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Article

A Simple Spectral Observer

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Abstract: The principal aim of a spectral observer is twofold: the reconstruction of a signal of time via state estimation and the decomposition of such a signal into the frequencies that make it up. This paper proposes a novel spectral observer with an adjustable constant gain for reconstructing a given signal by means of the recursive identification of the coefficients of a Fourier series. The reconstruction or estimation of a signal in the context of this work means to find the coefficients of a linear combination of sines a cosines that fits a signal such that it can be reproduced. The design procedure of the spectral observer is presented along with the following applications: (1) the reconstruction of a simple periodical signal, (2) the approximation of both a square and a triangular signal, (3) the edge detection in signals by using the Fourier coefficients and (4) the fitting of the historical Bitcoin market data from 2014-12-01 to 2018-01-08.

Keywords: Signal processing; Fourier series; state observer

1. Introduction

The term spectral observer was proposed by Gene H. Hostetter in his pioneering work [1] to name the algorithm that permits the recursive calculation of the Fourier transform (FT) of a band-limited signal via state estimation. Since the presentation of such a work, several designs of spectral observers with improved features has been proposed either to deal with noise [2], disturbances, lack of data [3] or to estimate other parameters such as frequency [4]. The main goals of a spectral observer are both the estimation of a given signal and the transformation of such a signal to the frequency domain by means of the recursive identification of the coefficients of a Fourier series [5]. The estimation of a signal in the context of this work means to find the coefficients of a linear combination of functions —sines a cosines functions in our case— that approximates a signal of interest such that it can be reconstructed [6]. Spectral observers are useful in a wide number of applications, e.g., for determining the source of harmonic pollution in power systems [7], for the simulation of the sea surface [8], for fault diagnosis in motors [9], [10] or in vibrating structures, such as aerospace and mechanical structures, marine structures, buildings, bridges and offshore platforms. The observer that we propose in this contribution is designed from a dynamical system which is constructed from the N derivatives of a n-th order Fourier series. To perform the estimation, the observer solely requires: (1) The measurement of the signal to be approximated, $y(t)$, which actually is used to compute the observation error $e(t) = y(t) - \hat{y}(t)$, where $\hat{y}(t)$ is the observer output. (2) A frequency step $\Delta\omega = 2\pi/\Delta T$, where ΔT is a period step that must satisfy the Nyquist-Shannon

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sampling theorem. The estimation provided by the observer are both a signal that approximates the original signal and Fourier coefficients to compute the magnitude and phase spectrums.

This paper is organized as follows: Section 2 presents the core of the proposed method which is the formulation of the spectral observer from the Fourier series. Section 3 presents some examples with test results of the proposed method utilized in different applications. In Section 4 the main results are discussed. Finally, in Section 5 some concluding thoughts are given.

2. The Proposed Method

To construct the proposed observer, we formulate a dynamical synthetic system in state space representation by considering, firstly, that a given signal expressed as $s(t)$ can be approximated by a Fourier series, and secondly, that the Fourier series is the first state of the system and the rest of the states are the N first-order derivatives of the Fourier series expressed by Eq. (1), where n is the series order.

$$y(t) = \frac{a_0}{2} + \sum_{k=1}^{n} \left[a_k \cos(k\omega t) + b_k \sin(k\omega t) \right], \; k \in z. \tag{1}$$

where $a_0, a_1, b_1, ..., a_n, b_n$ are the Fourier coefficients and ω is the fundamental angular frequency of the signal to be estimated.

Before formulating the dynamical system, we remove the term a_0 from Eq. (1) since the offset can be estimated through the coefficients $a_0, a_1, b_1, ..., a_n, b_n$ as a constant component of them. Then, the series for approximating the time function can be expressed as follows

$$y(t) = \sum_{k=1}^{n} \left[a_k \cos(k\omega t) + b_k \sin(k\omega t) \right]. \tag{2}$$

If the order of the Fourier series is $n = 1$, we need to formulate a dynamical system with $N = 2$ states, each one to recover each coefficient (a_1 and b_1). Thus, the two first states are the Fourier series and its first derivative.

