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TOPSIS Based Algorithm for Solving Multi-objective Multi-level Programming Problem with Fuzzy Parameters

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Abstract: The paper proposes TOPSIS method for solving multi-objective multi-level programming problem (MO-MLPP) with fuzzy parameters via fuzzy goal programming (FGP). At first, λ - cut method is used to transform the fuzzily described MO-MLPP into deterministic MO-MLPP. Then, for specific λ, we construct the membership functions of distance functions from positive ideal solution (PIS) and negative ideal solution (NIS) of all level decision makers (DMs). Thereafter, FGP based multi-objective decision model is established for each level DM for obtaining individual optimal solution. A possible relaxation on decisions for all DMs is taken into account for satisfactory solution. Subsequently, two FGP models are developed and compromise optimal solutions are found by minimizing the sum of negative deviational variables. To recognize the better compromise optimal solution, the concept of distance functions is utilized. Finally, a novel algorithm for MO-MLPP involving fuzzy parameters is provided and an illustrative example is solved to verify the proposed procedure.

Keywords: multi-objective multi-level programming; fuzzy parameters; TOPSIS; fuzzy goal programming; multi-objective decision making

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1. Introduction

Multi-level programming (MLP) technique is a powerful analytical device for describing decentralized planning problems involving several decision makers (DMs) in a hierarchical organization. MLP has diverse practical applications in such fields as agricultural economics [1], conflict resolution [2], network design [3], pollution control policies [4], warfare [5], and so on. In a multi-level programming problem (MLPP), there exists a single and independent DM at each level and each level DM attempts to optimize its objective function over a common feasible region but the decision of each level DM is exaggerated by the actions and reactions of the other DMs. Consequently, decision deadlock may occur in the decision making circumstances. However, it has been observed that each level DM should have a motivation to cooperate with each other, and a minimal level of satisfaction of all level DMs must be considered for overall profit of the hierarchical structure.

Using the idea of tolerance membership function of fuzzy set theory [6] to MLPPs for satisfactory decisions, Lai [7] incorporated an efficient fuzzy approach at first in 1996. Shih et al. [8]

and Shih and Lee [9] applied non-compensatory max-min aggregation operator and compensatory fuzzy operator respectively for solving MLPPs. Sakawa et al. [10] criticized Lai et al.'s method [7] and claimed that the method discussed in [7] may produce undesirable solution to the MLPPs when the fuzzy goals of objective function and decision variables of upper level DM are inconsistent. In order to get rid of such situations, Sakawa et al. [10] proposed an interactive fuzzy programming (IFP) for MLPPs by removing the fuzzy goals of the decision variables. Sinha [11, 12] proposed fuzzy mathematical programming for solving MLPPs through a supervised search procedure. Pramanik and Roy [13] proposed fuzzy goal programming (FGP) models to MLPPs by taking into consideration of the relaxation of decision of the upper DMs for proper allocation of decision powers to the DMs within the hierarchical organization. Baky [14] presented alternative FGP models for solving multi-objective MLPP (MO-MLPP) to get the highest degree of each of the membership goals by minimizing over and under deviational variables.

In 2000, Sakawa et al. [15] presented IFP for obtaining satisfactory solution to MLPPs with fuzzy parameters by updating the satisfactory degree of DMs in view of of the overall satisfactory balance among all DMs. Zhang et al. [16] derived an approximation branch and bound algorithm for solving decentralized multi-objective bi-level decision making with fuzzy demands. Gao et al. [17] developed λ-cut and goal programming based approach for solving fuzzy linear multi-objective bi-level decision problems and presented a case study on a newsborn problem. Pramanik [18] formulated three novel and effective FGP models in order to solve bi-level programming problem (BLPP) with fuzzy parameters by considering preference bounds of upper and lower level DMs and distance function is used to select better compromise optimal solution. Pramanik [19] also presented λ- cut and FGP based models for MLPP with fuzzy parameters by extending the concept discussed in [18]. Pramanik and Dey [20] solved multi-objective BLPP (MO-BLPP) involving fuzzy parameters based on FGP approach due to Pramanik and Dey [21] and presented an algorithm with termination criteria. Baky et al. [22] extended the concept of Pramanik and Dey [20] and proposed an alternative FGP approach for solving MO-BLPP with fuzzy demands by taking into consideration of the relaxation on decision of upper level DM and obtained solutions of both level DMs by minimizing over and under deviational variables. Baky and Sayed [23] proposed a hybrid approach of TOPSIS and FGP for MO-BLPP with fuzzy parameters. Baky and Sayed [24] studied FGP method to solve MO-BLPP with fuzzy parameters using TOPSIS and modified TOPSIS techniques.

TOPSIS is a familiar multi-attribute decision making method which was developed by Hwang and Yoon [25] is based on the principle that a DM selects an alternative which is nearest from positive ideal solution (PIS) and farthest from negative ideal solution (NIS). Abo- Sinna et al. [26] and Abo- Sinna and Amer [27] investigated TOPSIS for multi-objective large scale non-linear programming problems with block angular structure and max-min operator is used to resolve the conflict between new criteria. Abo- Sinna and Abou-El-Enien [28] presented a TOPSIS based interactive algorithm for large scale multi-objective programming problem involving fuzzy parameters. Baky [29] proposed two interactive TOPSIS algorithms for solving non-linear MO-MLPPs. Recently, Dey et al. [30] investigated TOPSIS scheme for linear fractional MO-BLPP through FGP approach by assigning preference bounds on the decision variables to reach the optimal solution.

In this paper, we have extended the concept of Dey et al. [25] for solving MO-MLPP with fuzzy parameters using FGP procedure. The remainder of the paper is structured in the following way. Section 2 is devoted to present some basic definitions concerning fuzzy set theory. In section 3, the

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formulation of MO-MLPP with fuzzy parameters is exhibited. Section 4 provides deterministic formulation of MO-MLPP with fuzzy parameters using λ- cut technique. Some basic concepts relating to distance measures are briefly stated in section 5. TOPSIS based FGP approach for solving MO-MLPP with fuzzy parameters is developed in the next section. Distance functions for obtaining compromise optimal solution are discussed in section 7. In section 8, TOPSIS based algorithm for solving MO-MLPP with fuzzy parameters through FGP method is provided. A MO-MLPP with fuzzy parameters is solved to demonstrate the validity and efficiency of the proposed approach in section 9. Finally the last section concludes the paper with some future scope of research.

2. Preliminaries

In this section, some basic definitions regarding fuzzy set theory are provided.

Definition 2.1 Fuzzy set [6] A fuzzy set $\tilde{\Theta}$ in U is defined by $\tilde{\Theta} = \{\langle x, \mu_{\tilde{\Theta}}(x)\rangle \mid x \in U\}$, where $\mu_{\tilde{\Theta}}(x) : U \rightarrow [0, 1]$ is called the membership function of $\tilde{\Theta}$ and $\mu_{\tilde{\Theta}}(x)$ is the degree of membership to which $x \in \tilde{\psi}$.

Definition 2.2 Normal fuzzy set [31] $\tilde{\Theta}$ is said to be a normal fuzzy set if there exists a point x in U such that $\mu_{\tilde{\Theta}}(x) = 1$.

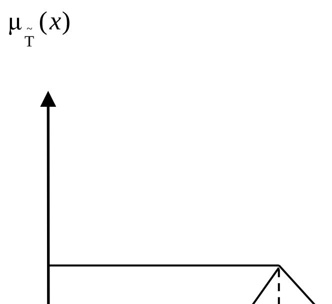
Definition 2.3 Convex fuzzy set [31] $\tilde{\Theta}$ is called a convex fuzzy set if and only if for any $x_1, x_2 \in U$ and $\lambda \in [0, 1]$, $\mu_{\tilde{\Theta}}[\lambda x_1 + (1 - \lambda) x_2] \geq \text{Min}[\mu_{\tilde{\Theta}}(x_1), \mu_{\tilde{\Theta}}(x_2)]$.

Definition 2.4 λ- cut [31] The λ-cut of a fuzzy set $\tilde{\Theta}$ of U is a non-fuzzy set denoted by ${}^{\lambda}\Theta$ defined by a subset of all elements $x \in U$ such that their membership functions exceed or identical to a real number $\lambda \in [0, 1]$, i.e. ${}^{\lambda}\Theta = \left[x : \mu_{\tilde{\Theta}}(x) \geq \lambda, \lambda \in [0,1], \forall x \in U\right]$.

Definition 2.5 Triangular fuzzy number [31] Triangular fuzzy number is both convex and normal fuzzy set in U which is defined by $\tilde{T} = \langle x, \mu_{\tilde{T}}(x)\rangle$ where

$$\mu_{\tilde{T}}(x) = \begin{cases} \dfrac{x-r}{s-r}, & if\ r \leq x \leq s \\ \dfrac{t-x}{t-s}, & if\ s \leq x \leq t \\ 0, & if\ Otherwise \end{cases}$$

Generally, a triangular fuzzy number is represented as (r, s, t) (see **Fig. 1**).



