Some new results associated with the generalized Lommel-Wright function

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Abstract

The aim of this article is to establish a new class of unified integrals associated with the generalized Lommel-Wright functions, which are expressed in terms of Wright Hypergeometric function.Some integrals involving trigonometric,generalized Bessel function and Struve functions are also indicated as special cases of our main results.further, with the help of our main results and their special cases, we obtain two reduction formulas for the Wright hypergeometric function.

keywords: Gamma function, generalized Wright hypergeometric function ${}_p\psi_q$, Hypergeometric function ${}_pF_q$, generalized Lommel-Wright functions $J^{\mu\, m}_{\nu,\lambda}(z)$, Lavoie-Trottier integral formula.

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1 Introduction

The Wright hypergeometric function defined by the series [13]:

$$ {}_p\psi_q\left[\begin{array}{c|}(\alpha_1, A_1), .., (\alpha_p, A_p);\\ \\ (\beta_1, B_1)..., (\beta_q, B_q)\end{array}\; z\right] = \sum_{k=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(\alpha_j + A_j k)z^k}{\prod_{j=1}^{q}\Gamma(\beta_j + B_j k)k!}, \tag{1.1}$$

where the coefficients $A_1, ...A_p$ and $B_1, ...B_q$ are positive real numbers such that

$$1 + \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j \geq 0. \tag{1.2}$$

can be slightly generalized (1.1) as given below.

$$ {}_p\psi_q\left[\begin{array}{c}(\alpha_1, 1), .., (\alpha_p, 1);\\ \\ (\beta_1, 1)..., (\beta_q, 1)\end{array}\; z\right] = \frac{\prod_{j=1}^{p}\Gamma(\alpha_j)}{\prod_{j=1}^{q}\Gamma(\beta_j)}\, {}_pF_q\left[\begin{array}{c}\alpha_1, .., \alpha_p;\\ \\ \beta_1, ..., \beta_q\end{array}\; z\right], \tag{1.3}$$

where ${}_pF_q$ is the generalized hypergeometric function defined by [13, 11]

$$ {}_pF_q\left[\begin{array}{c} \alpha_1,..,\alpha_p; \\ \\ \beta_1,...,\beta_q \end{array} z\right] = \sum_{k=0}^{\infty}\frac{(\alpha_1)_n,....,(\alpha_p)_n z^n}{(\beta_1)_n,....,(\beta_q)_n n!} = {}_pF_q(\alpha_1,...,\alpha_p;\beta_1,....,\beta_q;z), \tag{1.4} $$

where $(\lambda)_n$ is the well known Pochhammer symbol [13].

The series representation of the generalized Lommel Wright function as [5];

$$ J^{\mu,m}_{\nu,\lambda}(z) = \sum_{k=0}^{\infty}\frac{(-1)^k\Gamma(k+1)(\frac{z}{2})^{2k+\nu+2\lambda}}{\Gamma(\lambda+k+1)^m\Gamma(\nu+k\mu+\lambda+1)k!}, \tag{1.5} $$

$(z\in\mathbb{N}/(-\infty,0]\ m\in\mathbb{N},\ \nu,\lambda\in\mathbb{C},\mu>0)$.

Also, we have the following relations of generalized Lommel Wright functions with trigonometric functions and the generalized Bessel function and Struve function:

$$ J^{1,1}_{1/2,0}(z) = \sqrt{(\frac{2}{\pi z})}\sin(z) \tag{1.6} $$

$$ J^{1,1}_{-1/2,0}(z) = \sqrt{(\frac{2}{\pi z})}\cos(z) \tag{1.7} $$

$$ J^{\mu,1}_{\nu,\lambda}(z) = \mathbb{J}^{\mu}_{\nu,\lambda}(z) \tag{1.8} $$

$$ J^{1,1}_{\nu,1/2}(z) = H_\nu(z) \tag{1.9} $$

Further, we recall the following known result of Lavoie and Trottier [8].

$$ \int_0^1 t^{\rho-1}(1-t)^{2\sigma-1}\left(1-\frac{t}{3}\right)^{2\rho-1}\left(1-\frac{t}{4}\right)^{\sigma-1}dt = \left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma(\rho)\Gamma(\sigma)}{\Gamma(\rho+\sigma)}, \tag{1.10} $$

where $\Re(\rho)>0$ and $\Re(\sigma)>0$.

