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Article

(Non-)Thermal Production of WIMPs during Kination

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Abstract: Understanding the nature of the Dark Matter (DM) is one of the current challenges in modern astrophysics and cosmology. Knowing the properties of the DM particle would shed light on physics beyond the Standard Model and even provide us with details of the early Universe. In fact, the detection of such a relic would bring us information from the pre-Big Bang Nucleosynthesis (BBN) period, an epoch from which we have no data, and could even hint at inflationary physics. In this work, we assume that the expansion rate of the Universe after inflationary is governed by the kinetic energy of a scalar field ϕ, in the so-called "kination" model. We assume that the ϕ field decays into both radiation and DM particles, which we take to be Weakly Interacting Massive Particles (WIMPs). The present abundance of WIMPs is then fixed during the kination period through either a thermal "freeze-out" or "freeze-in" mechanism, or through a non-thermal process governed by the decay of ϕ. We explore the parameter space of this theory with the requirement that the present WIMP abundance provides the correct DM relic budget. Requiring that BBN occurs during the standard cosmological scenario sets a limit on the temperature at which the kination period ends. Using this limit and assuming the WIMP has a mass $m_\chi = 100\,\mathrm{GeV}$, we obtain that the thermally-averaged WIMP annihilation cross section has to satisfy the constraints $3.5 \times 10^{-16}\,\mathrm{GeV}^{-2} \lesssim \langle\sigma v\rangle \lesssim 1.4 \times 10^{-5}\,\mathrm{GeV}^{-2}$ in order for having at least one of the production mechanism to yield the observed amount of DM. This result shows how the properties of the WIMP particle, if ever measured, can yield information on the pre-BBN content of the Universe.

Keywords: Cosmology; Dark Matter; Early Universe

1. Introduction

The existence of a Dark Matter (DM) component in the Universe has long been established [1, 2], with a Weakly Interacting Massive Particle (WIMP) being among the best motivated particle candidates [3,4]. In the simplest scenario of the early Universe, WIMPs of mass m_χ interact with the Standard Model (SM) particles at a sufficiently high rate so that the chemical equilibrium is attained. Owing to the expansion rate of the Universe, when the temperature falls below $T_{\mathrm{chem}} \approx m_\chi/20$ WIMPs chemically decouple from the plasma and "freeze-out" of the equilibrium distribution [5–12]. After freeze-out, the number of WIMPs in a comoving volume is fixed and the WIMP relic abundance is preserved to present day, assuming that there is no subsequent change in the entropy of the matter-radiation fluid. Coincidentally, the thermally-averaged WIMP annihilation cross section needed to explain the observed DM is of the same order of magnitude as that obtained for a process mediated by weakly interactions. For a WIMP of mass $m_\chi = 100\,\mathrm{GeV}$, the annihilation cross section that provides the observed amount of DM satisfies $\langle\sigma v\rangle_{\mathrm{std}} \approx 2 \times 10^{-9}\,\mathrm{GeV}^{-2}$. WIMPs continue to exchange momentum through elastic collisions with the plasma even after chemical decoupling, until this second mechanism also becomes inefficient and WIMPs decouple kinetically at a temperature T_{kd}. Typically, T_{kd} ranges between 10 MeV and a few GeV [13].

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Even when considering this thermal production in the standard cosmological scenario, many caveats allow to alter the predicted relic density. Besides co-annihilation [14,15], annihilation into forbidden channels [14,16,17], a momentum- or spin-dependent cross section [18–20], or Sommerfeld enhancement [21–24], one possibility is that the annihilation cross section into the SM sector is so low that WIMPs never reach thermal equilibrium, effectively "freezing-in" to the present relic density. Examples include models of Feebly Interacting Massive Particles [25–27].

WIMPs might also be produced non-thermally, through the decay of a parent particle ϕ [28], whose existence is motivated by the post-inflationary reheating scenario. In facts, assume an early inflationary stage at an energy scale H_I driven by one massive scalar field ρ (the inflaton), of mass m_ρ. When slow-roll is violated, around $m_\rho \sim H_I$, inflation ends and the inflaton field reheats the Universe by decaying into lighter degrees of freedom [29,30]. We are not entering the details of the reheating mechanism here. In the standard picture, a radiation-dominated period begins as soon as the inflaton field has decayed and reheated the Universe.

However, in some reheating models the inflaton might also decay into one or more additional hypothetical fields, say for example a field ϕ of mass m_ϕ which comes to dominate the post-inflationary stage. The inclusion of a non-standard cosmology between the reheating epoch right after inflation and the standard radiation-dominated scenario is motivated in realistic models of inflation, and might sensibly alter the WIMP relic density. A non-standard period might have lasted for a considerable amount of time, namely since the end of inflation down to a temperature which, using considerations on the Big Bang Nucleosynthesis (BBN) mechanism [31–35], can be as low as $\sim 5\,\mathrm{MeV}$. In these modified cosmologies, various properties of the WIMPs like their free-streaming velocity and the temperature at which the kinetic decoupling occurs have been investigated [36–39].

In the Low Temperature Reheat Scenario (LTRS) [40–43], ϕ is a massive modulus which drives an early matter-dominated epoch, eventually decaying into Standard Model particles and, possibly, WIMPs. In the pre-BBN LTRS, the thermal (both freeze-out and freeze-in) and non-thermal production of WIMPs have both been extensively studied [36,44–60].