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0 r s t x

Figure 1. Triangular fuzzy number

3. Formulation of MO-MLPP with fuzzy parameters

Consider a MO-MLPP where the objective functions at each level are maximization type with fuzzy parameters and common constraints are linear functions with fuzzy parameters. Let, DM_i denotes the DM at the i-th level (i = 1, 2, …, p) which controls the variable $x_i = (x_{i1}, x_{i2}, \ldots, x_{iN_i}) \in \Omega^{N_i}$, (i = 1, 2, …, p) where $x = (x_1, x_2, \ldots, x_p)$ and $N = N_1 + N_2 + \ldots + N_p$ and further suppose that $\tilde{Y}_i(x_1, x_2, \ldots, x_p) = \tilde{Y}_i(x) : (\Omega^{N_1} \times \Omega^{N_2} \times \ldots \times \Omega^{N_p}) \to \Omega^{M_i}$, (i = 1, 2, …, p) are the vector of objective functions of the DM_i, (i = 1, 2, …, p) at the i-th level. Mathematically, a p-level MO-MLPP with fuzzy parameters is presented as follows:

[First Level]:

$$\operatorname*{Max}_{x_1} \tilde{Y}_1(x) = \operatorname*{Max}_{x_1} (\tilde{Y}_{11}(x), \tilde{Y}_{12}(x), \ldots, \tilde{Y}_{1M_1}(x)), \tag{3.1}$$

[Second Level]:

$$\operatorname*{Max}_{x_2} \tilde{Y}_2(x) = \operatorname*{Max}_{x_2} (\tilde{Y}_{21}(x), \tilde{Y}_{22}(x), \ldots, \tilde{Y}_{2M_2}(x)), \tag{3.2}$$

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[p^th Level]:

$$\operatorname*{Max}_{x_p} \tilde{Y}_p(x) = \operatorname*{Max}_{x_p} (\tilde{Y}_{p1}(x), \tilde{Y}_{p2}(x), \ldots, \tilde{Y}_{pM_p}(x)), \tag{3.3}$$

Subject to

$$\tilde{P}_1 x_1 + \tilde{P}_2 x_2 + \ldots + \tilde{P}_p x_p \ (\le, =, \ge) \tilde{Q}, \tag{3.4}$$

$$x_1 \ge 0, \quad x_2 \ge 0, \ldots, x_p \ge 0. \tag{3.5}$$

where $\tilde{Y}_{ij}(x) = \tilde{H}^{ij}_1 x_1 + \tilde{H}^{ij}_2 x_2 + \ldots + \tilde{H}^{ij}_p x_p = \tilde{H}^{ij}_{11} x_{11} + \tilde{H}^{ij}_{12} x_{12} + \ldots + \tilde{H}^{ij}_{1N_1} x_{1N_1} + \tilde{H}^{ij}_{21} x_{21} + \tilde{H}^{ij}_{22} x_{22} + \ldots + \tilde{H}^{ij}_{2N_2} x_{2N_2} + \ldots + \tilde{H}^{ij}_{p1} x_{p1} + \tilde{H}^{ij}_{p2} x_{p2} + \ldots + \tilde{H}^{ij}_{pN_p} x_{pN_p}$, (i = 1, 2, …, p), (j = 1, 2, …, M_i) (3.6)

Here, $\tilde{P}_i$ is M × N_i matrix, (i = 1, 2, …, p), $\tilde{Q}$ is the M component column vector. $\tilde{H}^{ij}_k = \left(\tilde{H}^{ij}_{k1}, \tilde{H}^{ij}_{k2}, ..., \tilde{H}^{ij}_{kN_k}\right)$, (i = 1, 2, …, p), (j = 1, 2, …, M_i) are constants, $x = x_1 \cup x_2 \cup \ldots \cup x_p$ is the set of decision vector, $N = N_1 + N_2 + \ldots + N_p$ = total number of decision variables in the system and M is the total number of system constraints. Here, $\tilde{Y}_1(x)$, $\tilde{Y}_2(x)$, …, $\tilde{Y}_p(x)$ are linear and bounded with fuzzy coefficients and let us represent the system constraints (3.4) & (3.5) as J ($\neq \Phi$).

4. Deterministic formulation of MO-MLPP with fuzzy parameters

At first, we convert the fuzzily described objectives and constraints to deterministic objectives and constraints for a specific value of λ. Now, for specific value of λ, maximization-type objective function $\tilde{Y}_{ij}(x)$, (i = 1, 2, …, p), (j = 1, 2, …, M_i) can be replaced by the upper bound of its λ-cut i.e.,

$$^{\lambda}\left(\tilde{Y}_{ij}(x)\right)^U = {}^{\lambda}(\tilde{H}^{ij}_1)^U x_1 + {}^{\lambda}(\tilde{H}^{ij}_2)^U x_2 + \ldots + {}^{\lambda}(\tilde{H}^{ij}_p)^U x_p, \text{ (i = 1, 2, …, p), (j = 1, 2, …, } M_i) \quad (4.1)$$

Similarly, minimization-type objective function $\tilde{Y}_{ij}(x)$, (i =1, 2, …, p), (j = 1, 2, …, M_i) can be replaced by the lower bound of its λ-cut i.e.,

$$^{\lambda}\left(\tilde{Y}_{ij}(x)\right)^L = {}^{\lambda}(\tilde{H}^{ij}_1)^L x_1 + {}^{\lambda}(\tilde{H}^{ij}_2)^L x_2 + \ldots + {}^{\lambda}(\tilde{H}^{ij}_p)^L x_p, \text{ (i = 1, 2, …, p), (j = 1, 2, …, } M_i) \quad (4.2)$$

The inequality constraints

$$\sum_{j=1}^{N} \tilde{P}_{ij} x_j \geq \tilde{Q}_i, \text{ (i = 1, 2, …, } m_1) \quad (4.3)$$

$$\sum_{j=1}^{N} \tilde{P}_{ij} x_j \leq \tilde{Q}_i, \text{ (i = } m_1\text{+1, } m_1\text{+2, …, } m_2) \quad (4.4)$$

can be modified by the following constraints:

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^U x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^L, \text{ (i = 1, 2, …, } m_1) \quad (4.5)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^L x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^U, \text{ (i = } m_1\text{+1, } m_1\text{+2, …, } m_2) \quad (4.6)$$

The fuzzy equality constraints

$$\sum_{j=1}^{N} \tilde{P}_{ij} x_j = \tilde{Q}_i, \text{ (i = } m_2\text{+1, } m_2\text{+2, …, M)} \quad (4.7)$$

can be replaced by two equivalent inequality constraints as given below.

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^U x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^L, \text{ (i = } m_2\text{+1, } m_2\text{+2, …, M)} \quad (4.8)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^L x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^U, \text{ (i = } m_2\text{+1, } m_2\text{+2, …, M)} \quad (4.9)$$

Lee and Li [32] proved that the Eq. (4.7) is equivalent to the Eq. (4.8) and Eq. (4.9).

Then, for a prescribed value of λ, the MO-MLPP reduces to the following problem as given below.

$$\underset{x_1}{\text{Max}}\ {}^{\lambda}(\tilde{Y}_1(x))^{U} = \underset{x_1}{\text{Max}}\left({}^{\lambda}(\tilde{Y}_{11}(x))^{U}, {}^{\lambda}(\tilde{Y}_{12}(x))^{U}, ..., {}^{\lambda}(\tilde{Y}_{1M_1}(x))^{U}\right),$$

$$\underset{x_2}{\text{Max}}\ {}^{\lambda}(\tilde{Y}_2(x))^{U} = \underset{x_2}{\text{Max}}\left({}^{\lambda}(\tilde{Y}_{21}(x))^{U}, {}^{\lambda}(\tilde{Y}_{22}(x))^{U}, ..., {}^{\lambda}(\tilde{Y}_{2M_2}(x))^{U}\right),$$

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$$\underset{x_p}{\text{Max}}\ {}^{\lambda}(\tilde{Y}_p(x))^{U} = \underset{x_p}{\text{Max}}\left({}^{\lambda}(\tilde{Y}_{p1}(x))^{U}, {}^{\lambda}(\tilde{Y}_{p2}(x))^{U}, ..., {}^{\lambda}(\tilde{Y}_{pM_p}(x))^{U}\right)$$

Subject to

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{U} x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^{L}, \ (i = 1, 2, \ldots, m_1, m_2+1, m_2+2, \ldots, M)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{L} x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^{U}, \ (i = m_1+1, \ldots, m_2, m_2+1, m_2+2, \ldots, M)$$

$$x_1 \geq 0, \quad x_2 \geq 0, \ldots, x_p \geq 0. \qquad (4.10)$$

5. Some basic concepts concerning distance measures

Basic concept of distance measure is presented in this section, for additional details see [26, 27, 28]. Let, $\tilde{Y}(x) = (\tilde{Y}_1(x), \tilde{Y}_2(x), \ldots, \tilde{Y}_M(x))$ be the vector of the objective functions with fuzzy parameters. For a prescribed value of λ, we assume that $Y^* = (Y_1^*, Y_2^*, ..., Y_M^*)$ be the PIS of the objective functions such that $Y_i^* = \underset{x\in J}{\text{Max}}\ {}^{\lambda}(\tilde{Y}_i(x))^{U}$, (i = 1, 2, ..., M) and $Y^- = (Y_1^-, Y_2^-, ..., Y_M^-)$ be the NIS of the objective functions such that $Y_i^- = \underset{x\in J}{\text{Min}}\ {}^{\lambda}(\tilde{Y}_i(x))^{L}$, (j = 1, 2, ..., M). B_K-metric is used to attain the measure of "closeness". B_K-metric defines the distance between $\tilde{Z}(x)$ and Z^* which is presented as follows:

$$B_k = \left\{\sum_{j=1}^{M} \varepsilon_j^k \left(Y_j^* - {}^{\lambda}(\tilde{Y}_j(x))^{U}\right)^k\right\}^{\frac{1}{k}}, \ k = 1, 2, ..., \infty. \qquad (5.1)$$

Here, ε_j^k, (j = 1, 2, ..., M; k = 1, 2, ..., ∞) denotes the relative weight of the j-th objective function.

