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Article

The Optimal Power Allocation with Hybrid Relaying Based on Channel Condition

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Abstract: This paper considers a hybrid relay network consisting of the source, the amplify-and-forward (AF) relay, the decode-and-forward (DF) relay, and the destination. We propose the optimal power allocation schemes between two different relays which maximize the achievable rate under a sum relay power constraint for given channel gains and transmit power from source. By solving the optimization problem to maximize the achievable rate for each relay network, the transmit power values in closed-form are derived. When the channel gains are the same, the optimal power allocation scheme for AF-DF relay network proves that a more power should be allocated at the first relay to maximize the achievable rate. In case of the DF-AF relay network, we derive the optimal power allocation scheme for the possible four cases. Under the same SNR condition at the first hop, we show that the achievable rate of AF-DF relay network is greater than that of DF-AF relay network when the channel gain between two relays is higher than that between the second relay and destination. Simulation results show that the proposed power allocation schemes provide a higher achievable rate than the equal power allocation schemes.

Keywords: Power allocation; hybrid relay network; amplify-and-forward (AF); decode-and-forward (DF); achievable rate

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1. Introduction

Cooperative comunication has recently attracted method to improve the performace [1–7]. In multi-hop cooperative communications, the source transmits signal to relays that forward signal to the destination or other relays.

There are several relaying strategies: amplify-and-forward (AF) and decode-and-forward (DF). In the AF relaying scheme, a relay simply amplifies the received signals from the source and retransmits them to the destination without performing any signal regeneration, which may lead to the propagation of noise and interference. For the DF relaying scheme, a relay decodes the received signals and retransmits the recovered signals to the destination. Although the DF relaying scheme achieves extra coding gain, the error propagation is caused by decoding errors at the relay.

To get advantages of both the AF and the DF, hybrid relaying schemes were studied in [8,9]. In [8], the authors analyzed the bit error probability for both the AF relaying and the DF relaying with respect to signal-to-noise ratio (SNR) and proposed a hybrid relaying scheme which changes the relaying scheme based on analyzed bit error probability. Like [8], [9] calculated the symbol error probability for both homogeneous relaying and hybrid relaying networks and simulated the symbol error rate (SER) according to the location of relay. The hybrid relaying schemes in [8,9] have higher bit error rate (BER) and SER performances than the simple homogeneous relaying schemes. These hybrid relaying networks obtain more gains than the homogeneous relaying schemes.

Recently, the power allocation problem in a cooperative system has also attracted lots of research attention. In [10] and [11], power allocation schemes have been proposed to maximize the capacity

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under a sum transmit power constraint for AF relay and DF relay, respectively. The optimal power allocation schemes for hybrid network have not been analyzed for a two-hop AF and DF cooperative relay system employing outage probability as the optimization criterion in [12]. In [13], optimal power allocation based on average end-to-end symbol error probability (SEP) as the optimization criterion is performed for a two-hop DF cooperative relay system. In [14] and [15], the instantaneous received signal-to-noise ratio (SNR) and its approximate expression are exploited to obtain optimal power allocation for an AF multi-hop relaying system. The optimal power allocation based on outage probability in a DF multi-hop system is discussed in [16]. In [17], the power allocation scheme that minimizes a bit error rate (BER) at the destination for uncoded AF with Rayleigh fading channel under a sum transmit power consumption was proposed. The optimal power allocation strategy is proposed in [18] to maximize achievable secrecy rates under an overall transmit power constraint assuming that a single relay is located at each individual hop.

In hybrid relaying networks, the error performance is mainly affected by received SNR which is changed by transmit power, channel power and noise power. This paper proposes optimal power allocation schemes for hybrid relay networks within limited total power. The proposed scheme has higher achievable rate than the equal power allocation scheme and can approach the maximum achievable rate with lower power than the equal power allocation scheme. Also, after applying our power allocation schemes, the achievable rates for two hybrid relay networks, i.e. AF-DF and DF-AF, are different according to channel state from a relay to another relay and from a relay to a destination. So, a proper relaying scheme can be selected adaptively based on estimated channel state. In inter-cell communication system which is more common than intra-cell communication system, three-hop relaying transmission is sufficient to achieve optimal throughput and to find the optimal relay node [19]. The simulation results in [19] show that three-hop relaying transmission has better throughput performance than two-hop and four-hop relaying transmission. For transmission with two-hop relaying network, the transmission range is short and it causes a decrease of achievable throughput in the inter-cell communication. Due to the short range, two-hop system does not select better relaying node in terms of throughput performance than three-hop system. Also, for transmission with four-hop or more relaying hops, the throughput is severely decreased because the routing with four-hop or more relaying hops increases forwarding delay and it causes many overheads and signal processing delay for overall systems. Therefore, the number of hops in the relay network is confined to three. The proposed schemes enable the achievable rate to maximize by adaptively allocating the power to the first and the second relay nodes. For adaptive power allocation of each relay node, we derive the transmit power values in closed-form for each relay network according to channel condition. Analytical solutions are derived, and the proposed power allocation schemes are compared with the equal power allocation scheme. In addition, we compare the achievable rates of the proposed power allocation schemes when SNR of the first hop is the same. The simulation results show that the proposed optimal power allocation scheme requires lower transmit power to achieve a specific achievable rate than the equal power allocation scheme. Therefore, this paper contributes to reduction for the lower limit of transmit power consumption to satisfy the achievable rate in a cooperative communication. Also, in the next generation system, green communication has attracted more and more attention. The reduction for the lower limit of transmit power consumption by the proposed scheme contributes to the implementation of green communication in next generation system.

The remainder of this paper is organized as follows. In Section II, the system model of three-hop relay networks is presented. In Section III, the optimal power allocation schemes are proposed for three-hop AF-DF and DF-AF relay networks, respectively. Section IV shows the simulation results, and the conclusion is drawn in Section V.