In the Kination Scenario (KS) [61–66], ϕ is a "fast-rolling" field whose kinetic energy governs the expansion rate of the post-inflationary Universe, with an equation of state relating the pressure p_ϕ and energy density ρ_ϕ of the fluid as $p_\phi = \rho_\phi$. Owing to the scaling of the energy density in radiation with the scale factor $\rho_R \sim a^{-4}$, which is slower than the scaling of the energy density in the ϕ field $\rho_\phi \sim a^{-6}$, the contribution from the radiation energy density in determining the expansion rate eventually becomes more important than that from the ϕ field. When the ϕ field redshifts away, the standard radiation-dominated cosmology takes place. Since the scalar field ϕ dominates the expansion rate for some period, KS differs from the superWIMP model of Refs. [67,68]. Thermal production of WIMPs in the KS has been discussed in Refs. [65,69–73], and has recently been investigated in Ref. [74], in light of recent data. Ref. [75] discussed the "relentless" thermal freeze-out in models where the ϕ field has a pressure $p_\phi > \rho_\phi/3$, thus including KS as an important sub-case. Ref. [76] discusses an intermediate model between LTRS and KS, in which a sub-dominant massive scalar field reheats the Universe during a kination period governed by an additional field.

In this paper, we assume that the ϕ field driving kination might decay into both radiation and WIMPs, with a decay rate Γ_ϕ and a branching ratio into WIMPs equal to b. Contrarily to previous KS models, kination ends when Γ_ϕ is equal to the expansion rate of the Universe, so when the ϕ field has decayed instead of being redshifted away. WIMP production proceeds by assuming that, before the Universe gets to be dominated by radiation at a temperature $T_{\rm kin} \gtrsim 5\,\mathrm{MeV}$, the KS occurs. In the following, the subscript "kin" labels a quantity evaluated at $T_{\rm kin}$. We consider the thermal freeze-out and freeze-in mechanisms of WIMP production, governed by the thermal-averaged annihilation cross section times velocity $\langle\sigma v\rangle$. We also include the non-thermal WIMP production from the decay of the ϕ field. We check that the WIMP population is always under-abundant with respect to other forms of energy. In summary, we show that in the model the present WIMP relic abundance can be reached

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through four different methods, namely the thermal ($b = 0$) or non-thermal ($b \neq 0$) production, either with or without ever reaching chemical equilibrium, as occurs in the LTRS [50,51].

2. Boltzmann equations for the model

We follow the evolution of the energy components by modeling the system through a set of coupled Boltzmann equations

$$\dot{\rho}_\phi + 6H\rho_\phi = -\Gamma_\phi \rho_\phi, \tag{1}$$

$$\dot{\rho}_R + 4H\rho_R = \left(1 - \frac{bm_\chi}{m_\phi}\right)\Gamma_\phi \rho_\phi + \epsilon_\chi \langle\sigma v\rangle \left(n_\chi^2 - n_{\rm EQ}^2\right), \tag{2}$$

$$\dot{n}_\chi + 3Hn_\chi = \frac{b\Gamma_\phi}{m_\phi}\rho_\phi - \langle\sigma v\rangle \left(n_\chi^2 - n_{\rm EQ}^2\right). \tag{3}$$

Here, n_χ is the WIMP number density, with a value $n_{\rm EQ}$ when in chemical equilibrium, ρ_R is the energy density in radiation, and we defined the WIMP energy through $\rho_\chi = \epsilon_\chi n_\chi$. The equation of state relating the energy density and the pressure of the ϕ field is given by $\rho_\phi = p_\phi$, and translates into the term $6H\rho_\phi$ appearing in Eq. (1). At any time, the temperature T is defined through the energy density in the relativistic component as $\rho_R = \alpha T^4$, with $\alpha = \pi^2 g_*(T)/30$ and where $g_*(T)$ is the number of relativistic degrees of freedom. We assume that WIMPs have g degrees of freedom, so that the equilibrium distribution is given by $(E^2 = p^2 + m_\chi^2)$

$$n_{\rm EQ} = \int f(p)\frac{g d^3p}{(2\pi)^3} = \frac{g}{2\pi^2}\int_{m_\chi}^{+\infty} \frac{\sqrt{E^2 - m_\chi^2}}{e^{E/T} + 1} E dE. \tag{4}$$

The set of Eqs. (1)-(3) is closed when solved together with the Friedmann equation

$$H^2 = \frac{8\pi}{3M_{\rm Pl}^2}\left(\rho_\phi + \rho_R + m_\chi n_\chi\right), \tag{5}$$

where $M_{\rm Pl}$ is the Planck mass. We assume that the mass of the ϕ field is larger than both ϵ_χ and bm_χ. In this limit, Eqs. (1)-(3) simplify as

$$\dot{\rho}_\phi + 6H\rho_\phi = -\Gamma_\phi \rho_\phi, \tag{6}$$

$$\dot{\rho}_R + 4H\rho_R = \Gamma_\phi \rho_\phi, \tag{7}$$

$$\dot{n}_\chi + 3Hn_\chi = \frac{b\Gamma_\phi}{m_\phi}\rho_\phi - \langle\sigma v\rangle \left(n_\chi^2 - n_{\rm EQ}^2\right). \tag{8}$$