Various generalizations and cases of Lommel-Wright function have been investigated (see, for details, [1, 12, 7]). For more details of integral involving various special functions, one may be referred to the recent research papers [4, 9]

Integral formulas involving Lommel-Wright functions have been developed by many authors (see, e.g., [2, 3, 6, 10]). In this sequel, here, we aim at establishing certain new generalized integral formula involving the generalized Lommel-Wright function $J^{\mu,m}_{\nu,\lambda}(z)$ interesting integral formulas which are derived as special cases.

2 Main Results

This section deals with some integral formulas involving Lommel-Wright function.

Theorem 1. *For $\eta,\theta\in\mathbb{C}$ and $t>0$ with $\Re(\eta)>0,\Re(\theta)>0$, the following integral formula holds true*

$$ \int_0^1 x^{\eta-1}(1-x)^{2\theta-1}\left(1-\frac{x}{3}\right)^{2\eta-1}\left(1-\frac{x}{4}\right)^{\theta-1} J^{\mu,m}_{\nu,\lambda}\left(y\left(1-\frac{t}{4}\right)(1-t)^2\right)dt $$

$$= \Gamma(\eta)\left(\frac{2}{3}\right)^{2\eta}\left(\frac{y}{2}\right)^{\nu+2\lambda}$$

$$\times\, {}_2\psi_{m+2}\left[\begin{array}{c} (1,1),(\theta+\nu+2\lambda,2); \\ (\lambda+1,1),...,(\lambda+1,1),(\eta+\theta+\nu+2\lambda,2),(\nu+\lambda+1,\mu); \end{array} -\frac{y^2}{4}\right]. \tag{2.1}$$

Proof. On using (1.5) in the integrand of (2.1) and then interchanging the order of integral sign and summation which is verified by uniform convergence of the involved series under the given conditions we get

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1} J_{\nu,\lambda}^{\mu,m}\left(y\left(1-\frac{t}{4}\right)(1-t)^2\right)dt$$
$$= \left(\frac{y}{2}\right)^{\nu+2\lambda}\sum_{k=0}^{\infty}\frac{\Gamma(k+1)(-y^2/4)^k}{\Gamma(\lambda+k+1)^m\Gamma(\nu+k\mu+\lambda+1)k!}$$
$$\times \int_0^1 t^{\eta-1}(1-t)^{2(\theta+2k+\nu+2\lambda)-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{(\theta+2k+\nu+2\lambda)-1} dt. \tag{2.2}$$

Now using (1.10) in the above equation we get

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1} J_{\nu,\lambda}^{\mu,m}\left(y\left(1-\frac{t}{4}\right)(1-t)^2\right)dt = \left(\frac{2}{3}\right)^{2\eta}$$
$$\times\, \Gamma(\eta)\left(\frac{y}{2}\right)^{\nu+2\lambda}\sum_{k=0}^{\infty}\frac{\Gamma(k+1)\Gamma(\theta+\nu+2\lambda+2k)(\frac{-y^2}{4})^k}{\Gamma(\lambda+k+1)^m\Gamma(\eta+\theta+\nu+2\lambda+2k)\Gamma(\nu+k\mu+\lambda+1)k!}. \tag{2.3}$$

Finally, using (1.1) in the above equation, we get our assertion (2.1). This completes the proof of Theorem 1. □

Theorem 2. *The following integral formula holds true: For $\eta,\theta \in C$ and $t > 0$ with $\Re(\theta) > 0, \Re(\eta) > 0$,*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1} J_{\nu,\lambda}^{\mu,m}\left(ty\left(1-\frac{t}{3}\right)^2\right)dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\nu+2\lambda)}\left(\frac{y}{2}\right)^{\nu+2\lambda}\Gamma(\theta)$$
$$\times\, {}_2\psi_{m+2}\left[\begin{array}{c} (1,1),(\eta+\nu+2\lambda,2); \\ (\lambda+1,1),...,(\lambda+1,1)(1+\nu+\lambda,\mu),(\eta+\theta+\nu+2\lambda,2); \end{array} \frac{-4y^2}{81}\right]. \tag{2.4}$$

