

A Novel Design Framework for Structures/Materials with Enhanced Mechanical Performances

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Abstract

Structure/material requires simultaneous consideration of both its design and manufacturing processes to dramatically enhance its manufacturability, assembly and maintainability. In this work, we present a novel design framework for structure/material with requested mechanical performances in virtue of the compelling properties of topological design and origami techniques. The framework comprises four procedures, including *topological design*, *unfold*, *reduction manufacturing*, and *fold*. Topological design method, i.e. Solid Isotropic Material Penalization (SIMP) method, serves to optimize the structure to achieve preferred mechanical characteristics and origami technique is exploited to make the structure rapidly and easily fabricated. Topological design and unfold procedures can be conveniently completed in a computer; then, reduction manufacturing, i.e. cutting, is performed to remove materials from the unfolded flat plate; the final structure is finally obtained by folding the plate of the previous procedure. A series of cantilevers, consisting of origami with parallel creases and Miura-ori (usually regarded as a metamaterial), made of paperboard are designed with least weight and required stiffness by using the proposed framework. The findings here furnish an alternative design framework for engineering structures which could be better than 3D printing technique, especially for large structures made of thin metal materials.

Keywords: design and fabrication framework; origami; topological design

1. Introduction

The product design process is normally divorced from the manufacturing process, leading to the extremely poor manufacturability, assembly and maintainability of the product. Thus, simultaneously considering the design and manufacturing processes is desired in actual engineering.

The specific functionalities and mechanical performances of the product need to be taken into account according to its service environment during the design process. This can be rather a simple task for structural optimization techniques [1]. Structural optimization methods can be broadly divided into three categories, namely, size optimization [2], shape optimization [3,4], and topology optimization [5-11] methods. The main difference among the three methods is the design variables and the design freedom. Compared with the former two methods, topology optimization is a challenging and active research field, which can produce various innovative candidates with expected mechanical properties. Since the inception of the homogenization method [5], this field has received a growing level of attention, emerging a series of methods, including density-based methods, hard-kill methods, boundary variation methods, and so forth. Among them, Solid Isotropic Material Penalization (SIMP) method [5], Evolutionary Structural Optimization (ESO) method [7] and its improved version Bi-directional Evolutionary Structural Optimization (BESO) method [12,13], and Level-set method [8-10] are recognized as the most widely used, which have been applied in various fields, covering aerospace [14], multifunctional materials [15-17], biomedical design [18,19], uncertain design [20-22], etc.

Traditionally, engineers utilize reduction manufacturing method to fabricate engineering structures, which will significantly waste materials in most cases. To this end, 3D printing [23], as a kind of rapid prototyping technology, based on a digital model file and the use of powder metal or plastic bonding material, through layer by layer printing method to construct objects, has again gotten engineers' attention recently due to its myriad merits, such as saving in material, producing structures with highly complex geometries, et al. Although 3D printing has been successfully applied in automotive, aerospace, medical industries, civil engineering, and other fields [24-26], it has its limitations at the present, i.e., being relatively too expensive and inefficient, the limited available material and manufacturing size, having accuracy and quality problems. Hence, 3D printing cannot replace the traditional manufacturing

industry and reduction manufacturing method is still the mainstream in the future.

Alternatively, one structure/material can be created by using origami technique [27,28]. Origami is an ancient art such that it transforms a flat sheet of paper into a finished sculpture through folding along predefined creases. Inspired by its compelling and extraordinary features, origami technique has been imitated and utilized to design metamaterials [29,30], self-folding structures [31], sandwich structures [32], mechanisms [33], and energy absorbing structures [34-35], etc. Research into topological design for rigid foldable origami structures [36,37] mainly determine where to put the crease lines on the initial flat plane on the basis of the displacement response. Size optimization for the origami-based structures has also been observed. However, layout design for origami-based structures to look for the optimal material distribution has rarely been studied.

In this study, we try to tap the virtues of topological design and origami techniques to propose a novel design framework for structures. This framework can provide a fast design method for structures with predominantly mechanical performances, especially for large structures made from thin metal plates. We here first investigate the design of simple origami-based cantilevers, and then use the framework to design more complex origami-based cantilevers, i.e. Miura-ori-based cantilevers.

