

# Testing for causality-in-mean and variance between the UK housing and stock markets

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## Abstract

*This paper employs the two-step procedure developed by Cheung and Ng (1996) to analyze the causality-in-mean and causality-in-variance between the housing and stock markets of the UK. The empirical findings make two key contributions. First, although previous studies have indicated a one-way causal relation from the housing market to the stock market in the UK, this paper discovered a two-way causal relation between them. Second, a causality-in-variance as well as a causality-in-mean was detected from the housing market to the stock market.*

Running head: Testing for causality-in-mean and variance

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## Introduction

Although major financial institutions experienced the subprime mortgage crisis and Lehman Brothers went out of business, the market for real estate has grown steadily in the last decade. As indicated in Figure 1, the UK is one of the largest markets in the world, followed by the US, Japan, Australia, and France. In addition, since the UK decided to withdraw from the European Union (“Brexit”), based on a referendum conducted on June 23, 2016, market participants and macroeconomic policymakers have focused more on its impact on the UK real estate market. Therefore, examining the relation between the UK real estate and other financial markets is useful for both practitioners and academic researchers. Many previous empirical studies have explored the relation between the real estate and stock markets. Regarding this relation, we need to understand the following two effects. First, researchers who support the “wealth effect” claim that households benefiting from unanticipated gains in stock prices tend to increase housing demand. Second, researchers who support the “credit price effect” claim that an increase in real estate prices can stimulate economic activity and the future profitability of companies by raising the value of collateral and reducing the cost of borrowing for both companies and households. Thus, identifying the direction of causality between the real estate and stock markets as well as the number of lags is

essential.

As mentioned above, many previous empirical studies have analyzed the relation between the real estate and stock markets (e.g., Gyourko and Keim (1992); Ibbotson and Siegel (1984); Ibrahim (2010); Kapapoulos and Siokis (2005); Lin and Fuerst (2014); Liow (2006); Liow (2012); Liow and Yang (2005); Louis and Sun (2013); Okunev and Wilson (1997); Okunev, Wilson, and Zurbruegg (2000); Quan and Titman (1999); Su (2011); and Tsai, Lee, and Chiang (2012)). To the best of our knowledge, no studies have analyzed the causality-in-variance between the real estate and stock markets. As indicated by Ross (1989), volatility provides useful data on the flow of information. For institutional investors such as banks, life insurance companies, hedge funds, and pension funds, deeper knowledge of spillover mechanisms for volatility can be useful to diversify investments and hedge risk.

Table 1 summarizes the previous studies. Academic research on the relation between the real estate and stock markets has been undertaken since the 1980s. In this research, almost all studies have focused on the cointegration relation between the two markets. In recent years, not only a linear cointegration method but also a nonlinear cointegration method has been undertaken (e.g., Liow and Yang (2005); Okunev, Wilson, and Zurbruegg (2000); Su (2011); and Tsai, Lee, and Chiang (2012)). Using data from

four major Asian countries (Japan, Hong Kong, Singapore, and Malaysia), Liow and Yang (2005) analyzed the relation between the securitized real estate and stock markets. Moreover, they conducted a fractional cointegration analysis of two asset markets. Furthermore, they revealed that fractional cointegration exists between the securitized real estate and stock markets of Hong Kong and Singapore. Okunev, Wilson, and Zurbruegg (2000) examined the dynamic relation between the US real estate and S&P 500 stock index from 1972 to 1998 by conducting both linear and nonlinear causality tests. While the linear test results generally indicate a unidirectional relation from the real estate market to the stock market, nonlinear causality tests indicate a strong unidirectional relation from the stock market to the real estate market. Su (2011) used nonparametric rank test to empirically investigate the long-run nonlinear equilibrium relation within Western European countries. Nonlinear causality test results demonstrated that unidirectional causality from the real estate market to the stock market exists in the Germany, Netherlands, and UK. Unidirectional causality from the stock market to the real estate market was observed in Belgium and Italy, and feedback effects were discovered in France, Spain, and Switzerland. Tsai, Lee, and Chiang (2012) used nonlinear models to analyze the long-term relation between the US housing and stock markets. Empirical results demonstrated that the wealth effect between the stock

and housing markets is more significant when the stock price outperforms the housing price by an estimated threshold level.