2. System model

Fig. 1 shows the system model consisting of a source s, the first relay r_1, the second relay r_2, and a destination d. The nodes operate in the half-duplex mode, i.e., they are not able to receive and transmit

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at the same time and same frequency. We assume that the channel gains are acquired from channel state information (CSI) of a system such as reference signal (RS) of 3GPP LTE and that relays know the used power for transmission of a source. In case of this three hop relay network, three RS should be allocated and destination feedbacks the CSI through reverse links of the relay network. For channel estimation, various schemes can be concerned [20,21]. However, the perfect CSI is assumed to compare the maximum performance with other power allocation schemes. In addition, it is assumed that the total power for relaying is fixed. Since the relays also need power to transmit their own signal, this condition is needed. In case of equal power allocation scheme, the fixed total power is allocated to all relays equally. However, since channel condition is not concerned, the equal allocation scheme allocates the burden inefficiently. Therefore, optimization should be applied for power efficiency. This fixed total power can be normalized for comparison with other power allocation schemes [22,23].

An aim of hybrid relay network is that it achieves both high throughput performance and simple implementation. The different property between AF-DF and DF-AF relay network is that these two schemes have different performances with respect to achievable throughput and implementation according to channel condition. Generally, because the AF scheme severely amplifies the noise power, the error performance for the AF scheme is lower than the error performance for the DF scheme. The error performance for the AF scheme is almost the same as error performance for the DF scheme when the channel condition is good. However, because the DF scheme always know channel state information (CSI) to decode the received signals, one of main disadvantage for the DF scheme is that the real-time implementation is harder than the AF scheme in multi-hop transmission system. The AF scheme which simply amplifies the received signal does not require CSI. The AF-DF scheme is adequate when a communication link between the source and the first relay is good with simple implementation by amplifying the received signal and has high throughput performance when a communication link between the first relay and the second relay is not good. Also, the DF-AF scheme has high throughput performance when a communication link between the source and the first relay is not good and is adequate when a communication link between the first relay and the second relay is good with simple implementation. In this paper, both the system models of the AF-DF and DF-AF are represented for general analysis of hybrid relay network in various channel conditions.

2.1. AF and DF Relay Network

In this subsection, we assume that the first relay r_1 considers AF protocol and the second relay r_2 considers DF protocol.

In the first time slot, the source transmits the signal $x_{ad,s}$ with transmit power P_s to the first relay. The received signal $y_{ad,1}$ at the first relay can be expressed as

$$y_{ad,1} = h_1 x_{ad,s} + n_1, \tag{1}$$

where h_1 is the channel coefficient from source to the first relay and n_1 is the zero-mean additive white Gaussian noise (AWGN) with unit variance at the first relay.

In the second time slot, the first relay transmits the signal $x_{ad,1}$ with transmit power $P_{ad,1}$ to the second relay. The transmitted signal $x_{ad,1}$ at the first relay is

$$x_{ad,1} = \beta_{ad} y_{ad,1}, \tag{2}$$

where β_{ad} is the amplification factor for AF relay and it is given by

$$\beta_{ad} = \sqrt{\frac{P_{ad,1}}{P_s |h_1|^2 + 1}}. \tag{3}$$

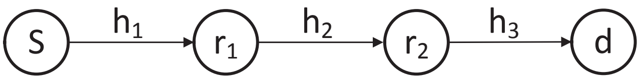

Figure 1. Hybrid three-hop relay network.

The received signal $y_{ad,2}$ at the second relay can be expressed as

$$\begin{aligned} y_{ad,2} &= h_2 x_{ad,1} + n_2, \\ &= \beta_{ad} h_2 (h_1 x_{ad,s} + n_1) + n_2, \end{aligned} \tag{4}$$

where h_2 is the channel coefficient from the first relay to the second relay and n_2 is the zero-mean AWGN with unit variance at the second relay.

The second relay decodes the received signal. In the third time slot, the second relay transmits the signal $x_{ad,2}$ with transmit power $P_{ad,2}$ to the destination. The received signal $y_{ad,d}$ at destination can be expressed as

$$y_{ad,d} = h_3 x_{ad,2} + n_d, \tag{5}$$

120 where h_3 is the channel coefficient from the second relay to destination and n_d is the zero-mean AWGN with unit variance at the destination.

The achievable rate is given by

$$R_{ad}(P_{ad,1}, P_{ad,2}) = \log_2(1 + \gamma_{ad}), \tag{6}$$

where γ_{ad} is the SNR for AF-DF relay network and it is given by

$$\gamma_{ad} = \min\left(\frac{P_s P_{ad,1} |h_1|^2 |h_2|^2}{P_s |h_1|^2 + P_{ad,1} |h_2|^2 + 1}, P_{ad,2} |h_3|^2 \right). \tag{7}$$

2.2. DF and AF Relay Network

In this subsection, we assume that the first relay r_1 considers DF protocol and the second relay r_2 considers AF protocol. In the first time slot, the source transmits the signal $x_{da,s}$ with transmit power P_s to the first relay. The received signal $y_{da,1}$ at first relay can be expressed as

$$y_{da,1} = h_1 x_{da,s} + n_1, \tag{8}$$

where h_1 is the channel coefficient from source to the first relay and n_1 is the zero-mean additive white Gaussian noise (AWGN) with unit variance at the first relay.

The first relay decodes the received signal. In the second time slot, the first relay transmits the signal $x_{da,1}$ with transmit power $P_{da,1}$ to the second relay. The received signal $y_{da,2}$ at the second relay can be expressed as

$$y_{da,2} = h_2 x_{da,1} + n_2, \tag{9}$$

125 where h_2 is the channel coefficient from the first relay to the second relay and n_2 is the zero-mean AWGN with unit variance at the second relay.

In the third time slot, the second relay transmits the signal $x_{da,2}$ with transmit power $P_{da,2}$ to the destination. The transmitted signal $x_{da,2}$ at the second relay is

$$x_{da,2} = \beta_{da} y_{da,2}, \tag{10}$$

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where β_{da} is the amplification factor for AF relay and it is given by

$$\beta_{da} = \sqrt{\frac{P_{da,2}}{P_{da,1}|h_2|^2 + 1}}. \tag{11}$$

The received signal $y_{da,d}$ at the destination can be expressed as

$$\begin{aligned} y_{da,d} &= h_3 x_{da,2} + n_d, \\ &= \beta_{da} h_3 (h_2 x_{da,1} + n_2) + n_d, \end{aligned} \tag{12}$$

where h_3 is the channel coefficient from the second relay to destination and n_d is the zero-mean AWGN with unit variance at the destination.