Proof. On using (1.5) in the integrand of (2.4) and then interchanging the order of integral sign and summation which is verified by uniform convergence of the involved series under the given conditions we get

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1} J_{\nu,\lambda}^{\mu,m}\left(ty\left(1-\frac{t}{3}\right)^2\right)dt$$
$$= (y/2)^{\nu+2\lambda}\sum_{k=0}^{\infty}\frac{\Gamma(k+1)\left(-\frac{y^2}{4}\right)^k}{\Gamma(\lambda+k+1)^m\Gamma(\nu+k\mu+\lambda+1)k!}$$

$$\times \int_0^1 t^{\eta+\nu+2\lambda+2k-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2(\eta+\nu+2\lambda+2k)-1}\left(1-\frac{t}{4}\right)^{\theta-1} dt. \tag{2.5}$$

Now using (1.10) in the above equation we get

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1} J_{\nu,\lambda}^{\mu,m}\left(y\left(1-\frac{t}{4}\right)\right)dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\nu+2\lambda)} \Gamma(\theta)\left(\frac{y}{2}\right)^{\nu+2\lambda}$$
$$\times \sum_{k=0}^{\infty} \frac{\Gamma(k+1)\Gamma(\eta+\nu+2\lambda+2k)\left(\frac{-4y^2}{81}\right)^k}{\Gamma(\lambda+k+1)^m\Gamma(\nu+k\mu+\lambda+1)\Gamma(\eta+\theta+\nu+2\lambda+2k)k!}. \tag{2.6}$$

Finally, using (1.1) in the above equation, we get our assertion (2.4). This completes the proof of Theorem 2. □

Next we consider other variations of theorem 1 and theorem 2 in the form of corollaries.

Corollary 2.1. *In (2.3), on separating the hypergeometric series into its even and odd terms,we get the following integral formulas:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1} J_{\nu,\lambda}^{\mu,m}\left(y\left(1-\frac{t}{4}\right)(1-t)^2\right)dt = \left(\frac{2}{3}\right)^{2\eta}\sqrt{\pi}$$
$$\times \left(\frac{y}{2}\right)^{\nu+2\lambda}\Gamma(\eta)\ {}_2\psi_{m+3}\left[\begin{matrix}(1,2),(\theta+\nu+2\lambda,4);\\ (\lambda+1,2),...,(\lambda+1,2)(1/2,1),(\eta+\theta+\nu+2\lambda,4),(\nu+\lambda+1,2\mu);\end{matrix}\ \frac{y^4}{64}\right]$$
$$-\left(\frac{2}{3}\right)^{2\eta}\frac{\sqrt{\pi}}{2}\left(\frac{y}{2}\right)^{\nu+2\lambda+2}\Gamma(\eta)$$
$$\times\ {}_2\Psi_{m+3}\left[\begin{matrix}(2,2),(\theta+\nu+2\lambda+2,4);\\ (\lambda+2,2),...,(\lambda+2,2)(\eta+\theta+\nu+2\lambda+2,4),(3/2,1),(\mu+\nu+\lambda+1,2\mu);\end{matrix}\ \frac{y^4}{64}\right], \tag{2.7}$$

where $\Re(\eta)>0$ *and* $\Re(\theta)>0$.