This paper uses the cross-correlation function (CCF) approach developed by Cheung and Ng (1996) to examine the causal relation between the housing and stock markets in the UK. This empirical technique has been widely applied in the examination of stock, fixed income, and commodities markets, business cycles, and derivatives.<sup>1</sup>

While the Granger causality test examines the causality-in-mean, the CCF approach evaluates both the causality-in-mean and causality-in-variance.<sup>2</sup> The CCF approach can detect the direction of causality as well as the number of leads/lags involved.<sup>3</sup>

Furthermore, it permits flexible specification of the innovation process and nondependence on normality.<sup>4</sup>

The remainder of this paper is organized as follows. The next section presents the CCF approach. In the following sections, we discuss the data, descriptive statistics, and results of the unit root tests and provide a description of the autoregressive- exponential generalized autoregressive conditional heteroskedasticity (AR-EGARCH) specification.

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<sup>1</sup> Some examples include studies by Hamori (2003), Alaganar and Bhar (2003), Bhar and Hamori (2005, 2008), Hoshikawa (2008), and Toyoshima and Hamori (2012).

<sup>2</sup> See Hafner and Herwartz (2008) for the causality-in-variance analysis using multivariate GARCH models.

<sup>3</sup> One purpose of this paper is to detect the number of leads/lags, so we do not adopt Hong's (2001) approach.

<sup>4</sup> See also Hamori (2003).

Thereafter, we present the empirical results and discuss the findings. Finally, a summary and conclusion are presented in the closing section.

## Empirical Techniques

Following Cheung and Ng (1996), suppose there are two stationary and ergodic time series,  $X_t$  and  $Y_t$ . When  $I_{1,t}$ ,  $I_{2,t}$ , and  $I_t$  are three information sets defined by  $I_{1,t} = (X_t, X_{t-1}, \dots)$ ,  $I_{2,t} = (Y_t, Y_{t-1}, \dots)$ , and  $I_t = (X_t, X_{t-1}, \dots, Y_t, Y_{t-1}, \dots)$ ,  $Y$  is said to cause  $X$  in the mean if

$$E[X_t | I_{1,t-1}] \neq E[X_t | I_{t-1}] \quad (1)$$

Similarly,  $X$  is said to cause  $Y$  in the mean if

$$E[Y_t | I_{2,t-1}] \neq E[Y_t | I_{t-1}] \quad (2)$$

Feedback effect in the mean occurs if  $Y$  causes  $X$  in the mean and  $X$  causes  $Y$  in the mean. On the other hand,  $Y$  is said to cause  $X$  in the variance if

$$E[(X_t - \mu_{X,t})^2 | I_{1,t-1}] \neq E[(X_t - \mu_{X,t})^2 | I_{t-1}] \quad (3)$$

where  $\mu_{X,t}$  denotes the mean of  $X_t$  conditioned on  $I_{1,t-1}$ . Similarly,  $X$  is said to cause

$Y$  in the variance if

$$E[(Y_t - \mu_{Y,t})^2 | I_{2,t-1}] \neq E[(Y_t - \mu_{Y,t})^2 | I_{t-1}] \quad (4)$$

where  $\mu_{Y,t}$  denotes the mean of  $Y_t$  conditioned on  $I_{2,t-1}$ . Feedback effect in the variance occurs if  $X$  causes  $Y$  in the variance and  $Y$  causes  $X$  in the variance.

We impose an additional structure in equations (1) through (4) to test for causality-in-mean and causality-in-variance. Suppose  $X_t$  and  $Y_t$  are written as

$$X_t = \mu_{X,t} + \sqrt{h_{X,t}} \varepsilon_t, \quad (5)$$

$$Y_t = \mu_{Y,t} + \sqrt{h_{Y,t}} \zeta_t, \quad (6)$$

where  $\{\varepsilon_t\}$  and  $\{\zeta_t\}$  are two independent white noise processes with zero mean and unit variance.

For the causality-in-mean test, we have the standardized innovation as follows:

$$u_t = (X_t - \mu_{X,t})^2 / h_{X,t} = \varepsilon_t^2, \quad (7)$$

$$v_t = (Y_t - \mu_{Y,t})^2 / h_{Y,t} = \zeta_t^2, \quad (8)$$

with  $\varepsilon_t$  and  $\zeta_t$  being the standardized residuals. Since these residuals are unobservable, we use their estimates. Next, using their estimates, we calculate the sample cross-correlation of the squared standardized residual series,  $r_{uv}(k)$ , and the sample cross-correlation of the standardized residual series,  $r_{\varepsilon\zeta}(k)$ , at time lag  $k$ .