The achievable rate is given by

$$R_{da}(P_{da,1}, P_{da,2}) = \log_2(1 + \gamma_{da}), \tag{13}$$

where γ_{da} is the SNR for DF-AF relay network and it is given by

$$\gamma_{da} = \frac{\min(P_s|h_1|^2, P_{da,1}|h_2|^2) P_{da,2}|h_3|^2}{\min(P_s|h_1|^2, P_{da,1}|h_2|^2) + P_{da,2}|h_3|^2 + 1}. \tag{14}$$

3. Optimal Power allocation schemes

130 We propose the optimal power allocation schemes for hybrid three-hop relay networks which maximize the achievable rate under a sum relay power constraint for given channel gains and transmit power from source.

3.1. AF and DF relay network

In this subsection, we propose the optimal power allocation scheme for three-hop AF and DF
135 relay network.

The optimization problem to maximize the achievable rate under a sum relay power constraint can be written as

$$\max_{P_{ad,1}, P_{ad,2}} R_{ad}(P_{ad,1}, P_{ad,2}), \quad s.t. \;\; P_{ad,1} + P_{ad,2} = P. \tag{15}$$

We define the ratio of $P_{ad,1}$ to P as α_{ad}. By using α_{ad}, the optimization problem is rewritten as

$$\max_{\alpha_{ad}} \min\{\gamma_{ad,1}(\alpha_{ad}), \gamma_{ad,2}(\alpha_{ad})\}, \quad s.t. \;\; 0 < \alpha_{ad} < 1, \tag{16}$$

where $\gamma_{ad,1}(\alpha_{ad})$ and $\gamma_{ad,2}(\alpha_{ad})$ are given, respectively, as

$$\gamma_{ad,1}(\alpha_{ad}) = \frac{P_s \alpha_{ad} P |h_1|^2 |h_2|^2}{P_s|h_1|^2 + \alpha_{ad} P |h_2|^2 + 1}, \tag{17}$$

$$\gamma_{ad,2}(\alpha_{ad}) = -\alpha_{ad} P |h_3|^2 + P|h_3|^2. \tag{18}$$

As α_{ad} increases, $\gamma_{ad,1}(\alpha_{ad})$ increases and $\gamma_{ad,2}(\alpha_{ad})$ decreases. Therefore, by solving the equation $\gamma_{ad,1}(\alpha_{ad}) = \gamma_{ad,2}(\alpha_{ad})$, the optimal α_{ad} is obtained as

$$\alpha_{ad} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \tag{19}$$

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where a, b, and c are given, respectively, as

$$\begin{aligned} a &= -P^2|h_2|^2|h_3|^2, \\ b &= -P_sP|h_1|^2(|h_2|^2 + |h_3|^2) + P|h_3|^2(P|h_2|^2 - 1), \\ c &= P|h_3|^2(1 + P_s|h_1|^2). \end{aligned} \tag{20}$$

When the channel gains are the same, α_{ad} in (19) is represented as

$$\alpha_{ad} = \frac{1}{2} + \frac{\sqrt{\lambda_{ad}^2 + P^2|h_{ad}|^4 + 2P|h_{ad}|^2} - \lambda_{ad}}{2P|h_{ad}|^2}, \tag{21}$$

where $|h_{ad}|^2 = |h_1|^2 = |h_2|^2 = |h_3|^2$ and λ_{ad} is given by

$$\lambda_{ad} = 2P_s|h_{ad}|^2 + 1. \tag{22}$$

From (21), we can know that α_{ad} is greater than $1/2$. In other words, we should allocate more power to the first relay than the second relay to maximize the achievable rate when the channel gains are the same.

When $|h_{ad}|^2 = |h_1|^2 = |h_2|^2 = |h_3|^2$, the SNR for AF-DF relay network in (7) is rewritten as

$$\gamma_{ad} = \min\left(\gamma_{ad,s2}, \gamma_{ad,2d}\right), \tag{23}$$

where $\gamma_{ad,s2}$ and $\gamma_{ad,2d}$ are given, respectively, as

$$\gamma_{ad,s2} = \frac{P_sP_{ad,1}|h_{ad}|^4}{P_s|h_{ad}|^2 + P_{ad,1}|h_{ad}|^2 + 1}, \tag{24}$$

$$\gamma_{ad,2d} = P_{ad,2}|h_{ad}|^2. \tag{25}$$

Because $\gamma_{ad,s2}$ has a similar form of the harmonic mean of $P_s|h_{ad}|^2$ and $P_{ad,1}|h_{ad}|^2$, the increment of $\gamma_{ad,s2}$ is less than that of $P_{ad,1}$ as $P_{ad,1}$ increases. On the other hand, the increment of $\gamma_{ad,2d}$ is equal to that of $P_{ad,2}$ as $P_{ad,2}$ increases. Therefore, the increment of $\gamma_{ad,s2}$ is less than that of $\gamma_{ad,2d}$ when the increments of $P_{ad,1}$ and $P_{ad,2}$ are the same. To maximize the minimum value between $\gamma_{ad,s2}$ and $\gamma_{ad,2d}$ in (23), it is necessary to further increase $\gamma_{ad,s2}$ which does not increase as much as $\gamma_{ad,2d}$. In addition, to increase $\gamma_{ad,s2}$ more than $\gamma_{ad,2d}$, we should allocate more power at the first relay than the second relay.

3.2. DF and AF relay network

In this subsection, we propose the optimal power allocation scheme for three-hop DF and AF relay network.