Corollary 2.2. *On expanding the r.h.s of (2.6) in series form and then separating the resulting series into its even and odd terms, we obtain.*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1} J_{\nu,\lambda}^{\mu,m}\left(ty\left(1-\frac{t}{3}\right)^2\right)dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\theta+2\lambda)}\sqrt{\pi}\left(\frac{y}{2}\right)^{\nu+2\lambda}\Gamma(\theta)$$
$$\times\ {}_2\psi_{m+3}\left[\begin{matrix}(1,2),(\eta+\theta+2\lambda,4);\\ (\lambda+1,2),...,(\lambda+1,2)(1/2,1),(\eta+\theta+\nu+2\lambda,4),(\nu+\lambda+1,2\mu);\end{matrix}\ \frac{y^4}{6561}\right]$$
$$+\left(\frac{2}{3}\right)^{2(\eta+\theta+2\lambda)}\frac{\sqrt{\pi}}{2}\left(-\frac{4y^2}{81}\right)\left(\frac{y}{2}\right)^{\nu+2\lambda}\Gamma(\theta)$$

$$\times\, {}_2\Psi_{m+3}\left[\begin{matrix}(2,2),(\eta+\theta+2\lambda+2,4);\\ (\lambda+2,2),...,(\lambda+2,2)(3/2,1),(\eta+\theta+\nu+2\lambda+2,4),(\nu+\lambda+\mu+1,2\mu);\end{matrix}\ \frac{y^4}{6561}\right], \tag{2.8}$$

where $\Re(\eta)>0$ *and* $\Re(\theta)>0$.

3 Special Cases

In this section, we derive some integral formulas involving trigonometric function and generalized Lommel-Wright function as follows:

Corollary 3.1. *If we take* $m=1,\mu=1,\lambda=0$ *and* $\nu=1/2$ *in* (2.1) *and then by using* (1.6), *we derive the following integral formula:*

$$\int_0^1 t^{\eta-1}(1-t)^{2(\theta-1/2)-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{(\theta-1/2)-1}\sin\left(y(1-t/4)(1-t)^2\right)dt$$
$$=\left(\frac{2}{3}\right)^{2\eta}\sqrt{\pi}(\frac{y}{2})\ \Gamma(\eta)_1\psi_2\left[\begin{matrix}(\theta+1/2,2);\\ (\eta+\theta+1/2,2),(\frac{3}{2},1);\end{matrix}\ -\frac{y^2}{4}\right] \tag{3.1}$$

where $\Re(\eta)>0$ *and* $\Re(\theta)>0$.

Corollary 3.2. *Again by taking* $m=1,\mu=1,\lambda=0$ *and* $\nu=1/2$ *in* (2.4) *and then by using* (1.6), *we deduce the following integral formula:*

$$\int_0^1 t^{\left(\eta-\frac{1}{2}\right)-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\left(\eta-\frac{1}{2}\right)-1}\left(1-\frac{t}{4}\right)^{\theta-1}\sin\left(ty\left(1-\frac{t}{3}\right)^2\right)dt$$
$$=\left(\frac{2}{3}\right)^{2\left(\eta+\theta\right)}\sqrt{\pi}\ (\frac{y}{2})\ \Gamma(\theta)_1\psi_2\left[\begin{matrix}(\eta+\theta,2);\\ (\eta+\theta+\frac{1}{2},2),(\frac{3}{2},1);\end{matrix}\ -\frac{4y^2}{81}\right] \tag{3.2}$$

where $\Re(\eta)>0$ *and* $\Re(\theta)>0$.

Corollary 3.3. *If we put* $m=1,\mu=1,\lambda=0$ *and* $\nu=1/2$ *in* (2.7) *and then by using* (1.6), *we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2(\theta-\frac{1}{2})-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{(\theta-\frac{1}{2})-1}\sin\left(y\left(1-\frac{t}{4}\right)(1-t)^2\right)dt$$
$$=\left(\frac{2}{3}\right)^{2\eta}(\frac{y\pi}{2})\Gamma(\eta)_1\psi_3\left[\begin{matrix}(\theta+\frac{1}{2},4);\\ (\eta+\theta+\frac{1}{2},4),(\frac{3}{2},1),(1/2,1);\end{matrix}\ \frac{y^4}{64}\right]$$
$$-\left(\frac{2}{3}\right)^{2\eta}(\frac{y^3\pi}{16})\Gamma(\eta)_1\psi_3\left[\begin{matrix}(\theta+\frac{5}{2},4);\\ (\eta+\theta+\frac{5}{2},4),(\frac{3}{2},1),(\frac{5}{2},2);\end{matrix}\ \frac{y^4}{64}\right], \tag{3.3}$$

where $\Re(\eta)>0$ *and* $\Re(\theta)>0$.