The quantities  $r_{\xi^c}(k)$  and  $r_{uv}(k)$  are used to test for causality-in-mean and causality-in-variance, respectively, within the framework offered by the CCF approach.

First, we can test the null hypothesis of no causality-in-mean using the CCF statistic, which is given by

$$CCF = \sqrt{T} \cdot r_{\xi^c}(k). \quad (9)$$

If the test statistic is larger than the critical value of standard normal distribution, then we reject the null hypothesis.

Second, we can test the null hypothesis of no causality-in-variance using the CCF statistic, which is given by

$$CCF = \sqrt{T} \cdot r_{uv}(k). \quad (10)$$

If the test statistic is larger than the critical value of standard normal distribution, then we reject the null hypothesis.

The CCF approach is divided into two steps. First, we estimate univariate time-series models that consider the time variation in both conditional means and conditional variances. In our case, we consider the AR-EGARCH specification to model the time-varying variance.<sup>5</sup> Second, from the estimated AR-EGARCH model, we obtain the standardized squared residuals of each estimated model and construct the

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<sup>5</sup> See Nelson (1991).

series of squared residuals standardized by conditional variances. As mentioned above, we use the CCF of these standardized residuals to test the null hypotheses of no causality-in-mean and no causality-in-variance.

### **Data, Descriptive Statistics, and Results of an Augmented Dickey-Fuller Test**

We employ monthly data on the UK housing and stock markets from January 1991 to August 2016. This sample period was chosen based on the availability of data obtained from *The Nationwide Building Society*.<sup>6</sup>

Table 2 presents the descriptive statistics for the monthly change rate in stock and housing prices. As indicated in Figure 2, the volatility of the stock market is higher than that of the housing market. The measure for skewness and kurtosis, together with the Jarque–Bera statistics, are also reported to test whether the housing and the stock monthly change rates are normally distributed.<sup>7</sup> The Jarque–Bera statistics reject normality at a 10% significance level in both variables.

Table 3 indicates the results of Augmented Dickey–Fuller test. The results reveal that while the null hypothesis that the variables have a unit root is accepted in both

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<sup>6</sup> We obtained the data from the URL below:

<http://www.nationwide.co.U.K./about/house-price-index/download-data#tab:Downloaddata>

<sup>7</sup> See Jarque and Bera (1978).

variables in level, the null hypothesis is rejected in a first difference.

### Estimation of an AR-EGARCH Model

The first step of the CCF approach is to model the monthly change rates in the housing and stock prices. We estimate the AR( $k$ )-EGARCH( $p, q$ ) model as follows:

$$y_t = a_0 + \sum_{i=1}^k a_i y_{t-i} + \varepsilon_t, \quad \varepsilon_{t/t-1} \sim N(0, \sigma_t^2), \quad (11)$$

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^q (\alpha_i |z_{t-i}| + \gamma_i z_{t-i}) + \sum_{i=1}^p \beta_i \log(\sigma_{t-i}^2), \quad (12)$$

where  $z_t = \varepsilon_t / \sigma_t$ . Note that the left-hand side of equation (12) is the log of the conditional variance. The log form of the EGARCH( $p, q$ ) model guarantees the non-negativity of the conditional variance, without the need to constrain the coefficients of the model. The asymmetric effect of positive and negative shocks is represented by the inclusion of the term  $z_{t-i}$ . If  $\gamma_i > 0$ , then  $z_{t-i} = \varepsilon_{t-i} / \sigma_{t-i}$  is positive. The persistence of shocks to the conditional variance is given by  $\sum_{i=1}^p \beta_i$ . Since negative coefficients are not precluded, EGARCH models allow for the possibility of cyclical behavior in volatility.

Equation (11), which is the conditional mean, is specified as an autoregressive process of order  $k$ . To determine the optimal lag length  $k$  for each variables, we use the

Schwartz–Bayesian Information Criterion (SBIC).<sup>8</sup> The SBIC is also applied in equation (12) to determine the optimal lag length  $p$  and  $q$ .<sup>9</sup>

Table 4 presents the estimates for the AR( $k$ )-EGARCH( $p,q$ ) model. Regarding the standard error, this paper accepts the robust standard error developed by Bollerslev and Wooldridge (1992). First, the EGARCH (1,1) model is chosen for both variables. While all parameters of the EGARCH model in the monthly change rate in the stock price are significant, all parameters excluding  $\gamma_1$  in the monthly change rate in the housing price are significant at the conventional significance levels.