The optimization problem to maximize the achievable rate under a sum relay power constraint can be written as

$$\max_{P_{da,1}, P_{da,2}} R_{da}(P_{da,1}, P_{da,2}), \quad s.t. \; P_{da,1} + P_{da,2} = P. \tag{26}$$

We define the ratio of $P_{da,1}$ to P as α_{da}. By using α_{da}, the optimization problem is rewritten as

$$\max_{\alpha_{da}} \gamma_{da}(\alpha_{da}), \quad s.t. \; 0 < \alpha_{da} < 1, \tag{27}$$

where $\gamma_{da}(\alpha_{da})$ is given as

$$\gamma_{da}(\alpha_{da}) = \frac{\min(P_s|h_1|^2, \alpha_{da}P|h_2|^2)P(1-\alpha_{da})|h_3|^2}{\min(P_s|h_1|^2, \alpha_{da}P|h_2|^2) + P(1-\alpha_{da})|h_3|^2 + 1}. \tag{28}$$

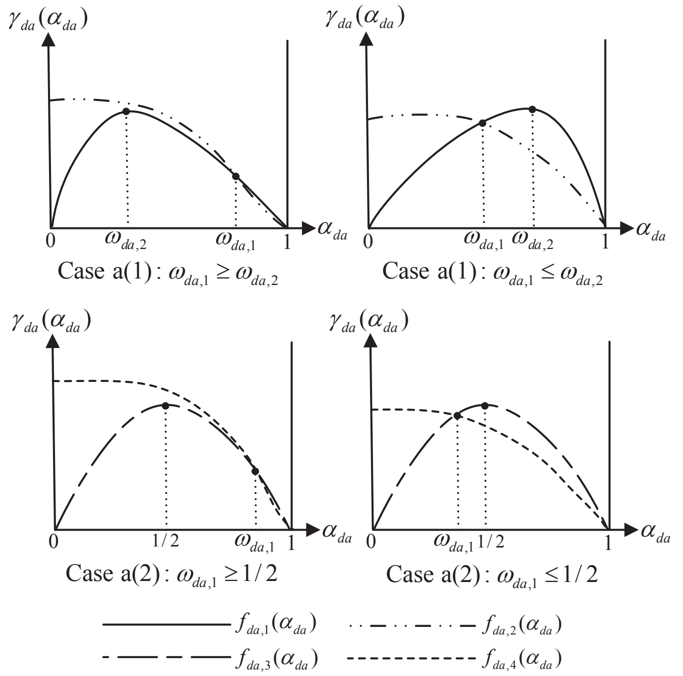

Figure 2. Possible cases of α_{da} in Case a for DF-AF relay network.

To determine $\min(P_s|h_1|^2, \alpha_{da}P|h_2|^2)$ in (28), we take into account two cases as follows.

3.2.1. Case a ($P_s|h_1|^2 < P|h_2|^2$)

We define $\omega_{da,1}$ as

$$\omega_{da,1} = \frac{P_s|h_1|^2}{P|h_2|^2}. \tag{29}$$

When $0 < \alpha_{da} \le \omega_{da,1}$, we can obtain the relation $\alpha_{da}P|h_2|^2 < P_s|h_1|^2$. Therefore, $\min(P_s|h_1|^2, \alpha_{da}P|h_2|^2)$ is determined as $\alpha_{da}P|h_2|^2$. In addition, when $\omega_{da,1} \le \alpha_{da} < 1$, we can obtain the relation $\alpha_{da}P|h_2|^2 > P_s|h_1|^2$. Therefore, $\min(P_s|h_1|^2, \alpha_{da}P|h_2|^2)$ is determined as $P_s|h_1|^2$. In other words,

$$\min(P_s|h_1|^2, \alpha_{da}P|h_2|^2) = \begin{cases} \alpha_{da}P|h_2|^2 & \text{for } 0 < \alpha_{da} \le \omega_{da,1} \\ P_s|h_1|^2 & \text{for } \omega_{da,1} \le \alpha_{da} < 1 \end{cases} \tag{30}$$

Firstly, we consider the Case a(1) when $|h_2|^2 \neq |h_3|^2$.
By using (30), the optimization problem in (27) is rewritten as

$$\max_{\alpha_{da}} \gamma_{da}(\alpha_{da}) = \begin{cases} f_{da,1}(\alpha_{da}) & \text{for } 0 < \alpha_{da} \le \omega_{da,1} \\ f_{da,2}(\alpha_{da}) & \text{for } \omega_{da,1} \le \alpha_{da} < 1 \end{cases} \quad s.t. \ 0 < \alpha_{da} < 1, \tag{31}$$

where $f_{da,1}(\alpha_{da})$ and $f_{da,2}(\alpha_{da})$ are given by

$$f_{da,1}(\alpha_{da}) = \frac{P^2|h_2|^2|h_3|^2(\alpha_{da} - \alpha_{da}^2)}{\alpha_{da}P(|h_2|^2 - |h_3|^2) + P|h_3|^2 + 1}, \tag{32}$$

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Algorithm 1 Three-hop DF and AF relay network

Input: P_s, P, $|h_1|^2$, $|h_2|^2$, $|h_3|^2$
Output: α_{da}
Calculate $\omega_{da,1} = \frac{P_s|h_1|^2}{P|h_2|^2}$.
Calculate $\omega_{da,2} = \frac{\sqrt{\lambda_{da}} - P|h_3|^2 - 1}{P(|h_2|^2 - |h_3|^2)}$,
where $\lambda_{da} = P^2(|h_2|^2|h_3|^2 + |h_2|^2 - |h_3|^2) + 2P|h_3|^2 + 1$.
if $P_s|h_1|^2 < P|h_2|^2$ **then**
  if $|h_2|^2 \neq |h_3|^2$ **then**
    if $\omega_{da,1} \leq \omega_{da,2}$ **then**
      $\alpha_{da} = \omega_{da,1}$.
    else if $\omega_{da,1} \geq \omega_{da,2}$ **then**
      $\alpha_{da} = \omega_{da,2}$.
    end if
  else if $|h_2|^2 = |h_3|^2$ **then**
    if $\omega_{da,1} \leq 1/2$ **then**
      $\alpha_{da} = \omega_{da,1}$.
    else if $\omega_{da,1} \geq 1/2$ **then**
      $\alpha_{da} = 1/2$.
    end if
  end if
else if $P_s|h_1|^2 \geq P|h_2|^2$ **then**
  if $|h_2|^2 \neq |h_3|^2$ **then**
    $\alpha_{da} = \omega_{da,2}$.
  else if $|h_2|^2 = |h_3|^2$ **then**
    $\alpha_{da} = 1/2$.
  end if
end if

$$f_{da,2}(\alpha_{da}) = \frac{(1-\alpha_{da})PP_s|h_1|^2|h_3|^2}{(1-\alpha_{da})P|h_3|^2 + P_s|h_1|^2 + 1}. \tag{33}$$