Corollary 3.4. *Further we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = 1/2$ in (2.8) and then by using (1.6), we obtain:*

$$\int_0^1 t^{(\eta-\frac{1}{2})-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2(\eta-\frac{1}{2})-1}\left(1-\frac{t}{4}\right)^{\theta-1}\sin\left(ty\left(1-\frac{t}{3}\right)^2\right)dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\theta)}\left(\frac{\pi y}{2}\right)\Gamma(\theta)\,{}_1\psi_3\left[\begin{array}{c} (\eta+\theta,4); \\ (\eta+\theta+\frac{1}{2},4),(\frac{1}{2},1),(\frac{3}{2},2); \end{array} \frac{4y^4}{6561}\right]$$
$$+\left(\frac{2}{3}\right)^{2(\eta+\theta)}\Gamma(\theta)\left(\frac{4y^3}{81}\right){}_1\psi_3\left[\begin{array}{c} (\eta+\theta+2,4); \\ (\eta+\theta+\frac{5}{2},4),(\frac{3}{2},1),(\frac{5}{2},2); \end{array} \frac{4y^4}{6561}\right], \tag{3.4}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

Corollary 3.5. *If we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = -1/2$ in (2.1)and then by using (1.7),we get the following integral formula:*

$$\int_0^1 t^{\eta-1}(1-t)^{2(\theta-\frac{1}{2})-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{(\theta-\frac{1}{2})-1}\cos\left(y\left(1-\frac{t}{4}\right)(1-t^2\right)dt$$
$$= \left(\frac{2}{3}\right)^{2\eta}\sqrt{\pi}\,\Gamma(\eta)\,{}_1\psi_2\left[\begin{array}{c} (\theta-\frac{1}{2},2); \\ (\eta+\theta-\frac{1}{2},2),(\frac{1}{2},1); \end{array} -\frac{y^2}{4}\right], \tag{3.5}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

Corollary 3.6. *Again we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = -1/2$ in (2.4) and then by using (1.7), we get the following integral formula:*

$$\int_0^1 t^{\left(\eta-\frac{1}{2}\right)-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\left(\eta-\frac{1}{2}\right)-1}\left(1-\frac{t}{4}\right)^{\theta-1}\cos\left(t\;y\left(1-\frac{t}{3}\right)^2\right)dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\theta)}\sqrt{\pi}\,\Gamma(\theta)\,{}_1\psi_2\left[\begin{array}{c} (\eta+\theta,2); \\ (\eta+\theta-\frac{1}{2},2),(\frac{1}{2},1); \end{array} -\frac{4y^2}{81}\right], \tag{3.6}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

Corollary 3.7. *If we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = -1/2$ in (2.7) and then by using (1.7), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2(\theta-\frac{1}{2})-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{(\theta-\frac{1}{2})-1}\cos\left(y\left(1-\frac{t}{4}\right)(1-t^2)\right)dt$$
$$= \left(\frac{2}{3}\right)^{2\eta}\pi\Gamma(\eta)\,{}_1\psi_3\left[\begin{array}{c} (\theta-\frac{1}{2},4); \\ (\eta+\theta-\frac{1}{2},4),(\frac{1}{2},2),(1/2,1); \end{array} \frac{y^4}{64}\right]$$
$$+\left(\frac{2}{3}\right)^{2\eta}\left(\frac{\pi y^2}{16}\right)\Gamma(\eta)\,{}_1\psi_3\left[\begin{array}{c} (\theta+\frac{3}{2},4); \\ (\eta+\theta+\frac{3}{2},4),(\frac{3}{2},1),(\frac{3}{2},2); \end{array} \frac{y^4}{64}\right], \tag{3.7}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