$$v_1(t) = y(t) = a_1 \cos(\omega t) + b_1 \sin(\omega t), \quad v_2(t) = \dot{y}(t) = -\omega a_1 \sin(\omega t) + \omega b_1 \cos(\omega t). \tag{3}$$

where v_i are the states of the synthetic system. Consequently, the dynamical system that results from the a change of coordinates, gives:

$$\dot{v}_1(t) = v_2(t), \quad \dot{v}_2(t) = -\omega^2 v_1(t), \tag{4}$$

which basically is the dynamical model of an harmonic oscillator. Now, what happens if the order of the Fourier series increases? If the order increases to $n = 2$, then $N = 4$, since we need to recover four coefficients.

$$\begin{aligned}
v_1(t) &= y(t) = a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_2 \cos(2\omega t) + b_2 \sin(2\omega t), \\
v_2(t) &= \dot{y}(t) = -\omega a_1 \sin(\omega t) + \omega b_1 \cos(\omega t) - 2\omega a_2 \sin(2\omega t) + 2\omega b_2 \cos(2\omega t), \\
v_3(t) &= \ddot{y}(t) = -\omega^2 a_1 \cos(\omega t) - \omega^2 b_1 \sin(\omega t) - 4\omega^2 a_2 \cos(2\omega t) - 4\omega^2 b_2 \sin(2\omega t), \\
v_4(t) &= y^{(3)}(t) = \omega^3 a_1 \sin(\omega t) - \omega^3 b_1 \cos(\omega t) + 8\omega^3 a_2 \sin(2\omega t) - 8\omega^3 b_2 \cos(2\omega t), \\
\dot{v}_4(t) &= y^{(4)}(t) = \omega^4 a_1 \cos(\omega t) + \omega^4 b_1 \sin(\omega t) + 16\omega^4 a_2 \cos(2\omega t) + 16\omega^4 b_2 \sin(2\omega t).
\end{aligned} \tag{5}$$

The dynamical system is then formulated as in Eq. (5) which, after some algebraic manipulations, it becomes

$$\dot{v}_1(t) = v_2(t); \quad \dot{v}_2(t) = v_3(t), \dot{v}_3(t) = v_4(t), \dot{v}_4(t) = -4\omega^4 v_1(t) - 5\omega^2 v_3(t), \tag{6}$$

in $v-$coordinates.

By generalizing Eq. (5) for order n, the dynamical system becomes:

$$
\begin{aligned}
\dot{v}_1(t) &= v_2(t), \\
\dot{v}_2(t) &= v_3(t), \\
\vdots &= \vdots, \\
\dot{v}_N(t) &= (-1)^{n(\mathrm{mod}\ 2)}\omega^{2n}, \begin{bmatrix} 1 & 2^{2n} & \cdots & n^{2n} \end{bmatrix} A_k^{-1} A_\omega^{-1} v(t),
\end{aligned}
\tag{7}
$$

where $N = 2n$, A_ω and A_k are expressed by Eq. (8) and Eq. (9), respectively.

$$
A_\omega \doteq \begin{bmatrix}
1 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\
0 & \omega & 0 & 0 & \cdots & 0 & \cdots & 0 \\
0 & 0 & -\omega^2 & 0 & \cdots & 0 & \cdots & 0 \\
0 & 0 & 0 & -\omega^3 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & (-1)^{(m(\mathrm{mod}\ 4) - m(\mathrm{mod}\ 2))/2}\,\omega^m & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & \cdots & (-1)^{(2n(\mathrm{mod}\ 4) - 2)/2}\,\omega^{2n-1}
\end{bmatrix},
\tag{8}
$$

$$
A_k \doteq \begin{bmatrix}
1 & 0 & 1 & 0 & \cdots & 1 & 0 \\
0 & 1 & 0 & 2 & \cdots & 0 & n \\
1 & 0 & 4 & 0 & \cdots & n^2 & 0 \\
0 & 1 & 0 & 8 & \cdots & 0 & n^3 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
1 & 0 & 2^{2n-2} & 0 & \cdots & n^{2n-2} & 0 \\
0 & 1 & 0 & 2^{2n-1} & \cdots & 0 & n^{2n-1}
\end{bmatrix}.
\tag{9}
$$

Before presenting the state observer for system (7), it is necessary to analyze its observability conditions. A dynamical system is said to be observable if it is possible to determine its initial state by knowledge of the input and output over a finite time interval. In this way, a state observer or estate estimator is a system that estimate the internal states of a system from the measured of its inputs and outputs. There are several ways to determine if a given system is observable, one of them is the observability rank condition which can be defined as:

Observability rank condition. A system

$$
\begin{aligned}
\dot{\varsigma}(t) &= A(t)\varsigma(t) + \varphi(y(t)) \\
y(t) &= C\varsigma(t)
\end{aligned}
\tag{10}
$$

is said to satisfy the observability rank condition if $\forall \varsigma(t), \mathrm{rank}\left(O(\varsigma(t))\right) = N$, where N is the state dimension of (10) and $O(\varsigma(t))$ is the observability matrix defined as

$$
O(\varsigma(t)) = \frac{\partial T(\varsigma(t))}{\partial \varsigma(t)},
\tag{11}
$$

where the observability mapping $T(\varsigma(t))$ is defined as follows

$$
T(\varsigma(t)) = (C, CA, CA^2, ..., CA^{(N-1)}) = (y(t), \dot{y}(t), \ddot{y}(t), ..., y(t)^{(N-1)})^T.
\tag{12}
$$

Thus, according to the definition, the observability matrix for system (7), which in fact can be set as system (10) with

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & & \ddots & \vdots \\ \vdots & & & & 1 \\ \gamma_1(\omega) & & \dots & & \gamma_n(\omega) \end{bmatrix}, \tag{13}$$

and $C = [1, 0, ..., 0]$, is given by Eq. (14) and has full rank. Notice that $\blacktriangle \triangleq \cos$, $\blacktriangledown \triangleq \sin$ and $\Gamma \triangleq n-1$.

$$O(v(t)) = \begin{pmatrix} \blacktriangle(\omega x) & \blacktriangledown(\omega x) & \blacktriangle(2\omega x) & \blacktriangledown(2\omega x) & \dots & \blacktriangle(n\omega x) & \blacktriangledown(n\omega x) \\ -\omega\blacktriangledown(\omega x) & \omega\blacktriangle(\omega x) & -2\omega\blacktriangledown(2\omega x) & 2\omega\blacktriangle(2\omega x) & \dots & -n\omega\blacktriangledown(n\omega x) & n\omega\blacktriangle(n\omega x) \\ -\omega^2\blacktriangle(\omega x) & -\omega^2\blacktriangledown(\omega x) & -4\omega^2\blacktriangle(2\omega x) & -4\omega^2\blacktriangledown(2\omega x) & \dots & -n^2\omega\blacktriangle(n\omega x) & -n^2\omega\blacktriangledown(n\omega x) \\ \vdots & & & & & & \vdots \\ \omega^\Gamma\blacktriangledown(\omega x) & -\omega^\Gamma\blacktriangle(\omega x) & 2^\Gamma\omega^\Gamma\blacktriangledown(2\omega x) & -2^\Gamma\omega^\Gamma\blacktriangle(2\omega x) & \dots & n^\Gamma\omega^\Gamma\blacktriangledown(n\omega x) & -n^\Gamma\omega^\Gamma\blacktriangle(n\omega x) \end{pmatrix} \tag{14}$$

Since system (7) is observable, a spectral observer can be designed as follows:

$$\dot{\hat{v}}(t) = A\hat{v}(t) + K(y(t) - C\hat{v}(t)), \quad \hat{y}(t) = C\hat{v} = \hat{v}_1. \tag{15}$$

where $\hat{}$ means estimation, $\hat{y}(t)$ is the estimated signals and A is given by (13), with constant coefficients expressed by $\gamma_i(\omega)$. The gain of the state observer K, involved in the correction term of Eq. (15), can be calculated as $K = S^{-1}C^T$, where S is the unique solution of the following algebraic Lyapunov equation:

$$-\lambda S - A^T S - SA + C^T C = 0 \tag{16}$$

and λ is a parameter that can be used to tune the convergence rate of the observer. The Fourier coefficients can be recovered from the new coordinates by the relation $\hat{c}_k(t) = O^{-1}(v(t))\hat{v}(t)$, where

$$\hat{c}_k(t) = [\hat{a}_1(t) \;\; \hat{b}_1(t) \;\; ... \;\; \hat{a}_n(t) \;\; \hat{b}_n(t)]^T.$$

Check Fig. 1 to see a schema of the estimation.