Taking partial derivative of (32) with respect to α_{da} and equating it to zero, we can obtain $\omega_{da,2}$ which maximizes $f_{da,1}(\alpha_{da})$ as

$$\omega_{da,2} = \frac{\sqrt{\lambda_{da}} - (P|h_3|^2 + 1)}{P(|h_2|^2 - |h_3|^2)}, \tag{34}$$

where $\lambda_{da} = P^2(|h_2|^2|h_3|^2 + |h_2|^2 - |h_3|^2) + 2P|h_3|^2 + 1$.

The $f_{da,1}(\alpha_{da})$ increases for $0 < \alpha_{da} \leq \omega_{da,2}$ and decreases for $\omega_{da,2} \leq \alpha_{da} < \omega_{da,1}$. Since $\frac{\partial f_{da,2}(\alpha_{da})}{\partial \alpha_{da}}$ is less than zero, $f_{da,2}(\alpha_{da})$ decreases as α_{da} increases for $\omega_{da,1} < \alpha_{da} < 1$. As a result, the optimal α_{da} is $\omega_{da,2}$ when $\omega_{da,1} \geq \omega_{da,2}$ and $\omega_{da,1}$ when $\omega_{da,1} \leq \omega_{da,2}$.

Secondly, we consider the Case a(2) when $|h_2|^2 = |h_3|^2$.

By using (30) and $|h_{da}|^2 = |h_2|^2 = |h_3|^2$, the optimization problem in (27) is rewritten as

$$\max_{\alpha_{da}} \ \gamma_{da}(\alpha_{da}) = \begin{cases} f_{da,3}(\alpha_{da}) & \text{for} \quad 0 < \alpha_{da} \leq \omega_{da,1} \\ f_{da,4}(\alpha_{da}) & \text{for} \quad \omega_{da,1} \leq \alpha_{da} < 1 \end{cases} \qquad s.t. \ \ 0 < \alpha_{da} < 1, \tag{35}$$

where $f_{da,3}(\alpha_{da})$ and $f_{da,4}(\alpha_{da})$ are given by

$$f_{da,3}(\alpha_{da}) = \frac{P^2|h_{da}|^4(\alpha_{da} - \alpha_{da}{}^2)}{P|h_{da}|^2 + 1}, \tag{36}$$

$$f_{da,4}(\alpha_{da}) = \frac{(1-\alpha_{da})PP_s|h_1|^2|h_{da}|^2}{(1-\alpha_{da})P|h_{da}|^2 + P_s|h_1|^2 + 1}. \tag{37}$$

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By solving the equation $\frac{\partial f_{da,3}(\alpha_{da})}{\partial \alpha_{da}} = 0$, we can know that $f_{da,3}(\alpha_{da})$ has a maximum value when $\alpha_{da} = 1/2$. Therefore, $f_{da,3}(\alpha_{da})$ increases for $0 < \alpha_{da} \leq 1/2$ and decreases for $1/2 \leq \alpha_{da} < \omega_{da,1}$. Since $\frac{\partial f_{da,4}(\alpha_{da})}{\partial \alpha_{da}}$ is less than zero, $f_{da,4}(\alpha_{da})$ decreases as α_{da} increases for $\omega_{da,1} \leq \alpha_{da} < 1$. As a result, the optimal α_{da} is $1/2$ when $\omega_{da,1} \geq 1/2$ and $\omega_{da,1}$ when $\omega_{da,1} \leq 1/2$.

Fig. 2 shows the possible cases of α_{da} in Case a.

3.2.2. Case b ($P_s|h_1|^2 \geq P|h_2|^2$)

Since $0 < \alpha_{da} < 1$, we can derive the relation $P_s|h_1|^2 \geq \alpha_{da}P|h_2|^2$. Therefore, $\min(P_s|h_1|^2, \alpha_{da}P|h_2|^2)$ in (28) is determined as $\alpha_{da}P|h_2|^2$.

Firstly, we consider the Case b(1) when $|h_2|^2 \neq |h_3|^2$.

The optimization problem in (27) is rewritten as

$$\max_{\alpha_{da}} \ f_{da,5}(\alpha_{da}), \quad s.t. \ 0 < \alpha_{da} < 1, \tag{38}$$

where $f_{da,5}(\alpha_{da})$ is given by

$$f_{da,5}(\alpha_{da}) = \frac{P^2|h_2|^2|h_3|^2(\alpha_{da} - \alpha_{da}{}^2)}{\alpha_{da}P(|h_2|^2 - |h_3|^2) + P|h_3|^2 + 1}. \tag{39}$$

By solving the equation $\frac{\partial f_{da,5}(\alpha_{da})}{\partial \alpha_{da}} = 0$, the optimal α_{da} is obtained as

$$\alpha_{da} = \frac{\sqrt{\lambda_{da}} - (P|h_3|^2 + 1)}{P(|h_2|^2 - |h_3|^2)}, \tag{40}$$

where $\lambda_{da} = P^2(|h_2|^2|h_3|^2 + |h_2|^2 - |h_3|^2) + 2P|h_3|^2 + 1$.

From (34), we can know that α_{da} is the same as $\omega_{da,2}$.

Secondly, we consider the Case b(2) when $|h_2|^2 = |h_3|^2$.