2. Materials and Methods

2.1 Testing

Dynamic mechanical analysis is performed to characterize the paperboard, making up the origami-based cantilevers, by using a DMA Q800 machine. During the test, the temperature is about 24 °C and the humidity is close to 50% RH. Stress scan method is used with force increasing from 0.05 to 10 N, with a logarithmic scale and 1 Hz force modulation. The stress-strain curve reported in Fig. 1a indicates that the paperboard with thickness $t=0.32\text{mm}$ is characterized by a Young's modulus $E_p=27.05\text{GPa}$ (Fig. 1A). It should be pointed out that for all the paperboard a Poisson's ratio $\nu_p=0.38$ is assumed.

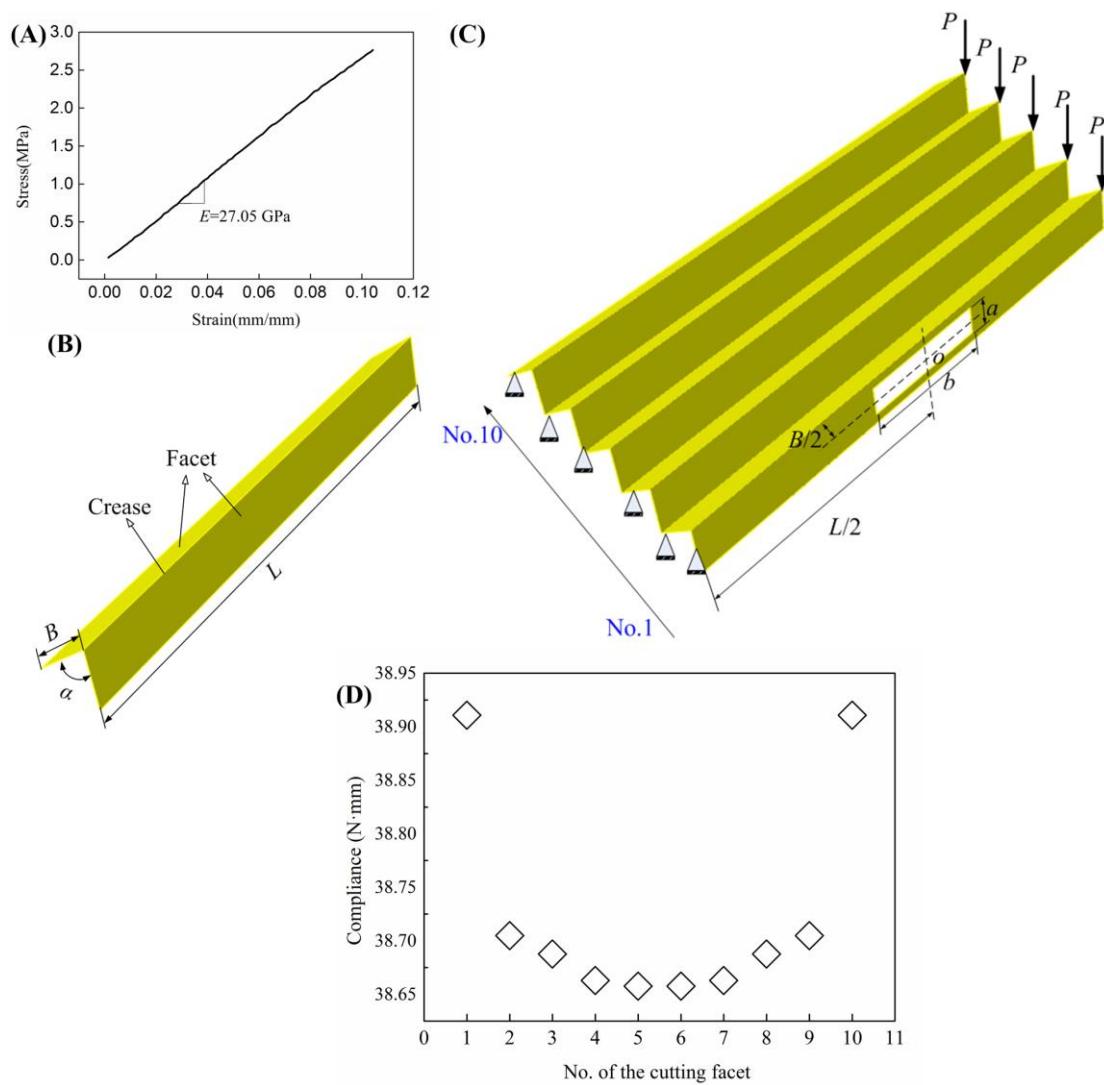


Fig. 1. (A) The stress-strain curve for the paperboard, (B) the geometry for one unit cell of the origami with parallel crease lines, (C) the geometry, boundary, and loading conditions for the origami-based cantilever, and (D) the corresponding compliance of cutting holes in different facet.