At early times $t \ll 1/\Gamma_\phi$, the Boltzmann Eq. (6) and the Friedmann Eq. (5) predict that, during KS, the energy density of the ϕ field and time scale as $\rho_\phi \sim a^{-6}$ and $t \sim a^3$, respectively. Contrarily to what found in kination models with negligible decay rate, for which $a \sim 1/T$ [63–66], in the model we study temperature depends on the scale factor as $T \propto \rho_\phi^{1/8} \propto a^{-3/4}$. In more details, Eq. (7) can be reformulated as a differential equation describing the evolution of the entropy per comoving volume $s = (p_R + \rho_R)/T$, as

$$\frac{ds}{dt} + 3Hs = \frac{\Gamma_\phi}{T}\rho_\phi. \tag{9}$$

As a consequence, entropy is not conserved in the model we consider because of the appearance of the dissipative term on the right hand side in Eq. (9), coming from the decay of the ϕ field. We later confirm these results by numerically solving the set of the Boltzmann equations, see Fig. 1 below.

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We switch to the independent coordinates $x = m_\phi a$ and $\tau = \Gamma_\phi t$, while we write the dependent quantities in terms of the fields

$$\Phi = A x^6 \rho_\phi / m_\phi, \quad R = A x^4 \rho_R / m_\phi, \quad X = A x^3 n_\chi. \tag{10}$$

Fixing the constant A through the Friedmann equation,

$$\mathcal{H} = \frac{1}{x}\frac{dx}{d\tau} = \frac{H}{\Gamma_\phi} = \frac{\sqrt{\Phi + R x^2 + x^3 X}}{x^3}, \tag{11}$$

we obtain that Eq. (5) is recovered when

$$A = \frac{8\pi m_\phi}{3 M_{\rm Pl}^2 \Gamma_\phi^2} \equiv \frac{m_\phi}{\rho_{\rm kin}}, \tag{12}$$

where we have defined the temperature $T_{\rm kin}$ in the instantaneous thermalization approximation, $\rho_{\rm kin} = \rho_R(T_{\rm kin})$, so that $\Gamma_\phi = H(T_{\rm kin})$. With this definition, we rewrite the system of Eqs. (6)-(8) as

$$\Phi' = \frac{-x^2 \Phi}{\sqrt{\Phi + x^2 R + x^3 X}}, \tag{13}$$

$$R' = \frac{\Phi}{\sqrt{\Phi + x^2 R + x^3 X}}, \tag{14}$$

$$X' = \frac{b\Phi - s\left(X^2 - X_{\rm EQ}^2\right)}{x\sqrt{\Phi + x^2 R + x^3 X}}, \tag{15}$$

where $s = \langle\sigma v\rangle / \Gamma_\phi A = \rho_{\rm kin}\langle\sigma v\rangle / \Gamma_\phi m_\phi$. This dimensionless form of the system has never been shown in the literature, and can be easily extended to cosmologies other than KS. We assume that the relativistic species is always at equilibrium, so that temperature is related to R as $T = T_{\rm kin}\, R^{1/4}/x$. At chemical equilibrium, the quantity $X_{\rm EQ} = A x^3\, n_{\rm EQ}$ is then

$$X_{\rm EQ} = \frac{g m_\phi}{\rho_{\rm kin}} \left(\frac{m_\chi T_{\rm kin} x R^{1/4}}{2\pi}\right)^{3/2} \exp\left(\frac{-m_\chi x}{T_{\rm kin}\, R^{1/4}}\right). \tag{16}$$

The set of Eqs. (13)-(16) possesses a scaling symmetry,

$$x \to \beta x, \quad \Phi \to \beta^6 \Phi \quad R \to \beta^4 R \quad X \to \beta^3 X, \tag{17}$$

for any value of β, thanks to which the solution to the set of Boltzmann equations is independent on choice of the initial value $\Phi(x_I) = \Phi_I$ at $x = x_I$ [45,46]. We fix the initial condition by requiring that the Hubble rate at $x = x_I$ be $H_I = \sqrt{8\pi\rho_I/3M_{\rm Pl}^2}$, where ρ_I is the value of the energy density in the inflaton field at the inflation scale. Indeed, assuming that $\rho_R(x_I) \ll \rho_\phi(x_I) \equiv \rho_I$ gives

$$\Phi(x) = \Phi_I \equiv \frac{\rho_I}{\rho_{\rm kin}}\, x_I^6, \quad R(x) = \sqrt{\Phi_I}\,(x - x_I). \tag{18}$$

Since the energy density ρ_ϕ during ϕ-domination satisfies $\rho_\phi \sim a^{-6}$, the transition to the standard radiation-dominated scenario occurs when

$$\rho_{\phi,I}\left(\frac{x_I}{x_{\rm kin}}\right)^6 = \rho_{\rm kin} \equiv \alpha\, T_{\rm kin}^4. \tag{19}$$

Given the value of $T_{\rm kin}$, Eq. (19) defines the moment at which the ϕ field decays. For the illustrative purpose, we solve Eqs. (13) and (14) for $T_{\rm kin} = 0.1\,$GeV, fixing the masses $m_\chi = 100\,$GeV and $m_\phi =$

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1000 TeV and assuming that we can safely neglect the contribution of WIMPs to the total energy density. We have plot Φ (blue solid line) and R (red dashed line) in units of Φ_I, as well as the temperature T/T_I (black dot-dashed line). The behavior of Φ confirms the scaling $\rho_\phi \sim a^{-6}$ and $\rho_R \sim a^{-3}$ at early times $x < x_{\rm kin}$, while for $x > x_{\rm kin}$ we obtain the scaling $\rho_R \sim a^{-4}$ for a radiation-dominated cosmology. The dashed vertical line marks the value of $x_{\rm kin}$ solution to Eq. (19).