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Corollary 3.8. *Further if we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = -1/2$ in (2.8) and then by using (1.7), we obtain:*

$$\int_0^1 t^{(\eta-\frac{1}{2})-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2(\eta-\frac{1}{2})-1}\left(1-\frac{t}{4}\right)^{\theta-1}\cos\left(ty\left(1-\frac{t}{3}\right)^2\right)dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\theta)} \pi\Gamma(\theta)\,{}_1\psi_3\left[\begin{array}{c} (\eta+\theta,4); \\ (\eta+\theta-\frac{1}{2},4),(\frac{1}{2},1),(\frac{1}{2},2); \end{array} \frac{y^4}{6561}\right]$$
$$+\left(\frac{2}{3}\right)^{2(\eta+\theta)}\left(-\frac{8\pi y^2}{81}\right)\Gamma(\theta)\,{}_1\psi_3\left[\begin{array}{c} (\eta+\theta+2,4); \\ (\eta+\theta+\frac{3}{2},4),(\frac{3}{2},1),(\frac{3}{2},2); \end{array} \frac{y^4}{6561}\right], \tag{3.8}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

Corollary 3.9. *If we take $m = 1$ in (2.1) and then by using (1.8), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1}\mathbb{J}^{\mu}_{\nu,\lambda}(y(1-t/4)(1-t)^2)dt = \left(\frac{2}{3}\right)^{2\eta}\left(\frac{y}{2}\right)^{\nu+2\lambda}\Gamma(\eta)$$
$$\times\,{}_2\psi_3\left[\begin{array}{c} (\theta+\nu+2\lambda,2),(1,1); \\ (\lambda+1,1),(\eta+\theta+\nu+2\lambda,2),(\nu+\lambda+1,\mu); \end{array} -\frac{y^2}{4}\right], \tag{3.9}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

Corollary 3.10. *Further if we take $m = 1$ in (2.3) and then by using (1.8), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1}\mathbb{J}^{\mu}_{\nu,\lambda}ty(1-t/3)^2dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\theta+2\lambda)}\left(\frac{y}{2}\right)^{\nu+2\lambda}\Gamma(\theta)\,{}_2\psi_3\left[\begin{array}{c} (\eta+\theta+2\lambda,2),(1,1); \\ (\lambda+1,1),(\eta+\theta+\nu+2\lambda,2),(1+\nu+\lambda,\mu); \end{array} -\frac{4y^2}{81}\right], \tag{3.10}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

Corollary 3.11. *If we take $m = 1$ in (2.7) and then by using (1.8), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1}\mathbb{J}^{\mu}_{\nu,\lambda}y(1-t/4)(1-t)^2dt$$
$$= \left(\frac{2}{3}\right)^{2\eta}\sqrt{\pi}\left(\frac{y}{2}\right)^{\nu+2\lambda}\Gamma(\eta)\,{}_2\psi_4\left[\begin{array}{c} (1,2),(\theta+\nu+2\lambda,4); \\ (\lambda+1,2),(1/2,1),(\eta+\theta+\nu+2\lambda,4),(\nu+\lambda+1,2\mu); \end{array} \frac{y^4}{64}\right]$$
$$+\left(\frac{2}{3}\right)^{2\eta}\sqrt{\frac{\pi}{2}}\left(\frac{y}{2}\right)^{\nu+2\lambda+2}\Gamma(\eta)$$
$$\times\,{}_2\psi_4\left[\begin{array}{c} (2,2),(\theta+\nu+2\lambda+2,4); \\ (\lambda+2,2),(\eta+\theta+\nu+2\lambda+2,4),(3/2,1),(\mu+\theta+\lambda+1,2\mu); \end{array} \frac{y^4}{64}\right], \tag{3.11}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