2.2 Geometrical dimensions

Origami can be constructed by periodic arranged units. A unit consists of two identical rectangles (facet) with length, $L=250$ mm, and width, $B=20$ mm, and one crease (see Fig. 1B). The dihedral angle is formed between two facets, $\alpha=60^\circ$. For simplicity and without loss of generality, we consider an origami with five units and mark the facets from No.1 to No. 10 (see Fig. 1C).

2.3 Finite element simulations

Finite element (FE) simulations are performed to investigate the deflection behaviors of the origami-based cantilevers by using the commercial package Abaqus\Standard 6.14. In all simulations, the models are divided with 3D shell

elements (S4R). The size of the shell element is 2×2mm. For the finite element model, its left boundary is fixed and five concentrated forces with equal magnitude, $P=5$ N, are applied to its right side (see Fig. 1C). The analysis method in Abaqus is static.

2.4 Topology optimization

In actual engineering, structure with high stiffness-weight ratio is always the pursuit of the engineers, especially in the aviation field. Here, we use the computational approach, continuum topology optimization method, to design the layout of patterns to yield a structure with the aforementioned mechanical performance. Specially, the weight of the origami-based cantilever is minimized and its stiffness is restrained. For each design problem, two non-design domains, D_{non} , highlighted in Fig.2A, are defined to maintain the integrity of the final optimal structure. The rest of the origami-based structure is defined as design domain, D_{des} . This design domain can be occupied by solid elements or void elements.

Normally, continuum topology optimization problem can be mathematically formulated as

$$\begin{aligned}
 & \text{Minimize : } f(\rho) \\
 & \text{Subject to : } g_i(\rho) \leq 0, \quad i = 1, \dots, m \\
 & \quad h_j(\rho) = 0, \quad j = 1, \dots, t \\
 & \quad \rho = [\rho_1, \dots, \rho_e, \dots, \rho_N]^T \\
 & \quad 0 < \rho_{\min} \leq \rho_e \leq 1
 \end{aligned} \tag{1}$$

where $f(\cdot)$ is the objective function. $g(\cdot)$ and $h(\cdot)$ are the inequality and equality constraints, respectively. m and t represent the number of the inequality and equality constraints, respectively. ρ_e stands for the design density which ranges from ρ_{\min} (normally, $\rho_{\min} = 0.001$) to 1. N is the number of the elements occupying the design domain, D_{des} .

In this study, the minimum weight is desired and the maximal deflection is restrained. Since homogeneous material is used, minimizing structural weight is equivalent to minimizing the structural volume and limiting the maximal deflection is equivalent to imposing a restriction on the structural compliance. The equality constraint is the structural static equilibrium equation. Thus, the objective function, $f(\cdot)$, the inequality constraint, $g(\cdot)$, and the equality constraint, $h(\cdot)$, can be respectively written as

$$f(\rho) = V(\rho) \tag{2}$$

$$g(\rho) = C^* - C(\rho) \leq 0 \quad (3)$$

$$h(\rho) = \mathbf{P} - \mathbf{K}(\rho)\mathbf{U}(\rho) = 0 \quad (4)$$

where $V(\cdot)$ is the volume. $C(\cdot)$ and C^* are the structural compliance and a predefined limit for the structural compliance, respectively, and $C(\rho) = \mathbf{P}\mathbf{U}(\rho)$. \mathbf{P} indicates the vector of the applied load. \mathbf{K} and \mathbf{U} are the structural stiffness matrix and the vector of the displacement, respectively.

Using the penalty scheme [5], the Young's modulus of the e th element can be expressed as

$$E_e = \rho_e^p E_s \quad (5)$$

where E_s is the Young's modulus of the solid element and p is the penalization factor which usually has a value of 3.