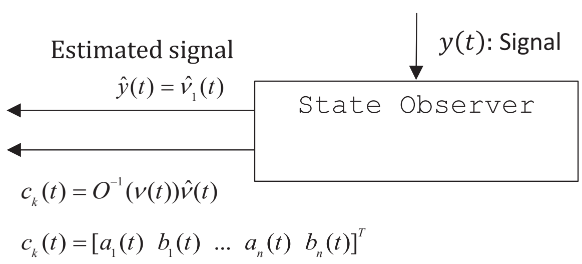

Figure 1. Schema of the reconstruction of a signal by using the spectral observer

Before presenting some possible applications of the spectral observer it is important make a point: Notice that matrix A of the spectral observer depends on the fundamental frequency ω. This means that this variable must be known. In case we want to approximate a periodical signal with a known fundamental frequency, we just need to use it in matrix A. In case we want to fit a periodical signal with unknown fundamental frequency or a non-periodical signal, we must assume, as in the Fourier transform deduction from the Fourier series, that the period of the signal tends to infinity, which means that the fundamental frequency tends to zero. As consequence of this assumption, ω

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have to be chosen depending on the accuracy desired in the recovery of the frequency components. In other words ω is the frequency step that determines the resolution of the discretized frequency domain, such that we have to choose ω thinking how close we want the frequency components.

3. Application examples

This section presents four examples of possible applications of the spectral observer, which were conceived such that the reader can be able to reproduce them.

3.1. A simple example

Let be the signal $s(t) = 4\cos t + \sin t + 2\cos 2t + 5\sin 2t$. It is obvious that the series order to reproduce the signal is $n = 2$, then the order of system (7) for the conception of the observer should be $N = 4$. Fig. 2 shows the estimation of the coefficients that was performed by the observer with a gain $\lambda = 8$, which actually was initialized with $\hat{v}(0) = 0$. The step time to perform the estimation in Simulink was $\Delta t = 0.005$ [s] and the used solver was ODE3. Before presenting the next examples, it

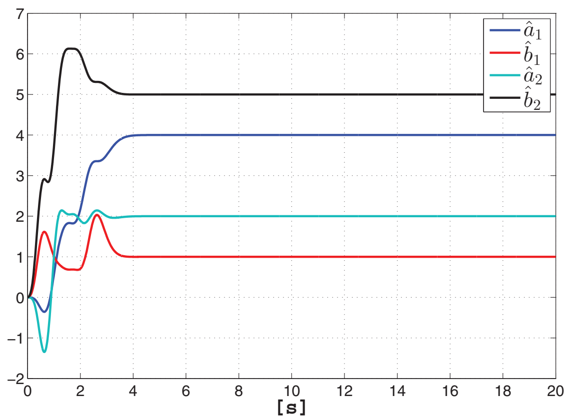

Figure 2. Example 1. Estimated coefficients

is important to make a point: the parameter λ permits to manipulate the speed of the convergence, if λ is large, the spectral observer will estimate the signal faster. However if the signal is corrupted with noise, this will be amplified.

3.2. Reconstruction of basic signals

This example aims to show the estimation of the coefficients for basic signals such as square and sawtooth waves. The first signal to be estimated is a square wave with angular frequency $\omega = 1$ [rad/s]. The observer was tuned with $\lambda = 15$. The order of the series was set $n = 2$, i.e. $N = 4$. The frequency step was set $\omega = 5$ [rad/s]. The step time to perform the estimation in Simulink was set $\Delta t = 0.01$ [s] and the used solver was ODE3. Fig. 3 shows the signal reconstruction performed by the spectral observer and the estimated coefficients. Firstly, notice that the coefficients do not converge towards a constant value, the reason is the number of coefficients used to approximate the signal, which is not enough to represent each harmonic that compose it. Even though the coefficients are not constant, the signal is estimated. Notice too that all the coefficients change abruptly at each discontinuity. This feature can be used for edge detection as will be seen in the next example.

Both the observer and conditions that were used to reconstruct the square wave were used to reconstruct the sawtooth signal shown in Fig. 4. Notice that the convergence time is less to one second and the coefficients become greater at the discontinuities.