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Corollary 3.12. *Further if we take $m = 1$, in (2.8) and then by using (1.8), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1}\mathbb{J}^{\mu}_{\nu,\lambda}ty(1-t/3)^2dt$$
$$= \left(\frac{2}{3}\right)^{2(\eta+\theta+2\lambda)}\sqrt{\pi}\,(\frac{y}{2})^{\nu+2\lambda}\Gamma(\theta)_2\psi_4\left[\begin{matrix}(1,2),(\eta+\theta+2\lambda,4);\\ (\lambda+1,2),(\frac{1}{2},1),(\eta+\theta+\nu+2\lambda,4),(\nu+\lambda+1,2\mu);\end{matrix}\ \frac{y^4}{6561}\right]$$
$$+\left(\frac{2}{3}\right)^{2(\eta+\theta+2\lambda)}\sqrt{\frac{\pi}{2}}\left(-\frac{16y^2}{81}\right)(\frac{y}{2})^{\theta+2\lambda}\Gamma(\theta)$$
$$\times\,_2\psi_4\left[\begin{matrix}(2,2),(\eta+\theta+2\lambda+2,4);\\ (\lambda+2,2),(\frac{3}{2},1),(\eta+\theta+\nu+2\lambda+2,4),(\nu+\lambda+\mu+1,2\mu);\end{matrix}\ \frac{y^4}{6561}\right], \tag{3.12}$$

where $\Re(\eta) > 0$ *and* $\Re(\theta) > 0$.

Corollary 3.13. *If we take $m = 1, \mu = 1$ and $\lambda = 1/2$ in (2.1) and then by using (1.9), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1}\mathbb{H}_{\nu}(y(1-t/4)(1-t)^2)dt$$
$$= \left(\frac{2}{3}\right)^{2\eta}(\frac{y}{2})^{\nu+1}\Gamma(\eta)_2\psi_3\left[\begin{matrix}(\theta+\nu+1,2),(1,1);\\ (\eta+\theta+\nu+1,2),(\nu+3/2,1),(3/2,1);\end{matrix}\ -\frac{y^2}{4}\right], \tag{3.13}$$

where $\Re(\eta) > 0$ *and* $\Re(\theta) > 0$.

Corollary 3.14. *Further if we take $m = 1, \mu = 1$ and $\lambda = 1/2$ in (2.3) and then by using (1.9), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1}\mathbb{H}_{\nu}ty(1-t/3)^2dt = \left(\frac{2}{3}\right)^{2(\eta+\theta+1)}(\frac{y}{2})^{\nu+1}\Gamma(\theta)$$
$$\times\,_2\psi_3\left[\begin{matrix}(\eta+\theta+1,2),(1,1);\\ (\eta+\theta+\nu+1,2),(\nu+3/2,1),(3/2,1);\end{matrix}\ -\frac{4y^2}{81}\right], \tag{3.14}$$

where $\Re(\eta) > 0$ *and* $\Re(\theta) > 0$.

Corollary 3.15. *If we take $m = 1, \mu = 1$ and $\lambda = 1/2$ in (2.7) and then by using (1.9), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\left(1-\frac{t}{3}\right)^{2\eta-1}\left(1-\frac{t}{4}\right)^{\theta-1}\mathbb{H}_{\nu}y(1-t/4)(1-t)^2dt$$
$$= \left(\frac{2}{3}\right)^{2\eta}\sqrt{\pi}\,(\frac{y}{2})^{\nu+1}\Gamma(\eta)_2\psi_4\left[\begin{matrix}(1,2),(\theta+\nu+1,4);\\ (\eta+\theta+\nu+1,4),(\nu+3/2,2),(3/2,2),(1/2,1);\end{matrix}\ \frac{y^4}{64}\right]$$
$$+\left(\frac{2}{3}\right)^{2\eta}\sqrt{\frac{\pi}{2}}(\frac{y}{2})^{\nu+3}\Gamma(\eta)_2\psi_4\left[\begin{matrix}(2,2),(\theta+\nu+3,4);\\ (\eta+\theta+\nu+3,4),(\nu+5/2,2),(5/2,2),(3/2,1);\end{matrix}\ \frac{y^4}{64}\right], \tag{3.15}$$

where $\Re(\eta) > 0$ *and* $\Re(\theta) > 0$.