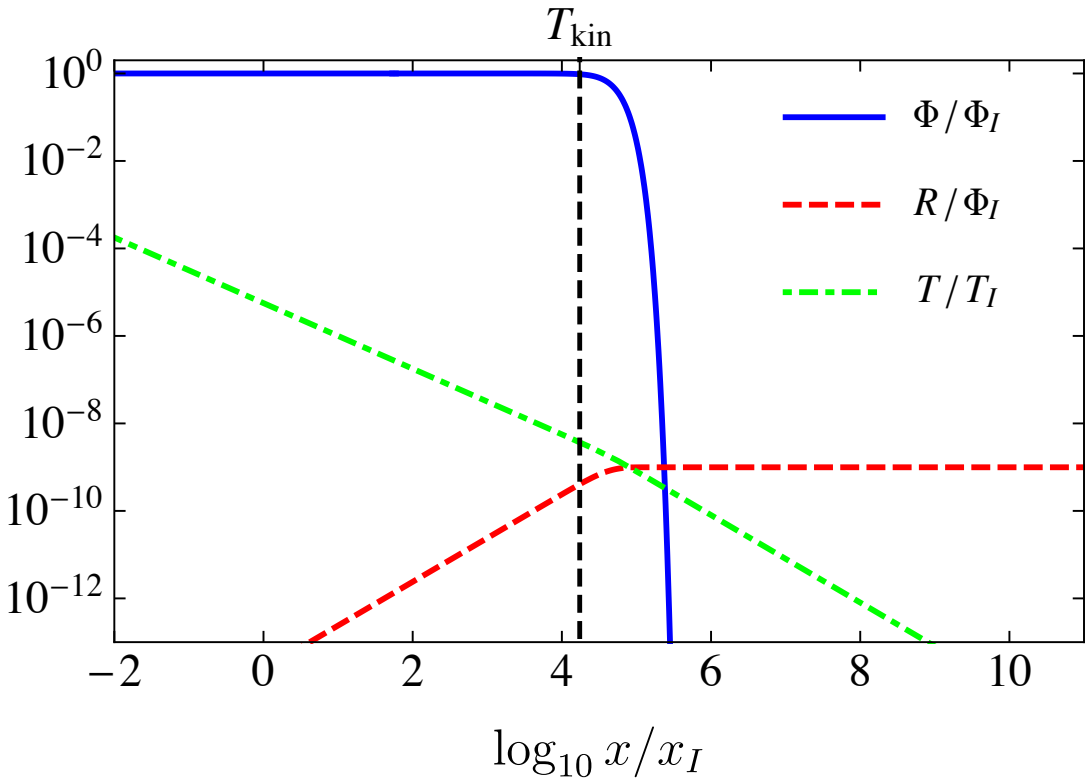

Figure 1. The quantities Φ and R, defined in Eq. (10) and related to ρ_ϕ and ρ_R respectively, in units of the initial value Φ_I. The vertical dashed line marks the moment at which the transition to the standard scenario occurs, according to Eq. (19). The green dot-dashed line shows the temperature $T(x)$, in units of its initial value $T(x_I)$.

3. Production of WIMPs during kination

With this framework, we solve the set of Boltzmann Eqs. (13)-(15) for different values of $T_{\rm kin}$, to obtain the WIMP relic abundance $n_{\rm kin} \equiv n_\chi(T_{\rm kin})$ when the ϕ field decays, after which the WIMP number density in a comoving volume is fixed. The present WIMP energy density in units of the critical density ρ_c is then

$$\Omega_\chi = \frac{m_\chi}{m_\phi}\frac{\rho_{\rm kin}}{\rho_c}\frac{g_S(T_0)}{g_S(T_{\rm kin})}\left(\frac{T_0}{T_{\rm kin}}\right)^3 \frac{X_{\rm kin}}{\sqrt{\Phi_I}}, \tag{20}$$

where $X_{\rm kin} = A x_{\rm kin}^3 n_{\rm kin}$, T_0 is the present temperature of the radiation bath and $g_S(T)$ is the number of entropy degrees of freedom at temperature T. We show different values of the abundance $\Omega_\chi h^2$ in Fig. 2, as a function of the temperature $T_{\rm kin}$, the annihilation cross section $\langle\sigma v\rangle$, and the branching ratio b. We have used three different values of $b = 10^{-10}$, 10^{-5}, and 10^{-1}, as well as five different values of $\langle\sigma v\rangle$. The value $\langle\sigma v\rangle = \langle\sigma v\rangle_{\rm std}$ gives the correct amount of DM when $T_{\rm kin} \gtrsim m_\chi/20$, since in such scenario the chemical decoupling of WIMPs occurs in the standard cosmology. The extra dashed line at $T_{\rm kin} = 0.1\,$GeV tracks the specific solution of the Boltzmann equations later used in Fig. 3. The value

