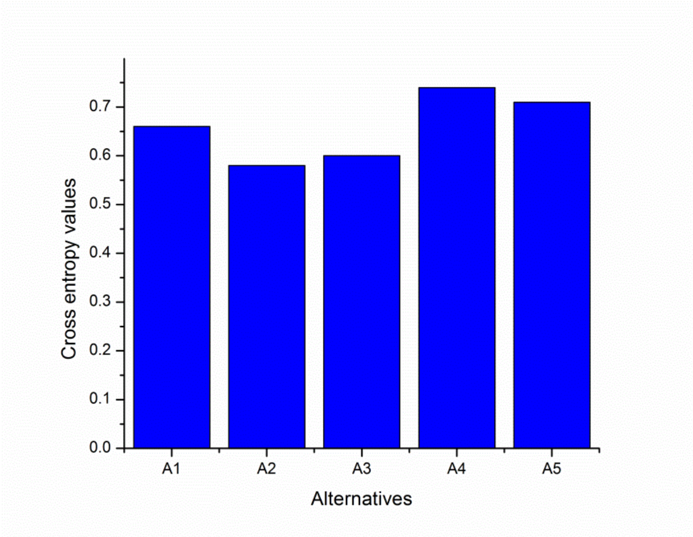

Fig.2. Bar diagram of alternatives versus cross entropy values of alternatives

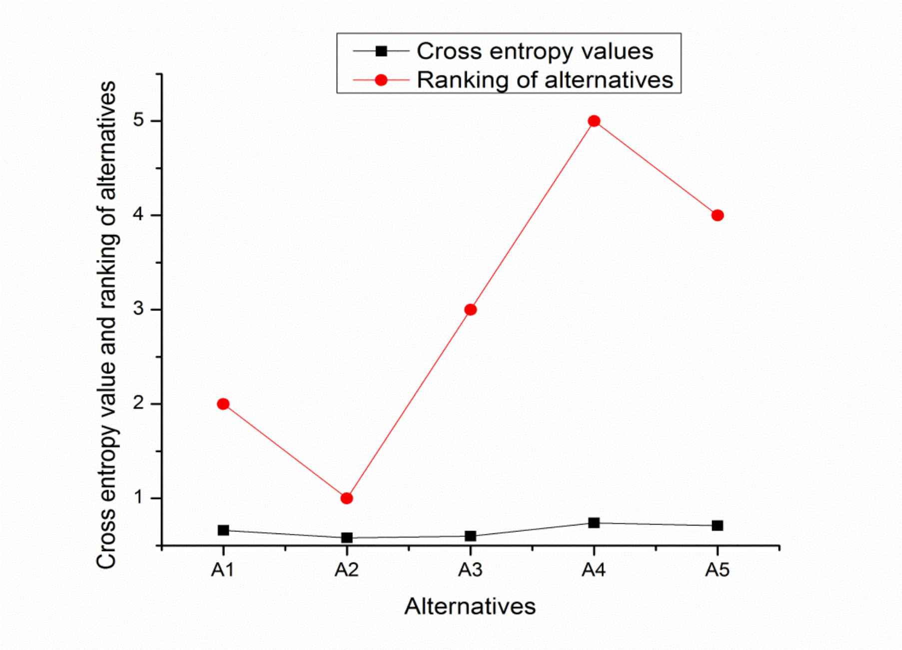

Fig.3. Graphical representation of cross entropy values and ranking of alternatives

6. Conclusion

In this paper we have introduced NC-cross entropy measure in NCS environment. We have proved the basic properties of the cross entropy measure. We have also introduced weighted NC- cross entropy measure and proved its basic properties. Based on the weighted NC-cross entropy measure, we proposed a novel MADM strategy. Finally, we solve a MADM problem to show the feasibility and efficiency of the proposed decision making strategy. The proposed NC-cross entropy based MADM strategy can be employed to solve a variety of problems such as fault diagnosis [15], logistics center selection [60], Weaver selection [61], teacher selection [62], brick selection [63] renewable energy selection [64], etc. The proposed NC-cross entropy based MADM strategy can also be extended to MAGDM strategy using suitable aggregation operators.

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Conflicts of Interest: The authors declare that there is no conflict of interest for publication of the article.

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