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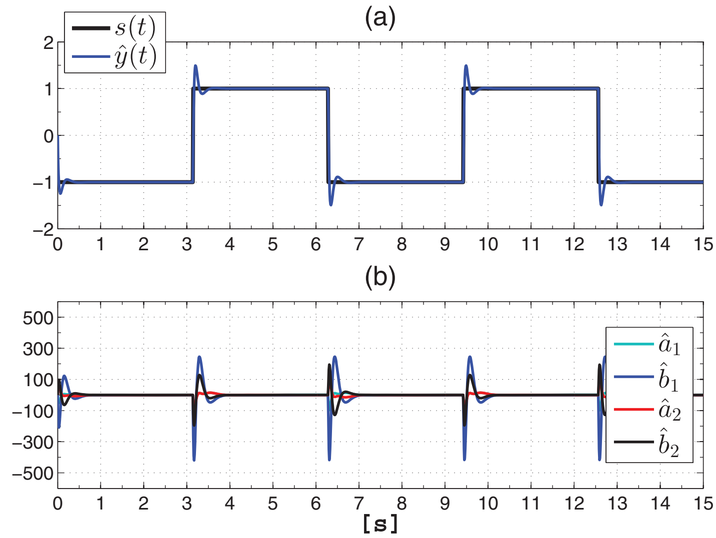

Figure 3. Example 2. (a) Square wave reconstruction and (b) Estimated coefficients

3.3. Edge detection by using the Fourier coefficients

Edge detection and the detection of discontinuities are important in many fields. In image processing, for example, one often needs to determine the boundaries of the items of which a picture is composed, [11] or in applications that utilizes time-domain reflectometry (TDR), which is a measurement technique used to determine the characteristics of transmission lines by observing reflected waveforms. TDR analysis begins with the propagation of a step or impulse of energy into a system and the subsequent observation of the energy reflected by the system. By analyzing the magnitude, duration and shape of the reflected waveform, the nature of the transmission system can be determined. TDR is a common method used to localize faults in transmission lines —such a leaks in pipelines or faults with small impedance in wires— because faults in transmission lines cause discontinuities in the reflected waveforms. For this reason, methodologies to detect discontinuities are required in order to localize the nature and position of the faults.

In order to shows how the spectral observer (15) can be used to detect discontinuities in a function, we present the following example: Let us consider $s(t) = \text{sign}(\sin(0.5t)) - 0.05\text{sign}(\sin(2t))$, which is plot in Fig. 5 (a). The aim of this test is to detect the discontinuities in the principal signal with period $T = 2$ [s]. To identify the discontinuities, the coefficients provided by the spectral observer are used to calculate the following indicator function:

$$r_k = \ln\left(\sqrt{\hat{a}_k^2 + \hat{b}_k^2}\right) \tag{17}$$

The observer to perform the estimation was tuned with $\lambda = 15$. The order of the series was set $n = 2$, i.e. $N = 4$. The frequency step was set $\omega = 1$ [rad/s]. The step time to perform the estimation in Simulink was set $\Delta t = 0.01$ [s] and the used solver was ODE3.

Fig. 5 (a) shows $s(t)$ and its reconstruction $\hat{y}(t)$. Fig 5 (b) shows the index $r_1(t)$ and $r_2(t)$ that becomes greater at the discontinuities indicating where they are.

3.4. Fittig complex signal: the Bitcoin price

Bitcoin is the longest running and most well known cryptocurrency in the world. It was released as open source in 2009 by the anonymous Satoshi Nakamoto. Bitcoin serves as a decentralized medium of digital exchange, with transactions verified and recorded in a public

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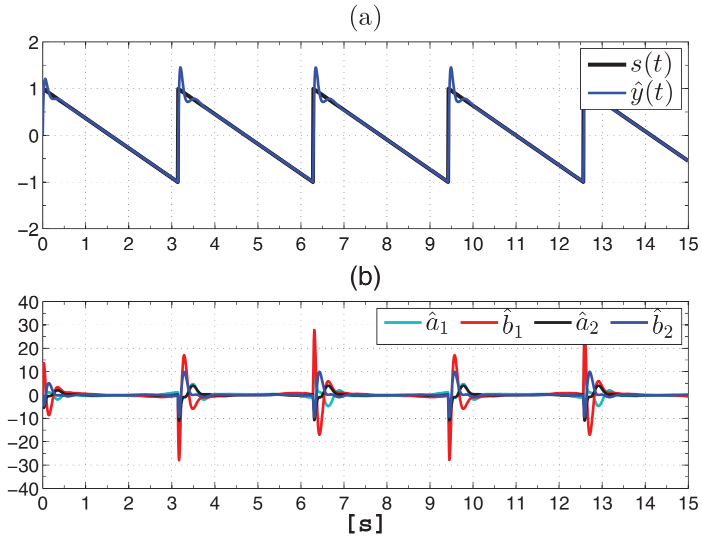