Corollary 3.16. *Further if we take $m = 1, \mu = 1$ and $\lambda = 1/2$ in (2.8) and then by using (1.9), we obtain:*

$$\int_0^1 t^{\eta-1}(1-t)^{2\theta-1}\Big(1-\frac{t}{3}\Big)^{2\eta-1}\Big(1-\frac{t}{4}\Big)^{\theta-1}\mathbb{H}_\nu ty(1-t/3)^2dt$$
$$= \Big(\frac{2}{3}\Big)^{2(\eta+\theta+1)}\sqrt{\pi}\,(\frac{y}{2})^{\nu+1}\Gamma(\theta)_2\psi_4\left[\begin{array}{c} (1,2),(\eta+\theta+1,4); \\ (\eta+\theta+\nu+1,4),(\nu+3/2,2),(3/2,2),(1/2,1); \end{array}\frac{y^4}{6561}\right]$$
$$+\Big(\frac{2}{3}\Big)^{2(\eta+\theta+1)}\sqrt{\frac{\pi}{2}}\Big(-\frac{16y^2}{81}\Big)(\frac{y}{2})^{\theta+1}\Gamma(\theta)$$
$$\times\,{}_2\psi_4\left[\begin{array}{c} (2,2),(\eta+\theta+3,4); \\ (\eta+\theta+\nu+3,4),(\nu+5/2,2),(5/2,2),(3/2,1); \end{array}\frac{y^4}{6561}\right], \tag{3.16}$$

where $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

4 Reducibility of the Wright Hypergeometic Function

Now we state two reduction formulas for the Wright hypergeometric function as follows:

$${}_2\psi_{m+2}\left[\begin{array}{c} (\theta+\nu+2\lambda,2)(1,1); \\ (\lambda+1,1),...(\lambda+1,1)(\eta+\theta+\nu+2\lambda,2)(\nu+\lambda+1,\mu); \end{array} -\frac{y^2}{4}\right]$$
$$=\sqrt{\pi}{}_2\psi_{m+3}\left[\begin{array}{c} (\theta+\nu+2\lambda,4)(1,2); \\ (\lambda+1,2),...(\lambda+1,2)(1/2,1)(\eta+\theta+\nu+2\lambda,4),(\nu+\lambda+1,2\mu); \end{array}\frac{y^4}{64}\right]+\sqrt{\frac{\pi}{8}}y^2$$
$${}_2\psi_{m+3}\left[\begin{array}{c} (\theta+\nu+2\lambda+2,4)(2,2); \\ (\lambda+2,2),...(\lambda+2,2)(\eta+\theta+\nu+2\lambda+2,4)(\frac{3}{2},1)(\mu+\nu+\lambda+1,2\mu); \end{array}\frac{y^4}{64}\right] \tag{4.1}$$

$${}_2\psi_{m+2}\left[\begin{array}{c} (\eta+\theta+2\lambda,2)(1,1); \\ (\lambda+1,1),...(\lambda+1,1),(\eta+\theta+\nu+2\lambda,2),(1+\nu+\lambda,\mu); \end{array}\frac{-4y^2}{81}\right]$$
$$=\sqrt{\pi}\;{}_2\psi_{m+3}\left[\begin{array}{c} (\eta+\theta+2\lambda,4)(1,2); \\ (\lambda+1,2),...(\lambda+1,2)(1/2,1),(\eta+\theta+\nu+2\lambda,4)(\nu+\lambda+1,2\mu); \end{array}\frac{y^4}{6561}\right]$$
$$+\sqrt{\pi}\Big(-\frac{4y^2}{81}\Big)$$
$$\times\,{}_2\psi_{m+3}\left[\begin{array}{c} (\eta+\theta+2\lambda+2,4)(2,2); \\ (\lambda+2,2),...(\lambda+2,2)(3/2,1)(\eta+\theta+\nu+2\lambda+2,4)(\nu+\lambda+\mu+1,2\mu); \end{array}\frac{y^4}{6561}\right] \tag{4.2}$$

By comparing (2.1) and (2.7),results (4.1) can be established and by comparing (2.4) and (2.8),result (4.2) can be established.

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