The optimization model defined in Eq. (1) is solved by general optimization algorithm implemented in Abaqus. The filter technique [38] is employed to prevent the checkerboard problem. In order to make the optimal structure easily fabricated, the minimum and maximum thickness of the member size is controlled, which can be realized by using geometric restriction in Abaqus. The optimization iteration procedure will be terminated when either the change of adjacent element densities or objective functions meets a prescribed convergence criterion.

2.5 Design and Fabrication framework

Topological design method, in conjunction with origami techniques forms a fast and efficient design and manufacturing strategy which can yield structures with desired mechanical performances. The strategy mainly consists of four procedures, namely, *topological design*, *unfold*, *reduction manufacture*, and *fold*, as shown in Fig. 2B. Topological design and unfold procedures can be completed via using computer, together named virtual design; while reduction manufacturing and fold are real manufacturing procedures.

3. Results

3.1 Deflection behavior of the origami-based cantilever

To study the influence of removing materials on the deflection behavior of the origami-based cantilever, we cut small holes in each facet. The holes may have different shapes and numbers. For the sake of simplification, we only focus on the small rectangle hole at the middle of the each facet. The rectangle hole has a length

$b=60\text{mm}$ and a width $a=12\text{mm}$ (see Fig. 1C).

The deflection behavior may be the primary consideration for the engineering structures. Generally, the deflection is determined by the structural stiffness in the elastic stage and the structural stiffness is the reciprocal of the structural compliance. Thus, we employ structural compliance, C , to characterize the deflection behavior of the origami-based cantilever.

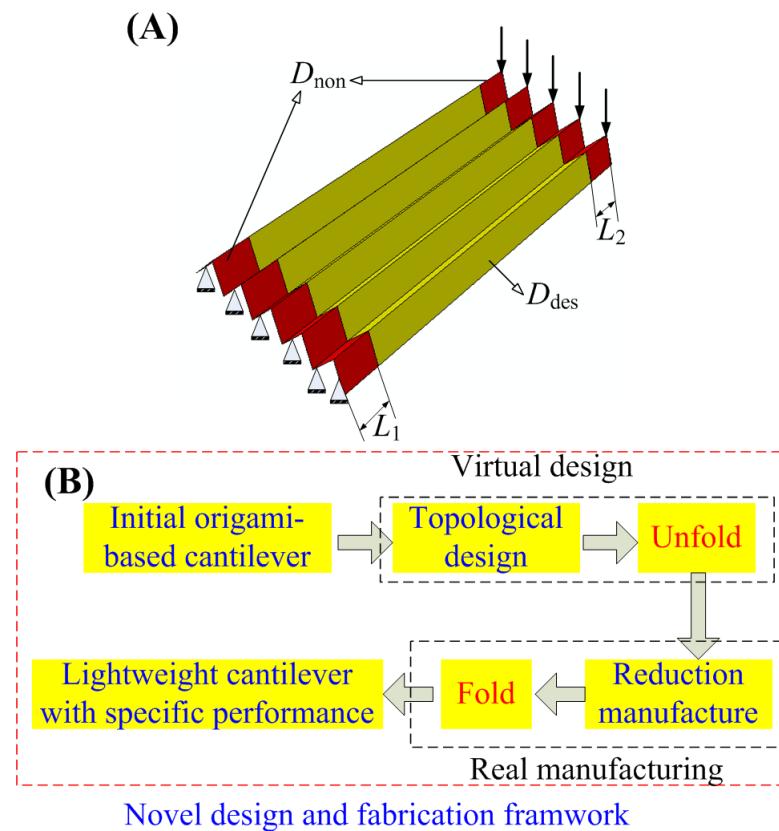


Fig.2. (A) The initial design and non-design domain for the origami-based cantilever, and (B) the proposed design and fabrication framework.

Fig. 1D shows the structural compliance of origami-based cantilevers with rectangle hole cutting in various facets, highlighted from No.1 to No.10. The corresponding compliance is 38.9108 N·mm, 38.7049 N·mm, 38.6878 N·mm, 38.663 N·mm, 38.6577 N·mm, 38.6577 N·mm, 38.663 N·mm, 38.6878 N·mm, 38.7049 N·mm, and 38.9108 N·mm respectively, manifesting that cutting materials from the structure may have great influence on structural stiffness. Hence, we can attempt to find ways to restrict the deflection of the structure by removing materials quantificationally and directionally. Topology optimization can control the structural maximal deflection at the same time yield structure with desired performance, i.e. least weigh.