Figure 4. Example 2. (a) Triangular wave reconstruction and (b) Estimated coefficients

distributed ledger (the blockchain) without the need for a trusted record keeping authority or central intermediary. Hereafter, we will use the proposed spectral observer for fitting the historical Bitcoin market close data every 1000 [min]. The records were downloaded from the website: https://www.kaggle.com/neelneelpurk/bitcoin/data.

The observer to perform the estimation was tuned with $\lambda = 1$. The order of the series was set $n = 20$, i.e. $N = 40$. The frequency step was set $\omega = 10$ [rad/s]. The step time to perform the estimation in Simulink was set $\Delta t = 0.01$ [s] and the used solver was ODE8.

In Fig. 6, the Bitcoin fitting performed by the spectral observer is shown. Fig. 7 shows the estimated coefficients which are not constant and look as they were enveloped by exponential functions. In order to have a model that represents the behavior of the Bitcoin in the specified interval, we can fit each coefficients by means of polynomials after calculating the natural logarithm of each one. In Fig. 8, $\ln(|a_1|)$ is plotted versus a cubic polynomial calculated to interpolate it.

$$\ln(|a_1|) = 1.2 \times 10^{-9} t^3 + 1.5 \times 10^{-6} t^2 - 0.0021t - 1.4, \tag{18}$$

We can perform the same procedure for each coefficient to obtain a series with the following form:

$$\begin{aligned} \hat{y}(t) = &\left(e^{|(\alpha_{c1}t^3+\beta_{c1}t^2+\gamma_{c1}t+\delta_{c1})|}\right)\cos(\omega t) + \left(e^{|(\alpha_{s1}t^3+\beta_{s1}t^2+\gamma_{s1}t+\delta_{s1})|}\right)\sin(\omega t) \\ &+ \left(e^{|(\alpha_{c2}t^3+\beta_{c2}t^2+\gamma_{c2}t+\delta_{c2})|}\right)\cos(2\omega t) + \left(e^{|(\alpha_{s2}t^3+\beta_{s2}t^2+\gamma_{s2}t+\delta_{s2})|}\right)\sin(2\omega t) \\ &+ \ldots + \left(e^{|(\alpha_{cn}t^3+\beta_{cn}t^2+\gamma_{cn}t+\delta_{cn})|}\right)\cos(n\omega t) + \left(e^{|(\alpha_{sn}t^3+\beta_{sn}t^2+\gamma_{sn}t+\delta_{sn})|}\right)\sin(n\omega t) \end{aligned} \tag{19}$$

where α_{ck}, β_{ck}, γ_{ck}, δ_{ck}, α_{sk}, β_{sk}, γ_{sk}, δ_{sk} are the coefficients of the polynomial that approximates the natural logarithms of the coefficients.

4. Conclusions

In this paper, we presented the design of a novel spectral observer, which can be used to estimate periodical and non-periodical signals via state estimation. To design the spectral observer, we constructed a synthetic system in state space representation from the Fourier series. We presented

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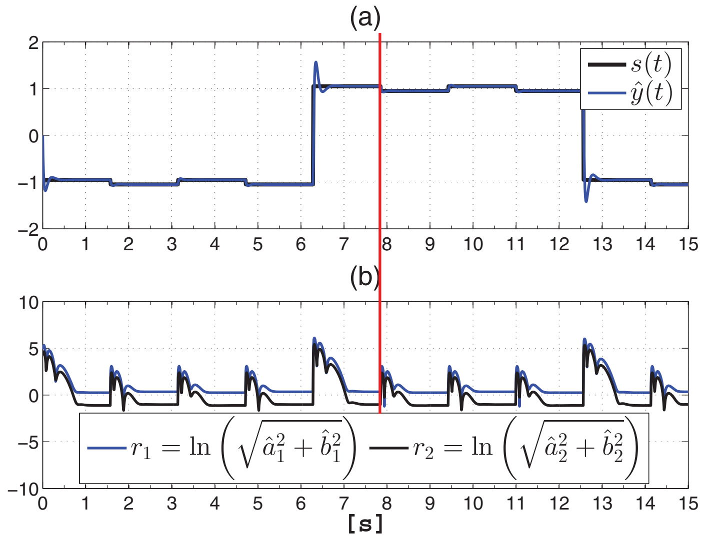