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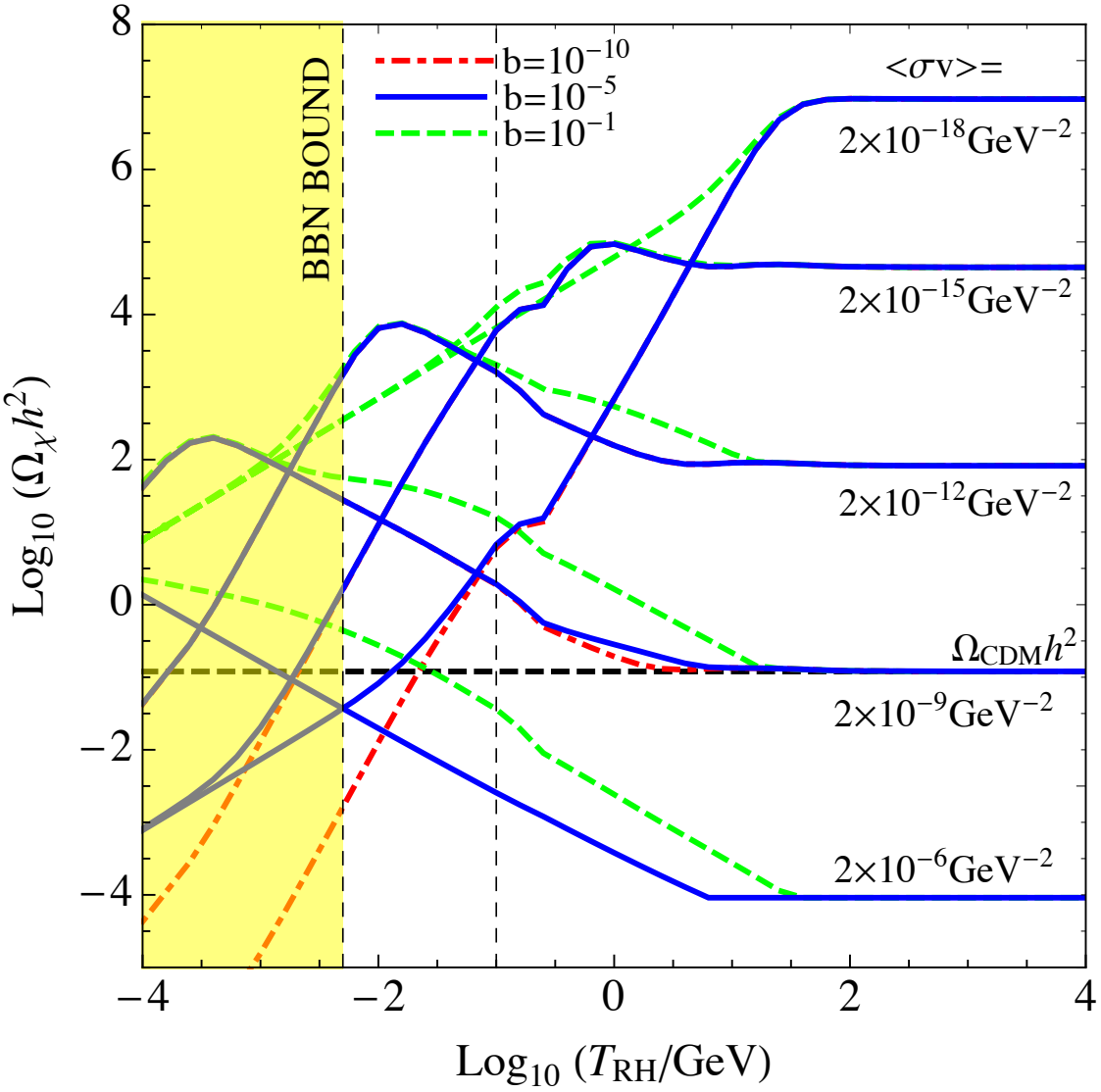

Figure 2. The present WIMP relic abundance for different values of $\langle\sigma v\rangle$ (see figure labels), and for different values of the branching ratio: $b = 10^{-10}$ (red dot-dashed line), $b = 10^{-5}$ (blue solid line), and $b = 10^{-1}$ (green dashed line), as a function of $T_{\rm kin}$. The horizontal dashed line shows the measured dark matter abundance $\Omega_{\rm DM}h^2 \sim 0.12$. The vertical yellow band defines the region excluded by BBN considerations, $T_{\rm kin} \geq 5\,$MeV. The vertical line at $T_{\rm kin} = 0.1\,$GeV marks the solutions to the Boltzmann Eq. (15) later used for Fig. 3, using $b = 0.1$.

of $n_{\rm kin}$ depends on the specific mechanism that dominates the production of the WIMP population. According to the values of b and $\langle\sigma v\rangle$, four possibilities appear, namely:

- Thermal production with chemical equilibrium ("freeze-out", Mechanism 1). We first focus on the case $b = 0$, corresponding to the negligible decay with respect to the annihilation into standard model particles and approximated by the red dot-dashed and blue solid lines in Fig. 2.