However, if ${}^{\lambda}(\tilde{Y}_j(x))^{U}$, (j = 1, 2, ..., M) is not expressed in commensurable unit, then we can employ the modified metric as follows:

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$$B_k = \left\{ \sum_{j=1}^{M} \varepsilon_j^k \left(\frac{Y_j^* - {}^{\lambda}(\tilde{Y}_j(x))^U}{Y_j^* - Y_j^-} \right)^k \right\}^{\frac{1}{k}}, \ k = 1, 2, \ldots, \infty. \tag{5.2}$$

In order to find the compromise solution of the multi-objective decision making (MODM) problem we solve the following problem:

$$\text{Max}\ \tilde{Y}(x) = (\tilde{Y}_1(x),\ \tilde{Y}_2(x),\ \ldots,\ \tilde{Y}_M(x))$$

Subject to

$$\tilde{P}_1 x_1 + \tilde{P}_2 x_2 + \ldots + \tilde{P}_p x_p \ (\le, =, \ge)\ \tilde{Q},$$

$$x_1 \ge 0,\ \ x_2 \ge 0, \ldots, x_p \ge 0. \tag{5.3}$$

According to Lai et al. [33], the above problem (5.3) is transformed into the following auxiliary problem as given below.

$$\text{Min } B_k = \left\{ \sum_{j=1}^{M} \varepsilon_j^k \left(\frac{Y_j^* - {}^{\lambda}(\tilde{Y}_j(x))^U}{Y_j^* - Y_j^-} \right)^k \right\}^{\frac{1}{k}}, \ k = 1, 2, \ldots, \infty$$

Subject to

$$\tilde{P}_1 x_1 + \tilde{P}_2 x_2 + \ldots + \tilde{P}_p x_p \ (\le, =, \ge)\ \tilde{Q},$$

$$x_1 \ge 0,\ \ x_2 \ge 0, \ldots, x_p \ge 0. \tag{5.4}$$

The parameter 'k' is known as the 'balancing factor' between the group utility and maximal individual regret. It is to be noted that if the value of 'k' increases, the group utility i.e. B_k decreases [33].

6. TOPSIS based FGP approach for MO-MLPP with fuzzy parameters

For specific value of of λ, consider the deterministic MODM problem at i-th level is expressed as follows:

$$\underset{x_i}{\text{Max}}\ {}^{\lambda}(\tilde{Y}_i(x))^U = \underset{x_i}{\text{Max}} \left({}^{\lambda}(\tilde{Y}_{i1}(x))^U, {}^{\lambda}(\tilde{Y}_{i2}(x))^U, \ldots, {}^{\lambda}(\tilde{Y}_{iM_i}(x))^U \right), \ (i = 1, 2, \ldots, p)$$

Subject to

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^U x_j \ \ge\ {}^{\lambda}\left(\tilde{Q}_i\right)^L, \ (i = 1, 2, \ldots, m_1, m_2+1, m_2+2, \ldots, M)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^L x_j \ \le\ {}^{\lambda}\left(\tilde{Q}_i\right)^U, \ (i = m_1+1, \ldots, m_2, m_2+1, m_2+2, \ldots, M)$$

$$x_1 \ge 0,\ \ x_2 \ge 0, \ldots, x_p \ge 0. \tag{6.1}$$

TOPSIS model for i-th level DM can be formulated as follows:

$$\text{Min}\ {}^{\lambda}(d_k^{PIS_i}(x)), \ (i = 1, 2, \ldots, p)$$

$$\text{Max}\ {}^{\lambda}(d_k^{NIS_i}(x)), \ (i = 1, 2, \ldots, p)$$

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Subject to

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{U} x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^{L}, \ (i = 1, 2, \ldots, m_1, m_2+1, m_2+2, \ldots, M)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{L} x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^{U}, \ (i = m_1+1, \ldots, m_2, m_2+1, m_2+2, \ldots, M)$$

$$x_0 \geq 0, \quad x_1 \geq 0, \ldots, x_p \geq 0. \qquad (6.2)$$

where $${}^{\lambda}(g_k^{PIS_i}(x)) = \left\{ \sum_{j=1}^{M_i} \varepsilon_j^k \left(\frac{{}^{\lambda}(Y_{ij})^* - {}^{\lambda}(\tilde{Y}_{ij}(x))^U}{{}^{\lambda}(Y_{ij})^* - {}^{\lambda}(Y_{ij})^-} \right)^k \right\}^{\frac{1}{k}}, \ (i = 1, 2, \ldots, p);$$

$${}^{\lambda}(g_k^{NIS_i}(x)) = \left\{ \sum_{j=1}^{M_i} \varepsilon_j^k \left(\frac{{}^{\lambda}(\tilde{Y}_{ij}(x))^U - {}^{\lambda}(Y_{ij})^-}{{}^{\lambda}(Y_{ij})^* - {}^{\lambda}(Y_{ij})^-} \right)^k \right\}^{\frac{1}{k}}, \ (i = 1, 2, \ldots, p).$$

Here, ${}^{\lambda}(Y_{ij})^* = \underset{x\in J}{\mathrm{Max}}\ {}^{\lambda}(\tilde{Y}_{ij}(x))^U$ and ${}^{\lambda}(Y_{ij})^- = \underset{x\in J}{\mathrm{Min}}\ {}^{\lambda}(\tilde{Y}_{ij}(x))^U$, (i = 1, 2, …, p) are the PIS and NIS for i-th level DM respectively.

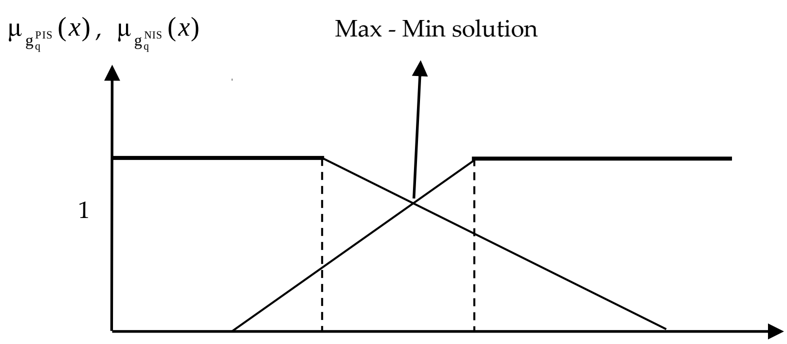
Let, ${}^{\lambda}\left(g_k^{PIS_i}\right)^* = \underset{x\in J}{\mathrm{Min}}\ {}^{\lambda}(g_k^{PIS_i}(x))$ and ${}^{\lambda}\left(g_k^{PIS_i}\right)^- = \underset{x\in J}{\mathrm{Max}}\ {}^{\lambda}(g_k^{PIS_i}(x))$;

${}^{\lambda}\left(g_k^{NIS_i}\right)^* = \underset{x\in J}{\mathrm{Max}}\ {}^{\lambda}(g_k^{NIS_i}(x))$ and ${}^{\lambda}\left(g_k^{NIS_i}\right)^- = \underset{x\in J}{\mathrm{Min}}\ {}^{\lambda}(g_k^{NIS_i}(x))$, (i = 1, 2, …, p).

The membership functions for ${}^{\lambda}(g_k^{PIS_i}(x))$ and ${}^{\lambda}(g_k^{NIS_i}(x))$ (see Fig.2) can be constructed as follows:

$${}^{\lambda}(\mu_{g_k^{PIS_i}}(x)) = \begin{cases} 0, & \text{if } {}^{\lambda}\left(g_k^{PIS_i}\right)^- \leq {}^{\lambda}(g_k^{PIS_i}(x)) \\ \dfrac{{}^{\lambda}\left(g_k^{PIS_i}\right)^- - {}^{\lambda}(g_k^{PIS_i}(x))}{{}^{\lambda}\left(g_k^{PIS_i}\right)^- - {}^{\lambda}\left(g_k^{PIS_i}\right)^*}, & \text{if } {}^{\lambda}\left(g_k^{PIS(F)}\right)^* \leq {}^{\lambda}(g_k^{PIS_i}(x)) \leq {}^{\lambda}\left(g_k^{PIS_i}\right)^-, \ (i = 1, 2, \ldots, p) \\ 1, & \text{if } {}^{\lambda}(g_k^{PIS_i}(x)) \leq {}^{\lambda}\left(g_k^{PIS_i}\right)^* \end{cases} \qquad (6.3)$$

$${}^{\lambda}(\mu_{g_k^{NIS_i}}(x)) = \begin{cases} 0, & \text{if } {}^{\lambda}(g_k^{NIS_i}(x)) \leq {}^{\lambda}\left(g_k^{NIS_i}\right)^- \\ \dfrac{{}^{\lambda}(g_k^{NIS_i}(x)) - {}^{\lambda}\left(g_k^{NIS_i}\right)^-}{{}^{\lambda}\left(g_k^{NIS_i}\right)^* - {}^{\lambda}\left(g_k^{NIS_i}\right)^-}, & \text{if } {}^{\lambda}\left(g_k^{NIS_i}\right)^- \leq {}^{\lambda}(g_k^{NIS}(x)) \leq {}^{\lambda}\left(g_k^{NIS_i}\right)^*, \ (i = 1, 2, \ldots, p) \\ 1, & \text{if } {}^{\lambda}(g_k^{NIS_i}(x)) \geq {}^{\lambda}\left(g_k^{NIS_i}\right)^* \end{cases} \qquad (6.4)$$