By using $|h_{da}|^2 = |h_2|^2 = |h_3|^2$, the optimization problem in (27) is rewritten as

$$\max_{\alpha_{da}} \ f_{da,6}(\alpha_{da}), \quad s.t. \ 0 < \alpha_{da} < 1, \tag{41}$$

where $f_{da,6}(\alpha_{da})$ is given by

$$f_{da,6}(\alpha_{da}) = \frac{P^2|h_{da}|^4(\alpha_{da} - \alpha_{da}{}^2)}{P|h_{da}|^2 + 1}. \tag{42}$$

By solving the equation $\frac{\partial f_{da,6}(\alpha_{da})}{\partial \alpha_{da}} = 0$, the optimal α_{da} is obtained as $1/2$.

From (29) and (34), we can know that $\omega_{da,1}$ does not depend on $|h_3|^2$ and $\omega_{da,2}$ does not depend on P_s and $|h_1|^2$.

The Algorithm 1 explains the procedure to find α_{da} for DF and AF relay network.

4. Simulation Results

This section presents the achievable rates of the proposed and equal power allocation schemes for hybrid three-hop relay networks. For the equal power allocation schemes, α_{ad} and α_{da} are fixed to $1/2$.

Fig. 3(a) and Fig. 3(b) show the achievable rates and α_{ad} for AF-DF relay network when $P_s = 10$ dB. From Fig. 3(a), it is observed that the achievable rate of the optimal power allocation scheme is greater than that of the equal power allocation scheme regardless of channel gains. 1. Among the results in Fig. 3(a), the achievable data rate expressed by dashed line of optimal power allocation and equal power allocation converges after the power constraint of 10dB. This can be understood from (17) and (18). After the power constraint of 10dB, γ_{ad} of the two allocation schemes is determined

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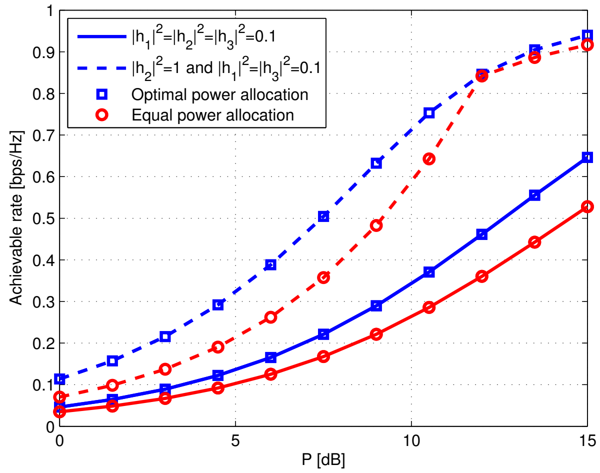
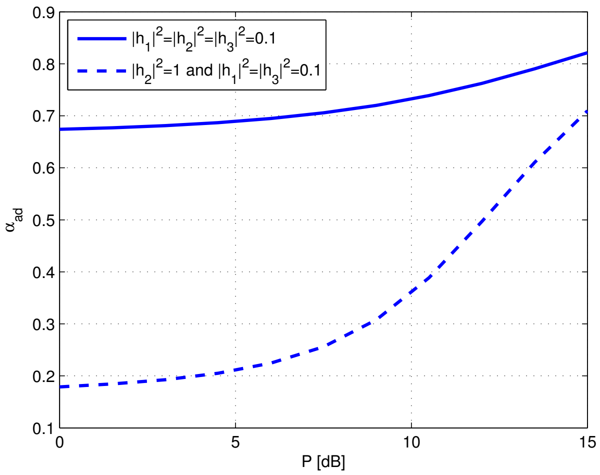


(a) Achievable rates for the power allocation schemes.



(b) α_{ad} for the optimal power allocation scheme.

Figure 3. Performance for AF-DF relay network when $P_s = 10$ dB.

by (17) and is hardly subject to α_{ad}. From Fig. 3(b), we can know that α_{ad} is greater than 1/2 when
185 $|h_1|^2 = |h_2|^2 = |h_3|^2 = 0.1$. As mentioned, the SNR at destination γ_{ad} is determined by minumum value between SNR at the second relay $\gamma_{ad,s2}$ and SNR of third hop $\gamma_{ad,2d}$. In addition, the increment of $\gamma_{ad,s2}$ is less than that of $\gamma_{ad,2d}$ when increments of the transmit power from each relay are the same. To maximize γ_{ad}, we need to further increase $\gamma_{ad,s2}$ which does not increase as much as $\gamma_{ad,2d}$. Therefore, we should allocate more power at the first relay than the second relay to increase $\gamma_{ad,s2}$ more than
190 $\gamma_{ad,2d}$.

Fig. 4(a) and Fig. 4(b) show the achievable rates and α_{da} for DF-AF relay network when $P_s = 10$ dB. The Case a(1) an a(2) are described when $|h_2|^2 \neq |h_3|^2$ and $|h_2|^2 = |h_3|^2$, respectively. Then, the Case b(1) is described when $|h_2|^2 \neq |h_3|^2$. From Fig. 4(a), it is observed that the optimal power allocation schemes for Case a(1) and b(1) provide a higher achievable rate than the equal power
195 allocation scheme. The α_{da} decreases for Case a(1) and increases for Case b(1) as P increases. As mentioned in Case a(2), α_{da} is 1/2 for $\frac{P_s|h_1|^2}{P|h_2|^2} \geq 1/2$ and $\frac{P_s|h_1|^2}{P|h_2|^2}$ for $\frac{P_s|h_1|^2}{P|h_2|^2} \leq 1/2$. In other words, α_{da} is 1/2 for $P \leq \frac{2P_s|h_1|^2}{|h_2|^2}$ and $\frac{P_s|h_1|^2}{P|h_2|^2}$ for $P \geq \frac{2P_s|h_1|^2}{|h_2|^2}$. When $P_s = 10$ dB and $|h_1|^2 = |h_2|^2 = 0.2$, $\frac{2P_s|h_1|^2}{|h_2|^2}$ is 13.0103 dB. Therefore, as shown in Fig. 4(b), α_{da} for Case a(2) is 1/2 for $P \leq 13.0103$ dB and $10^{(1-0.1P)}$