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We assume that the temperature $T_{\rm kin}$ is lower than the freeze-out temperature $T_{\rm f.o}$, which is defined as the temperature at which $n\langle\sigma v\rangle = H$, or

$$n_{\rm EQ}(T_{\rm f.o}) = \left(\frac{T_{\rm f.o}}{T_{\rm kin}}\right)^4 \frac{H(T_{\rm kin})}{\langle\sigma v\rangle}. \tag{21}$$

However, contrary to what obtained in the standard radiation-dominated scenario, the WIMP number density in the kination cosmology is not fixed at $T_{\rm f.o}$ and annihilation continues until the temperature drops to $T_{\rm kin}$ and the expansion rate transitions to that of a radiation-dominated one [75]. We assume that the freeze-out is reached at $x_{\rm f.o.}$, for which $X(x_{\rm f.o.}) = X_{\rm f.o.}$. For later times, using the approximation in Eq. (19), Eq. (15) reduces to

$$X' = -\frac{\rho_{\rm kin}\langle\sigma v\rangle}{\Gamma_\phi m_\phi}\frac{X^2}{x\sqrt{\Phi_I}}, \tag{22}$$

whose solution at $x > x_{\rm f.o.}$ give the abundance of thermally produced WIMPs at $T_{\rm kin}$, which reads

$$X_{\rm kin,th} = \left[\frac{1}{X_{\rm f.o.}} + \frac{\langle\sigma v\rangle T_{\rm kin}^2}{\sqrt{\Phi_I}}\frac{M_{\rm Pl}}{m_\phi}\ln\frac{x_{\rm kin}}{x_{\rm fo}}\right]^{-1}. \tag{23}$$

Neglecting $X_{\rm f.o.}$, the present abundance in Eq. (20) for the standard freeze-out mechanism gives

$$\Omega_\chi = \frac{g_S(T_0)}{g_S(T_{\rm kin})}\frac{m_\chi}{M_{\rm Pl}}\frac{\rho_{\rm kin}}{\rho_c}\frac{T_0^3}{\langle\sigma v\rangle T_{\rm kin}^5}\left(\ln\frac{x_{\rm kin}}{x_{\rm fo}}\right)^{-1} \propto \frac{1}{T_{\rm kin}}. \tag{24}$$

The solution describes the lines with negative slopes in Fig. 2, for $b = 10^{-10}$ and $b = 10^{-5}$ and for the cross sections $\langle\sigma v\rangle = 2\times10^{-6}\,{\rm GeV}^{-2}$, $\langle\sigma v\rangle = 2\times10^{-9}\,{\rm GeV}^{-2}$, and $\langle\sigma v\rangle = 2\times10^{-12}\,{\rm GeV}^{-2}$.

- Thermal production without ever reaching chemical equilibrium ("freeze-in", Mechanism 2). If the cross section is sufficiently low [74], WIMPs never reach thermal equilibrium and their number density freezes in at a fixed quantity. Since the number density of particles is always smaller than their value at thermal equilibrium, we neglect $X \ll X_{\rm EQ}$ so Eq. (15) with $b = 0$ reads

$$X' = \frac{\rho_{\rm kin}\langle\sigma v\rangle}{\Gamma_\phi m_\phi}\frac{\left(X_{\rm EQ}^2\right)}{x\sqrt{\Phi_I}} = c_1 x^{11/4}\exp\left(-2c_2 x^{3/4}\right), \tag{25}$$

where

$$c_1 = \frac{g^2\langle\sigma v\rangle m_\phi m_\chi^3 T_{\rm kin}^3}{(2\pi)^3\Gamma_\phi\rho_{\rm kin}\Phi_I^{1/8}}, \quad \text{and} \quad c_2 = \frac{m_\chi}{T_{\rm kin}\Phi_I^{1/8}}. \tag{26}$$

The solution to Eq. (25) reaches the asymptotic value of X at freeze-in

$$X_{\rm kin,fi} = \frac{c_1}{c_2^5} = \frac{g^2\langle\sigma v\rangle m_\phi T_{\rm kin}^8\sqrt{\Phi_I}}{(2\pi)^3\Gamma_\phi\rho_{\rm kin}m_\chi^2}, \tag{27}$$

which is reached when $x_{\rm fi} = (6/11c_2)^{4/3}$. The present abundance is then

$$\Omega_\chi = \frac{g_S(T_0)}{g_S(T_{\rm kin})}\frac{g^2\langle\sigma v\rangle T_0^3 T_{\rm kin}^5}{(2\pi)^3\Gamma_\phi\rho_c m_\chi} \propto T_{\rm kin}^3. \tag{28}$$

The solution describes the lines with positive slopes in Fig. 2, for $b = 10^{-10}$ and $b = 10^{-5}$ and for the cross sections $\langle\sigma v\rangle = 2\times10^{-12}\,{\rm GeV}^{-2}$, $\langle\sigma v\rangle = 2\times10^{-15}\,{\rm GeV}^{-2}$, and $\langle\sigma v\rangle = 2\times10^{-18}\,{\rm GeV}^{-2}$.