$g_q^{PIS}(x)$, $g_q^{NIS}(x)$

0 $\left(g_q^{NIS}\right)^-$ $\left(g_q^{PIS}\right)^+$ $\left(g_q^{NIS}\right)^+$ $\left(g_q^{PIS}\right)^-$

Figure 2. The membership functions of $g_q^{PIS}(x)$, $g_q^{NIS}(x)$

Convert the non-linear membership functions ${}^{\lambda}(\mu_{g_k^{PIS_i}}(x))$ and ${}^{\lambda}(\mu_{g_k^{NIS_i}}(x))$, (i = 1, 2, …, p) into equivalent linear membership functions ${}^{\lambda}(\tilde{\mu}_{g_k^{PIS_i}}(x))$ and ${}^{\lambda}(\tilde{\mu}_{g_k^{NIS_i}}(x))$, (i = 1, 2, …, p) respectively using first order Taylor polynomial series approximation as given below.

$$ {}^{\lambda}(\mu_{g_k^{PIS_i}}(x)) \tilde{=} {}^{\lambda}(\mu_{g_k^{PIS_i}}(x^{PIS^*})) + \sum_{j=1}^{N_i}(x_{ij} - x_{ij}^{PIS^*}) \left(\frac{\partial^{\lambda}(\mu_{g_k^{PIS_i}}(x))}{\partial x_{ij}} \right)_{at\ x = x^{PIS^*}} = {}^{\lambda}(\tilde{\mu}_{g_k^{PIS_i}}(x)),\ (i = 1, 2, \ldots, p) \qquad (6.5) $$

where $x^{PIS^*} = (x_1^{PIS^*}, x_2^{PIS^*}, \ldots, x_p^{PIS^*})$ is such that ${}^{\lambda}(\mu_{g_k^{PIS_i}}(x^{PIS^*})) = \underset{x\in J}{Max}\ {}^{\lambda}(\mu_{g_k^{PIS_i}}(x))$, (i = 1, 2, …, p);

$$ {}^{\lambda}(\mu_{g_k^{NIS_i}}(x)) \tilde{=} {}^{\lambda}(\mu_{g_k^{NIS_i}}(x^{NIS^*})) + \sum_{j=1}^{N_i}(x_{ij} - x_{ij}^{NIS^*}) \left(\frac{\partial^{\lambda}(\mu_{g_k^{NIS_i}}(x))}{\partial x_{ij}} \right)_{at\ x = x^{NIS^*}} = {}^{\lambda}(\tilde{\mu}_{g_k^{NIS_i}}(x)),\ (i = 1, 2, \ldots, p) \qquad (6.6) $$

where $x^{NIS^*} = (x_1^{NIS^*}, x_2^{NIS^*}, \ldots, x_p^{NIS^*})$ is such that ${}^{\lambda}(\mu_{g_k^{NIS_i}}(x^{NIS^*})) = \underset{x\in J}{Max}\ {}^{\lambda}(\mu_{g_k^{NIS_i}}(x))$, (i = 1, 2, …, p).

Due to Stanojević [29], we normalize ${}^{\lambda}(\tilde{\mu}_{g_k^{PIS_i}}(x))$ and ${}^{\lambda}(\tilde{\mu}_{g_k^{NIS_i}}(x))$ as follows:

$$ {}^{\lambda}(\bar{\mu}_{g_k^{PIS_i}}(x)) = \frac{{}^{\lambda}(\tilde{\mu}_{g_k^{PIS_i}}(x)) - \alpha_i^{PIS^*}}{\beta_i^{PIS^*} - \alpha_i^{PIS^*}}, \qquad (6.7) $$

$$ {}^{\lambda}(\bar{\mu}_{g_k^{NIS_i}}(x)) = \frac{{}^{\lambda}(\tilde{\mu}_{g_k^{NIS_i}}(x)) - \alpha_i^{NIS^*}}{\beta_i^{NIS^*} - \alpha_i^{NIS^*}},\ (i = 1, 2, \ldots, p); \qquad (6.8) $$

where $\beta_i^{PIS^*}$ and $\alpha_i^{PIS^*}$ are the maximal and minimal values of ${}^{\lambda}(\tilde{\mu}_{g_k^{PIS_i}}(x))$, (i = 1, 2, …, p); $\beta_i^{NIS^*}$ and $\alpha_i^{NIS^*}$ are the maximal and minimal values of ${}^{\lambda}(\tilde{\mu}_{d_k^{NIS_i}}(x))$, (i = 1, 2, …, p) respectively.

If $\beta_i^{PIS^*} > 1$, then we consider $\beta_i^{PIS^*} = 1$, (i = 1, 2, ..., p) since the value of the membership function cannot be superior than one. Also if $\alpha_i^{PIS^*} < 0$, then we consider $\alpha_i^{PIS^*} = 0$, (i = 1, 2, ..., p) because the value of the membership function cannot be less than zero [30]. The results also hold for $\beta_i^{NIS^*}$ and $\alpha_i^{NIS^*}$, (i = 1, 2, ..., p).

Solve the MODM model to achieve the satisfactory solution of i-th level DM as follows:

$$\text{Max}\ {}^{\lambda}(\bar{\mu}_{g_k^{PIS_i}}(x)),\ (i = 1, 2, ..., p)$$

$$\text{Max}\ {}^{\lambda}(\bar{\mu}_{g_k^{NIS_i}}(x)),\ (i = 1, 2, ..., p)$$

Subject to

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{U} x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^{L},\ (i = 1, 2, \ldots, m_1, m_2+1, m_2+2, \ldots, M)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{L} x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^{U},\ (i = m_1+1, \ldots, m_2, m_2+1, m_2+2, \ldots, M)$$

$$x_1 \geq 0,\quad x_2 \geq 0, \ldots, x_p \geq 0. \qquad (6.9)$$

According to Pramanik and Dey [22], the flexible membership goals of with aspiration level unity can be expressed as follows:

$${}^{\lambda}(\bar{\mu}_{g_k^{PIS_i}}(x)) + d^{-}_{PIS^i} = 1,\ (i = 1, 2, ..., p) \qquad (6.10)$$

$${}^{\lambda}(\bar{\mu}_{g_k^{NIS_i}}(x)) + d^{-}_{NIS^i} = 1,\ (i = 1, 2, ..., p) \qquad (6.11)$$

where $d^{-}_{PIS^i} \in [0, 1]$ and $d^{-}_{NIS^i} \in [0, 1]$, (1, 2, …, p) are the negative deviational variables corresponding to PIS and NIS respectively. The following MODM model is solved based on FGP method to achieve the optimal decision of each level DM as follows:

MODM Model:

Min ζ

Subject to

$${}^{\lambda}(\bar{\mu}_{g_k^{PIS_i}}(x)) + d^{-}_{PIS^i} = 1,\ (i = 1, 2, ..., p)$$

$${}^{\lambda}(\bar{\mu}_{g_k^{NIS_i}}(x)) + d^{-}_{NIS^i} = 1,\ (i = 1, 2, ..., p)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{U} x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^{L},\ (i = 1, 2, \ldots, m_1, m_2+1, m_2+2, \ldots, M)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{L} x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^{U},\ (i = m_1+1, \ldots, m_2, m_2+1, m_2+2, \ldots, M)$$

$$\zeta \geq d^{-}_{PIS^i},\ \zeta \geq d^{-}_{NIS^i},\ (i = 1, 2, ..., p)$$

$$d^{-}_{PIS^i} \in [0, 1],\ d^{-}_{NIS^i} \in [0, 1],\ (i = 1, 2, ..., p)$$

$$x_1 \geq 0,\quad x_2 \geq 0, \ldots, x_p \geq 0. \qquad (6.12)$$

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Solving the above Eq. (6.12), let $x^{i+} = (x_1^{i+}, x_2^{i+}, \ldots, x_p^{i+})$ be the optimal solution of i-th level DM.

To avoid any unwanted circumstance i.e decision deadlock, the level DMs should offer some relaxation on decision by assigning preference upper and lower bounds on the decision variables under their control [18, 22, 30, 35, 36, 37] for ovallall benefit and smooth functioning of the organization and these preference bounds are included in the constraints set.

Consider γ_i^i and δ_i^i, (i = 1, 2, ..., p) be the lower and upper tolerance values on the decision vector considered by i-th level DM such that

$$x_i^{i+} - \gamma_i^i \leq x_i^i \leq x_i^{i+} + \delta_i^i, \ (i = 1, 2, \ldots, p) \tag{6.13}$$

Therefore, the new hybrid models of FGP and TOPSIS for MO- MLPP for a specific λ can be formulated as follows:

Model (I):

Minimize ρ

Subject to

$${}^{\lambda}(\bar{\mu}_{g_k^{PIS_i}}(x)) + D^-_{PIS^i} = 1, \ (i = 1, 2, \ldots, p)$$

$${}^{\lambda}(\bar{\mu}_{g_k^{NIS_i}}(x)) + D^-_{NIS^i} = 1, \ (i = 1, 2, \ldots, p)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^U x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^L, \ (i = 1, 2, \ldots, m_1, m_2+1, m_2+2, \ldots, M)$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^L x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^U, \ (i = m_1+1, \ldots, m_2, m_2+1, m_2+2, \ldots, M)$$

$$x_i^{i+} - \gamma_i^i \leq x_i^i \leq x_i^{i+} + \delta_i^i, \ (i = 1,2, \ldots,p)$$

$$\rho \geq D^-_{PIS^i}, \ \rho \geq D^-_{NIS^i}, \ (i = 1,2, \ldots,p)$$

$$D^-_{PIS^i} \in [0, 1], \ D^-_{NIS^i} \in [0, 1], \ (i = 1,2, \ldots,p)$$

$$x_1 \geq 0, \quad x_2 \geq 0, \ \ldots, \ x_p \geq 0. \tag{6.14}$$

Model (II):