Furthermore, Table 4 reports the estimates of the coefficient  $\beta_1$ , which measures the degree of volatility persistence. We find that  $\beta_1$  is significant at conventional significance levels, and the value of  $\beta_1$  is close to 1. These estimates lead to the conclusion that the persistence in shocks to volatility is relatively large. Table 2 also indicates the diagnostics of the empirical results of the AR-EGARCH model. While  $Q(24)$  is a test statistic for the null hypothesis that there is no autocorrelation up to order 24 for standardized residuals,  $Q^2(24)$  is a test statistic for the null hypothesis that there is no autocorrelation up to order 20 for standardized residuals squared.<sup>10</sup> As

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<sup>8</sup> See Schwarz (1978).

<sup>9</sup> We selected the final models from EGARCH (1,1), EGARCH (1,2), EGARCH (2,1), and EGARCH (2,2).

<sup>10</sup> See Ljung and Box, (1978).

shown in these tables, both statistics are within 0.01 or 0.05 for all cases. Thus, the null hypothesis of no autocorrelation up to order 20 for standardized residuals and the standardized residuals squared is accepted. These results empirically support the specification of the AR-EGARCH model.

### **Testing for Causality-in-Variance**

The second step of the CCF approach is to test for causality-in-mean and causality-in-variance, using the sample cross-correlations of standardized residuals and standardized residuals squared. Table 4 indicates the empirical results. Lags are measured in months, which range from 1 to 24. For example, in the case of “housing and stock ( $-k$ ),” the significance of positive lags implies that the causal direction is from the stock market to the housing market.

Table 5 presents significance lags causality for both cases. First, in the case of “housing and stock ( $-k$ ),” the causality-in-mean exists in lag 6 and the causality-in-variance exists in lags 5 and 9. Second, in the case of “housing and stock ( $+k$ ),” the causality-in-mean exists in lags 4, 21, and 23 and the causality-in-variance exists in lags 4 and 12. The above results provide two interesting findings. First, although Su (2011) indicated a one-way causal relation from the housing market to the

stock market in the UK, this paper discovered a two-way causal relation between them. This supports the idea that both a wealth effect and a credit price effect exist between the housing and stock markets. Second, both causality-in-mean and causality-in-variance are detected from the housing market to the stock market. This finding has never been referred to in previous studies and is useful for both practitioners and academic researchers.

### **Concluding Remarks**

This paper analyzes the causality-in-mean and causality-in-variance between the UK stock and housing markets using monthly data from January 1991 to August 2016. A CCF approach developed by Cheung and Ng (1996) and a causality-in-variance test applied to financial market prices are used as tests (Cheung and Ng, 1996). The empirical findings make two key contributions. First, although Su (2011) showed a one-way causal relation from the housing market to the stock market in the UK, this paper discovered a two-way causal relation between them. Thus, both a wealth effect and a credit price effect exist between the housing and stock markets. This paper also detected a causality-in-mean and causality-in-variance from the housing market to the stock market. This point has never been referred to in previous studies and is useful for

both practitioners and academic researchers.

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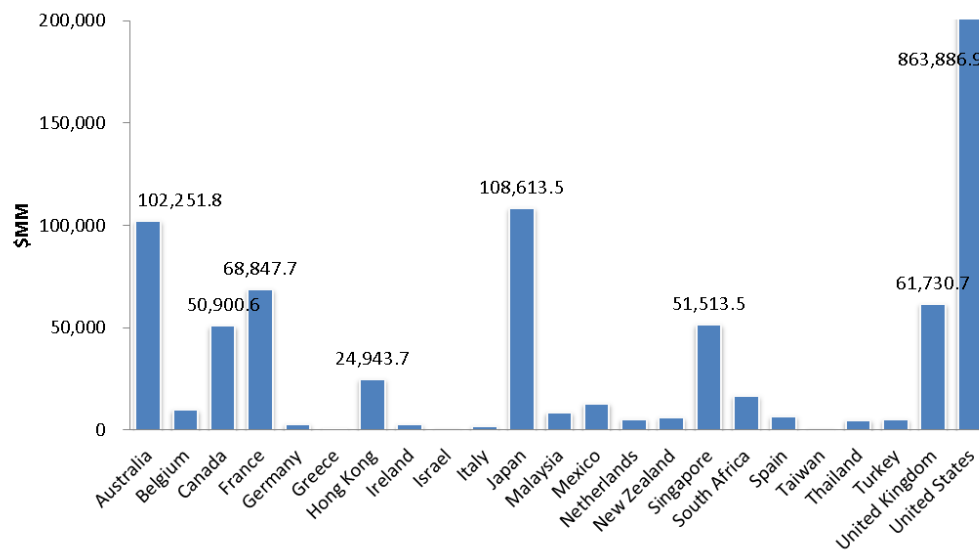