Figure 5. Example 3. (a) Signal reconstruction and (b) Estimated coefficients

some application examples to reconstruct periodical signals but also a well-know non-periodical one: the price of the Bitcoin from its genesis. As future work, we will present an analysis of the spectral observer face to perturbations and noise. We will explore some new applications and propose some improvements.

Author Contributions: Lizeth Torres conceived the spectral observer presented in this article. Javier Jiménez-Cabas and José Francisco Gómez-Aguilar conceived, designed and performed the simulation tests. Pablo Pérez-Alcazar advised the rest of the authors. All the author wrote the paper.

Conflicts of Interest: "The authors declare no conflict of interest."

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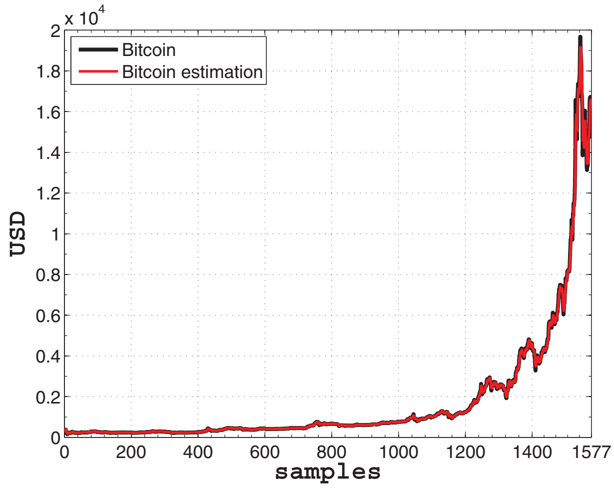

Figure 6. Example 4. Bitcoin fitting

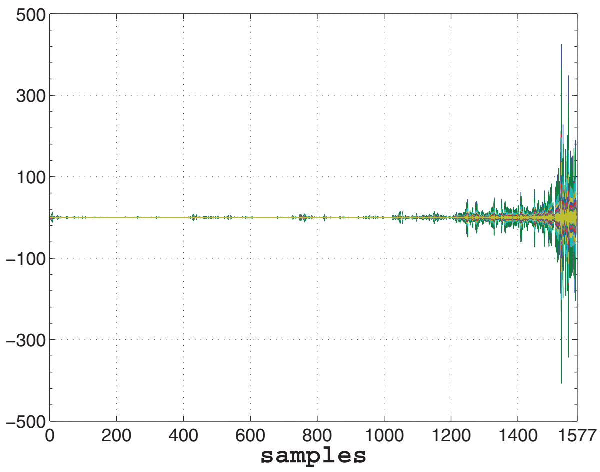

Figure 7. Example 4. Estimated coefficients

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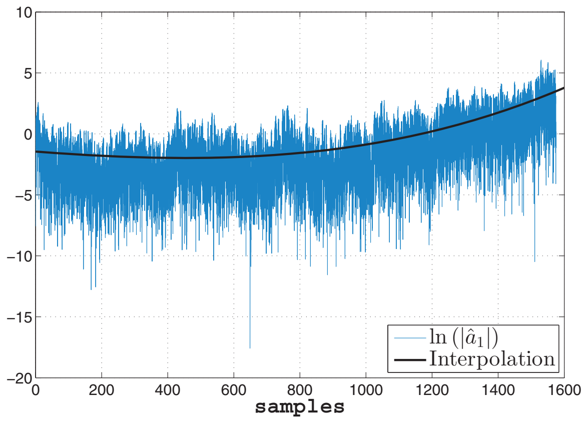

Figure 8. Example 4. $\log |a_1|$ vs. interpolation