Minimize $\sigma = w_{PIS^i} D^-_{PIS^i} + w_{NIS^i} D^-_{NIS^i}$, (i = 1, 2, ...,p)

Subject to

$${}^{\lambda}(\bar{\mu}_{g_k^{PIS_i}}(x)) + D^-_{PIS^i} = 1, \ (i = 1, 2, \ldots, p)$$

$${}^{\lambda}(\bar{\mu}_{g_k^{NIS_i}}(x)) + D^-_{NIS^i} = 1, \ (i = 1, 2, \ldots, p)$$

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$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{U} x_j \geq {}^{\lambda}\left(\tilde{Q}_i\right)^{L}, \text{ (i = 1, 2, …, } m_1, m_2+1, m_2+2, \text{ …, M)}$$

$$\sum_{j=1}^{N} {}^{\lambda}\left(\tilde{P}_{ij}\right)^{L} x_j \leq {}^{\lambda}\left(\tilde{Q}_i\right)^{U}, \text{ (i = } m_1+1, \text{ …, } m_2, m_2+1, m_2+2, \text{ …, M)}$$

$$x_i^{i+} - \gamma_i^{i} \leq x_i^{i} \leq x_i^{i+} + \delta_i^{i}, \text{ (i = 1, 2, ...,p)}$$

$$D^{-}_{PIS^i} \in [0, 1], D^{-}_{NIS^i} \in [0, 1], \text{ (i = 1,2, ...,p)}$$

$$x_1 \geq 0, \quad x_2 \geq 0, \ldots, x_p \geq 0. \tag{6.15}$$

The i-th level DM can take the normalized weight i.e. $w_{PIS^i} + w_{NIS^i} = 1$, (i = 1, 2, ..., p) or any preference weight in the decision making situation. $D^{-}_{PIS^i}$ and $D^{-}_{NIS^i} \in [0, 1]$, (i = 1, 2, ..., p) are negative deviational variables.

7. Selection of compromise optimal solution of MO-MLPP

For selecting compromise optimal solution, we consider a termination criteria based on distance functios. The family of distance functions defined by Zeleny [38] is expressed as given below.

$$L_{\Re}(\omega, q) = \left(\sum_{k=1}^{K} \omega_q^{\Re}(1-\varphi_q)^{\Re}\right)^{1/\Re} \tag{7.1}$$

where φ_q, (q = 1, 2, ..., Q) represents the measure of closeness of the preferred compromise solution to the optimal compromise solution vector regarding q - th objective function. Here, $\omega = (\omega_1, \omega_2, \ldots, \omega_Q)$ denotes the vector of attribute level and $\Re$ ($1 \leqq \Re \leqq \infty$) is the distance parameter. We consider $\Re = 2$, then the distance function becomes

$$L_2(\omega, q) = \left(\sum_{q=1}^{Q} \omega_q^2(1-\varphi_q)^2\right)^{1/2} \tag{7.2}$$

For maximization type of problem φ_q = (the preferred compromise solution/ the individual best solution). The solution for which $L_2(\omega, q)$ will be minimal would be the compromise optimal solution for each level DM.

8 TOPSIS based algorithm to MO-MLPP with fuzzy parameters

The proposed TOPSIS based algorithm (see Fig 3) for MO-MLPP with fuzzy parameters is provided below.

Step 1: For specified value of λ, the upper and lower bounds of the fuzzily described objective functions and constraints are defined at first.

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Step 2: Calculate the maximum and minimum values for the upper and lower λ- cuts of the objective functions for all level DMs separately subject to the common constraints.

Step 3: Compute PIS and NIS for i-th level DM and formulate distance functions for PIS and NIS $^{\lambda}(g_k^{PIS_i}(x))$ and $^{\lambda}(g_k^{NIS_i}(x))$, (i = 1,2, ...,p) respectively for i-th level DM.

Step 4: Request all the level DMs to select the value of k, (k = 1, 2, ..., ∞).

Step 5: Compute the maximum and minimum values of $^{\lambda}(g_k^{PIS_i}(x))$ and $^{\lambda}(g_k^{NIS_i}(x))$, (i = 1, 2, ..., p) subject to the common constraints and construct the membership functions $^{\lambda}(\mu_{g_k^{PIS_i}}(x))$ and $^{\lambda}(\mu_{g_k^{NIS_i}}(x))$, (i = 1, 2, ..., p).

Step 6: Transform the non-linear membership functions $^{\lambda}(\mu_{g_k^{PIS_i}}(x))$ and $^{\lambda}(\mu_{g_k^{NIS_i}}(x))$, (i = 1, 2, ..., p) into equivalent linear membership functions $^{\lambda}(\tilde{\mu}_{g_k^{PIS_i}}(x))$ and $^{\lambda}(\tilde{\mu}_{g_k^{NIS_i}}(x))$, (i = 1, 2, ..., p) respectively by using suitable transformation technique and then normalize equivalent linear membership functions.

Step 7: Formulate the MODM model (6.12) to identify the satisfactory solution $x^{i+} = (x_1^{i+}, x_2^{i+}, \ldots, x_p^{i+})$, (i = 1, 2, ..., p) of i-th level DM.

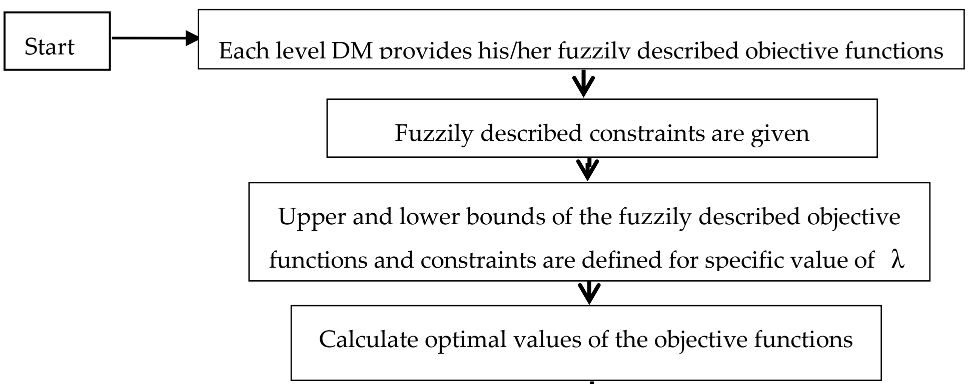
Step 8: DMs offer the lower and upper tolerance values γ_i^i and δ_i^i, (i = 1,2, ...,p) respectively on the decision vector $x^{i+} = (x_1^{i+}, x_2^{i+}, \ldots, x_p^{i+})$, (i = 1, 2, ..., p).

Step 9: Construct the TOPSIS based FGP Models (6.14) and (6.15).

Step 10: Solve the Models (6.14) and (6.15).

Step 11: $L_2(\omega, q)$ is employed to identify better compromise optimal solution of the problem.

Step 12: If the compromise optimal solution is acceptable to all level DMs then stop. Otherwise, adjust the lower and upper tolerance values of all level DMs and go to Step 8.



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A flowchart of the proposed algorithms

9. Numerical example

The following MO-MLPP with fuzzy parameters is considered to demonstrate the proposed procedure.

[First Level]

$$\underset{x_1}{\text{Max}}\ (\tilde{Y}_{11}(x) = \tilde{7}x_1 + x_2 + \tilde{2}x_3,\ \ \tilde{Z}_{12}(x) = \tilde{2}x_1 + \tilde{10}x_2 - \tilde{3}x_3)$$

[Second Level]

$$\underset{x_2}{\text{Max}}\ (\tilde{Y}_{21}(x) = -\tilde{2}x_1 + \tilde{4}x_2 + \tilde{4}x_3,\ \ \tilde{Z}_{22}(x) = -\tilde{6}x_1 + \tilde{7}x_2 + \tilde{4}x_3),$$

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[Third Level]

$$\underset{x_3}{\text{Max}}\ (\tilde{Y}_{31}(x) = -\tilde{3}\,x_1 + \tilde{2}\,x_2 + \tilde{10}\,x_3,\ \ \tilde{Z}_{32}(x) = -\tilde{5}\,x_1 + \tilde{7}\,x_2 + \tilde{12}\,x_3)$$

Subject to

$$\tilde{2}\,x_1 + \tilde{2}\,x_2 + x_3 \le \tilde{10},$$

$$x_1 + \tilde{5}\,x_2 - \tilde{2}\,x_3 \le \tilde{12},$$

$$\tilde{4}\,x_1 + x_2 - \tilde{3}\,x_3 \ge \tilde{5},$$

$$x_1 \ge 0,\ \ x_2 \ge 0,\ x_3 \ge 0. \qquad (9.1)$$

Here, we consider all the fuzzy numbers to be triangular fuzzy numbers and they are given by

$\tilde{2} = (0, 2, 3)$, $\tilde{3} = (2, 3, 4)$, $\tilde{4} = (2, 4, 5)$, $\tilde{5} = (4, 5, 6)$, $\tilde{6} = (5, 6, 8)$, $\tilde{7} = (5, 7, 8)$, $\tilde{10} = (9, 10, 12)$, $\tilde{12} = (11, 12, 14)$.