Figure 1. Market Capitalization of the S&P Global REIT Index in August 2016

Data Source: S&P Capital IQ

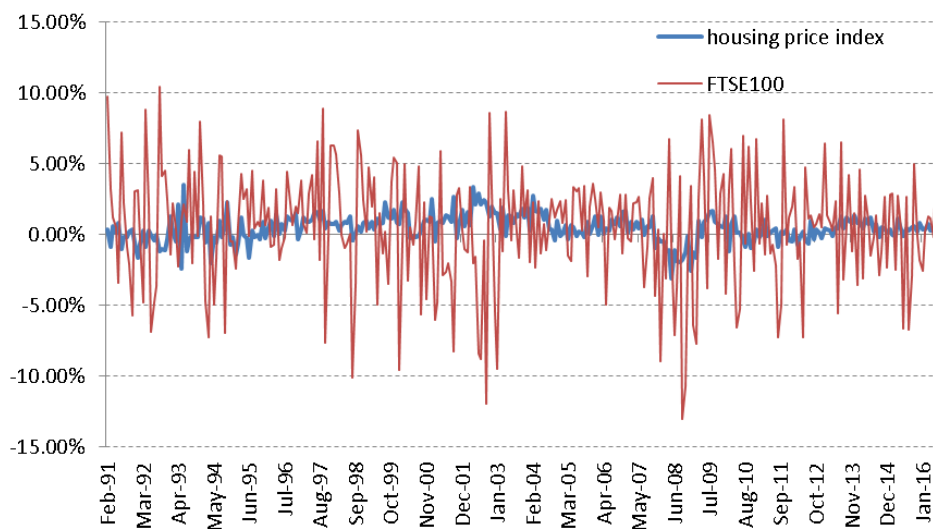


Figure 2. Rates of Change in the Stock and Housing Indexes

Data Source: Nationwide Building Society, Yahoo Finance

Table 1. Summaries of Previous Studies

Authors	Empirical technique	Country	Principal results
Gyourko and Keim (1992)	Market regression model	the US	Lagged equity REIT and stock return are predictors of property index.
Ibbotson and Siegel (1984)	Correlation, Regression	the US	Low correlation between the real estate and stocks, bonds is found.
Ibrahim (2010)	VAR model, Granger causality tests	Thailand	Unidirectional causality from stock prices to house prices is found.
Kapapoulos and Siokis (2005)	VAR model, Granger causality tests	Greece	Unidirectional causality from stock prices to house prices is found.
Lin and Fuerst (2014)	Johansen, Gregory–Hansen, Nonlinear cointegration tests	9 Asian countries	Market segmentation is observed in China, Japan, Thailand, Malaysia, Indonesia and South Korea.
Liow (2006)	ARDL cointegration tests	Singapore	Contemporaneous long-term relationship between the stock market, residential and office property prices is found.
Liow (2012)	Asymmetric DCC model	8 Asian countries	Conditional real estate–stock correlations are time varying and asymmetric in some cases.
Liow and Yang (2005)	FIVECM model, VEC model	4 Asian countries	FIVECM improves the forecasting performance over conventional VECM models.
Louis and Sun (2013)	Fama–MacBeth procedure	the US	Firms' long-term abnormal stock returns are negatively related to past growth in housing prices.
Okunev and Wilson (1997)	Cointegration tests	the US	Weak and nonlinear relationship between the stock and real estate markets is found.
Okunev, Wilson and Zurbrugg (2000)	Linear and nonlinear causality tests	the US	Strong uni-directional relationship from the stock market to the real estate market is found in nonlinear causality test.
Quan and Titman (1999)	Cross-sectional regression	17 countries	Positive relation between real estate values and stock returns is found.
Su (2011)	TEC model, Non-parametric rank test	8 Western European countries	Unidirectional causality from the real estate markets to the stock market is found in the Germany, Netherlands and the UK.
Tsai, Lee and Chiang (2012)	M-TAR cointegration model	the US	Threshold cointegration relationship between the housing market and the stock market is found.