3.2 Design and fabrication of the origami-based cantilevers

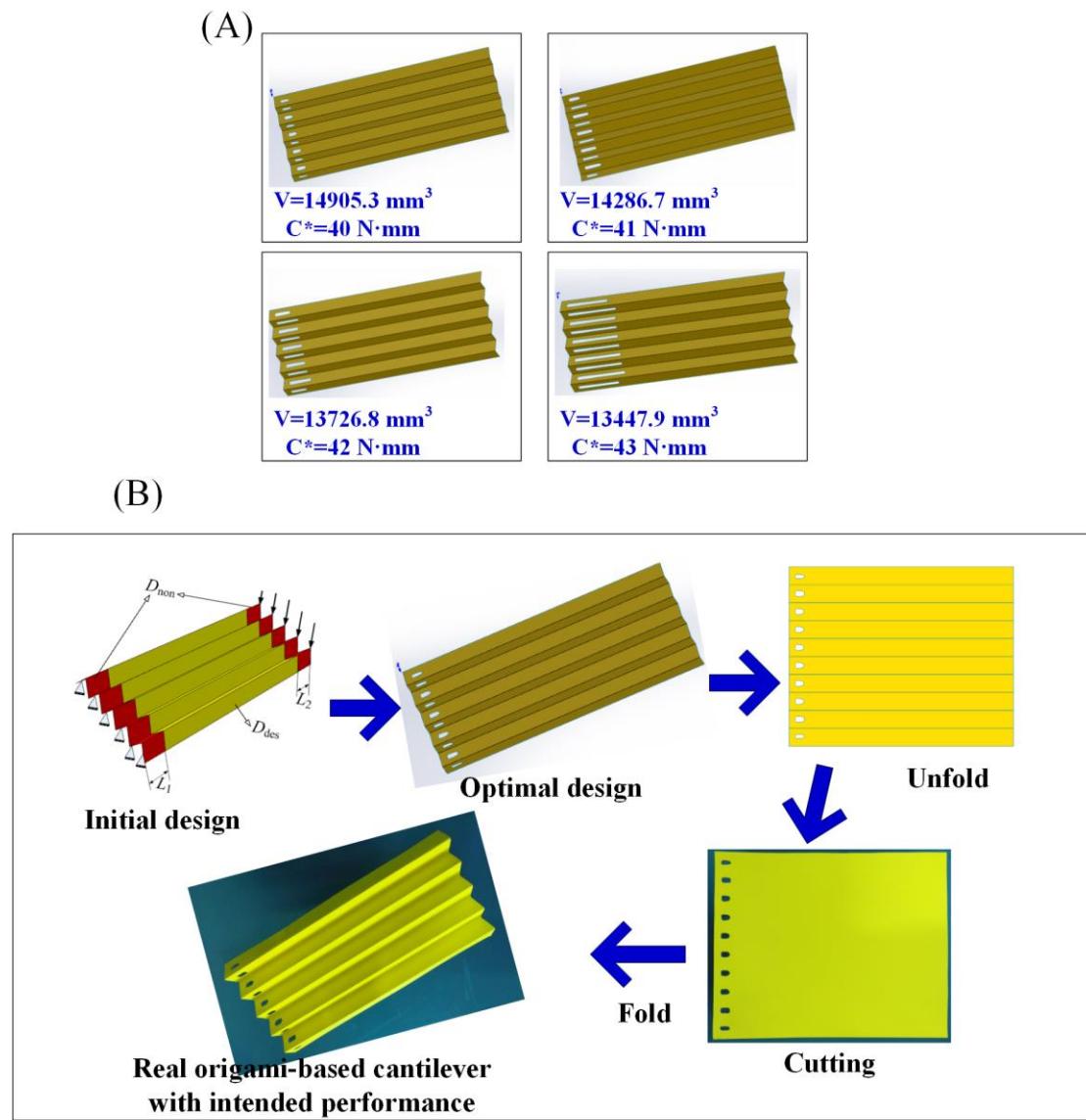


Fig.3. (A) Optimal designs for the origami-based cantilever under various constraints, and (B) applying the proposed framework to the origami-based cantilever.