Replacing the fuzzy coefficient by specified λ, the MO-MLPP can be represented as given below.

$$\underset{x_1}{\text{Max}}\ {}^{\lambda}\!\left(\tilde{Y}_{11}(x)\right)^{U} = (8 - \lambda)\,x_1 + x_2 + (3 - \lambda)\,x_3,$$

$$\underset{x_1}{\text{Max}}\ {}^{\lambda}\!\left(\tilde{Y}_{12}(x)\right)^{U} = (3 - \lambda)\,x_1 + (12 - 2\lambda)\,x_2 - (4 - \lambda)\,x_3,$$

$$\underset{x_2}{\text{Max}}\ {}^{\lambda}\!\left(\tilde{Y}_{21}(x)\right)^{U} = -(3 - \lambda)\,x_1 + (5 - \lambda)\,x_2 + (5 - \lambda)\,x_3,$$

$$\underset{x_2}{\text{Max}}\ {}^{\lambda}\!\left(\tilde{Y}_{22}(x)\right)^{U} = -(8 - 2\lambda)\,x_1 + (8 - \lambda)\,x_2 + (5 - \lambda)\,x_3,$$

$$\underset{x_3}{\text{Max}}\ {}^{\lambda}\!\left(\tilde{Y}_{31}(x)\right)^{U} = -(4 - \lambda)\,x_1 + (3 - \lambda)\,x_2 + (12 - 2\lambda)\,x_3,$$

$$\underset{x_3}{\text{Max}}\ {}^{\lambda}\!\left(\tilde{Y}_{32}(x)\right)^{U} = -(6 - \lambda)\,x_0 + (8 - \lambda)\,x_2 + (14 - 2\lambda)\,x_3,$$

Subject to

$$(2\lambda)\,x_1 + (2\lambda)\,x_2 + x_3 \le (12 - 2\lambda),$$

$$x_1 + (4 + \lambda)\,x_2 - (2\lambda)\,x_3 \le (14 - 2\lambda),$$

$$(5 - \lambda)\,x_1 + x_2 - (4 - \lambda)\,x_3 \ge (4 + \lambda),$$

$$x_1,\ x_2,\ x_3 \ge 0. \qquad (9.2)$$

For $\lambda = 0.5$, the above fuzzy MO-MLPP transforms itself into deterministic MO-MLPP as follows:

$$\underset{x_1}{\text{Max}}\ {}^{0.5}\!\left(\tilde{Y}_{11}(x)\right)^{U} = 7.5\,x_1 + x_2 + 2.5\,x_3,$$

$$\underset{x_1}{\text{Max}}\ {}^{0.5}\!\left(\tilde{Y}_{12}(x)\right)^{U} = 2.5\,x_1 + 11\,x_2 - 3.5\,x_3,$$

$$\underset{x_2}{\text{Max}}\ {}^{0.5}\!\left(\tilde{Y}_{21}(x)\right)^{U} = -2.5\,x_1 + 4.5\,x_2 + 4.5\,x_3,$$

$$\underset{x_2}{\text{Max}} \left(\tilde{Y}_{22}(x)\right)_{0.5}^{U} = -7x_1 + 7.5x_2 + 4.5x_3,$$

$$\underset{x_3}{\text{Max}} \left(\tilde{Y}_{31}(x)\right)_{0.5}^{U} = -3.5x_1 + 2.5x_2 + 11x_3,$$

$$\underset{x_3}{\text{Max}} \left(\tilde{Y}_{32}(x)\right)_{0.5}^{U} = -5.5x_0 + 7.5x_2 + 13x_3,$$

Subject to

$x_1 + x_2 + x_3 \leq 11,$

$x_1 + 4.5x_2 - x_3 \leq 13,$

$4.5x_1 + x_2 - 3.5x_3 \geq 4.5,$

$x_1, x_2, x_3 \geq 0.$ (9.3)

The individual best (maximal) solution $\left(\tilde{Y}_{ij}^{B}\right)_{0.5}^{U}$, (i = 1, 2, 3; j = 1, 2) of the objective functions of level DMs are presented in the Table 1.

Table 1. The individual best solution

	$\left(\tilde{Y}_{11}^{B}\right)_{0.5}^{U}$	$\left(\tilde{Y}_{12}^{B}\right)_{0.5}^{U}$	$\left(\tilde{Y}_{21}^{B}\right)_{0.5}^{U}$	$\left(\tilde{Y}_{22}^{B}\right)_{0.5}^{U}$	$\left(\tilde{Y}_{31}^{B}\right)_{0.5}^{U}$	$\left(\tilde{Y}_{32}^{B}\right)_{0.5}^{U}$
$\underset{x \in S}{\text{Max}} \left(\tilde{Y}_{ij}\right)_{0.5}^{U}$	82.5	32.357	23.8	18.403	43.062	58.421
	at (11, 0, 0)	at (10.429, 0.571, 0)	at (3.671, 3.029, 4.3)	at (0.377, 2.805, 0)	at (5.375, 0, 5.625)	at (3.671, 3.029, 4.3)

To obtain the individual worst (minimal) solutions, substitute the fuzzy coefficient by their λ-cuts as follows:

$$\underset{x_1}{\text{Min}} \left(\tilde{Y}_{11}(x)\right)_{\lambda}^{L} = (5 + 2\lambda)x_1 + x_2 + (2\lambda)x_3,$$

$$\underset{x_1}{\text{Min}} \left(\tilde{Y}_{12}(x)\right)_{\lambda}^{L} = (2\lambda)x_1 + (9 + \lambda)x_2 - (2 + \lambda)x_3,$$

$$\underset{x_2}{\text{Min}} \left(\tilde{Y}_{21}(x)\right)_{\lambda}^{L} = -(2\lambda)x_1 + (2 + 2\lambda)x_2 + (2 + 2\lambda)x_3,$$

$$\underset{x_2}{\text{Min}} \left(\tilde{Y}_{22}(x)\right)_{\lambda}^{L} = -(5 + \lambda)x_1 + (5 + 2\lambda)x_2 + (2 + 2\lambda)x_3,$$

$$\underset{x_3}{\text{Min}} \left(\tilde{Y}_{31}(x)\right)_{\lambda}^{L} = -(2 + \lambda)x_1 + (2\lambda)x_2 + (9 + \lambda)x_3,$$

$$\underset{x_3}{\text{Min}} \left(\tilde{Y}_{32}(x)\right)_{\lambda}^{L} = -(4 + \lambda)x_1 + (5 + 2\lambda)x_2 + (11 + \lambda)x_3,$$

Subject to

$x_1 + x_2 + x_3 \leq 11,$

$x_1 + 4.5\,x_2 - x_3 \le 13,$

$4.5\,x_1 + x_2 - 3.5\,x_3 \ge 4.5,$

$x_1, x_2, x_3 \ge 0.$ (9.4)

For $\lambda = 0.5$, the above problem (9.4) reduces to the problem as given below.

$$\underset{x_1}{\text{Min}}\ {}^{0.5}\left(\tilde{Y}_{11}(x)\right)^{L} = 6\,x_1 + x_2 + x_3,$$

$$\underset{x_1}{\text{Min}}\ {}^{0.5}\left(\tilde{Y}_{12}(x)\right)^{L} = x_1 + 9.5\,x_2 - 2.5\,x_3,$$

$$\underset{x_2}{\text{Min}}\ {}^{0.5}\left(\tilde{Y}_{21}(x)\right)^{L} = -\,x_1 + 3\,x_2 + 3\,x_3,$$

$$\underset{x_2}{\text{Min}}\ {}^{0.5}\left(\tilde{Y}_{22}(x)\right)^{L} = -\,5.5\,x_1 + 6\,x_2 + 3\,x_3,$$

$$\underset{x_3}{\text{Min}}\ {}^{0.5}\left(\tilde{Y}_{31}(x)\right)^{L} = -\,2.5\,x_1 + x_2 + 9.5\,x_3,$$

$$\underset{x_3}{\text{Min}}\ {}^{0.5}\left(\tilde{Y}_{32}(x)\right)^{L} = -\,4.5\,x_1 + 6\,x_2 + 11.5\,x_3,$$

Subject to

$x_1 + x_2 + x_3 \le 11,$

$x_1 + 4.5\,x_2 - x_3 \le 13,$

$4.5\,x_1 + x_2 - 3.5\,x_3 \ge 4.5,$

$x_1, x_2, x_3 \ge 0.$ (9.5)

The individual worst (minimal) solution ${}^{0.5}\left(\tilde{Y}^{W}_{ij}\right)^{L}$, $(i = 1, 2, 3;\ j = 1, 2)$ of the objective functions of level DMs are demonstrated in the Table 2.

Table 2. The individual worst solution

	${}^{0.5}\left(\tilde{Y}^{W}_{11}\right)^{L}$	${}^{0.5}\left(\tilde{Y}^{W}_{12}\right)^{L}$	${}^{0.5}\left(\tilde{Y}^{W}_{21}\right)^{L}$	${}^{0.5}\left(\tilde{Y}^{W}_{22}\right)^{L}$	${}^{0.5}\left(\tilde{Y}^{W}_{31}\right)^{L}$	${}^{0.5}\left(\tilde{Y}^{W}_{32}\right)^{L}$
$\underset{x \in J}{\text{Min}}\ {}^{0.5}\left(\tilde{Y}_{ij}\right)^{L}$	5.065	-8.687	-11	-60.5	-27.5	-49.5
	at (0.377, 2.805, 0)	at (5.357, 0, 5.625)	at (11, 0, 0)	at (11, 0, 0)	at (11, 0, 0)	at (11, 0, 0)

Assume that $\varepsilon_1 = \varepsilon_2 = 0.5$, and $k = 2$.