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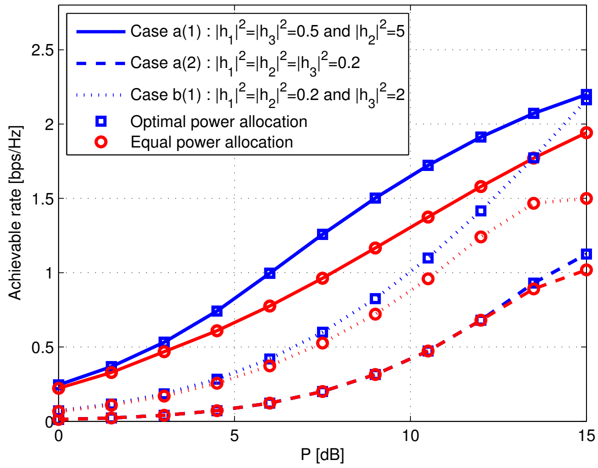
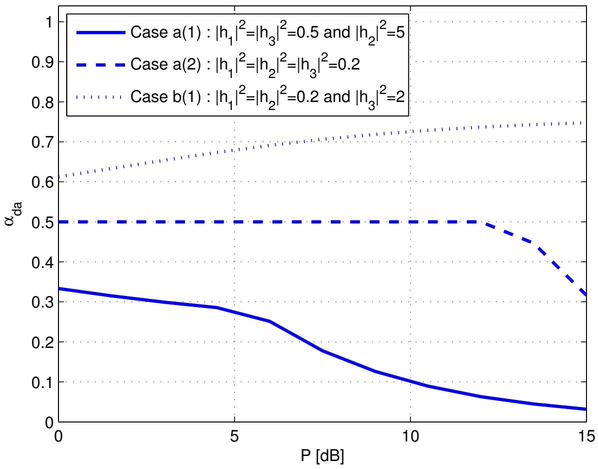


(a) Achievable rates for the power allocation schemes.



(b) α_{da} for the optimal power allocation scheme.

Figure 4. Performance for DF-AF relay network when $P_s = 10$ dB.

for $P \geq 13.0103$ dB. The achievable rate of the optimal power allocation scheme for Case a(2) is the same as that of the equal power allocation scheme when $P \leq 13.0103$ dB. Then, the optimal power allocation scheme for Case a(2) provides a higher achievable rate than the equal power allocation scheme when $P \geq 13.0103$ dB.

Fig. 5(a) and Fig. 5(b) show the achievable rates of the proposed power allocation schemes for hybrid three-hop relay networks when SNR of the first hop γ_{s1} is 0 dB, -3 dB and -6 dB. When $|h_3|^2$ has a greater value than $|h_2|^2$, it is observed that the achievable rate for DF-AF relay network is greater than that of AF-DF relay networks regardless of γ_{s1}. On the other hand, the achievable rate for AF-DF relay network is greater than that of DF-AF relay networks when $|h_2|^2$ has a greater value than $|h_3|^2$. As mentioned, the SNR at destination γ_{ad} for AF-DF relay network is determined as the minimum value between the SNR at the second relay $\gamma_{ad,s2}$ and the SNR of the third relay $\gamma_{ad,2d}$. Because $\gamma_{ad,s2}$ has a form similar to harmonic mean between γ_{s1} and SNR of the second hop, the increment of $\gamma_{ad,s2}$ is less than that of $\gamma_{ad,2d}$ when the increments in SNR of each hop are the same. Therefore, to maximize γ_{ad}, we need to further increase $\gamma_{ad,s2}$ which does not increase as much as $\gamma_{ad,2d}$. For a given γ_{s1}, $\gamma_{ad,s2}$ can be increased by increasing $|h_2|^2$. Unlike the AF-DF relay network, the SNR at destination γ_{da}

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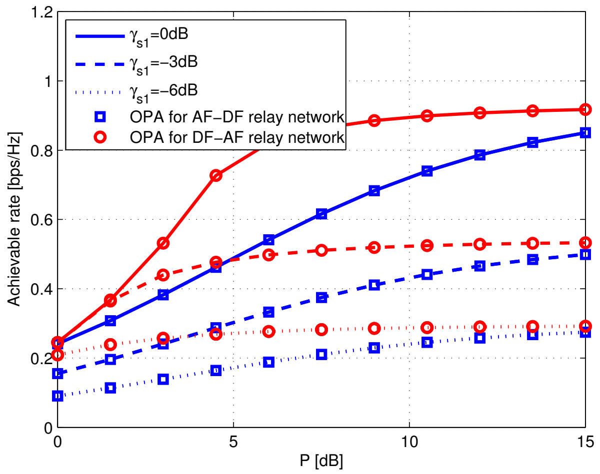
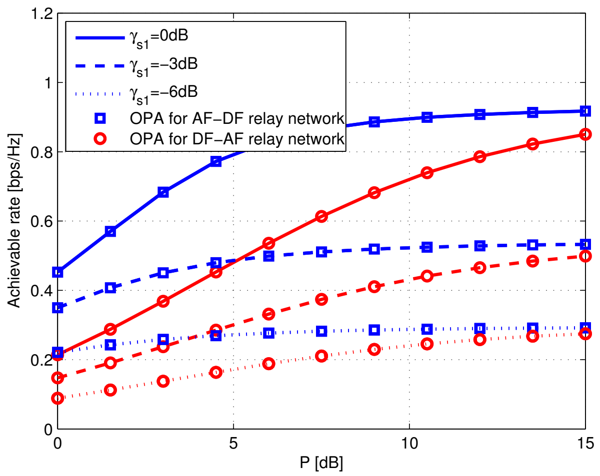


(a) Propoed power allocation schemes when $|h_2|^2 = 0.5$ and $|h_3|^2 = 5$.



(b) Propoed power allocation schemes when $|h_2|^2 = 5$ and $|h_3|^2 = 0.5$.