Fig. 3A shows a series of optimal origami-based cantilevers obtained from the topology optimization with various limits for the structural compliance. The minimum and maximum thickness of the member size is controlled as 6mm and 7mm, respectively. The convergence criterion for the adjacent element densities and objective functions is 0.005 and 0.001, respectively. For the non-design domain, $L_1=L_2=6\text{mm}$. It can be clearly found that the final layout and the final volume of the origami-based cantilever significantly differ from each other with different constraints. To be specific, the final volume is 14905.3 mm^3 , 14286.7 mm^3 , 13726.8 mm^3 , and 13447.9 mm^3 when the predefined limit of the structural compliance is $40 \text{ N}\cdot\text{mm}$, 41

N·mm, 42 N·mm, and 43 N·mm, respectively. It is easy to understand that more materials will be removed from the initial design domain when the restraint for the structural compliance becomes larger, leading to the lighter weight of the origami-based cantilever. We will choose the first design to demonstrate our proposed framework.

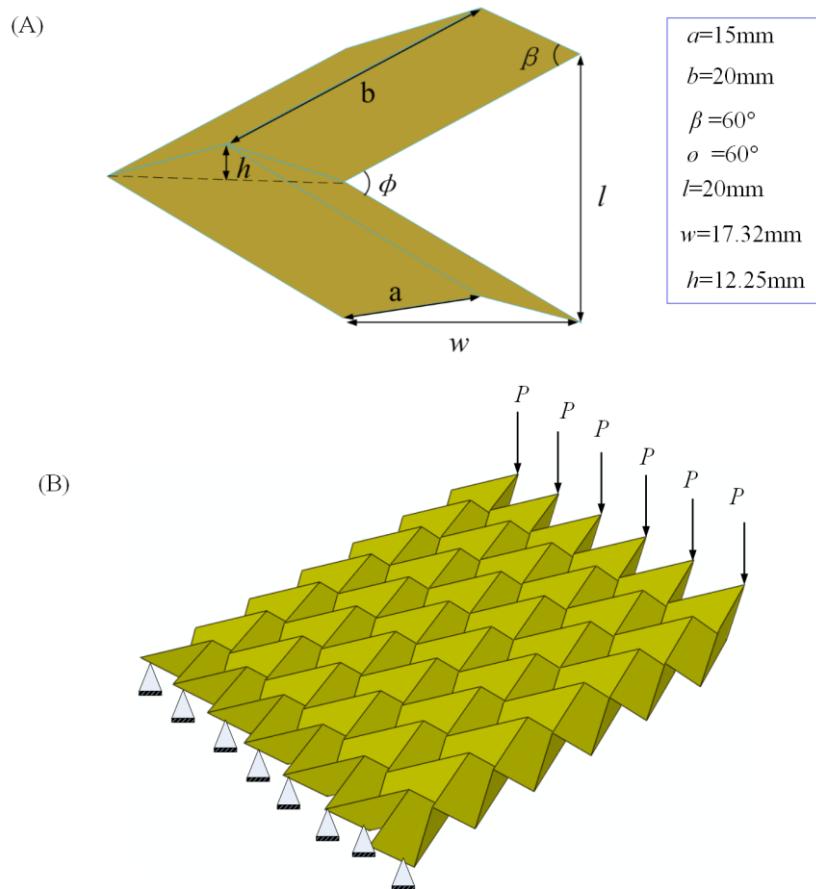


Fig.4. (A) Geometry for the unit cell of the Miura-ori, and (B) the geometry, boundaries, and loading conditions for the Miura-ori-based cantilever.

Fig. 3B presents the novel design and fabrication framework for the chosen origami-based cantilever. Apparently, the framework includes: 1) establish the finite element model of the origami-based cantilever in Abaqus; 2) perform the continuum topology method to optimize the cantilever, and get the optimal design; 3) unfold the optimal design with specific patterns; 4) Cut holes in the paperboard to obtain the patterns; 5) use the origami technique to fold the paperboard, achieving the real origami-based cantilever with intended mechanical performance. It should be noted that the first two procedures are completed in Abaqus, and the third procedure is finished in Solidworks, and the last two procedures are accomplished manually.