- Non-thermal production without chemical equilibrium (Mechanism 3). We now discuss the non-thermal production of dark matter, in the case in which the particle has never reached

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the chemical equilibrium. For a large branching ratio $b = O(0.1)$, and for $T_{\rm kin} \ll m_\chi$, the abundance of dark matter is set by the decay of the ϕ field, with a number density at $T_{\rm kin}$ given by [50,52,55,58,60,77]

$$n_{\rm kin} \approx b\, n_\phi(T_{\rm kin}) = b\frac{\rho_{\rm kin}}{m_\phi}. \tag{29}$$

Deriving the result from directly integrating Eq. (15) with $\langle\sigma v\rangle = 0$ and neglecting the contributions from R and X in the denominator gives an extra logarithmic dependence on $x_{\rm kin}$, as

$$X_{\rm kin,decay} = b\sqrt{\Phi_I}\ln\frac{x_{\rm kin}}{x_I}. \tag{30}$$

The present WIMP abundance when the non-thermal production dominates is given by Eq. (20) with $X_{\rm kin}$ as in Eq. (30), and predicts the behavior $\Omega_\chi \propto T_{\rm kin}$ corresponding to the green dashed line with $\langle\sigma v\rangle = 10^{-18}\,{\rm GeV}^{-2}$ in Fig. 2, for $T_{\rm kin} \lesssim 10\,{\rm GeV}$. Notice that, except for the logarithmic dependence which is present in the kination cosmology, the result in Eq. (30) is independent of the cosmology used, so the result in Eq. (29) also holds in the LTRS.

- Non-thermal production with chemical equilibrium (Mechanism 4). If the branching ratio b is sufficiently high, we find that the quantity X is fixed to the value at freeze-out, which is obtained by setting to zero the right-hand side of Eq. (15),

$$X_{\rm kin,ann} = \sqrt{\frac{b\Phi_I\Gamma_\phi m_\phi}{\rho_{\rm kin}\langle\sigma v\rangle}}. \tag{31}$$

The result in Eq. (31) can be alternatively derived by considering the balancing between the decay rate of the ϕ field into WIMPs and the annihilation rate of WIMPs, as

$$\langle\sigma v\rangle n_\chi^2 = b\Gamma_\phi\frac{\rho_\phi}{m_\phi}. \tag{32}$$

The value of $X_{\rm kin,ann}$ remains constant until $T_{\rm kin}$, without experiencing the additional depletion described in the freeze-out regime [74,75]. However, when the temperature of the plasma falls below $T_{\rm kin}$, the energy density in the ϕ field drops to zero, while WIMP annihilation is still maintained approximately until $\rho_\phi \approx \rho_R$, when the WIMP number density is

$$n_\chi = \frac{\Gamma_\phi}{\langle\sigma v\rangle}, \quad \text{or} \quad X_{\rm kin,nonTH} = \sqrt{\frac{b\rho_{\rm kin}\langle\sigma v\rangle}{\Gamma_\phi m_\phi}} \tag{33}$$

The present abundance is expressed as

$$\Omega_\chi = \frac{m_\chi\Gamma_\phi}{\rho_c\,\langle\sigma v\rangle}\frac{g_S(T_0)}{g_S(T_{\rm kin})}\left(\frac{T_0}{T_{\rm kin}}\right)^3 \propto \frac{1}{T_{\rm kin}}. \tag{34}$$

4. Discussion and summary

In Fig. 3, we summarize the results obtained by considering the behavior of the quantity X solution to the Boltzmann Eq. (15) for $b = 0.1$, the green dot-dashed line in Fig. 2, and for $T_{\rm kin} = 0.1\,{\rm GeV}$. In facts, for this choice of the parameters, the different values of $\langle\sigma v\rangle$ in Fig. 2 give the different production mechanisms discussed. In Fig. 3, the black solid line represents $X_{\rm EQ}$, while the different mechanisms of productions are all given for different values of $\langle\sigma v\rangle$, with $\langle\sigma v\rangle = 2\times10^{-9}\,{\rm GeV}^{-2}$ (blue line) describing non-thermal production with chemical equilibrium (Mechanism 4), $\langle\sigma v\rangle = 2\times10^{-18}\,{\rm GeV}^{-2}$ (green line) describing non-thermal production without chemical equilibrium (Mechanism 3), $\langle\sigma v\rangle = 2\times10^{-15}\,{\rm GeV}^{-2}$ (orange line) describing thermal production without chemical equilibrium (Mechanism 2), and $\langle\sigma v\rangle = 2\times10^{-12}\,{\rm GeV}^{-2}$ (red line) describing thermal production with chemical equilibrium

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(Mechanism 1). In addition to the thermal mechanisms of production recently discussed in Refs. [74,75],