First-level MODM problem:

$$g_2^{PIS^1}(x) = \left\{ (0.5)^2 \left[\frac{82.5 - 7.5x_1 - x_2 - 2.5x_3}{82.5 - 5.065} \right]^2 + (0.5)^2 \left[\frac{32.357 - 2.5x_1 - 11x_2 + 3.5x_3}{32.357 + 8.687} \right]^2 \right\}^{1/2},$$

$$g_2^{NIS^1}(x) = \left\{ (0.5)^2 \left[\frac{7.5x_1 + x_2 + 2.5x_3 - 5.065}{82.5 - 5.065} \right]^2 + (0.5)^2 \left[\frac{2.5x_1 + 11x_2 + 3.5x_3 + 8.687}{32.357 + 8.687} \right]^2 \right\}^{1/2}$$

We determine: $\left(g_2^{PIS^1}(x)\right)^* = \underset{x \in J}{\text{Min}}\; g_2^{PIS^1}(x) = 0.022$ at (10.509, 0.491, 0); $\left(g_2^{PIS^1}(x)\right)^- = \underset{x \in J}{\text{Max}}\; g_2^{PIS^1}(x) =$ 0.606 at (1, 0, 0); $\left(g_2^{NIS^1}(x)\right)^* = \underset{x \in J}{\text{Max}}\; g_2^{NIS^1}(x) = 0.69$ at (10.429, 0.571, 0); $\left(g_2^{NIS^1}(x)\right)^- = \underset{x \in J}{\text{Min}}\; g_2^{NIS^1}(x) = 0.134$ at (1.415, 0, 0.533).

The membership functions of $g_2^{PIS^1}(x)$ and $g_2^{NIS^1}(x)$ can be formulated as follows:

$$\mu_{g_2^{PIS^1}}(x) = \begin{cases} 0, & \text{if } 0.606 \le g_2^{PIS^1}(x) \\ \dfrac{0.606 - g_2^{PIS^1}(x)}{0.606 - 0.022}, & \text{if } 0.022 \le g_2^{PIS^1}(x) \le 0.606\,; \\ 1, & \text{if } g_2^{PIS^1}(x) \le 0.022 \end{cases}$$

$$\mu_{g_2^{NIS^1}}(x) = \begin{cases} 0, & \text{if } \left(g_2^{NIS^1}(x)\right) \le 0.134 \\ \dfrac{g_2^{NIS^1}(x) - 0.134}{0.69 - 0.134}, & \text{if } 0 \le g_2^{NIS^1}(x) \le 0.69 \\ 1, & \text{if } g_2^{NIS^1}(x) \ge 0.69 \end{cases}$$

Solve the following MODM Model to obtain the satisfactory solution of First-level DM:

Min α

Subject to

$((1 + (x_1 - 10.509) \times 0.096 + (x_2 - 0.491) \times 0.096 + (x_3 - 0) \times (-0.002) - 0.004)/(1-0.004)) + d^-_{PIS^1} = 1,$

$1 + (x_1 - 10.429) \times 0.1 + (x_2 - 0.571) \times 0.183 + (x_3 - 0) \times (-0.036) + d^-_{NIS^1} = 1,$

$\alpha \ge d^-_{PIS^1},\ \alpha \ge d^-_{NIS^1},$

$d^-_{PIS^1} \in [0, 1],\ d^-_{NIS^1} \in [0, 1],$

$x_1 + x_2 + x_3 \le 11,$

$x_1 + 4.5\,x_2 - x_3 \le 13,$

$4.5\,x_1 + x_2 - 3.5\,x_3 \ge 4.5,$

$x_1,\ x_2,\ x_3 \ge 0.$ (9.6)

The satisfactory solution of the First-level MODM problem is obtained as $x^{F^*} = (x_1^{F^*}, x_2^{F^*}, x_3^{F^*}) =$ (10.429, 0.571, 0). Suppose the First-level DM decides $x_1^{F^*} = 10.429$ with lower tolerance $\gamma_1^1 = 5.929$ and upper tolerance $\delta_1^1 = 0.571$ such that $10.429 - 5.929 \le x_1 \le 10.429 + 0.571$.

Second level MODM problem:

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$$g_2^{PIS^2}(x)=\left\{(0.5)^2\left[\frac{23.8+2.5x_1-4.5x_2-4.5x_3}{23.8+11}\right]^2+(0.5)^2\left[\frac{18.403+7x_1-7.5x_2-4.5x_3}{18.403+60.5}\right]^2\right\}^{1/2},$$

$$g_2^{NIS^1}(x)=\left\{(0.5)^2\left[\frac{-2.5x_1+4.5x_2+4.5x_3+11}{23.8+11}\right]^2+(0.5)^2\left[\frac{-7x_1+7.5x_2+4.5x_3+60.5}{18.403+60.5}\right]^2\right\}^{1/2}$$

We calculate: $\left(g_2^{PIS^2}(x)\right)^*=\underset{x\in J}{\mathrm{Min}}\ g_2^{PIS^2}(x)$ = 0.013 at (3.653, 3.027, 4.276); $\left(g_2^{PIS^2}(x)\right)^-=\underset{x\in J}{\mathrm{Max}}\ g_2^{PIS^2}(x)$ = 0.953 at (11, 0, 0); $\left(g_2^{NIS^2}(x)\right)^*=\underset{x\in J}{\mathrm{Max}}\ g_2^{NIS^2}(x)$ = 0.698 at (3.671, 3.029, 4.3); $\left(g_2^{NIS^1}(x)\right)^-$ = $\underset{x\in J}{\mathrm{Min}}\ g_2^{NIS^2}(x)$ = 0.054 at (8.96, 0, 2.04).

The membership functions $\mu_{g_2^{PIS^2}}(x)$ and $\mu_{g_2^{NIS^2}}(x)$ can be obtained as follows:

$$\mu_{g_2^{PIS^2}}(x)=\begin{cases}0, & \text{if } 0.953\le g_2^{PIS^1}(x)\\ \dfrac{0.953-g_2^{PIS^1}(x)}{0.953-0.013}, & \text{if } 0.013\le g_2^{PIS^1}(x)\le 0.953\\ 1, & \text{if } g_2^{PIS^1}(x)\le 0.013\end{cases};$$

$$\mu_{g_2^{NIS^2}}(x)=\begin{cases}0, & \text{if } \left(g_2^{NIS^2}(x)\right)\le 0.054\\ \dfrac{g_2^{NIS^1}(x)-0.054}{0.698-0.054}, & \text{if } 0.054\le g_2^{NIS^2}(x)\le 0.698\\ 1, & \text{if } g_2^{NIS^1}(x)\ge 0.698\end{cases}.$$

MODM model for Second-level DM for obtaing satisfactory solution is developed as given below.

Min α

Subject to

$1 + (x_1 - 3.653) \times (-0.05) + (x_2 - 3.027) \times 0.557 + (x_3 - 4.276) \times 0.036 + d^-_{PIS^2} = 1,$

$1 + (x_1 - 3.671) \times (-0.088) + (x_2 - 3.029) \times 0.123 + (x_3 - 4.3) \times 0.103 + d^-_{NIS^2} = 1,$

$\alpha \ge d^-_{PIS^2}, \alpha \ge d^-_{NIS^2},$

$d^-_{PIS^1} \in [0, 1], d^-_{NIS^2} \in [0, 1],$

$x_1 + x_2 + x_3 \le 11,$

$x_1 + 4.5 x_2 - x_3 \le 13,$

$4.5 x_1 + x_2 - 3.5 x_3 \ge 4.5,$

$x_1, x_2, x_3 \ge 0.$ (9.7)

The satisfactory solution of the Second-level MODM problem is determined as x^{S^*} = $(x_1^{S^*}, x_2^{S^*}, x_3^{S^*})$ = (3.672, 3.027, 4.3). Let the Second-level DM decides $x_2^{S^*}$ = 3.027 with lower tolerance γ_2^2 = 1.027 and upper tolerance δ_2^2 = 1.473 such that $3.027 - 1.027 \le x_2 \le 3.027 + 1.473$.

Third-level MODM problem:

$$g_2^{PIS^3}(x)=\left\{(0.5)^2\left[\frac{43.062+2.5x_1-x_2-9.5x_3}{43.062+27.5}\right]^2+(0.5)^2\left[\frac{58.421+5.5x_1-7.5x_2-12x_3}{58.421+49.5}\right]^2\right\}^{1/2},$$

$$g_2^{NIS^3}(x)=\left\{(0.5)^2\left[\frac{-2.5x_1+x_2+9.5x_3+27.5}{43.062+27.5}\right]^2+(0.5)^2\left[\frac{-5.5x_1+7.5x_2+12x_3+49.5}{58.421+49.5}\right]^2\right\}^{1/2}$$

Here, $\left(g_2^{PIS^3}(x)\right)^* = \underset{x\in J}{Min}\ g_2^{PIS^3}(x)$ = 0.062 at (3.849, 2.713, 4.438); $\left(g_2^{PIS^3}(x)\right)^- = \underset{x\in J}{Max}\ g_2^{PIS^3}(x)$ = 0.744 at (11, 0, 0); $\left(g_2^{NIS^3}(x)\right)^* = \underset{x\in J}{Max}\ g_2^{NIS^3}(x)$ = 0.625 at (3.671, 3.029, 4.3); $\left(g_2^{NIS^3}(x)\right)^-$ = $\underset{x\in J}{Min}\ d_2^{NIS^3}(x)$ = 0.02 at (10.241, 0.613, 0).