Figure 5. Achievable rates for the proposed power allocation schemes.

for DF-AF relay network has a similar form of the harmonic mean between γ_{s1} and the SNR of the third hop when γ_{s1} is less than the SNR of the second hop. For a given γ_{s1}, γ_{da} can be increased by increasing $|h_3|^2$. Therefore, γ_{ad} has a greater value than γ_{da} when $|h_2|^2$ is sufficiently larger than $|h_3|^2$. On the other hand, γ_{da} has a greater value than γ_{ad} when $|h_3|^2$ is sufficiently larger than $|h_2|^2$.

Fig. 6(a) and Fig. 6(b) show the achievable rates of the equal power allocation schemes for hybrid three-hop relay networks when SNR of the first hop γ_{s1} is 0 dB, -3 dB and -6 dB. As shown in Fig. 5(a) and Fig. 5(b), the achievable rate of DF-AF relay network is greater than that of AF-DF relay networks when $|h_3|^2$ has a greater value than $|h_2|^2$ and vice versa. It is observed that the achievable rates of the equal power allocation schemes are lower than that of the proposed power allocation schemes.

From Fig. 3 to Fig. 6, it is noticed that proposed optimal allocation uses less P than equal power allocation for keeping same achievable rate. Also, the results consider achievable rate per unit bandwidth. Therefore, the advantage increases linearly according to the bandwidth of systems. Since bandwidth of recent communication systems has been increased continuously to accommodate

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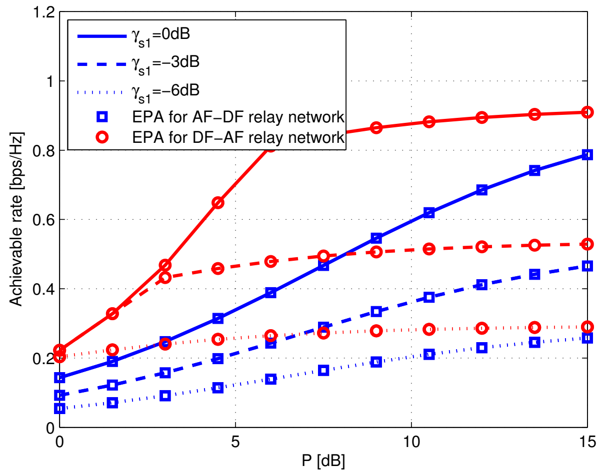
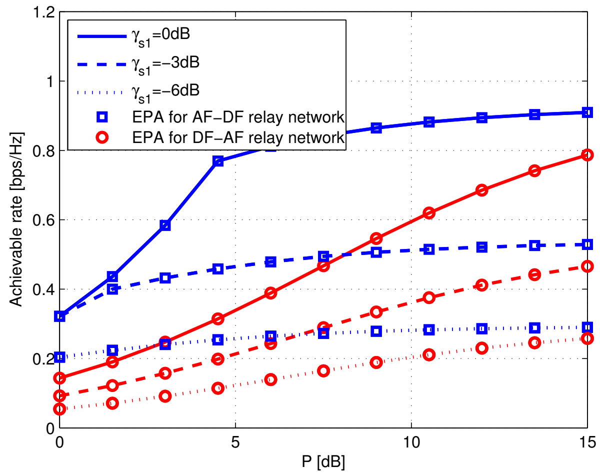


(a) Equal power allocation schemes when $|h_2|^2 = 0.5$ and $|h_3|^2 = 5$.



(b) Equal power allocation schemes when $|h_2|^2 = 5$ and $|h_3|^2 = 0.5$.

Figure 6. Achievable rates for the equal power allocation schemes.

future data traffic, proposed optimal allocation scheme can contribute to power efficiency of the recent wideband systems.

230 The simulation results in Fig. 5 and 6 show that the appropriate hybrid relaying according to the channel condition has significant performance improvement compared with power allocation scheme. For further comparisons, the power allocation schemes of [22] and [23] are referenced. Since the system model for each paper is different, we refer to the simulation results of each paper. In [22], the proposed power allocation is used to improve diversity gain by cooperative transmission in the hybrid 235 decode-amplify-forward cooperative communication system. Simulation results for the achievable rates according to power usage showed about 3dB performance improvement compared with equal power allocation. Also, in [23], the proposed power allocation was used to improve diversity gain by cooperative transmission with relay selection in the cooperative communication system with multiple relays. Simulation results for the achievable rates according to power usage showed the performance 240 improvement of less than 1dB. However, from the simulation results of Fig. 5 and 6, according to channel condition, we confirm the performance improvement over 5dB at in range of $5 \sim 10$dB of used power by using appropriate hybrid relaying compared with other types of hybrid relaying.

Therefore, the analysis of hybrid relaying and power allocation according to the channel condition yields meaningful results.

5. Conclusion

Under a sum relay power constraint, this paper proposed the optimal power allocation schemes to maximize the achievable rates for hybrid three-hop relay networks when the channel gains and the transmit power from source are given. By solving the optimization problem, we derived the transmit power value from the first relay in closed-form for AF-DF and DF-AF relay networks. When the channel gains are the same for the AF-DF relay network, we showed that more power should be allocated at the first relay than the second relay to maximize the achievable rate. In addition, we derived the optimal power allocation scheme for DF-AF relay network for the possible four cases. When the SNR of the first hop is the same, it is shown that the optimal power allocation scheme for AF-DF relay network provides a higher achievable rate than that for DF-AF relay network when the channel gain between two relays is higher than that between the second relay and destination. On the contrary, the achievable rate of DF-AF relay network is greater than that of AF-DF relay network when the channel gain between the second relay and destination is higher than that between two relays. We can choose the optimal power allocation scheme which can provide the best performace in a given environment. Both the analytical solutions and simulation results have shown that the proposed power allocation schemes outperform the equal power allocation schemes.

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Author Contributions: Junpyo Jeon and Yeonggyu Shim conceived and designed the experiments; Junpyo Jeon performed the experiments; Junpyo Jeon and Yeonggyu Shim analyzed the data; Junpyo Jeon wrote the paper; Hyuncheol Park has approved the submitted version.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

AF	amplify-and-forward
DF	decode-and-forward
SNR	signal-to-noise ratio
SER	symbol error rate
BER	bit error rate
CSI	channel state information
AWGN	additive white Gaussian noise

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