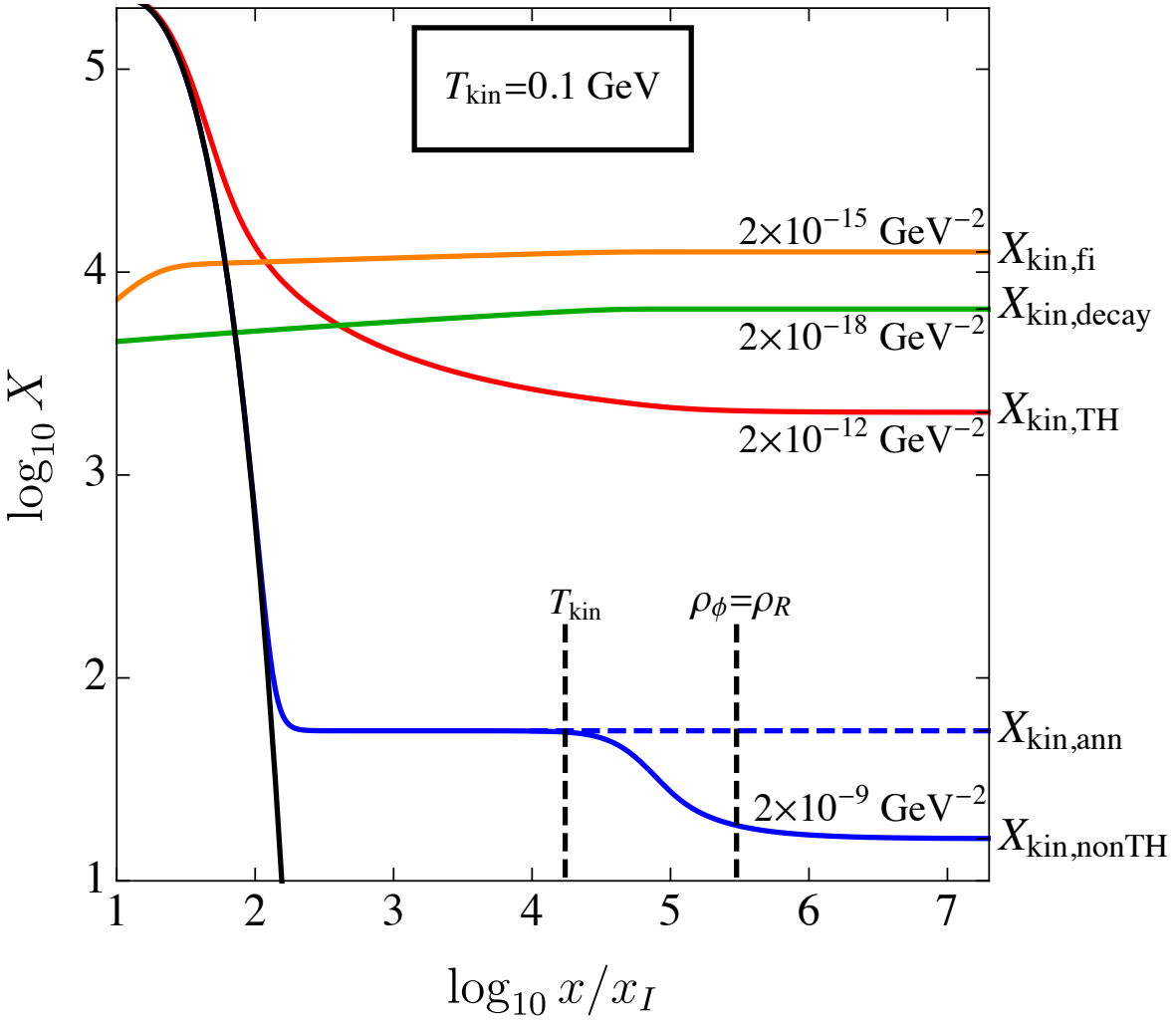

Figure 3. The solution to the Boltzmann Eq. (15) for different values of the annihilation cross section. We have set $T_{\rm kin} = 0.1\,{\rm GeV}$ and, when non-thermal production is considered, $b = 0.1$. See text for further details.

we have included the possibility that the field ϕ responsible for the kination period decays into radiation and WIMPs. We have then studied the non-thermal production of WIMPs in the kination cosmology, as summarized in Fig. 2. Given the value $\langle\sigma v\rangle_{\rm std} = 2\times10^{-9}\,{\rm GeV}^{-2}$ that gives the present abundance of DM from the freeze-out of WIMPs in the standard cosmology, Fig. 2 shows that larger values of $\langle\sigma v\rangle$ can still lead to the right DM abundance, if WIMPs are produced either through Mechanisms 1) or 4) during the KS. Similarly, we can have $\langle\sigma v\rangle$ smaller than its standard value and still have the correct amount of DM, if WIMPs are produced through Mechanism 2). Non-thermal production without chemical equilibrium (Mechanism 3) would lead to the correct amount of DM only for values of $T_{\rm kin}$ that are excluded by the BBN considerations. Using the bound $T_{\rm kin} \gtrsim 5\,{\rm MeV}$, we infer the possible range of the parameter

$$3.5\times10^{-16}\,{\rm GeV}^{-2} \lesssim \langle\sigma v\rangle \lesssim 1.4\times10^{-5}\,{\rm GeV}^{-2}, \tag{35}$$

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the lower bound being obtained by using Mechanism 2 and the upper bound being given by Mechanism 4 with $b = 1$. This result is valid for a WIMP of mass $m_\chi = 100\,\text{GeV}$ and $m_\phi = 1000\,\text{TeV}$.

To summarize, if the DM is a WIMP of mass $m_\chi = 100\,\text{GeV}$, and if the annihilation cross section is measured to lie outside of the bound in Eq. (35), then the kination model discussed would have to be discarded. The same analysis can be performed by varying the masses of the WIMP and of the ϕ field, which would lead to different values of the bounds in Eq. (35). If ever discovered, the properties of the WIMP could then shed light on the pre-BBN cosmology.

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Abbreviations

The following abbreviations are used in this manuscript:

DM	Dark Matter
BBN	Big Bang Nucleosynthesis
WIMP	Weakly Interacting Massive Particle
LTRS	Low Temperature Reheat Scenario
KS	Kination Scenario

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