3.3 Design and fabrication of Miura-ori-based structures

Using the proposed framework, we attempt to design and fabricate more complex origami-based cantilevers. Here, the famous Miura-ori is selected. Fig. 4 shows the geometry, boundaries, and loading conditions for the Miura-ori-based cantilever. For the geometry, the Miura-ori consists of 36 unit cells. One unit cell (see Fig. 4A) is formed by four parallelograms, and each parallelogram has sides a and b and acute angle γ . The dihedral angle between two parallelograms is ϕ . The outer dimensions, like l , w , and h can be determined by the equations in reference [39]. The models are divided with 3D shell elements (S4R). Each element has a size of 2×2mm. Its left boundary is fixed and six concentrated forces with equal magnitude, $P=5$ N, are applied to its right side (see Fig. 4B). It should be noted that Miura-ori-based structure is always considered to be rigid-foldable. Here, we consider the facet as deformable plate as the applied loads are assumed relatively small.

In the topological design, the minimum and maximum thickness of the member size is selected as 4.5mm and 6mm, respectively. The convergence criterion for the adjacent element densities and objective functions is set as 0.005 and 0.001, respectively. Fig. 5A shows the optimal designs of the Miura-ori-based cantilevers. Similarly, more materials have been omitted from the initial design domain when the restraint for the structural compliance becomes larger, resulting in lighter weight of the Miura-ori-based cantilever. Specifically, the final volume is 11535.7 mm³, 11400.6 mm³, 11226 mm³, and 11050.6 mm³ when the predefined limit of the structural compliance is 89.1 N·mm, 90 N·mm, 91 N·mm, and 92 N·mm, respectively.

We choose the first design of the Miura-ori-based cantilever to give evidence of our proposed design and fabrication framework. Fig. 5B depicts the whole procedures of our framework. After establishing the finite element modeling of the Miura-ori-based cantilever, the design goal and constraints are given and the topological design is performed. The optimal design is obtained, and later we unfold it in a 3D modeling software, i.e. Solidworks. Whereafter, we cut the holes in the paperboard to get the patterns the same with that obtained from the 3D modeling software. Finally, we use the origami technique to fold the paperboard at the predefined crease lines, and we achieve the real Miura-ori-based cantilever with expected mechanical performances.