Table 2. Descriptive Statistics: Rates of Change in the Stock and Housing Indexes

	housing	stock
Sample Size	307	307
Mean	0.4421%	0.4527%
Std.Dev.	0.9544%	4.0076%
Skewness	-0.2221	-0.4557
Kurtosis	1.1434	0.5011
Maximum	3.4912%	10.3952%
Minimum	-3.1084%	-13.0247%
Jarque-Bera	19.2472	13.8362
Probability	0.0066%	0.0990%

Table 3. Results of an Augmented Dickey–Fuller Test

variable		auxiliary model		
		const	const & trend	none
housing	Level	-0.2988	-2.3811	1.5701
	First difference	-4.5065 ***	-4.5019 ***	-3.8047 ***
stock	Level	-1.8598	-2.2418	0.7945
	First difference	-17.3975 ***	-17.4146 ***	-17.2370 ***

Notes: \*\*\* indicates significance at 1%.

Table 4. AR-EGARCH model

	housing		stock	
	AR(3)-EGARCH(1,1)		AR(1)-EGARCH(1,1)	
	Estimate	SE	Estimate	SE
$a_0$	0.0015 ***	(0.0005)	0.0067 ***	(0.002)
$a_1$	0.0426	(0.0593)	-0.0586	(0.0607)
$a_2$	0.4177 ***	(0.0504)		
$a_3$	0.2603 ***	(0.0585)		
$\omega$	-0.4338 *	(0.2364)	-1.4792 ***	(0.4902)
$\alpha_1$	0.2506 ***	(0.0805)	0.2974 **	(0.123)
$\gamma_1$	0.0087	(0.0481)	-0.1368 **	(0.0669)
$\beta_1$	0.9765 ***	(0.021)	0.8112 ***	(0.0665)
Log Likelihood	1073.3960		569.9130	
SBIC	-6.9114		-3.6127	
Q (24)	36.1810		11.7890	
P-value	0.0530		0.9820	
Q <sup>2</sup> (24)	21.0020		17.9710	
P-value	0.6390		0.8040	

Notes: \*\*\*, \*\*, \* indicate significance at 1%, 5%, and 10%, respectively.  $Q(24)$  and  $Q^2(24)$  are the Ljung–Box

statistics with 24 lags for the standardized residuals and their squares. In addition, we checked the lag of LB

statistics from 1 to 24.

Table 5. Cross-Correlation for Levels and Squares of the Standardized Residuals

Lag $k$	housing and stock(- $k$ )		housing and stock(+ $k$ )	
	Mean	Variance	Mean	Variance
1	-0.0189	0.0020	0.0077	0.0221
2	-0.0182	0.0666	0.0723	-0.0240
3	-0.0636	0.0273	0.0455	-0.0335
4	0.0122	0.0698	0.0811 *	0.1032 **
5	-0.0282	0.1413 ***	0.0375	-0.0427
6	0.1459 ***	-0.0293	-0.0723	0.0635
7	0.0050	0.0062	-0.0306	0.0191
8	-0.0343	0.0057	0.0182	-0.0429
9	0.0483	0.1392 ***	-0.0036	-0.0006
10	0.0414	0.0142	0.0225	0.0723
11	0.0336	0.0391	-0.0088	-0.0107
12	-0.0673	0.0104	-0.0167	0.1884 ***
13	-0.0643	-0.0109	0.0008	0.0321
14	-0.0624	0.0253	-0.1251	0.0492
15	-0.0781	0.0115	-0.0794	-0.0479
16	-0.0058	-0.0274	-0.0028	-0.1094
17	0.0380	0.0327	-0.0556	-0.0123
18	-0.0457	0.0386	-0.0296	0.0573
19	-0.0352	0.0026	-0.0535	0.0583
20	-0.0210	0.0244	0.0067	0.0373
21	-0.0200	-0.0633	0.0965 **	-0.0084
22	-0.0112	-0.0566	-0.0180	0.0580
23	-0.0849	-0.0444	0.0759 *	0.0084
24	-0.1082	-0.0077	-0.0452	-0.0307

Notes: \*\*\*, \*\*, \* indicate significance at 1%, 5%, and 10%, respectively.