The membership functions of $g_2^{PIS^3}(x)$ and $g_2^{NIS^3}(x)$ can be presented as given below.

$$\mu_{g_2^{PIS^3}}(x)=\begin{cases}0, & \text{if } 0.744\le g_2^{PIS^3}(x)\\ \dfrac{0.744-g_2^{PIS^3}(x)}{0.744-0.062}, & \text{if } 0.062\le g_2^{PIS^1}(x)\le 0.744\\ 1, & \text{if } g_2^{PIS^3}(x)\le 0.062\end{cases};$$

$$\mu_{g_2^{NIS^3}}(x)=\begin{cases}0, & \text{if } \left(g_2^{NIS^3}(x)\right)\le 0.02\\ \dfrac{g_2^{NIS^3}(x)-0.02}{0.652-0.02}, & \text{if } 0.02\le g_2^{NIS^3}(x)\le 0.652\\ 1, & \text{if } g_2^{NIS^3}(x)\ge 0.652\end{cases}$$

Next, in order achieve the satisfactory solution of Third-level DM, we solve the following MODM model:

Min α

Subject to

$((1 + (x_1 - 3.849) \times (-0.04) + (x_2 - 2.713) \times 0.032 + (x_3 - 4.438) \times 0.125) - 0.072)/(1-0.072) + d^-_{PIS^3} = 1,$

$1 + (x_1 - 3.671) \times (-0.049) + (x_2 - 3.029) \times 0.048 + (x_3 - 4.3) \times 0.137 + d^-_{NIS^3} = 1,$

$\alpha \ge d^-_{PIS^3}$, $\alpha \ge d^-_{NIS^3}$,

$d^-_{PIS^3} \in [0, 1]$, $d^-_{NIS^3} \in [0, 1]$,

$x_1 + x_2 + x_3 \le 11,$

$x_1 + 4.5\,x_2 - x_3 \le 13,$

$4.5\,x_1 + x_2 - 3.5\,x_3 \ge 4.5,$

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$ (9.8)

By solving the above Eq. (9.8), the satisfactory solution of the Third-level DM is obtained as x^{T^*} = $(x_1^{T^*}, x_2^{T^*}, x_3^{T^*})$ = (3.672, 3.028, 4.3). Suppose in the decision making situation, the Third -level DM

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decides $x_3^{T^*} = 4.3$ with lower tolerance $\gamma_3^3 = 2.3$ and upper tolerance $\delta_3^3 = 0.7$ such that $4.3 - 2.3 \le x_3 \le 4.3 + 0.7$.

Finally, the FGP models due to Dey et al. [30] for solving MO-MLPP involving fuzzy parameters based on TOPSIS method are formulated as follows:

Model (I)

Minimize ρ

Subject to

$$((1 + (x_1 - 10.509) \times 0.096 + (x_2 - 0.491) \times 0.096 + (x_3 - 0) \times (-0.002) - 0.004)/(1-0.004)) + D^-_{PIS^1} = 1,$$

$$1 + (x_1 - 10.429) \times 0.1 + (x_2 - 0.571) \times 0.183 + (x_3 - 0) \times (-0.036) + D^-_{NIS^1} = 1,$$

$$1 + (x_1 - 3.653) \times (-0.05) + (x_2 - 3.027) \times 0.557 + (x_3 - 4.276) \times 0.036 + D^-_{PIS^2} = 1,$$

$$1 + (x_1 - 3.671) \times (-0.088) + (x_2 - 3.029) \times 0.123 + (x_3 - 4.3) \times 0.103 + D^-_{NIS^2} = 1,$$

$$((1 + (x_1 - 3.849) \times (-0.04) + (x_2 - 2.713) \times 0.032 + (x_3 - 4.438) \times 0.125) - 0.072)/(1-0.072) + D^-_{PIS^3} = 1,$$

$$1 + (x_1 - 3.671) \times (-0.049) + (x_2 - 3.029) \times 0.048 + (x_3 - 4.3) \times 0.137 + D^-_{NIS^3} = 1,$$

$$\rho \ge D^-_{PIS^i},\ \rho \ge D^-_{NIS^i},\ (i = 1, 2, 3)$$

$$D^-_{PIS^i} \in [0, 1],\ D^-_{NIS^i} \in [0, 1],\ (i = 1, 2, 3)$$

$$x_1 + x_2 + x_3 \le 11,$$
$$x_1 + 4.5\, x_2 - x_3 \le 13,$$
$$4.5\, x_1 + x_2 - 3.5\, x_3 \ge 4.5,$$
$$10.429 - 5.929 \le x_1 \le 10.429 + 0.571,$$
$$3.027 - 1.027 \le x_2 \le 3.027 + 1.473,$$
$$4.3 - 2.3 \le x_3 \le 4.3 + 0.7,$$
$$x_1,\ x_2,\ x_3 \ge 0. \qquad (9.10)$$

The optimal solution of the Model (I) for MO-MLPP is shown in the Table 3.

Table 3. The optimal solution of Model (I)

Approach	Optimal solution	Optimal solution point	Objective values	Membership values
Model (I)	$\rho = 0.3550353$	4.963, 2.559, 3.478	48.475, 28.384, 14.759, 0.102, 27.285,37.11	0.561, 0.903, 0.74, 0.768, 0.776, 0.802

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Model (II)

Minimize $\sigma = 1/6(D^{-}_{PIS^i} + D^{-}_{NIS^i})$, $(i = 1, 2, 3)$

Subject to

$((1 + (x_1 - 10.509) \times 0.096 + (x_2 - 0.491) \times 0.096 + (x_3 - 0) \times 0.096) - 0.548)/ 0.452 + D^{-}_{PIS^1} = 1,$

$1 + (x_1 - 10.429) \times 0.1 + (x_2 - 0.571) \times 0.183 + (x_3 - 0) \times (-0.036) + D^{-}_{NIS^1} = 1,$

$1 + (x_1 - 3.653) \times (-0.05) + (x_2 - 3.027) \times 0.557 + (x_3 - 4.276) \times 0.036 + D^{-}_{PIS^2} = 1,$

$1 + (x_1 - 3.671) \times (-0.088) + (x_2 - 3.029) \times 0.123 + (x_3 - 4.3) \times 0.103 + D^{-}_{NIS^2} = 1,$

$((1 + (x_1 - 3.849) \times (-0.04) + (x_2 - 2.713) \times 0.032 + (x_3 - 4.438) \times 0.125) - 0.072)/ (1\text{-}0.072) + D^{-}_{PIS^3} =$
1,

$1 + (x_1 - 3.671) \times (-0.049) + (x_2 - 3.029) \times 0.048 + (x_3 - 4.3) \times 0.137 + D^{-}_{NIS^3} = 1,$

$D^{-}_{PIS^i} \in [0, 1], D^{-}_{NIS^i} \in [0, 1], (i = 1, 2, 3)$

$x_1 + x_2 + x_3 \leq 11,$
$x_1 + 4.5\, x_2 - x_3 \leq 13,$
$4.5\, x_1 + x_2 - 3.5\, x_3 \geq 4.5,$
$10.429 - 5.929 \leq x_1 \leq 10.429 + 0.571,$
$3.027 - 1.027 \leq x_2 \leq 3.027 + 1.473,$
$4.3 - 2.3 \leq x_3 \leq 4.3 + 0.7,$
$x_1, \quad x_2, x_3 \geq 0.$ (9.11)

Here, we consider the normalized weights associated with negative deviational variables. The optimal solution of Model (II) is shown in the Table 4.

Table 4. The optimal solution of Model (II)

Approach	Optimal solution	Optimal solution point	Objective values	Membership values
Model (II)	σ = 0.225927	4.5, 2.727, 3.773	45.91, 28.042, 18, 5.931, 32.57, 44.752	0.527, 0.895, 0.833, 0.842, 0.851, 0.873

Finally, the comparison of the optimal solutions obtained from the proposed models is presented in the Table 5.

Table 5. The comparison of the optimal solutions based on distance functions

Approach	Optimal solution point	Objective values	Membership values	L_2
Model (I)	4.963, 2.559, 3.478	48.475, 28.384, 14.759, 0.102, 27.285, 37.11	0.561, 0.903, 0.74, 0.768, 0.776, 0.802	0.629614

Model (II)	4.5, 2.727, 3.773	45.91, 28.042, 18, 5.931, 32.57, 44.752	0.527, 0.895, 0.833 0.842, 0.851, 0.873	0.4602425

On comparing L_2 (see the Table 5), we notice that proposed Model (II) provides better compromise optimal solution than the solution obtained by proposed Model (I). Therefore, the better compromise optimal solution of the problem is obtained as $x_1 = 4.5$, $x_2 = 2.727$, $x_3 = 3.773$.

Note: The models are solved by Lingo ver.11.0.

10. Conclusions

The paper proposes a new solution methodology for dealing with MO-MLPP involving fuzzy parameters. We examine how the hybrid approach of FGP and TOPSIS can be efficiently used to solve MO-MLPP with fuzzy parameters. TOPSIS based FGP models are developed in the paper to obtain satisfactory solutions of the problem and distance functions are used to identify the better compromise optimal solution. Our proposed hybrid approach is straightforward and effortless to apply in the practical decision making circumstances where each level DM has autonomy to control some preassigned decision variables to obtain minimum level of satisfaction of compromise decision. Also the computational burden of the proposed approach is obviously less because we do not require any positive deviational variables. We hope that the proposed method can be effective in dealing with practical decision making problems such as agriculture planning problems, conflict resolutions, economic systems, managements, network designs, logistics, and other real world problems with fuzzily described different parameters. The proposed approach can be extended to solve decentralized MO-MLPPs, chance constrained MO-MLPPs, etc involving fuzzy parameters.

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