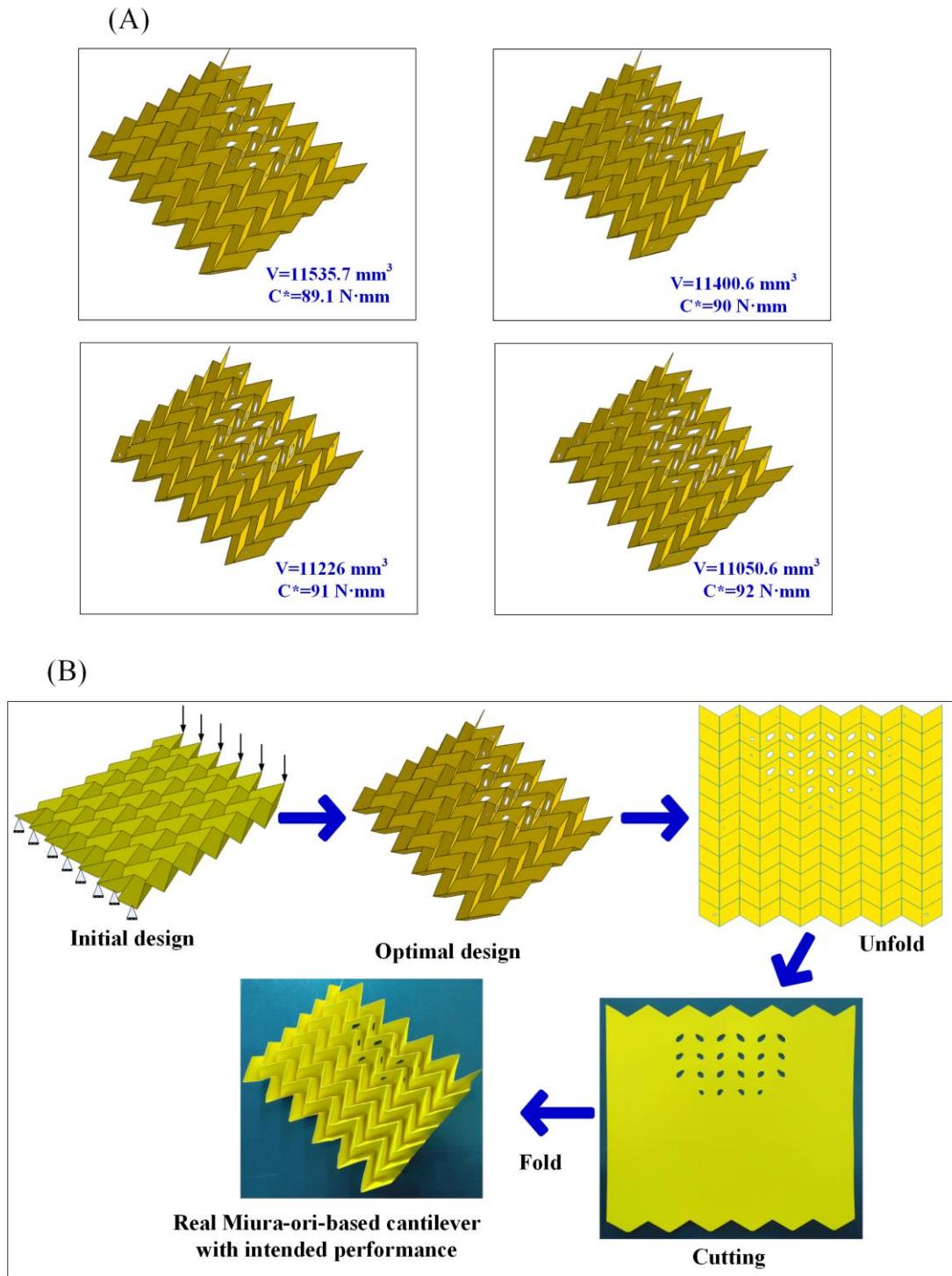


Fig.5. (A) Optimal designs for the Miura-ori-based cantilever under various constraints, and (B) applying the proposed framework to the Miura-ori-based cantilever.

4. Discussion and future work

The design and fabrication for the origami-based cantilevers successfully demonstrate the feasibility and effectiveness of our proposed framework. The cantilevers are inspired from a simple origami with parallel creases and a complex

Miura-ori. The framework is preferred for structures made of thin material. The desired mechanical performances are not restricted to the minimum weight and the limit on the structural stiffness. Mechanical characteristics like maximum natural frequency, largest stiffness, etc. can be also sought.

We here manually cut the materials from the paperboard. An alternative way to increase precision would be to use laser cutting that would produce accurate holes. To promote our framework for practical engineering applications, metal thin materials, i.e. aluminum, should be used to make the real origami-based cantilevers. Unfortunately, since the thickness of the metal affecting the folding process, the real metal origami-based cantilevers can be difficult to make. To facilitate folding, at the crease lines the material may be locally thinned by means of chemical etching. The real metal origami-based cantilevers could be manufactured by using a cold gas pressure folding technique [39]. However, this method may only be suitable for small scale structures as it is not an easy task to produce uniform pressure to fold large metal structures. Future work will attempt to use the framework to design and fabricate large engineering structures made of thin metal materials, in particular stainless steel.

5. Conclusion

In this paper, we have presented a novel framework for designing engineering structures/materials with intended mechanical performances by combining the compelling merits of topological design and origami techniques. The framework comprises four procedures, including *topological design*, *unfold*, *reduction manufacturing*, and *fold*. Specifically, topological design method serves to optimize the structure to achieve preferred mechanical characteristics and origami technique is exploited to make the structure rapidly and easily fabricated. We use one simple origami with parallel creases and one complex Miura-ori based cantilevers to validate the effectiveness of the proposed framework. The minimum weight of the structure with restrained stiffness is achieved via topological design and the real structure is easily made by using the folding technique. This framework can be applied to design and fabrication of large engineering structures made of thin metal materials, is inexpensive and quick to fabricate, compared with 3D printing techniques. It should be noted that the proposed framework is not restricted to design and make cantilevers; it can be applied to other kinds of structures as well.

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